Universidad de Granada

European Centre for Soft Computing

A Novel Framework to Design Fuzzy Rule-Based Ensembles Using Diversity Induction and Evolutionary Algorithms-Based Classifier Selection and Fusion

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Outline

1. **Introduction: Classifier Ensembles**
2. **Proposed Framework**
3. **Fuzzy Rule-based Classifier Ensemble Design from Classical Machine Learning Approaches**
   i. Bagging FURIA-based fuzzy classifier ensembles
   ii. Random Oracle-based Bagging FURIA fuzzy classifier ensembles
   iii. Experiments
4. **Evolutionary Multiobjective Selection of Component Classifiers**
5. **Classifier Selection and Fusion via an Interpretable Genetic Fuzzy System**
6. **Conclusions**

Problem description and objectives

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3. FRBCE Design from Classical ML Approaches

4. EMO Selection of Component Classifiers

5. Classifier Selection and Fusion via an Interpretable GFS

6. Conclusions

- Strong interest on classifier ensembles (CEs) in the classical machine learning field: High accuracy
- Fuzzy rule-based classification systems (FRBCSs) achieve good performance: Soft boundaries (and interpretability)
- Problems with high complexity data: **Curse of dimensionality**
- Fuzzy rule-based classification ensembles (FRBCEs) ability to deal with high complexity data

Problem description and objectives (II)

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- Well established and recent advanced CE design methods to increase accuracy by inducing diversity

- Existing mechanisms to look for the best accuracy-complexity tradeoff in CEs: overproduce-and-choose

- Evolutionary multiobjective optimization (EMO) ability to deal with conflicting optimization criteria

Our proposal:

A novel framework incorporating classical and advanced CE methodologies and evolutionary algorithms to design fuzzy rule-based classification ensembles (FRBCEs)

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Classifier ensembles

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- A CE is the result of the combination of the outputs of a group of individually trained classifiers to get a more accurate system that any of its components

- CEs are able not only to outperform a single classifier but also to deal with complex and high dimensional classification problems

- CE design is mainly based on two stages:
  - learning of the component classifiers
  - combination of the individual decisions provided into the global output

- The CE accuracy relies on the performance and the proper integration of these two tasks

1. Introduction
Classifier ensembles (II)

One person	Several people
QUESTION	QUESTION

CORRECT ANSWER?	CORRECT ANSWER?

Diversity helps to improve accuracy

1. Introduction
Classifier ensemble design issues: diversity induction

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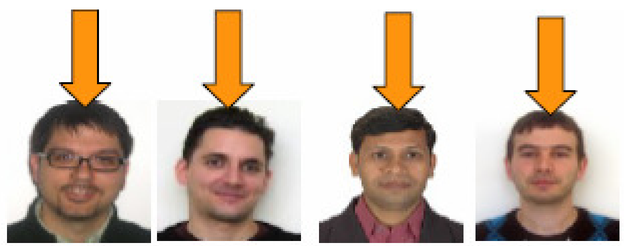
Diversity – An individual classifier must provide different generalization patterns to obtain a diverse set of classifiers

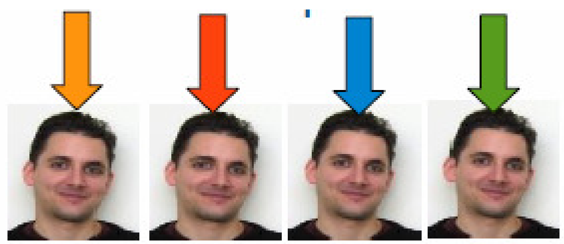
The best situation is that where the individual classifiers are both accurate and fully complementary (they make their errors on different parts of the problem space)

Different methods to induce diversity among the base classifiers (first stage):

Different classifiers:



Different "inputs"/features:



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Combination methods used in the second stage:

- not only consider the issue of aggregating the results provided by all the initial set of component classifiers (classifier fusion),
- but also can involve:
  - either locally selecting the best single/subgroup of classifier(s) to be used to provide a decision for each specific input pattern (dynamic classier selection),
  - or globally selecting the subgroup of classifiers to be considered for every input pattern (static classier selection)

Classifier ensemble design issues: classifier fusion

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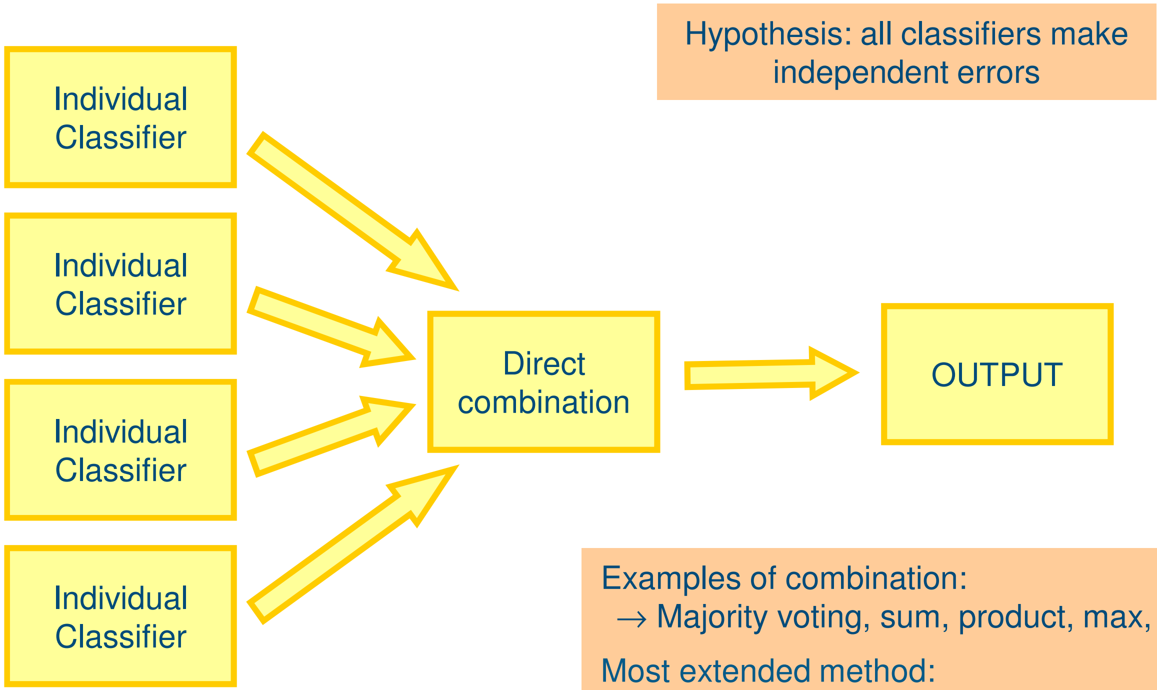
Two strategies to combine the results of individual classifiers:

1) Classifier fusion methods



Examples of combination:
→ Majority voting, sum, product, max, min, …

Most extended method:
→ weighted majority voting [Lam and Suen, 1997]

Classifier ensemble design issues: classifier selection

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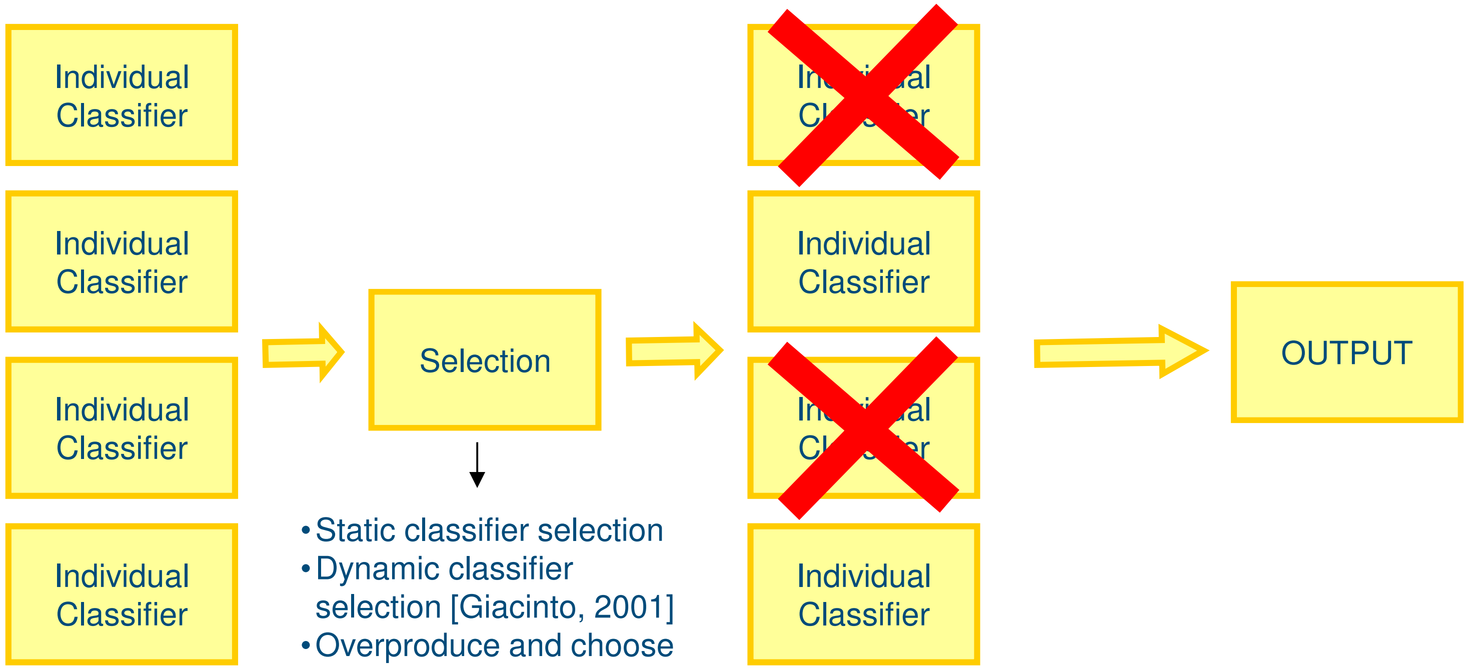
Two strategies to combine the results of individual classifiers:

2) Classifier selection methods

Hypothesis: only some classifiers have influence on the final result



- Static classifier selection
- Dynamic classifier selection [Giacinto, 2001]
- Overproduce and choose strategies [Partridge, 96]

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Our proposal involves a global methodology to design accurate, diverse and compact FRBCEs

Different independent specific methods are proposed for each of the FRBCE design stages:

- A quick and accurate fuzzy rule generation method (FURIA) including dimensionality (feature selection) is considered for the base classifier generation
- At this stage, diversity is induced by a classical data resampling approach (bootstrap aggregating, bagging) or
- An advanced method (random oracles) based on training data splitting can additionally be considered

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- The second stage can either involve only classifier selection or joint classifier selection and fusion

- Classifier selection is made by means of the classical overproduce-and-choose (OCS) strategy allowing us to both increase the accuracy and reduce the complexity

- Use of EMO ability to deal with conflicting optimization criteria to improve OCS (accuracy, complexity and diversity criteria)

- Advanced interpretable mechanism to combine component classifiers by means of a FRBCS (joint classifier fusion and classifier selection)

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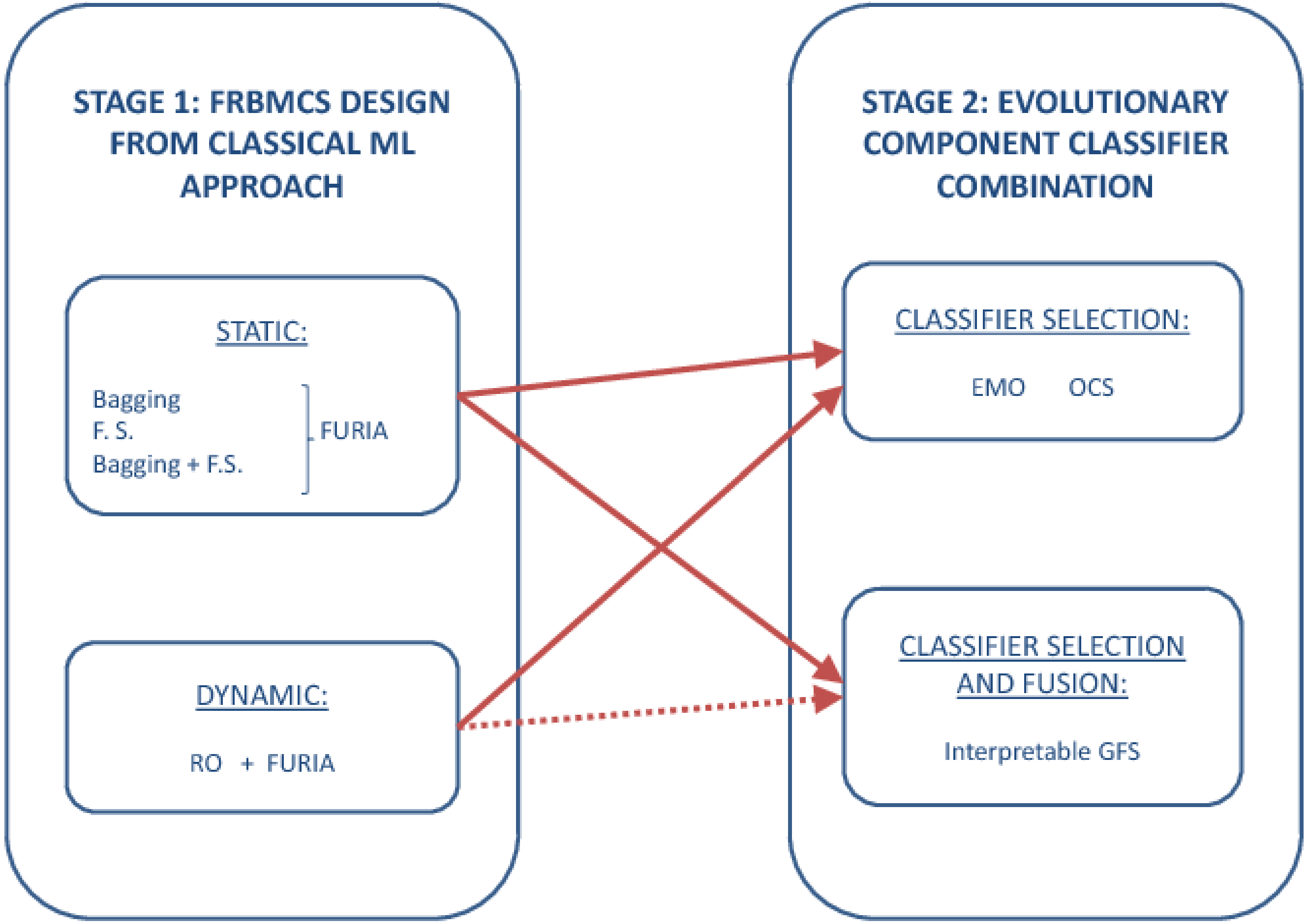

Proposed Framework Graphical Representation

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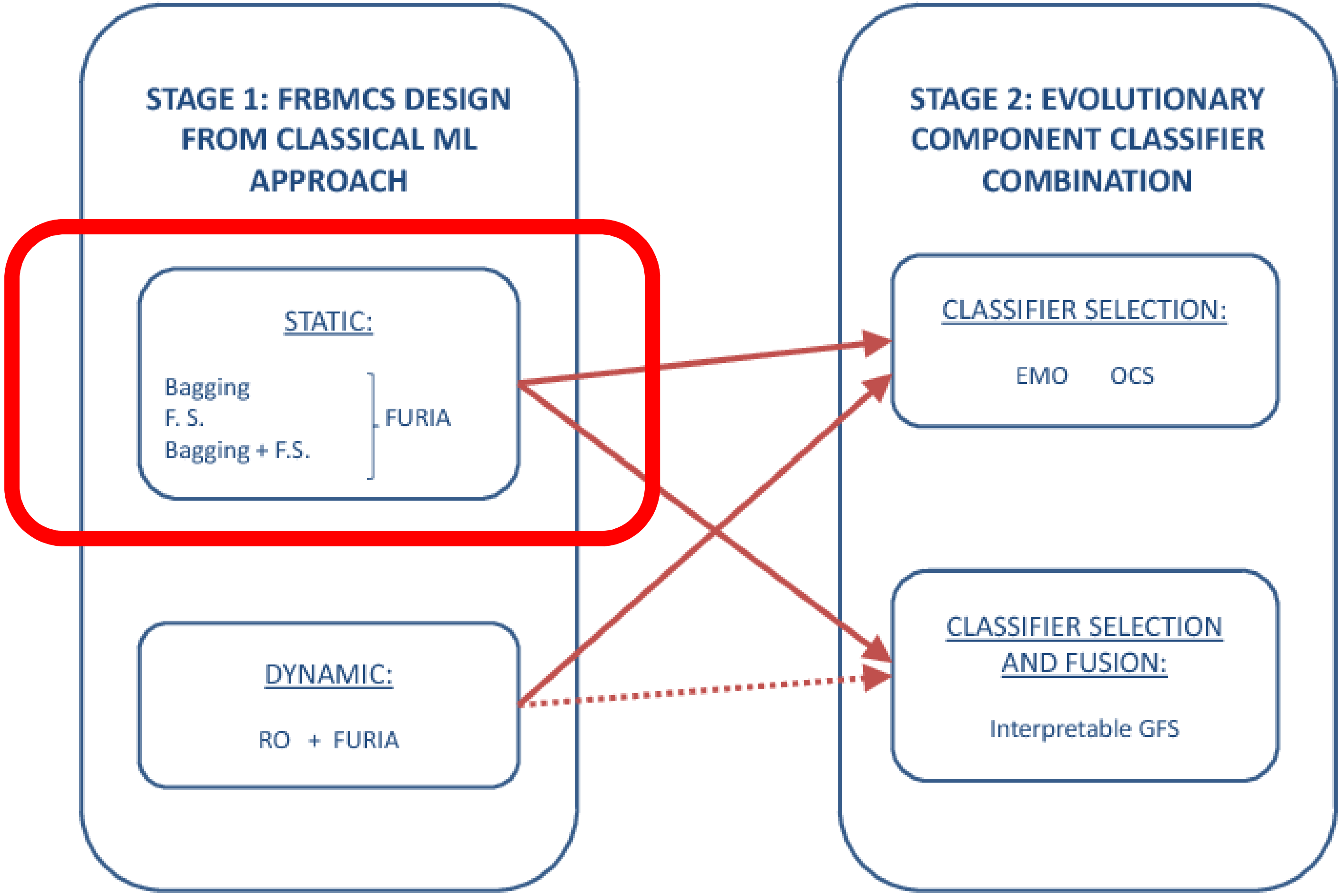

3.1. Bagging FURIA-based fuzzy classifier ensembles (Static)
Overall view

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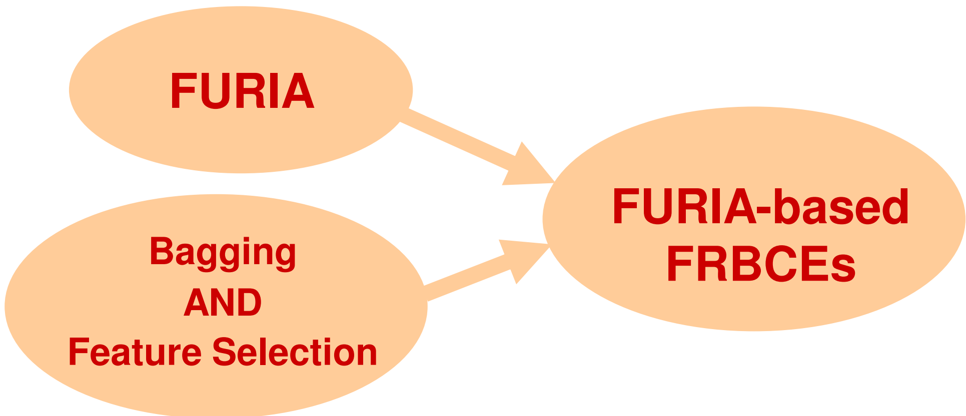
Method combining several classical techniques to quickly generate accurate and diverse base fuzzy classifiers:

- A parallel approach: bootstrap aggregating (bagging)
- A dimensionality reduction method (feature selection)
- A quick and accurate fuzzy rule generation method (FURIA)



Bagging + feature selection is a generic approach to design good performance CEs (Panov & al, 2007)

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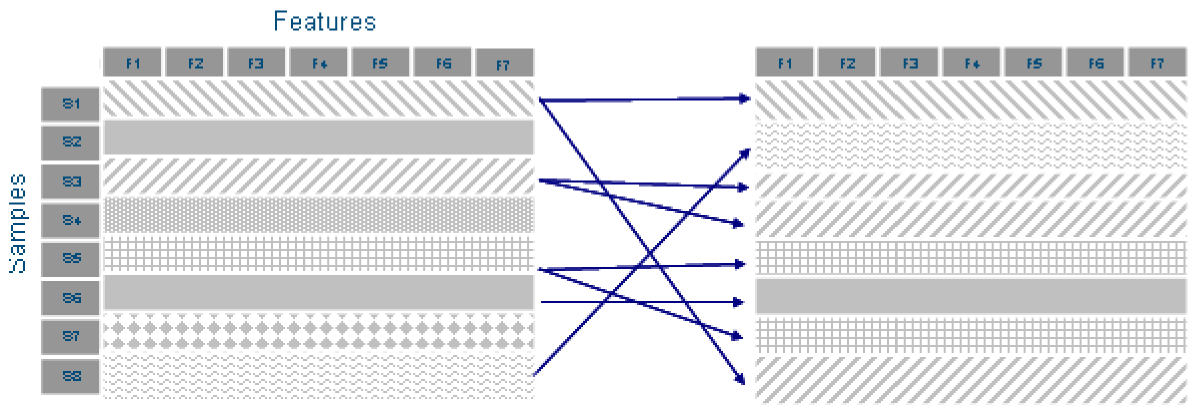
Bagging Predictors (Breiman, 1996):

- Boostrap AGGregatING: create multiple boostrap samples, train a classifier on each, and combine the classifier outputs by voting
- The individual classifiers (weak learners) are independently learnt from resampled training sets ("bags"), which are randomly selected with replacement from the original training data set



Good for unstable (large bias) classifiers (e.g. decision trees)

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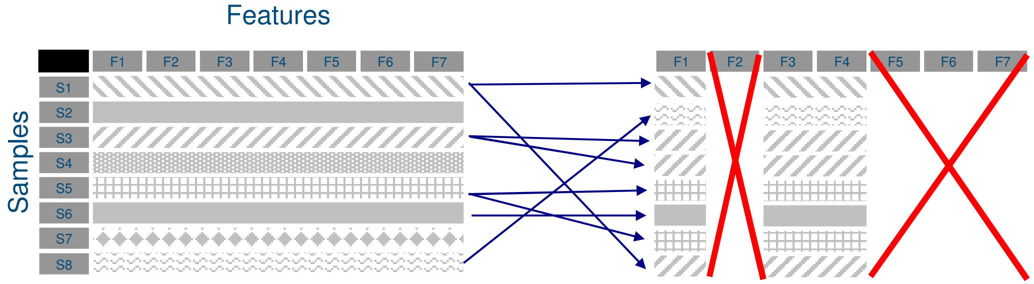
Feature selection:

- Another classical approach to induce diversity
- Good to use for not so unstable classifier design methods, such as many fuzzy rule generation methods
- Three different feature selection methods used:

Bagging
+ feature selection



- MIFS greedy method (Battiti, 1994)
- Randomly (random subspace, Breiman 2001)
- Random greedy (combines MIFS with RS)

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FURIA (Fuzzy Unordered Rule Induction Algorithm)
(Hüllermeier et al., 2009):

- A rule learning algorithm extending RIPPER
- Generates simple and compact fuzzy classification rules
- Decision tree-based learning approach: deals properly with high dimensional datasets and incorporates feature selection
- Very quick generation method
- Performs well comparing to C4.5 and RIPPER

AIM: Improve accuracy by embedding FURIA into the FRBCE framework

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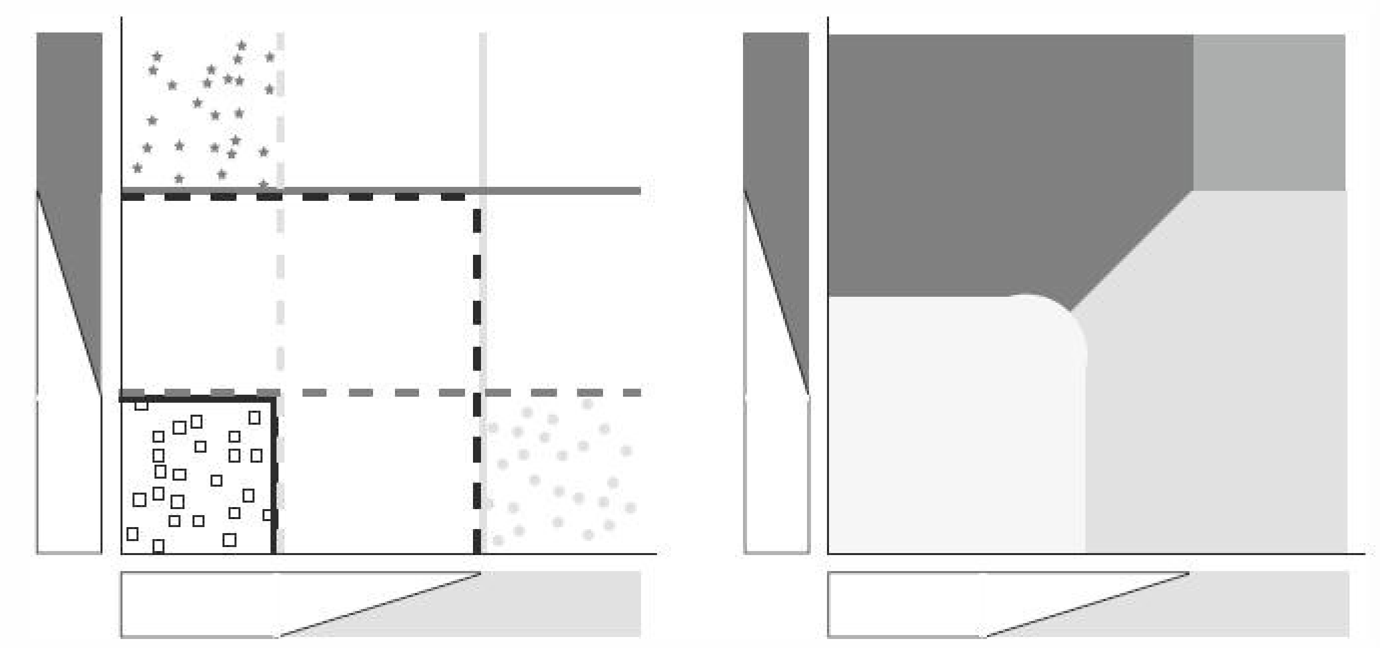
Fuzzy rules – soft boundaries



A. Crisp rules B. Fuzzy rules

Soft boundaries provided by fuzzy rules (Hüllermeier et al. 2009)

3.1. Bagging FURIA-based fuzzy classifier ensembles (Static)
General Scheme

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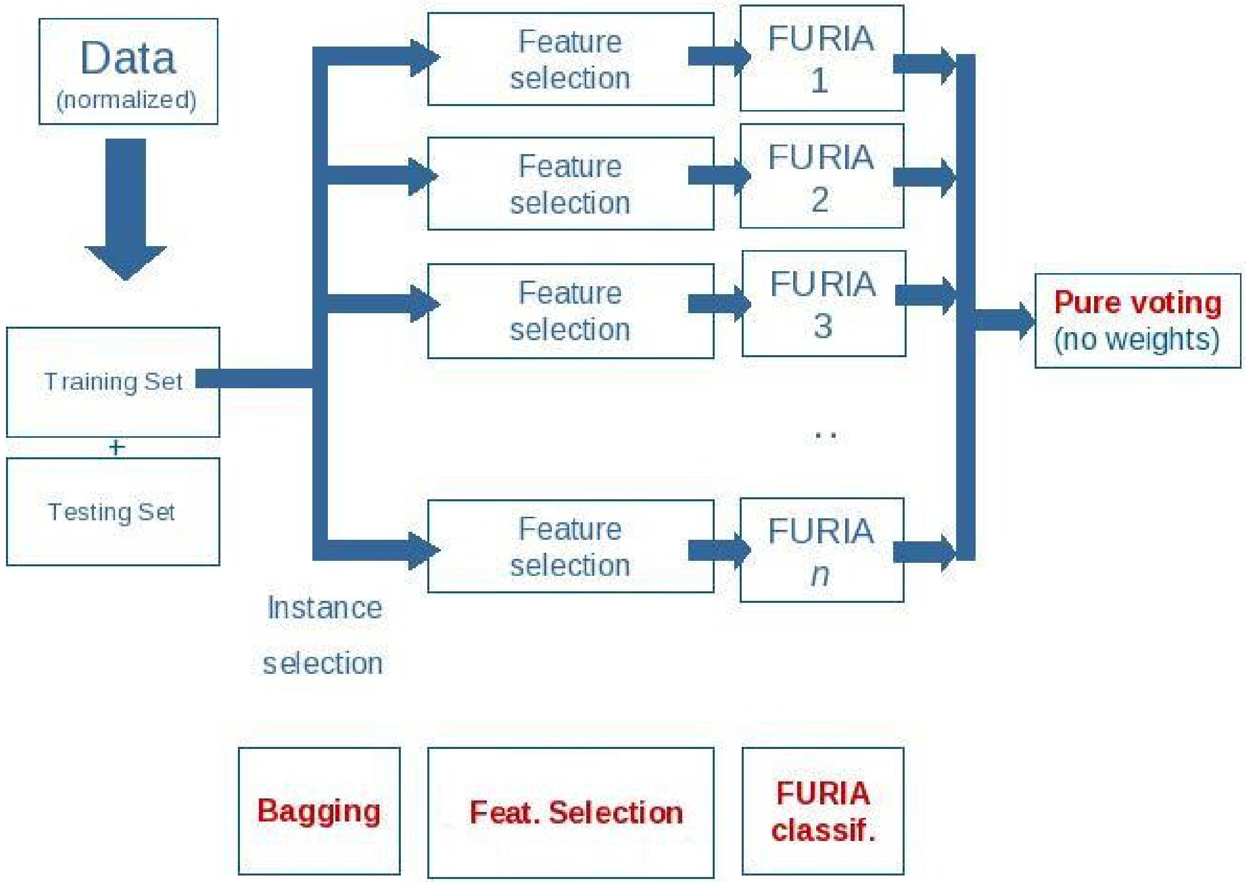

K. Trawinski, O. Cordón, A. Quirin. On designing fuzzy multiclassifier systems by combining FURIA with bagging and feature selection. International Journal of Uncertainty, Fuzziness, and Knowledge-based Systems 19:4 (2011) 589-633. *IF: 1.781. Cat: CS, AI. O: 31/111. Q2*

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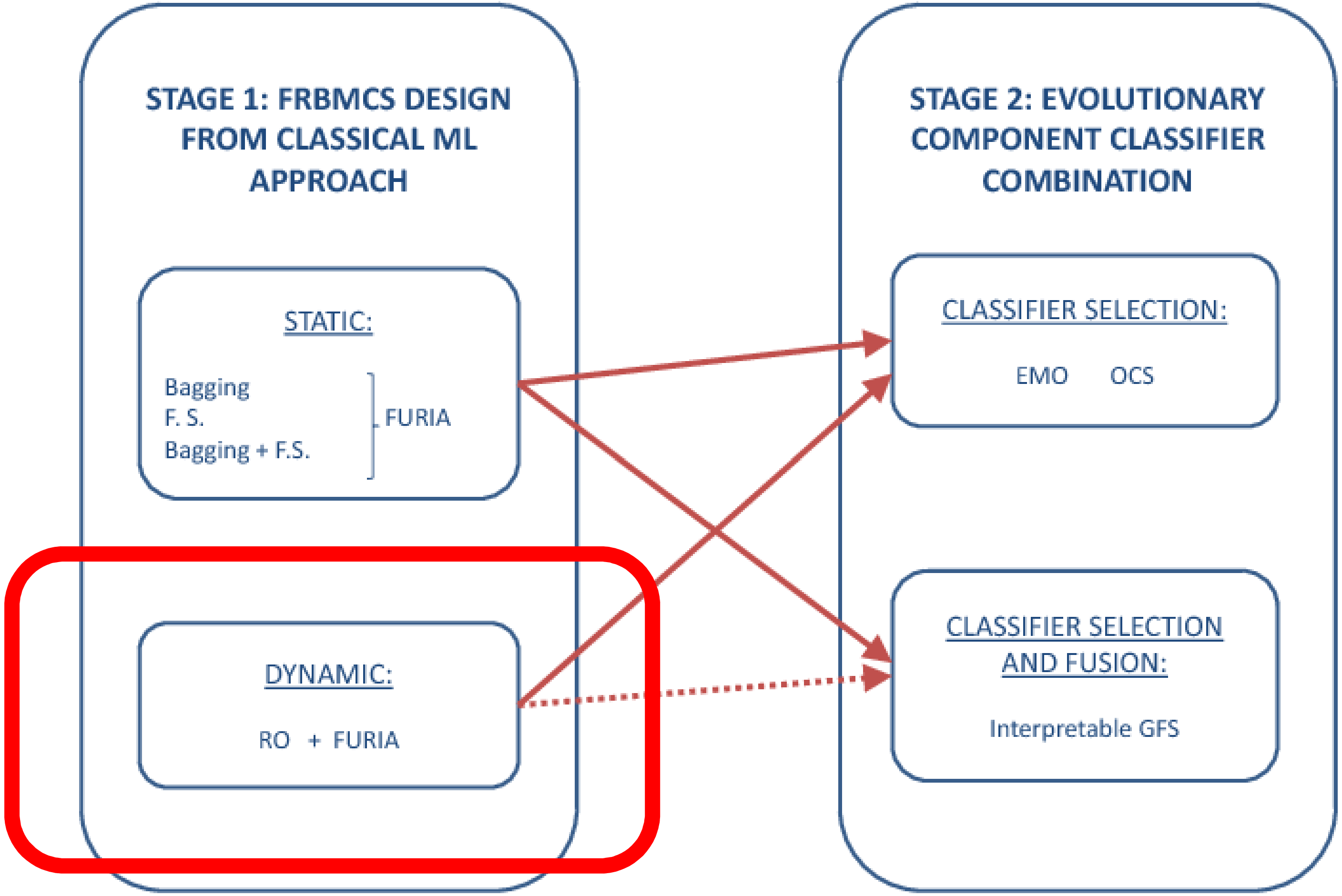

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- Random Oracles (ROs) is a recent proposal of a generic and fast CE design method introducing **additional diversity** and thus improving **accuracy**

- We aim to incorporate ROs into bagging FURIA-based FRBCEs:
  - The ensemble accuracy can be improved thanks to the dynamic approach and the additional diversity induced
  - The additional diversity can also be beneficial for an *a posteriori* global classifier selection process

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The RO algorithm:

- **Learning.** Each base classifier (k=1, …, K) is constructed as follows:
  - Draw a random hyperplane h_k in the feature space of problem P
  - Split the training set T_k into two parts, T^+_k and T^-_k, depending on which side of h_k the points lie
  - Train a classifier for each side/part, $D^+_k=D(T^+_k,C_j)$ and $D^-_k(T^-_k,C_j)$
- **Classification.** For each new data example x, assign the decision of each ensemble component by choosing D^+_k or D^-_k depending on which side of h_k x lays

Random Oracles (II)

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Two different RO variants according to the oracle plane generation:

Random Linear Oracles (RLOs) use a randomly generated hyperplane while Random Spherical Oracles (RSOs) consider a random hypersphere

- **RLO (hyperplane):**
  - Select randomly a pair of examples from the training set
  - Find the line segment between these points passing through the middle point M
  - Calculate the hyperplane perpendicular to the obtained line segment and containing M
- **RSO (hypersphere and feature selection):**
  - Select randomly at least the half (≥50%) of the features
  - Choose randomly a training set example to become the center
  - Calculate the distances from the center to E examples (chosen at random); the median of these distances is the hypersphere radius

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RO main features:

- Fast to train and evaluate
- Embeds only the base classifier. Thus,
  - any CE strategy can be applied;
  - any classifier learning algorithm (sub-classifier) can be used
- Combines *classifier fusion* and *classifier selection*
- Increases diversity and thus the final CE accuracy

General Scheme

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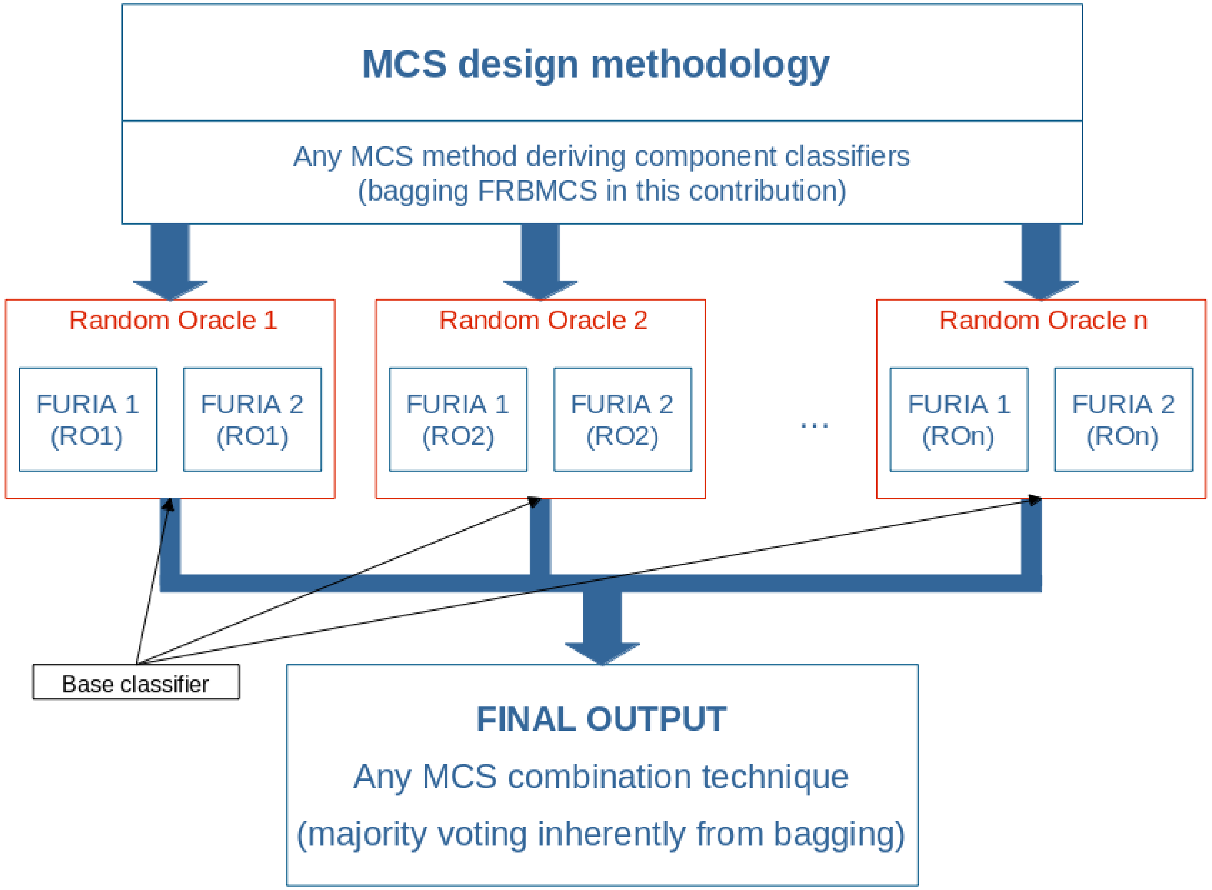

K. Trawinski, O. Cordón, A. Quirin. On Applying Random Oracles to Fuzzy Rule-Based Classifier Ensembles for High Complexity Datasets. Proc. EUSFLAT-2013, September 2013.

K. Trawinski, O. Cordón, A. Quirin. Random Oracles Fuzzy Rule-Based Multiclassifiers for High Complexity Datasets. Proc. FuzzIEEE2013, July 2013

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UCI and KEEL datasets:

- Large number of datasets considered: **29**
- Every attribute is continuous (abalone has one nominal, bioassay_688red has some binary attributes)
- Complex and high dimensional: large number of features (617), classes (28), and instances (58.000)
- Pentium i-5 3.1 GHz, 4 GB, 4 cores

TABLE I: Datasets considered

Dataset	#ex.	#attr.	(R/I/N)	cmpl.	#cl.
abalone	4178	8	(7/0/1)	3.3	28
bioassay_688red	27190	153	(27/126/0)	416.0	2
coil2000	9822	85	(0/85/0)	83.5	2
gas_sensor	13910	128	(128/0/0)	178.0	7
isolet	7797	617	(617/0/0)	481.1	26
letter	20000	16	(0/16/0)	32.0	26
magic	19020	10	(10/0/0)	19.0	2
marketing	6876	13	(0/13/0)	8.9	9
mfeat_fac	2000	216	(0/216/0)	43.2	10
mfeat_fou	2000	76	(76/0/0)	15.2	10
mfeat_kar	2000	64	(64/0/0)	12.8	10
mfeat_zer	2000	47	(47/0/0)	9.4	10
musk2	6598	166	(0/166/0)	109.5	2
optdigits	5620	64	(0/64/0)	36.0	10
pblocks	5474	10	(4/6/0)	5.5	5
pendigits	10992	16	(0/16/0)	17.6	10
ring_norm	7400	20	(20/0/0)	14.8	2
sat	6436	36	(0/36/0)	23.2	6
segment	2310	19	(19/0/0)	4.4	7
sensor_read_24	5456	24	(24/0/0)	13.1	4
shuttle	58000	9	(0/9/0)	52.2	7
spambase	4602	57	(57/0/0)	26.2	2
steel_faults	1941	27	(11/16/0)	5.2	7
texture	5500	40	(40/0/0)	22.0	11
thyroid	7200	21	(6/15/0)	15.1	3
two_norm	7400	20	(20/0/0)	14.8	2
waveform_noise	5000	40	(40/0/0)	20.0	3
waveform1	5000	21	(21/0/0)	10.5	3
wquality_white	4898	11	(11/0/0)	5.4	7

Experimental setup (II)

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Validation:

- Dietterich's **5x2-fold** cross validation
- Statistical tests:
  - **Friedman** and **Iman Davenport** tests for multiple comparison
  - **Holm** (1×n) **and Shaffer** (n×n) tests for pairwise comparison
- **Test accuracy** and **#rules** as the performance measures

Parameter values:

- **100 classifiers** for classical bagging FRBMCSs and **75** for RLO-based bagging FRBMCSs generated
- The confidence level for the null hypothesis rejection for all statistical tests = **5%**

3.3. Experiments
Fuzzy rule-based classifier ensemble results

*FRBCE **Accuracy** Benchmarking:*

RO-based bagging FRBCEs outperform bagging FRBCEs:

BAG 5 wins (1 tie);

RLO 8 wins (2 ties);

RSO 19 wins (2 ties)

Better average, lower std. dev.

Statistical tests:

Friedman and Iman Davenport

Shaffer

Dataset	BAG Test err.	BAG+RLO Test err.	BAG+RSO Test err.
abalone	0.7455	0.7452	**0.7480**
bioassay_688red	**0.0090**	**0.0090**	**0.0090**
coil2000	0.0602	**0.0601**	**0.0601**
gas_sensor	0.0086	0.0079	**0.0078**
isolet	0.0774	**0.0691**	0.0700
letter	0.0778	**0.0742**	0.0743
magic	0.1325	0.1314	**0.1299**
marketing	0.6749	0.6673	**0.6671**
mfeat_fac	0.0547	0.0434	**0.0431**
mfeat_fou	0.1992	0.1941	**0.1925**
mfeat_kar	0.0825	**0.0699**	0.0709
mfeat_zer	0.2231	**0.2169**	0.2181
musk2	0.0338	0.0328	**0.0320**
optdigits	0.0324	0.0283	**0.0282**
pblocks	**0.0335**	0.0353	0.0338
pendigits	0.0155	0.0137	**0.0132**
ring_norm	0.0432	0.0438	**0.0315**
sat	0.1013	0.1008	**0.1001**
segment	0.0309	0.0303	**0.0295**
sensor_read_24	**0.0222**	0.0227	0.0233
shuttle	**0.0008**	0.0009	0.0009
spambase	0.0663	0.0651	**0.0639**
steel_faults	0.2371	0.2367	**0.2361**
texture	0.0288	0.0278	**0.0274**
thyroid	**0.0212**	0.0215	0.0218
two_norm	0.0316	**0.0271**	0.0276
waveform_noise	0.1480	0.1461	**0.1457**
waveform	0.1480	**0.1451**	0.1453
wquality_white	0.3908	0.3840	**0.3803**
Avg.	0.1286	0.1259	**0.1252**
Std. Dev.	0.1833	0.1825	0.1829

Fuzzy rule-based classifier ensemble results (II)

*FRBCE **Accuracy** Benchmarking:*

RO-based bagging FRBCEs outperform bagging FRBCEs:

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RLO 8 wins (2 ties);

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Better average, lower std. dev.

Statistical tests:

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bioassay_688red	**0.0090**	**0.0090**	**0.0090**
coil2000	0.0602	**0.0601**	**0.0601**
gas_sensor	0.0086	0.0079	**0.0078**
isolet	0.0774	**0.0691**	0.0700
letter	0.0778	**0.0742**	0.0743
magic	0.1325	0.1314	**0.1299**
marketing	0.6749	0.6673	**0.6671**
mfeat_fac	0.0547	0.0434	**0.0431**
mfeat_fou	0.1992	0.1941	**0.1925**
mfeat_k[illegible]	[illegible]	[illegible]	0.0709
mfeat_z[illegible]	[illegible]	[illegible]	0.2181
musk2	[illegible]	[illegible]	**0.0320**
optdigi[illegible]	[illegible]	[illegible]	**0.0282**
pblocks	[illegible]	[illegible]	0.0338
pendigi[illegible]	[illegible]	[illegible]	**0.0132**
ring_no[illegible]	[illegible]	[illegible]	**0.0315**
sat	0.1013	0.1008	**0.1001**
segment	0.0309	0.0303	**0.0295**
sensor_read_24	**0.0222**	0.0227	0.0233
waveform	0.1480	**0.1451**	0.1453
wquality_white	0.3908	0.3840	**0.3803**
Avg.	0.1286	0.1259	**0.1252**
Std. Dev.	0.1833	0.1825	0.1829

Algorithm	Ranking
FURIA+BAG+RSO	1.552
FURIA+BAG+RLO	1.828
FURIA+BAG	2.621

Comparison	p-value
BAG+RSO vs BAG	+(**1.41e-4**)
BAG+RLO vs BAG	+(**0.002**)
BAG+RSO vs BAG+RLO	=(0.293)

Fuzzy rule-based classifier ensemble results (III)

*FRBCE **Complexity** Benchmarking:*

RO-based bagging FRBCEs outperform bagging FRBCEs:

BAG 2 wins;

RLO 25 wins;

RSO 2 wins

Better average, lower std. dev.

Statistical tests:

Friedman and Iman Davenport

Shaffer

Dataset	BAG # Rules	BAG+RLO # Rules	BAG+RSO # Rules
abalone	**8369.0**	8696.7	9382.8
bioassay_688red	5526.9	**4642.8**	4780.8
coil2000	4331.9	**3804.1**	4002.1
gas_sensor	8628.3	**7091.3**	7310.7
isolet	12215.7	**10523.6**	10828.5
letter	47109.1	**39410.5**	40972.9
magic	13770.8	**13143.0**	14556.9
marketing	**6418.5**	7252.0	7429.1
mfeat_fac	3479.9	**3050.2**	3110.3
mfeat_fou	5483.5	**4711.4**	4886.9
mfeat_kar	4953.3	**4448.4**	4581.0
mfeat_zer	5028.3	**4349.9**	4549.2
musk2	4332.2	**3581.1**	3582.7
optdigits	7167.3	**6352.4**	6511.1
pblocks	3201.7	2877.9	**2816.4**
pendigits	8788.6	**7348.0**	7491.6
ring_norm	7308.9	6205.7	**5961.4**
sat	8454.4	**6956.2**	7109.5
segment	2546.3	**2201.6**	2378.7
sensor_read_24	3430.8	**3340.4**	3428.3
shuttle	1826.2	**1723.8**	1737.5
spambase	3612.9	**3281.9**	4181.1
steel_faults	5467.3	**4799.0**	4857.0
texture	6537.2	**5305.7**	5542.8
thyroid	3299.5	**2831.7**	2959.8
two_norm	6147.5	**4973.3**	5307.8
waveform_noise	7932.6	**6729.9**	6850.6
waveform	8303.0	**7017.3**	7115.0
wquality_white	13429.3	**12134.0**	12564.4
Avg.	7831.1	6854.6	7130.6
Std. Dev.	8144.6	6857.3	7156.8
Avg. (Without Letter)	6428.3	5691.9	5921.9
Std. Dev. (Without Letter)	3100.2	2847.3	3030.4

Fuzzy rule-based classifier ensemble results (IV)

*FRBCE **Complexity** Benchmarking:*

RO-based bagging FRBCEs outperform bagging FRBCEs:

BAG 2 wins;

RLO 25 wins;

RSO 2 wins

Better average, lower std. dev.

Statistical tests:

Friedman and Iman Davenport

Shaffer

Dataset	BAG # Rules	BAG+RLO # Rules	BAG+RSO # Rules
abalone	**8369.0**	8696.7	9382.8
bioassay_688red	5526.9	**4642.8**	4780.8
coil2000	4331.9	**3804.1**	4002.1
gas_sensor	8628.3	**7091.3**	7310.7
isolet	12215.7	**10523.6**	10828.5
letter	47109.1	**39410.5**	40972.9
magic	13770.8	**13143.0**	14556.9
marketing	**6418.5**	7252.0	7429.1
mfeat_fac	3479.9	**3050.2**	3110.3
mfeat_fou	5483.5	**4711.4**	4886.9
mfeat_kar	4953.3	**4448.4**	4581.0
mfeat_zer	5028.3	**4349.9**	4549.2
musk2	[illegible]	[illegible]	[illegible]
optdigits	[illegible]	[illegible]	[illegible]
pblocks	[illegible]	[illegible]	[illegible]
pendigits	[illegible]	[illegible]	[illegible]
ring_norm	[illegible]	[illegible]	[illegible]
sat	[illegible]	[illegible]	[illegible]
segment	[illegible]	[illegible]	[illegible]
sensor_rea	[illegible]	[illegible]	[illegible]
shuttle	[illegible]	[illegible]	1737.5
spambase	3612.9	**3281.9**	4181.1
steel_faults	5467.3	**4799.0**	4857.0
texture	6537.2	**5305.7**	5542.8
thyroid	[illegible]	[illegible]	[illegible]
Std. Dev. (Without Letter)	3100.2	2847.3	3030.4

Algorithm	Ranking
FURIA+BAG+RLO	1.138
FURIA+BAG+RSO	2.069
FURIA+BAG	2.793

Comparison	p-value
BAG+RLO vs BAG	+(8.77e-10)
BAG+RSO vs BAG	+(0.006)
BAG+RLO vs BAG+RSO	=+(3.92e-4)

Fuzzy rule-based classifier ensemble results (V)

Classical CE-FRBCE **Accuracy** Benchmarking:

RSO-based bagging FRBCEs outperform classical CEs:

RSO 14 wins (2 ties);

BAG C4.5 10 wins (2 ties);

BAG NB 2 wins;

RF 7 wins (1 tie)

Better average, lower std. dev.

Statistical tests:

Friedman and Iman Davenport

Holm

Dataset	FURIA Test err.	C4.5 Test err.	NB Test err.	RF Test err.
abalone	**0.7480**	0.7681	0.7619	0.7536
bioassay_688red	**0.0090**	**0.0090**	0.0152	**0.0090**
coil2000	0.0601	0.0615	0.1847	**0.0597**
gas_sensor	**0.0078**	0.0089	0.2939	0.0092
isolet	**0.0700**	0.0788	0.1246	0.0766
letter	0.0743	**0.0615**	0.2927	0.0701
magic	0.1299	**0.1255**	0.2391	0.1314
marketing	0.6671	0.6735	0.6864	**0.6624**
mfeat_fac	**0.0431**	0.0498	0.0659	0.0475
mfeat_fou	0.1925	0.1902	0.2221	**0.1858**
mfeat_kar	0.0709	0.0818	**0.0593**	0.0597
mfeat_zer	**0.2181**	0.2273	0.2464	0.2330
musk2	0.0320	**0.0271**	0.1107	0.0375
optdigits	0.0282	**0.0276**	0.0709	0.0277
pblocks	0.0338	**0.0327**	0.0706	0.0332
pendigits	**0.0132**	0.0150	0.0864	0.0162
ring_norm	**0.0315**	0.0376	0.0199	0.0587
sat	0.1001	**0.0950**	0.1720	0.1027
segment	**0.0295**	0.0328	0.1180	0.0350
sensor_read_24	0.0233	0.0234	0.3710	**0.0224**
shuttle	**0.0009**	**0.0009**	0.0143	**0.0009**
spambase	0.0639	0.0651	0.1788	**0.0625**
steel_faults	0.2361	**0.2263**	0.3441	0.2517
texture	**0.0274**	0.0334	0.1384	0.0383
thyroid	**0.0218**	0.0222	0.0381	0.0221
two_norm	0.0276	0.0280	**0.0219**	0.0389
waveform_noise	**0.1457**	0.1643	0.1668	0.1556
waveform	**0.1453**	0.1588	0.1534	0.1587
wquality_white	0.3803	**0.3688**	0.5230	0.3864
Avg.	**0.1252**	0.1274	0.1997	0.1292
Std. Dev.	0.1829	0.1852	0.1890	0.1830

Fuzzy rule-based classifier ensemble results (VI)

Classical CE-FRBCE ***Accuracy*** *Benchmarking:*

RSO-based bagging FRBCEs outperform classical CEs:

RSO 14 wins (2 ties);

BAG C4.5 10 wins (2 ties);

BAG NB 2 wins;

RF 7 wins (1 tie)

Better average, lower std. dev.

Statistical tests:

Friedman and Iman Davenport

Holm

Dataset	FURIA Test err.	C4.5 Test err.	NB Test err.	RF Test err.
abalone	**0.7480**	0.7681	0.7619	0.7536
bioassay_688red	**0.0090**	**0.0090**	0.0152	**0.0090**
coil2000	0.0601	0.0615	0.1847	**0.0597**
gas_sensor	**0.0078**	0.0089	0.2939	0.0092
isolet	**0.0700**	0.0788	0.1246	0.0766
letter	0.0743	**0.0615**	0.2927	0.0701
magic	0.1299	**0.1255**	0.2391	0.1314
marketing	0.6671	0.6735	0.6864	**0.6624**
mfeat_fac	**0.0431**	0.0498	0.0659	0.0475
mfeat_fou	0.1925	0.1902	0.2221	**0.1858**
mfeat_kar	0.0709	0.0818	**0.0593**	0.0597
mfeat_zer				0.2330
musk2				0.0375
optdigits				0.0277
pblocks				0.0332
pendigits				0.0162
ring_norm				0.0587
sat				0.1027
segment				0.0350
sensor_read_24	0.0233	0.0234	0.3710	**0.0224**
shuttle	**0.0009**	**0.0009**	0.0143	**0.0009**
spambase	0.0639	0.0651	0.1788	**0.0625**
steel_faults	0.2361	**0.2263**	0.3441	0.2517
Avg.	**0.1252**	0.1274	0.1997	0.1292
Std. Dev.	0.1829	0.1852	0.1890	0.1830

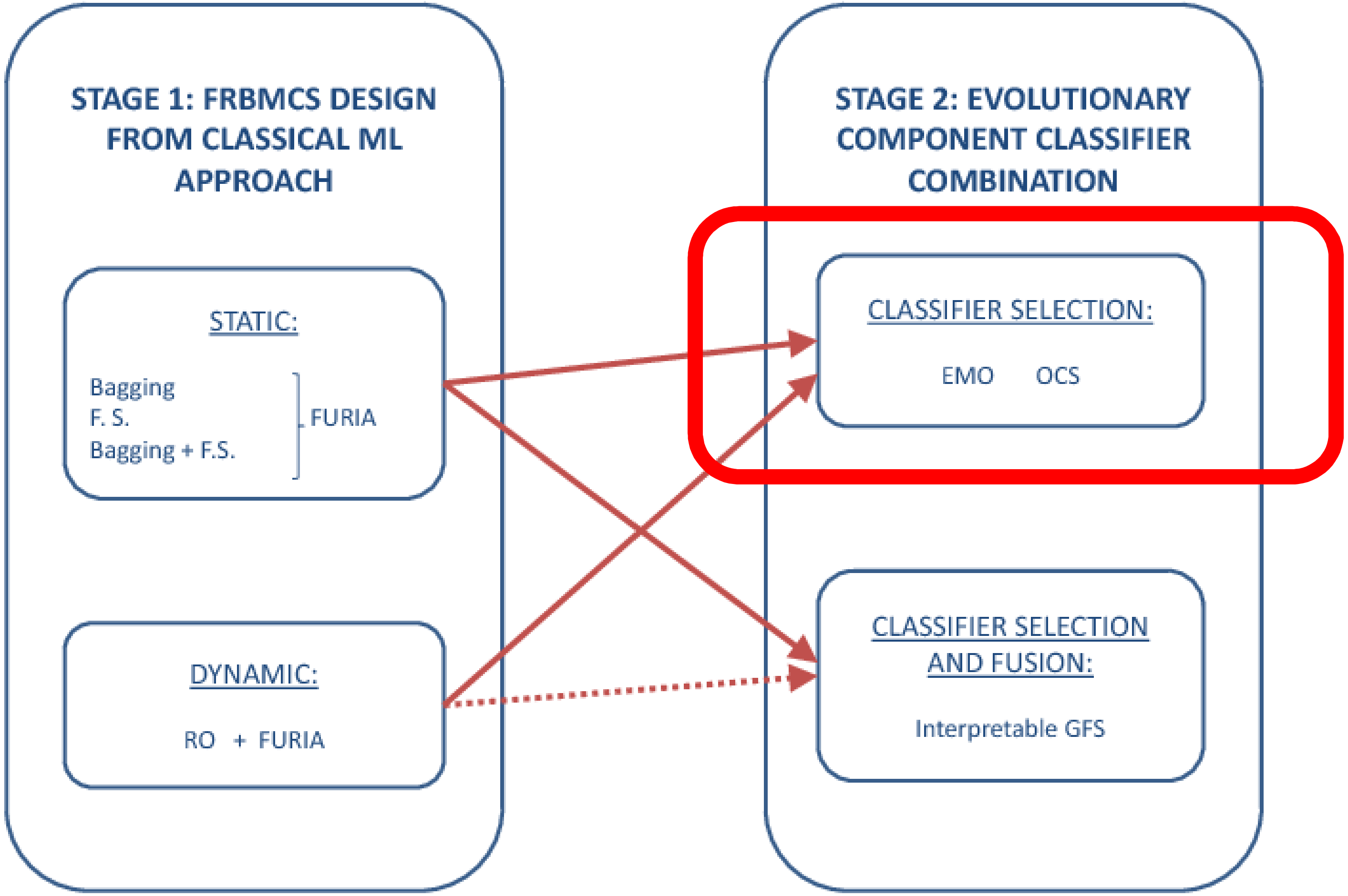
Algorithm	Ranking
FURIA+BAG+RSO	1.793
C4.5+BAG+RSO	2.276
RF	2.345
NB+BAG+RSO	3.586

Comparison	p-value
FURIA+BAG+RSO vs C4.5+BAG+RSO	=(0.207)
FURIA+BAG+RSO vs RF	=(0.207)
FURIA+BAG+RSO vs NB+BAG+RSO	**+(3.69e-7)**

Proposed Framework Graphical Representation

OVERVIEW

1. Introduction
2. Proposed Framework
3. FRBCE Design from Classical ML Approaches
4. **EMO Selection of Component Classifiers**
5. Classifier Selection and Fusion via an Interpretable GFS
6. Conclusions



Overall view

OVERVIEW

1. Introduction
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6. Conclusions

- OCS is an extended classifier selection strategy both reducing the complexity and improving the accuracy of CEs
- Accuracy is usually considered as the main optimization criterion but complexity and diversity are also interesting (not well known relation between accuracy and diversity)
- Evolutionary algorithms have been widely used for OCS. EMO shows a strong ability in the optimization of conflicting criteria
- We aim to propose an **EMO-based OCS strategy** as a component of our framework:
  - Joint optimization of up to three different kinds of criteria
  - Obtaining of a set of CE designs with different accuracy-complexity tradeoffs in a single algorithm run
  - **Specific:** Check how beneficial the additional diversity induced by ROs is for EMO OCS-based FRBCEs

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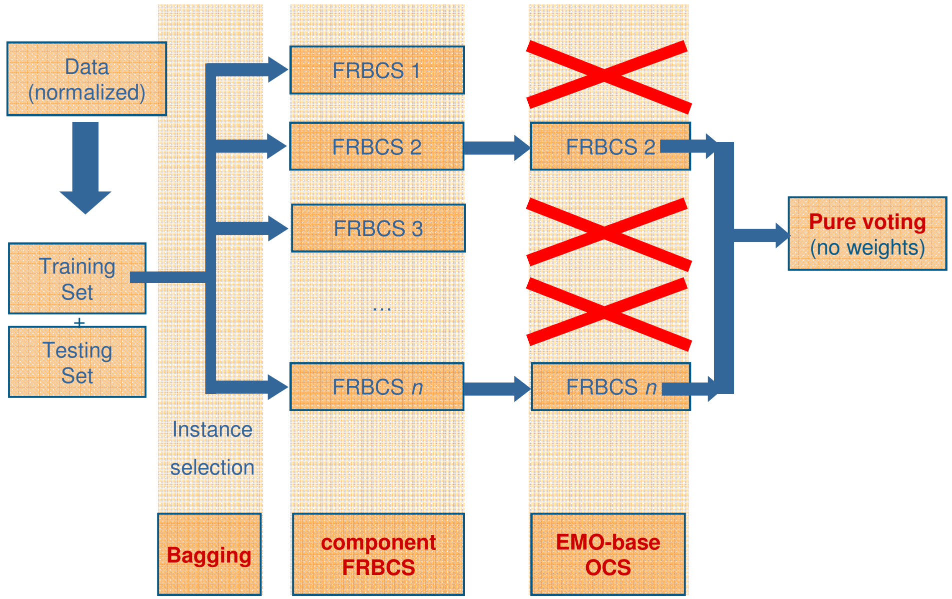
OCS strategy (Partridge and Yates, 1996) :

- Generate many classifiers and select the best cooperating subset
- Decrease complexity/eliminate useless classifiers to improve accuracy



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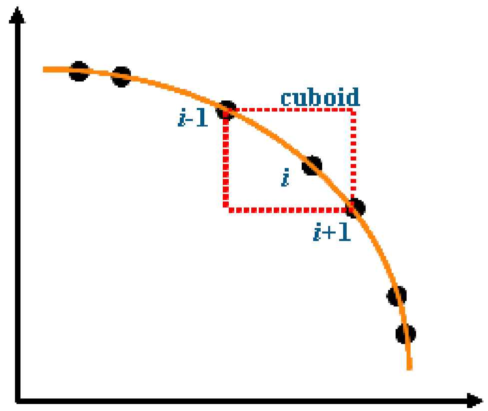
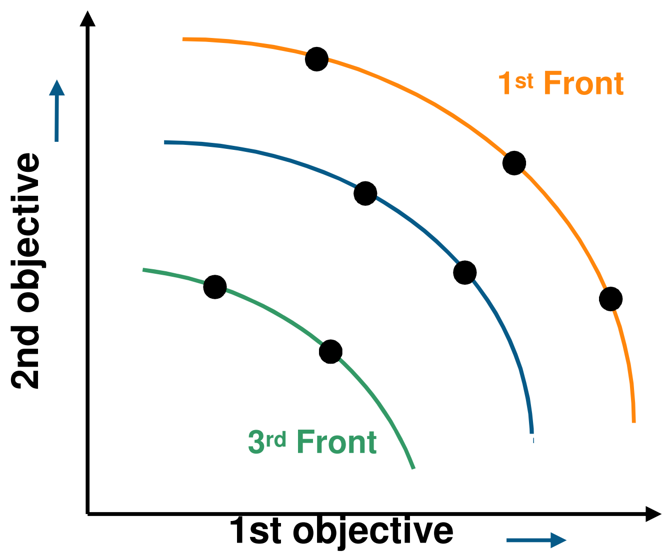
NSGA-II EMO algorithm (Deb et al., 2002):

- Produces a set of efficient solutions (Pareto-optimal set) in a single run
- Based on Pareto dominance depth approach, where population is divided into several fronts
- Solutions in the same front have the same fitness rank
- Crowding distance to promote Pareto front spreading





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NSGA-II-based EMO OCS method components:

- **Binary coding** –
  - General: a binary value is assigned to each component classifier (1 → selected classifier; 0 → discarded classifier)
  - RO-specific: a binary value is assigned to each RO subclassifier
- Generational approach and **elitist** replacement strategy
- **Binary tournament**
- Classical **two-point crossover** and **bit-flip mutation** or **biased** (towards smaller ensembles) **bit-flip mutation**

NSGA-II-based OCS method (II)

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Reparation operator (RO-specific coding scheme):

- As the oracle assigns only half-a-region of the feature space to each subclassifier, NSGA-II may select a subset of subclassifiers not covering the entire feature space
- To avoid that, at least one subclassifier in the RO pair is forced to be selected
- **Procedure:**
  - Generate all the possible combinations containing a single RO pair (to cover the entire feature space)
  - Evaluate them in the objective space and remove the dominated ones
  - Select one of the non-dominated solutions at random

NSGA-II-based OCS method (III)

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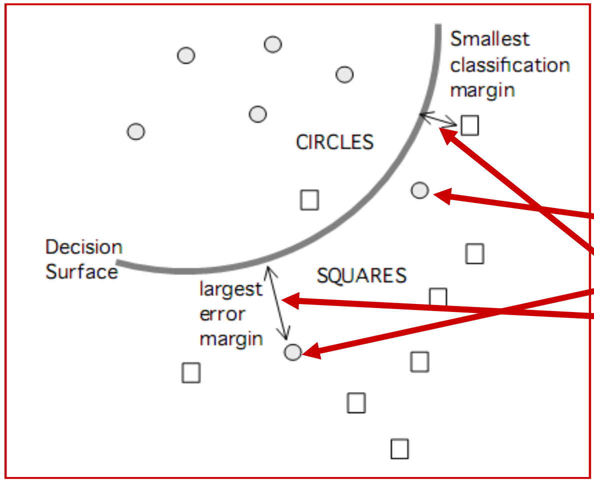
Objective functions: Three-objective fitness function designed from an **evaluation criteria** taken from each of the existing families:

- **accuracy**



A lexicographic order of:

1) Training error *e*

2) Error margin m_1

3) Classification margin m_2

- **complexity** (#classifiers)
- **diversity**: variance (θ)

General Scheme

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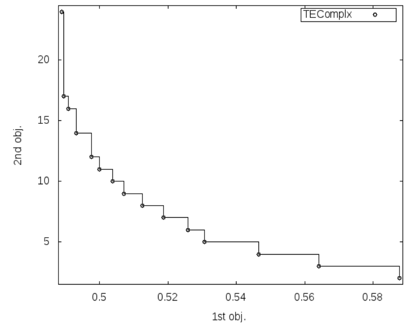

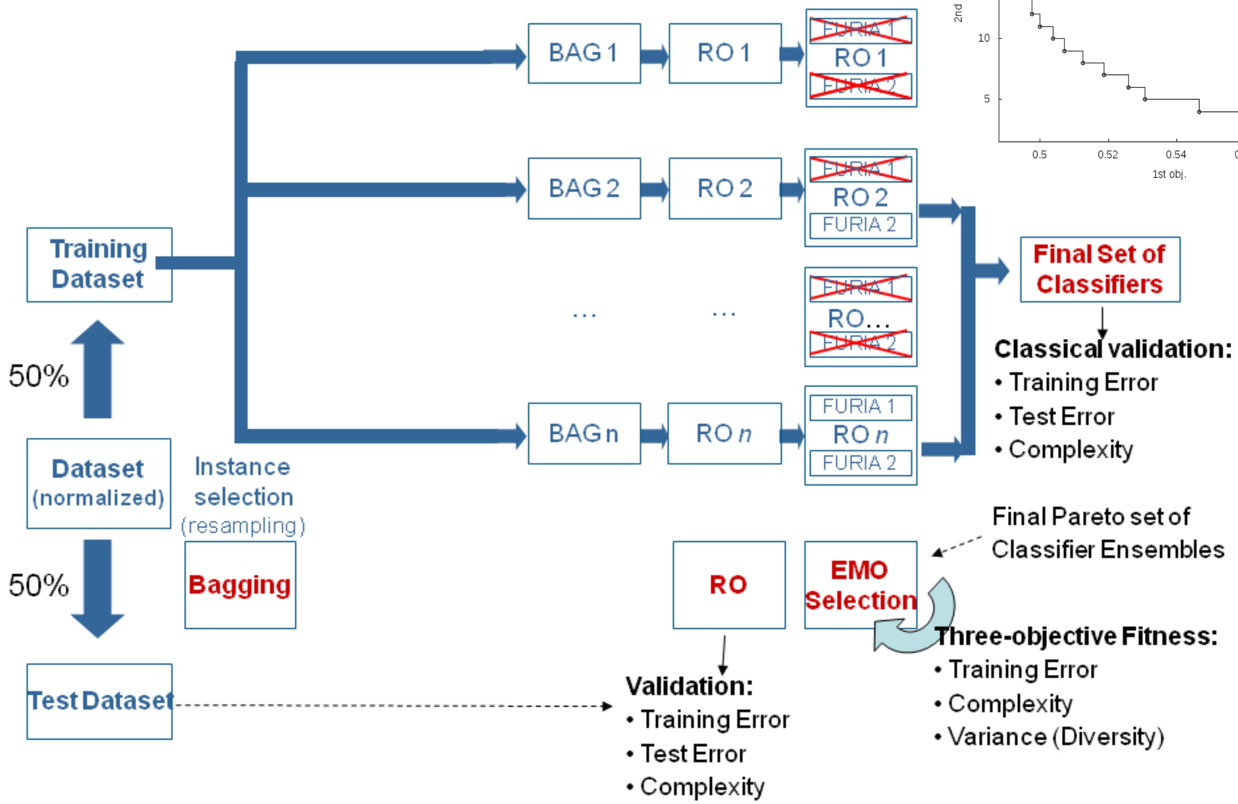

K. Trawinski, O. Cordón, A. Quirin. A Study on the Use of Multiobjective Genetic Algorithms for Classifier Selection in FURIA-based Fuzzy Multiclassifiers. Int. J. Comp. Intel. Syst. 5:2 (2012) 231-253

K. Trawinski, O. Cordón, A. Quirin. Multiobjective Genetic Classier Selection For Random Oracles Fuzzy Rule-Based Classier Ensembles: How Beneficial Is The Additional Diversity? Knowledge-based Systems (2013) **Submitted**. *IF 2011: 2.422. Cat: CS, AI. O: 15/111. Q1*

Experimental setup

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6. Conclusions

Same datasets and validation mechanism than in the previous experimental study:

Design choices and parameter values:

- Comparison between different fuzzy component classifier generation methods and EMO OCS strategy variants
- **Test accuracy** and **#rules** of each Pareto-optimal solution are measured to allow for a global comparison
- NSGA-II parameters: 200 individuals, 1000 generations, crossover prob. 0.6, mutation prob. 0.1
- Hypervolume Ratio (HVR) indicator considered to compare the obtained Pareto front approximations (PFAs)

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The FRBCE/EMO variants for the comparison purpose:

abbreviation	base classifier	CE methodology	OCS strategy	mut. type
BAS-BAG	FURIA	bagging	standard NSGA-II	standard
BAS-RLO	RLO (2×FURIA+oracle)	bagging+RLO	standard NSGA-II	standard
ADV-RLO	RLO (2×FURIA+oracle)	bagging+RLO	specific RO NSGA-II	standard
ADV-BI-RLO	RLO (2×FURIA+oracle)	bagging+RLO	specific RO NSGA-II	biased
BAS-RSO	RSO (2×FURIA+oracle)	bagging+RSO	standard NSGA-II	standard
ADV-RSO	RSO (2×FURIA+oracle)	bagging+RSO	specific RO NSGA-II	standard
ADV-BI-RSO	RSO (2×FURIA+oracle)	bagging+RSO	specific RO NSGA-II	biased

EMO-based OCS results

Comparison of PFAs using the HVR measure:

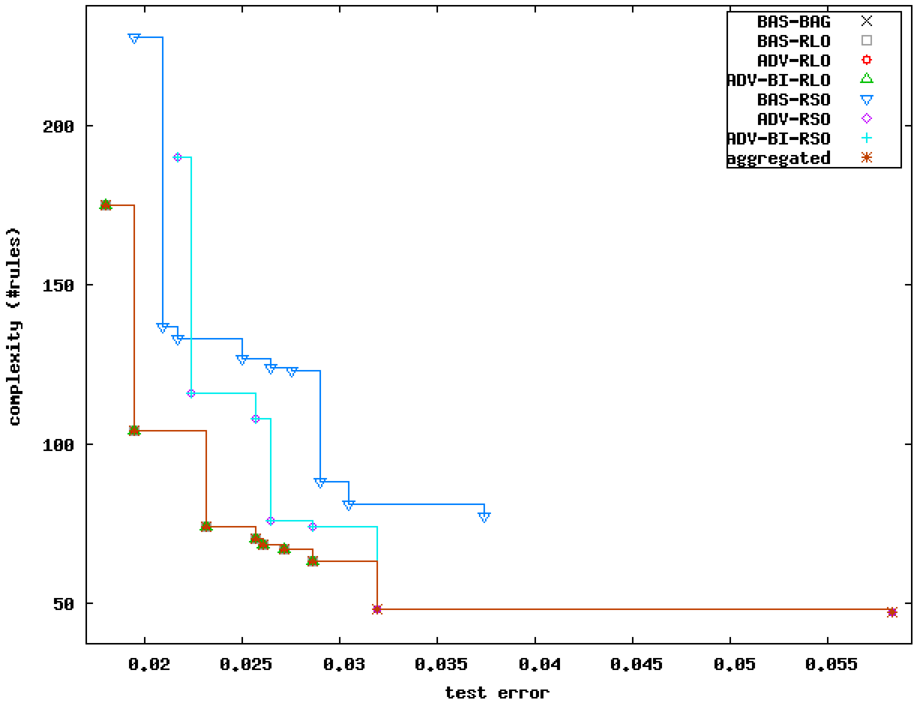
Reference PFAs considered (O1:Test Error, O2: #rules)

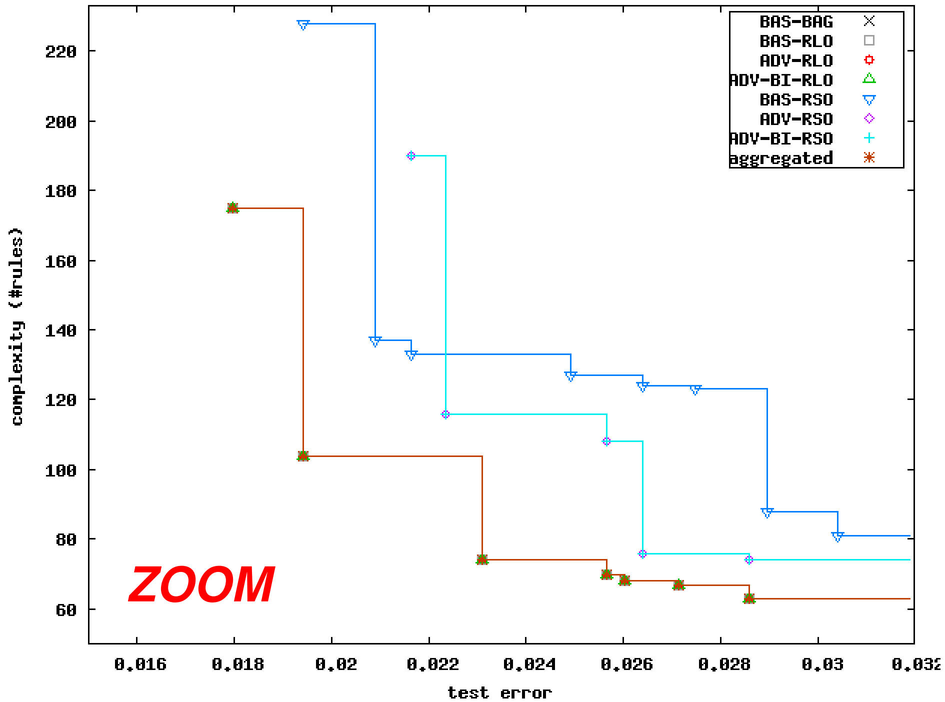
Variant ADV-BI-RLO clearly reports the best performance

	BAS-BAG	BAS-RLO	ADV-RLO	ADV-BI-RLO	BAS-RSO	ADV-RSO	ADV-BI-RSO
aba	0.8248	0.8594	0.6399	**0.8878**	0.8378	0.7305	0.8500
bio	0.8343	0.9073	0.8059	**0.9825**	0.9115	0.9118	0.9678
coi	0.6929	0.7419	0.5687	**0.7548**	0.7251	0.6497	0.6477
gas	0.8590	0.9404	0.6876	**0.9771**	0.9382	0.8435	0.9642
iso	0.8611	0.9118	0.7661	**0.9534**	0.9074	0.8571	0.9155
let	0.9127	0.9477	0.7961	0.9726	0.9626	0.8945	**0.9727**
mag	0.7970	0.8423	0.6444	**0.9061**	0.8433	0.8119	0.8737
mar	0.7214	0.8217	0.6569	**0.8689**	0.8170	0.7994	0.8225
mfa	0.8874	0.9463	0.7886	**0.9763**	0.9439	0.8717	0.9600
mfo	0.8373	0.8838	0.7145	**0.9322**	0.8809	0.8040	0.8931
mka	0.8661	0.9227	0.7643	**0.9631**	0.9091	0.8418	0.9211
mze	0.8041	0.8650	0.6498	**0.9183**	0.8560	0.7702	0.8660
mus	0.7112	0.8098	0.6161	**0.8779**	0.8172	0.7071	0.8122
opt	0.8721	0.9316	0.7662	**0.9669**	0.9322	0.8411	0.9415
pbl	0.7487	0.7794	0.6038	0.7231	0.8052	0.7764	**0.8421**
pen	0.8617	0.9375	0.6873	**0.9752**	0.9419	0.8106	0.9609
rin	0.8187	0.8526	0.6878	0.8803	0.9221	0.8954	**0.9222**
sat	0.8436	0.9219	0.7196	**0.9613**	0.9284	0.8296	0.9468
seg	0.8551	0.9081	0.7621	**0.9358**	0.9080	0.8172	0.8417
sen	0.8597	0.9234	0.6630	**0.9644**	0.9228	0.8043	0.9503
shu	0.9347	0.9192	0.7051	0.9645	0.9176	0.7858	**0.9661**
spa	0.8196	0.8932	0.6805	**0.9343**	0.8690	0.8535	0.9109
ste	0.8206	0.8836	0.6620	**0.9264**	0.8877	0.7998	0.9053
tex	0.8713	0.9308	0.7769	**0.9614**	0.9288	0.8388	0.9444
thy	0.8368	0.9084	0.6804	**0.9560**	0.9025	0.8303	0.9487
two	0.8774	0.9558	0.7478	**0.9814**	0.9392	0.8880	0.9565
wan	0.8566	0.8881	0.7335	**0.9397**	0.8873	0.8400	0.8890
wav	0.8426	0.9033	0.7163	**0.9300**	0.8989	0.8367	0.9192
wqu	0.7914	0.8567	0.6973	**0.9098**	0.8724	0.8119	0.8881
avg.	0.8317	0.8894	0.7031	**0.9269**	0.8901	0.8191	0.9035
dev.	0.0562	0.0522	0.0608	0.0618	0.0524	0.0564	0.0681

EMO-based OCS results (II)

***REFERENCE PFAs** (O1:Test Error, O2:Complexity)*
*obtained for **sensor_read_24** by all the EMO variants*





EMO-based OCS results (III)

Comparison of averaged performance of four single solutions selected from the obtained Pareto sets

		Best train			Best complx			Best trade-off			Best test		
		Tra	Tst	Cmpl	Tra	Tst	Cmpl	Tra	Tst	Cmpl	Tra	Tst	Cmpl
avg.	BAS-BAG	0.0506	0.1306	2002.6	0.0694	0.1399	444.7	0.0586	0.1287	854.4	0.0536	0.1263	1951.1
	BAS-RLO	0.0436	0.1301	1748.4	0.0983	0.1736	202.1	0.0575	0.1309	529.1	0.0478	0.1247	1819.4
	ADV-RLO	0.0435	0.1267	2663.0	0.0525	0.1314	1058.1	0.0490	0.1243	1598.6	0.0460	0.1225	2620.6
	ADV-BI-RLO	0.0426	0.1276	1268.5	0.0916	0.1680	90.2	0.0514	0.1287	343.0	0.0458	0.1232	1205.4
	BAS-RSO	0.0420	0.1292	1882.7	0.0926	0.1688	204.8	0.0554	0.1304	590.4	0.0447	0.1237	2100.6
	ADV-RSO	0.0409	0.1271	2453.8	0.0542	0.1326	932.7	0.0496	0.1242	1313.0	0.0451	0.1220	2216.0
	ADV-BI-RSO	0.0392	0.1308	1464.6	0.1410	0.2107	116.7	0.0609	0.1357	401.6	0.0438	0.1242	1492.4
dev.	BAS-BAG	0.1390	0.1832	3505.3	0.1500	0.1858	511.6	0.1456	0.1820	1174.6	0.1433	0.1814	3004.1
	BAS-RLO	0.1213	0.1821	2976.0	0.1511	0.1925	221.8	0.1363	0.1817	659.9	0.1303	0.1808	2770.6
	ADV-RLO	0.1206	0.1828	3508.0	0.1319	0.1845	1267.3	0.1294	0.1816	2082.4	0.1244	0.1809	3832.2
	ADV-BI-RLO	0.1196	0.1829	1644.9	0.1535	0.1928	95.6	0.1325	0.1821	389.2	0.1252	0.1811	1758.9
	BAS-RSO	0.1194	0.1820	3460.7	0.1472	0.1906	204.7	0.1342	0.1823	758.4	0.1239	0.1810	3398.5
	ADV-RSO	0.1156	0.1829	3169.7	0.1322	0.1851	971.9	0.1283	0.1815	1534.8	0.1224	0.1807	2866.2
	ADV-BI-RSO	0.1135	0.1839	2234.2	0.1467	0.1876	119.2	0.1358	0.1831	493.3	0.1217	0.1820	2117.4

EMO-based OCS results (IV)

Comparison of averaged performance of four single solutions selected from the obtained Pareto sets

Statistical tests:

Friedman and Iman Davenport

Holm

Algorithm	Ranking
ADV-RSO	2.207
ADV-RLO	2.310
ADV-BI-RLO	3.776
BAS-RSO	4.328
ADV-BI-RSO	4.776
BAS-RLO	5.155
BAS-BAG	5.448

Comparison	p-value
ADV-RSO vs BAS-BAG	**+(6.63e-8)**
ADV-RSO vs BAS-RLO	**+(1.01e-6)**
ADV-RSO vs ADV-BI-RSO	**+(2.38e-5)**
ADV-RSO vs BAS-RSO	**+(5.56e-4)**
ADV-RSO vs ADV-BI-RLO	**+(0.011)**
ADV-RSO vs ADV-RLO	=(0.855)

EMO-based OCS results (V)

Accuracy *Benchmarking of EMO OCS RSO-based FRBCEs versus static FURIA-based FRBCEs and classical CEs*

Dataset	ADV-RSO Test err.	FURIA+BAG+RSO Test err.	C4.5+BAG+RSO Test err.	RF Test err.
abalone	**0.7391**	0.7480	0.7681	0.7536
bioassay_688red	0.0091	**0.0090**	**0.0090**	**0.0090**
coil2000	0.0598	0.0601	0.0615	**0.0597**
gas_sensor	**0.0076**	0.0078	0.0089	0.0092
isolet	**0.0697**	0.0700	0.0788	0.0766
letter	0.0742	0.0743	**0.0615**	0.0701
magic	0.1274	0.1299	**0.1255**	0.1314
marketing	**0.6597**	0.6671	0.6735	0.6624
mfeat_fac	**0.0401**	0.0431	0.0498	0.0475
mfeat_fou	**0.1849**	0.1925	0.1902	0.1858
mfeat_kar	**0.0700**	0.0709	0.0818	0.0597
mfeat_zer	**0.2119**	0.2181	0.2273	0.2330
musk2	0.0299	0.0320	**0.0271**	0.0375
optdigits	**0.0272**	0.0282	**0.0276**	0.0277
pblocks	**0.0308**	0.0338	0.0327	0.0332
pendigits	**0.0124**	0.0132	0.0150	0.0162
ring_norm	**0.0282**	0.0315	0.0376	0.0587
sat	0.0969	0.1001	**0.0950**	0.1027
segment	**0.0239**	0.0295	0.0328	0.0350
sensor_read_24	**0.0210**	0.0233	0.0234	0.0224
shuttle	**0.0005**	0.0009	0.0009	0.0009
spambase	**0.0581**	0.0639	0.0651	0.0625
steel_faults	**0.2238**	0.2361	0.2263	0.2517
texture	**0.0261**	0.0274	0.0334	0.0383
thyroid	**0.0206**	0.0218	0.0222	0.0221
two_norm	**0.0271**	0.0276	0.0280	0.0389
waveform_noise	**0.1432**	0.1457	0.1643	0.1556
waveform	**0.1418**	0.1453	0.1588	0.1587
wquality_white	0.3720	0.3803	**0.3688**	0.3864
avg.	**0.1220**	0.1252	0.1274	0.1292
dev.	0.1807	0.1829	0.1852	0.1830

EMO-based OCS results (VI)

Accuracy *Benchmarking of EMO OCS RSO-based FRBCEs versus static FURIA-based FRBCEs and classical CEs*

Dataset	ADV-RSO Test err.	FURIA+BAG+RSO Test err.	C4.5+BAG+RSO Test err.	RF Test err.
abalone	**0.7391**	0.7480	0.7681	0.7536
bioassay_688red	0.0091	**0.0090**	**0.0090**	**0.0090**
coil2000	0.0598	0.0601	0.0615	**0.0597**
gas_sensor	**0.0076**	0.0078		
isolet	**0.0697**	0.0700		
letter	0.0742	0.0743		
magic	0.1274	0.1299		
		0.6671		
		0.0431		
		0.1925		
		0.0709		
		0.2181		
		0.0320	**0.0271**	0.0375
		0.0282	**0.0276**	0.0277
		0.0338	0.0327	0.0332
		0.0132	0.0150	0.0162
ring_norm	**0.0282**	0.0315		
sat	0.0969	0.1001		
segment	**0.0239**			
sensor_read_24	**0.0210**	0.0233		
shuttle	**0.0005**	0.0009		
spambase	**0.0581**	0.0639		
steel_faults	**0.2238**	0.2361	0.2263	0.2517
texture	**0.0261**	0.0274	0.0334	0.0383
thyroid	**0.0206**	0.0218	0.0222	0.0221
two_norm	**0.0271**	0.0276	0.0280	0.0389
waveform_noise	**0.1432**	0.1457	0.1643	0.1556
waveform	**0.1418**	0.1453	0.1588	0.1587
wquality_white	0.3720	0.3803	**0.3688**	0.3864
avg.	**0.1220**	0.1252	0.1274	0.1292
dev.	0.1807	0.1829	0.1852	0.1830

Statistical tests:

Friedman and Iman Davenport

Holm

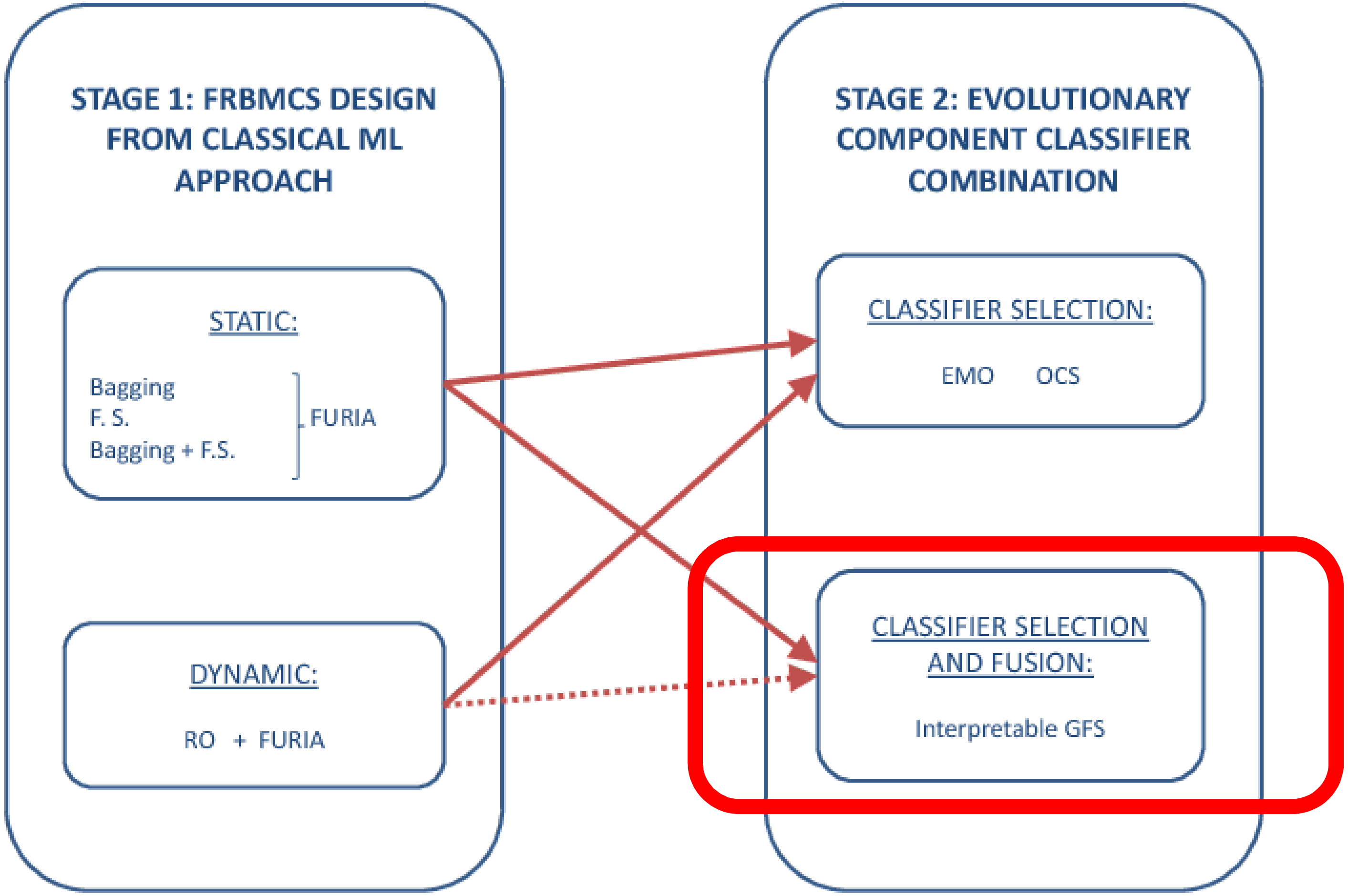
Algorithm	Ranking
ADV-RSO	1.379
FURIA+BAG+RSO	2.655
C4.5+BAG+RSO	2.900
RF	3.069

Comparison	p-value
ADV-RSO vs RF	**+(1.87e-6)**
ADV-RSO vs C4.5+BAG+RSO	**+(1.53e-5)**
ADV-RSO vs FURIA+BAG+RSO	**+(1.68e-4)**

Proposed Framework Graphical Representation

OVERVIEW

1. Introduction
2. Proposed Framework
3. FRBCE Design from Classical ML Approaches
4. EMO Selection of Component Classifiers
5. **Classifier Selection and Fusion via an Interpretable GFS**
6. Conclusions



Overall view

OVERVIEW

1. Introduction
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6. Conclusions

- Accuracy and complexity can be both improved by developing a combination method involving joint classifier selection and fusion
- Using a fuzzy linguistic system for this task would provide some interpretability about the classifier fusion method operation
- We will introduce the use of a **FRBCS-based combination method (FRBCS-CM)**:
  - Combining classifier fusion and classifier selection at class level
  - Working on any classifier with class certainty degrees
  - Showing a human-understandable structure
  - Being automatically learned from training data using a genetic algorithm (GA) → genetic fuzzy system
  - **FRBCE-specific:** Two-level hierarchical structure with component FRBCSs in the 1st level and the FRBCS-CM in the 2nd (stacking)

Component classifier output

OVERVIEW

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Component Cl. 1 | Component Cl. 2 | Component Cl. 3 | Component Cl. 4

… … …

RULE BASE

IF x is A1 THEN class C1 WITH CF1
IF x is A2 THEN class C1 WITH CF2
…
} All rules for class 1

IF x is Ai THEN class C2 WITH CFi
IF x is Ai+1 THEN class C2 WITH CFi+1
} All rules for class 2

…
} Etc.

TO CLASSIFY x ?

(for each individual classifier)

Certainty degree for class 1,

$$R_1(x) = \underbrace{\vee_j (\mathrm{CF}_j \cdot \mathrm{A}_j(x))}_{\text{Only from rules coding for class 1}}$$

Certainty degree for class 2,

$$R_2(x) = \underbrace{\vee_j (\mathrm{CF}_j \cdot \mathrm{A}_j(x))}_{\text{Only from rules coding for class 2}}$$

SO, WHICH CLASS ?

$$\arg\max_{i \in \{1,\ldots,\#classes\}} R_i(x)$$

Component classifier output fusion

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Component Cl. 1 | Component | Component | Component Cl. 4

Classifier selection:

IF (Classifier 1 says that class is 1)
or (Classifier 4 says that class is 1)
THEN class is 1
…

IF x is A2 THEN …
…
IF x is Ai THEN class C2 WITH CFi
IF x is Ai+1 THEN class C2 WITH CFi+1
…

All rules for class 2

Etc.

TO CLASSIFY x ?

(for each individual classifier)

Certainty degree for class 1,

$R_1(x) = \underset{j}{\vee}(\mathrm{CF}_j \cdot \mathrm{A}_j(x))$

Only from rules coding for class 1

For each class

Certainty degree for class 2,

$R_2(x) = \underset{j}{\vee}(\mathrm{CF}_j \cdot \mathrm{A}_j(x))$

Only from rules coding for class 2

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ \dots & \dots & \dots & \dots \end{bmatrix}$$

- **Binary matrix b_{ck}**
- **Size: #classes x #classifiers**
- **2nd level of our system**

$$R^{BIN}_{class1} = \vee(b_{ck} \cdot CF_j \cdot A_j(x))$$

SO, WHICH CLASS ?

$\arg\max_{i \in \{1,\dots,\#classes\}} R_i(x)$

Component classifier output fusion (II)

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Component Cl. 1

Component

Component

Component

Classifier selection and fusion:

IF (Classifier 1 says that class is 1 with w_{11}=0.5)
or (Classifier 4 says that class is 1 with w_{41}=0.3)
THEN class is 1

…

IF x is A1 …
IF x is A2 THEN cla…
…
IF x is Ai THEN class C2 WITH CFi
IF x is Ai+1 THEN class C2 WITH CFi+1
…

For each …

All rules for class 2

Etc.

- **Size: #classes x #classifiers**
- **2nd level of our system (also fuzzy inference)**

TO CLASSIFY x ?

(for each individual classifier)

Certainty degree for class 1,

$R_1(x) = \vee_j (CF_j \cdot A_j(x))$

Only from rules coding for class 1

For each class

Certainty degree for class 2,

$R_2(x) = \vee_j (CF_j \cdot A_j(x))$

Only from rules coding for class 2

$$\begin{bmatrix} \mathbf{0.5} & \mathbf{0.0} & \mathbf{0.0} & \mathbf{0.3} \\ \mathbf{0.1} & \mathbf{0.0} & \mathbf{0.2} & \mathbf{0.0} \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix}$$

$R_{class1}^{WEIGHT} = \vee (w_{ck} \cdot CF_j \cdot A_j(x))$

SO, WHICH CLASS ?

arg ma…

SELECTION + FUSION

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The weights are learnt by a GA:

$$\begin{bmatrix} 0.5 & 0.0 & 0.0 & 0.3 \\ 0.1 & 0.0 & 0.2 & 0.0 \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \Longleftrightarrow \boxed{\text{GA}}$$

Constraint: the percentage of $w_{ck} \neq 0$ is defined by the user, thus selecting the desired accuracy-complexity tradeoff

Chromosome representation: coding for a *sparse* matrix

$$\begin{bmatrix} 0.5 & 0.0 & 0.0 & 0.3 \\ 0.1 & 0.0 & 0.2 & 0.0 \\ \cdots & \cdots & \cdots & \cdots \end{bmatrix} \Longleftrightarrow$$

(1,1)	(1,4)	(2,1)	(2,3)	← Position
0.5	0.3	0.1	0.2	← Values w_{ck}

Operators and fitness function

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Initial population: totally random

Crossover:

(1,1)	(1,4)	(2,1)	(2,3)
0.5	0.3	0.1	0.2

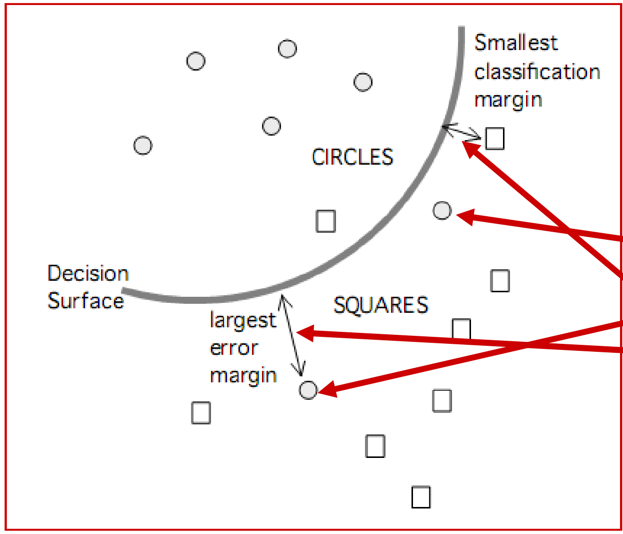
(1,3)	(2,1)	(2,2)	(3,4)
0.8	0.7	0.9	0.4

→

(1,1)	(1,4)	(2,1)	(2,3)
0.8	0.7	0.9	0.4

(1,3)	(2,1)	(2,2)	(3,4)
0.5	0.3	0.1	0.2

Mutation: random mutation of a value or the full chromosome



Fitness function:
a lexicographic order of:

1) Training error e
2) Error margin m_1
3) Classification margin m_2

General Scheme

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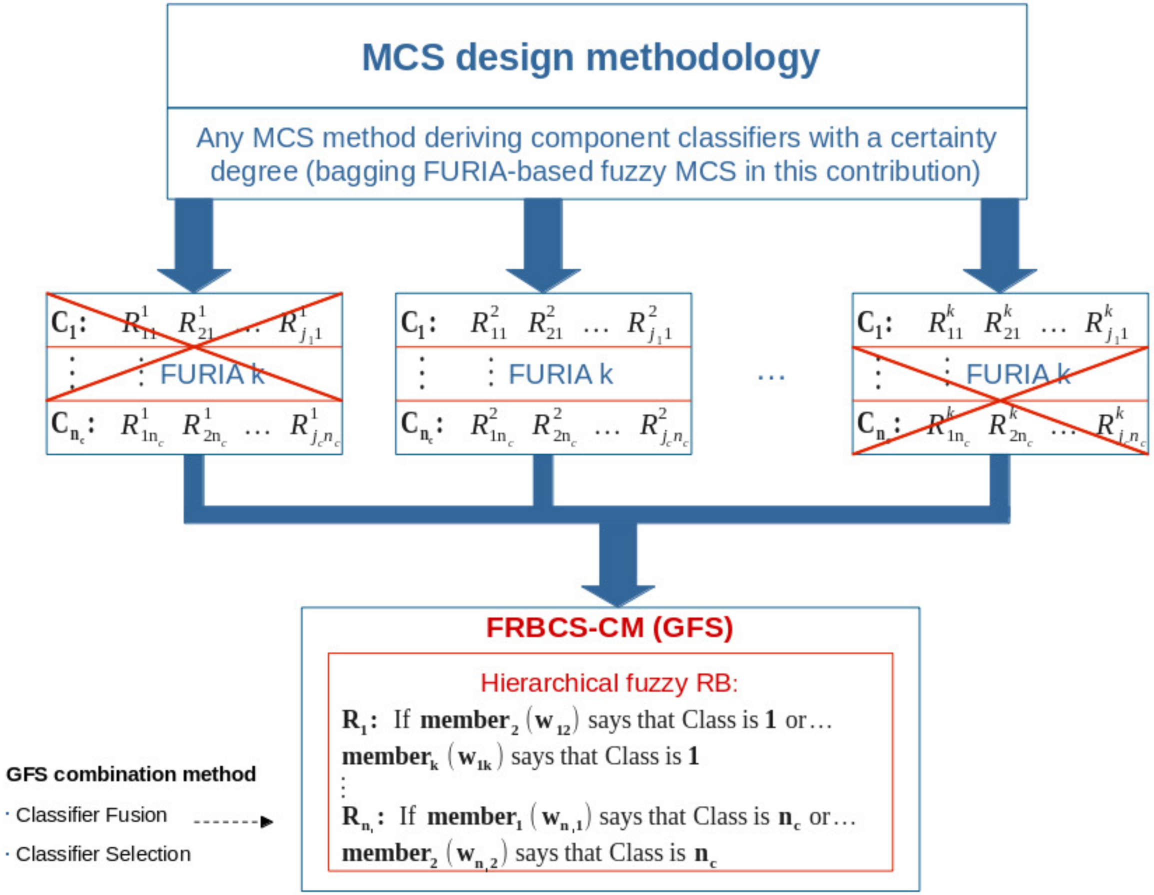

K. Trawinski, O. Cordón, L. Sánchez, A. Quirin. A Genetic Fuzzy Linguistic Combination Method for Fuzzy Rule-Based Multiclassifiers. IEEE Transactions on Fuzzy Systems (2013), to appear. IF 2011: 4.260. Cat: CS, AI. O: 5/111. Q1

Experimental setup

OVERVIEW

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6. Conclusions

Considered UCI datasets:

- Significant number of datasets considered: **20**
- Every attribute is continuous
- From small to large number of features (64), classes (28), instances (19020)
- Pentium 2.4 GHz, 2 GB, 2-4 cores (Granada cluster)

Data set	#examples	#attr.	#classes
Low dimensional:			
abalone	4178	7	28
breast	700	9	2
glass	214	9	7
heart	270	13	2
magic	19020	10	2
pblocks	5474	10	5
phoneme	5404	5	2
pima	768	8	2
wine	178	13	3
yeast	1484	8	10
High dimensional:			
ionosphere	352	34	2
optdigits	5620	64	10
pendigits	10992	16	10
sat	6436	36	6
segment	2310	19	7
sonar	208	60	2
spambase	4602	57	2
texture	5500	40	11
vehicle	846	18	4
waveform	5000	40	3

Validation: Dietterich's 5x2-fold cross validation

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Parameter values:

- **50 bagging FURIA component classifiers** generated
- Steady-state GA with parameters: 100 individuals, 1000 generations, crossover prob. 0.6, mutation prob. 0.1
- % of **weights** = {10%, 25%, 50%, 75%, 90%}
- **Wilcoxon test** to find statistical differences
- Test accuracy and #rules for a global comparison and some interpretability insights

Experimental setup (III)

OVERVIEW

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6. Conclusions

Methods considered for comparison:

- **Bagging FRBCE** full ensemble with standard Majority Voting fusion method

- Combination of state-of-the-art fusion:
  - **Majority Voting** (MV), **Average** (AVG), and **Decision Templates** (DT)

  and selection methods:
  - Greedy **Forward** (high reduction) and **Backward** selection (low reduction)

- **[Dimililer et al. 2009]**: recent GA-based proposal performing both classifier selection at class level and classifier fusion (mid reduction)

FRBCS-CM results (I)

Our approach is competitive in terms of ***accuracy***

Dataset	fuzzy MCSs	FRBCS-CM 10%	25%	50%	75%	90%	Greedy FS MV	AVG	DT	Greedy BS MV	AVG	DT	GA Dimil.
Low dim.:													
abalone	0.7458	0.7581	0.7537	0.7493	0.7470	0.7461	0.7524	0.7582	0.7610	0.7484	0.7524	0.7511	0.7494
breast	0.0409	0.0472	0.0469	0.0452	0.0438	0.0432	0.0455	0.0418	0.0398	0.0412	0.0386	0.0372	0.0409
glass	0.2822	0.3159	0.2879	0.2832	0.2692	0.2710	0.2981	0.3271	0.3000	0.2832	0.2720	0.2776	0.3131
heart	0.1822	0.1785	0.1733	0.1719	0.1696	0.1696	0.1859	0.2015	0.1874	0.1778	0.1770	0.1674	0.1726
magic	0.1346	0.1340	0.1314	0.1309	0.1302	0.1300	0.1329	0.1328	0.1323	0.1338	0.1326	0.1298	0.1336
pblocks	0.0288	0.0285	0.0265	0.0271	0.0268	0.0261	0.0282	0.0302	0.0296	0.0286	0.0269	0.0263	0.0402
phoneme	0.1332	0.1277	0.1252	0.1261	0.1256	0.1264	0.1260	0.1232	0.1258	0.1291	0.1271	0.1248	0.1301
pima	0.2385	0.2492	0.2484	0.2411	0.2432	0.2424	0.2503	0.2516	0.2596	0.2385	0.2375	0.2414	0.2398
wine	0.0393	0.0461	0.0382	0.0303	0.0404	0.0393	0.0629	0.0551	0.0607	0.0393	0.0371	0.0360	0.0348
yeast	0.4008	0.4155	0.4054	0.3985	0.4034	0.4013	0.4116	0.4142	0.4189	0.4011	0.3978	0.4018	0.4116
Avg. Low	0.2227	0.2301	0.2237	0.2204	0.2199	0.2196	0.2294	0.2336	0.2315	0.2221	0.2199	0.2193	0.2266
High dim.:													
ionosphere	0.1459	0.1527	0.1413	0.1458	0.1430	0.1430	0.1584	0.1532	0.1646	0.1476	0.1430	0.1413	0.1464
optdigits	0.0329	0.0337	0.0327	0.0327	0.0318	0.0313	0.0367	0.0352	0.0351	0.0329	0.0284	0.0279	0.0721
pendigits	0.0156	0.0174	0.0152	0.0140	0.0140	0.0138	0.0171	0.0150	0.0162	0.0156	0.0129	0.0126	0.0160
sat	0.1021	0.1067	0.1027	0.0997	0.0986	0.1005	0.1044	0.1010	0.1005	0.1022	0.0967	0.0971	0.1040
segment	0.0336	0.0334	0.0319	0.0304	0.0316	0.0302	0.0318	0.0326	0.0336	0.0330	0.0309	0.0306	0.0345
sonar	0.2269	0.2404	0.2183	0.2077	0.2077	0.2058	0.2163	0.2337	0.2452	0.2260	0.2183	0.2163	0.2231
spambase	0.0587	0.0569	0.0559	0.0555	0.0539	0.0546	0.0576	0.0573	0.0574	0.0579	0.0554	0.0549	0.0574
texture	0.0307	0.0343	0.0312	0.0304	0.0291	0.0285	0.0343	0.0330	0.0336	0.0308	0.0268	0.0270	0.0325
vehicle	0.2726	0.2773	0.2664	0.2690	0.2664	0.2674	0.2671	0.2690	0.2693	0.2723	0.2641	0.2600	0.2721
waveform	0.1492	0.1554	0.1490	0.1503	0.1489	0.1479	0.1508	0.1535	0.1533	0.1498	0.1468	0.1472	0.1532
Avg. High	0.1068	0.1108	0.1045	0.1036	0.1025	0.1023	0.1075	0.1084	0.1109	0.1068	0.1023	0.1015	0.1111
Avg. All	0.1647	0.1704	0.1641	0.1620	0.1612	0.1609	0.1684	0.1710	0.1712	0.1644	0.1611	0.1604	0.1689

FRBCS-CM results (II)

***Accuracy** benchmarking vs. original Bagging FRBCEs*

Dataset	fuzzy MCSs	FRBCS-CM 10%	FRBCS-CM 25%	FRBCS-CM 50%	FRBCS-CM 75%	FRBCS-CM 90%	Greedy FS MV	Greedy FS AVG	Greedy FS DT	Greedy BS MV	Greedy BS AVG	Greedy BS DT	GA Dimil.
Low dim.:													
abalone	0.7458	0.7581	0.7537	0.7493	0.7470	0.7461	0.7524	0.7582	0.7610	0.7484	0.7524	0.7511	0.7494
breast	0.0409	0.0472	0.0469	0.0452	0.0438	0.0432	0.0455	0.0418	0.0398	0.0412	0.0386	0.0372	0.0409
glass	0.2822	0.3159	0.2879	0.2832	0.2692	0.2710	0.2981	0.3271	0.3000	0.2832	0.2720	0.2776	0.3131
heart	0.1822	0.1785	0.1733	0.1719	0.1696	0.1696	0.1859	0.2015	0.1874	0.1778	0.1770	0.1674	0.1726
magic	0.1346	0.1340	0.1314	0.1309	0.1302	0.1300	0.1329	0.1328	0.1323	0.1338	0.1326	0.1298	0.1336
pblocks	0.0288	0.0285	0.0265	0.0271	0.0268	0.0261	0.0282	0.0302	0.0296	0.0286	0.0269	0.0263	0.0402
phoneme	0.1332	0.1277	0.1252	0.1261	0.1256	0.1264	0.1260	0.1232	0.1258	0.1291	0.1271	0.1248	0.1301
pima	0.2385	0.2492	0.2484	0.2411	0.2432	0.2424	0.2503	0.2516	0.2596	0.2385	0.2375	0.2414	0.2398
wine	0.0393	0.0461	0.0382	0.0303	0.0404	0.0393	0.0629	0.0551	0.0607	0.0393	0.0371	0.0360	0.0348
yeast	0.4008	0.4155	0.4054	0.3985	0.4034	0.4013	0.4116	0.4142	0.4189	0.4011	0.3978	0.4018	0.4116
Avg. Low	0.2227	0.2301	0.2237	0.2204	0.2199	0.2196	0.2294	0.2336	0.2315	0.2221	0.2199	0.2193	0.2266
High dim.:													
ionosphere	0.1459	0.1527	0.1413	0.1458	0.1430	0.1430	0.1584	0.1532	0.1646	0.1476	0.1430	0.1413	0.1464
optdigits	0.0329	0.0337	0.0327	0.0327	0.0318	0.0313	0.0367	0.0352	0.0351	0.0329	0.0284	0.0279	0.0721
pendigits	0.0156	0.0174	0.0152	0.0140	0.0140	0.0138	0.0171	0.0150	0.0162	0.0156	0.0129	0.0126	0.0160
sat	0.1021	0.1067	0.1027	0.0997	0.0986	0.1005	0.1044	0.1010	0.1005	0.1022	0.0967	0.0971	0.1040
segment	0.0336	0.0334	0.0319	0.0304	0.0316	0.0302	0.0318	0.0326	0.0336	0.0330	0.0309	0.0306	0.0345
sonar	0.2269	0.2404	0.2183	0.2077	0.2077	0.2058	0.2163	0.2337	0.2452	0.2260	0.2183	0.2163	0.2231
spambase	0.0587	0.0569	0.0559	0.0555	0.0539	0.0546	0.0576	0.0573	0.0574	0.0579	0.0554	0.0549	0.0574
texture	0.0307	0.0343	0.0312	0.0304	0.0291	0.0285	0.0343	0.0330	0.0336	0.0308	0.0268	0.0270	0.0325
vehicle	0.2726	0.2773	0.2664	0.2690	0.2664	0.2674	0.2671	0.2690	0.2693	0.2723	0.2641	0.2600	0.2721
waveform	0.1492	0.1554	0.1490	0.1503	0.1489	0.1479	0.1508	0.1535	0.1533	0.1498	0.1468	0.1472	0.1532
Avg. High	0.1068	0.1108	0.1045	0.1036	0.1025	0.1023	0.1075	0.1084	0.1109	0.1068	0.1023	0.1015	0.1111
Avg. All	0.1647	0.1704	0.1641	0.1620	0.1612	0.1609	0.1684	0.1710	0.1712	0.1644	0.1611	0.1604	0.1689

FRBCS-CM results (III)

Accuracy benchmarking vs. low reduction and existing fusion methods

Dataset	fuzzy MCSs	FRBCS-CM 10%	FRBCS-CM 25%	FRBCS-CM 50%	FRBCS-CM 75%	FRBCS-CM 90%	Greedy FS MV	Greedy FS AVG	Greedy FS DT	Greedy BS MV	Greedy BS AVG	Greedy BS DT	GA Dimil.
Low dim.:													
abalone	0.7458	0.7581	0.7537	0.7493	0.7470	0.7461	0.7524	0.7582	0.7610	0.7484	0.7524	0.7511	0.7494
breast	0.0409	0.0472	0.0469	0.0452	0.0438	0.0432	0.0455	0.0418	0.0398	0.0412	0.0386	0.0372	0.0409
glass	0.2822	0.3159	0.2879	0.2832	0.2692	0.2710	0.2981	0.3271	0.3000	0.2832	0.2720	0.2776	0.3131
heart	0.1822	0.1785	0.1733	0.1719	0.1696	0.1696	0.1859	0.2015	0.1874	0.1778	0.1770	0.1674	0.1726
magic	0.1346	0.1340	0.1314	0.1309	0.1302	0.1300	0.1329	0.1328	0.1323	0.1338	0.1326	0.1298	0.1336
pblocks	0.0288	0.0285	0.0265	0.0271	0.0268	0.0261	0.0282	0.0302	0.0296	0.0286	0.0269	0.0263	0.0402
phoneme	0.1332	0.1277	0.1252	0.1261	0.1256	0.1264	0.1260	0.1232	0.1258	0.1291	0.1271	0.1248	0.1301
pima	0.2385	0.2492	0.2484	0.2411	0.2432	0.2424	0.2503	0.2516	0.2596	0.2385	0.2375	0.2414	0.2398
wine	0.0393	0.0461	0.0382	0.0303	0.0404	0.0393	0.0629	0.0551	0.0607	0.0393	0.0371	0.0360	0.0348
yeast	0.4008	0.4155	0.4054	0.3985	0.4034	0.4013	0.4116	0.4142	0.4189	0.4011	0.3978	0.4018	0.4116
Avg. Low	0.2227	0.2301	0.2237	0.2204	0.2199	0.2196	0.2294	0.2336	0.2315	0.2221	0.2199	0.2193	0.2266
High dim.:													
ionosphere	0.1459	0.1527	0.1413	0.1458	0.1430	0.1430	0.1584	0.1532	0.1646	0.1476	0.1430	0.1413	0.1464
optdigits	0.0329	0.0337	0.0327	0.0327	0.0318	0.0313	0.0367	0.0352	0.0351	0.0329	0.0284	0.0279	0.0721
pendigits	0.0156	0.0174	0.0152	0.0140	0.0140	0.0138	0.0171	0.0150	0.0162	0.0156	0.0129	0.0126	0.0160
sat	0.1021	0.1067	0.1027	0.0997	0.0986	0.1005	0.1044	0.1010	0.1005	0.1022	0.0967	0.0971	0.1040
segment	0.0336	0.0334	0.0319	0.0304	0.0316	0.0302	0.0318	0.0326	0.0336	0.0330	0.0309	0.0306	0.0345
sonar	0.2269	0.2404	0.2183	0.2077	0.2077	0.2058	0.2163	0.2337	0.2452	0.2260	0.2183	0.2163	0.2231
spambase	0.0587	0.0569	0.0559	0.0555	0.0539	0.0546	0.0576	0.0573	0.0574	0.0579	0.0554	0.0549	0.0574
texture	0.0307	0.0343	0.0312	0.0304	0.0291	0.0285	0.0343	0.0330	0.0336	0.0308	0.0268	0.0270	0.0325
vehicle	0.2726	0.2773	0.2664	0.2690	0.2664	0.2674	0.2671	0.2690	0.2693	0.2723	0.2641	0.2600	0.2721
waveform	0.1492	0.1554	0.1490	0.1503	0.1489	0.1479	0.1508	0.1535	0.1533	0.1498	0.1468	0.1472	0.1532
Avg. High	0.1068	0.1108	0.1045	0.1036	0.1025	0.1023	0.1075	0.1084	0.1109	0.1068	0.1023	0.1015	0.1111
Avg. All	0.1647	0.1704	0.1641	0.1620	0.1612	0.1609	0.1684	0.1710	0.1712	0.1644	0.1611	0.1604	0.1689

FRBCS-CM results (IV)

Accuracy benchmarking vs. high reduction and existing fusion methods

Dataset	fuzzy MCSs	FRBCS-CM 10%	25%	50%	75%	90%	Greedy FS MV	AVG	DT	Greedy BS MV	AVG	DT	GA Dimil.
Low dim.:													
abalone	0.7458	0.7581	0.7537	0.7493	0.7470	0.7461	0.7524	0.7582	0.7610	0.7484	0.7524	0.7511	0.7494
breast	0.0409	0.0472	0.0469	0.0452	0.0438	0.0432	0.0455	0.0418	0.0398	0.0412	0.0386	0.0372	0.0409
glass	0.2822	0.3159	0.2879	0.2832	0.2692	0.2710	0.2981	0.3271	0.3000	0.2832	0.2720	0.2776	0.3131
heart	0.1822	0.1785	0.1733	0.1719	0.1696	0.1696	0.1859	0.2015	0.1874	0.1778	0.1770	0.1674	0.1726
magic	0.1346	0.1340	0.1314	0.1309	0.1302	0.1300	0.1329	0.1328	0.1323	0.1338	0.1326	0.1298	0.1336
pblocks	0.0288	0.0285	0.0265	0.0271	0.0268	0.0261	0.0282	0.0302	0.0296	0.0286	0.0269	0.0263	0.0402
phoneme	0.1332	0.1277	0.1252	0.1261	0.1256	0.1264	0.1260	0.1232	0.1258	0.1291	0.1271	0.1248	0.1301
pima	0.2385	0.2492	0.2484	[illegible]	0.2432	0.2424	0.2503	0.2516	0.2596	0.2385	0.2375	0.2414	0.2398
wine	[illegible]	[illegible]	[illegible]	0.0303	0.0404	0.0393	0.0629	0.0551	0.0607	0.0393	0.0371	0.0360	0.0348
yeast	0.4008	0.4155	0.4054	0.3985	0.4034	0.4013	0.4116	0.4142	0.4189	0.4011	0.3978	0.4018	0.4116
Avg. Low	0.2227	0.2301	0.2237	[illegible]	0.2199	0.2196	0.2294	0.2336	0.2315	0.2221	0.2199	0.2193	0.2266
High dim.:													
ionosphere	0.1459	0.1527	0.1413	0.1458	0.1430	0.1430	0.1584	0.1532	0.1646	0.1476	0.1430	0.1413	0.1464
optdigits	0.0329	0.0337	0.0327	0.0327	0.0318	0.0313	0.0367	0.0352	0.0351	0.0329	0.0284	0.0279	0.0721
pendigits	0.0156	0.0174	0.0152	0.0140	0.0140	0.0138	0.0171	0.0150	0.0162	0.0156	0.0129	0.0126	0.0160
sat	0.1021	0.1067	0.1027	0.0997	0.0986	0.1005	0.1044	0.1010	0.1005	0.1022	0.0967	0.0971	0.1040
segment	0.0336	0.0334	0.0319	0.0304	0.0316	0.0302	0.0318	0.0326	0.0336	0.0330	0.0309	0.0306	0.0345
sonar	0.2269	0.2404	0.2183	0.2077	0.2077	0.2058	0.2163	0.2337	0.2452	0.2260	0.2183	0.2163	0.2231
spambase	0.0587	0.0569	0.0559	0.0555	0.0539	0.0546	0.0576	0.0573	0.0574	0.0579	0.0554	0.0549	0.0574
texture	0.0307	0.0343	0.0312	0.0304	0.0291	0.0285	0.0343	0.0330	0.0336	0.0308	0.0268	0.0270	0.0325
vehicle	0.2726	0.2773	0.2664	0.2690	0.2664	0.2674	0.2671	0.2690	0.2693	0.2723	0.2641	0.2600	0.2721
waveform	0.1492	0.1554	0.1490	0.1503	0.1489	0.1479	0.1508	0.1535	0.1533	0.1498	0.1468	0.1472	0.1532
Avg. High	0.1068	0.1108	0.1045	0.1036	0.1025	0.1023	0.1075	0.1084	0.1109	0.1068	0.1023	0.1015	0.1111
Avg. All	0.1647	0.1704	0.1641	0.1620	0.1612	0.1609	0.1684	0.1710	0.1712	0.1644	0.1611	0.1604	0.1689

No statistical differences

FRBCS-CM results (V)

Accuracy benchmarking vs. joint classifier selection (mid red.) and fusion

Dataset	fuzzy MCSs	FRBCS-CM 10%	25%	50%	75%	90%	Greedy FS MV	AVG	DT	Greedy BS MV	AVG	DT	GA Dimil.
Low dim.:													
abalone	0.7458	0.7581	0.7537	0.7493	0.7470	0.7461	0.7524	0.7582	0.7610	0.7484	0.7524	0.7511	0.7494
breast	0.0409	0.0472	0.0469	0.0452	0.0438	0.0432	0.0455	0.0418	0.0398	0.0412	0.0386	0.0372	0.0409
glass	0.2822	0.3159	0.2879	0.2832	0.2692	0.2710	0.2981	0.3271	0.3000	0.2832	0.2720	0.2776	0.3131
heart	0.1822	0.1785	0.1733	0.1719	0.1696	0.1696	0.1859	0.2015	0.1874	0.1778	0.1770	0.1674	0.1726
magic	0.1346	0.1340	0.1314	0.1309	0.1302	0.1300	0.1329	0.1328	0.1323	0.1338	0.1326	0.1298	0.1336
pblocks	0.0288	0.0285	0.0265	0.0271	0.0268	0.0261	0.0282	0.0302	0.0296	0.0286	0.0269	0.0263	0.0402
phoneme	0.1332	0.1277	0.1252	0.1261	0.1256	0.1264	0.1260	0.1232	0.1258	0.1291	0.1271	0.1248	0.1301
pima	0.2385	0.2492	0.2484	0.2411	0.2432	0.2424	0.2503	0.2516	0.2596	0.2385	0.2375	0.2414	0.2398
wine	0.0393	0.0461	0.0382	0.0303	0.0404	0.0393	0.0629	0.0551	0.0607	0.0393	0.0371	0.0360	0.0348
yeast	0.4008	0.4155	0.4054	0.3985	0.4034	0.4013	0.4116	0.4142	0.4189	0.4011	0.3978	0.4018	0.4116
Avg. Low	0.2227	0.2301	0.2237	0.2204	0.2199	0.2196	0.2294	0.2336	0.2315	0.2221	0.2199	0.2193	0.2266
High dim.:													
ionosphere	0.1459	0.1527	0.1413	0.1458	0.1430	0.1430	0.1584	0.1532	0.1646	0.1476	0.1430	0.1413	0.1464
optdigits	0.0329	0.0337	0.0327	0.0327	0.0318	0.0313	0.0367	0.0352	0.0351	0.0329	0.0284	0.0279	0.0721
pendigits	0.0156	0.0174	0.0152	0.0140	0.0140	0.0138	0.0171	0.0150	0.0162	0.0156	0.0129	0.0126	0.0160
sat	0.1021	0.1067	0.1027	0.0997	0.0986	0.1005	0.1044	0.1010	0.1005	0.1022	0.0967	0.0971	0.1040
segment	0.0336	0.0334	0.0319	0.0304	0.0316	0.0302	0.0318	0.0326	0.0336	0.0330	0.0309	0.0306	0.0345
sonar	0.2269	0.2404	0.2183	0.2077	0.2077	0.2058	0.2163	0.2337	0.2452	0.2260	0.2183	0.2163	0.2231
spambase	0.0587	0.0569	0.0559	0.0555	0.0539	0.0546	0.0576	0.0573	0.0574	0.0579	0.0554	0.0549	0.0574
texture	0.0307	0.0343	0.0312	0.0304	0.0291	0.0285	0.0343	0.0330	0.0336	0.0308	0.0268	0.0270	0.0325
vehicle	0.2726	0.2773	0.2664	0.2690	0.2664	0.2674	0.2671	0.2690	0.2693	0.2723	0.2641	0.2600	0.2721
waveform	0.1492	0.1554	0.1490	0.1503	0.1489	0.1479	0.1508	0.1535	0.1533	0.1498	0.1468	0.1472	0.1532
Avg. High	0.1068	0.1108	0.1045	0.1036	0.1025	0.1023	0.1075	0.1084	0.1109	0.1068	0.1023	0.1015	0.1111
Avg. All	0.1647	0.1704	0.1641	0.1620	0.1612	0.1609	0.1684	0.1710	0.1712	0.1644	0.1611	0.1604	0.1689

FRBCS-CM results (VI)

*Our approach is competitive in terms of **complexity***

Dataset	fuzzy MCSs	FRBCS-CM 10%	25%	50%	75%	90%	Greedy FS MV	AVG	DT	Greedy BS MV	AVG	DT	GA Dimil.
Low dim.:													
abalone	3990.9	398.2	995.7	1996.9	2983.6	3578.4	1211.0	1047.6	1037.7	2711.3	3306.9	3398.7	2391.9
breast	435.2	46.1	110.9	217.0	326.2	391	33.0	25.7	24.1	415.9	426.6	427.4	221.1
glass	590.3	57.4	140.6	289.9	434.4	528	88.7	43.6	54.7	560.5	576.4	577.5	173.8
heart	466.0	49.4	120.3	235.3	352.6	421	48.9	35.7	33.4	444.6	455.7	454.7	221.1
magic	3882.1	421.0	968.3	1965.6	2969.9	3475.8	528.2	424.6	417.3	2247.8	3203.6	3319	2123.6
pblocks	1329.4	131.2	328.9	628.1	967.8	1182.2	248.2	108.9	106.1	1259	1288	1297.3	314.1
phoneme	2197.3	241.7	587.8	1132.5	1679.0	2000	493.2	381.1	339.4	1442.8	2046	2049.4	996.9
pima	1050.9	110.9	260.7	530.1	782.4	946	239.3	149.4	118.1	957	1025	1027.7	530
wine	231.4	23.7	57.9	116.4	172.7	208	9.1	6.8	6.2	222.4	226.9	226.9	71.2
yeast	2449.0	260.8	630.9	1198.4	1825.1	2198.4	511.5	389.5	434.9	1901.3	2296.7	2291.9	902.4
Avg. Low	1662.3	174.0	420.2	831.0	1249.4	1492.8	341.1	261.3	257.2	1216.3	1485.2	1507.1	794.6
High dim.:													
ionosphere	367.7	37.8	95.4	211.0	279.8	334	27.0	22.2	24.4	353.3	361.2	360.6	190.3
optdigits	3584.6	359.2	893.5	1787.7	2678.8	3227.2	652.7	428.7	423.7	3398.5	3513.8	3513.1	661.5
pendigits	4395.3	448.8	1098.1	2208.7	3299.9	3964.3	892.1	569.8	470.8	4167.2	4306.4	4307.5	1874.6
sat	4207.2	427.2	1046.9	2107.2	3128.1	3762.8	1214.0	728.7	800.6	3575.2	4006.8	4055	1431.9
segment	1175.3	130.1	290.9	593.4	876.9	1051.4	165.6	109.2	86.7	1100.5	1151.3	1151.4	414.2
sonar	319.3	32.4	80.4	162.0	240.0	288	24.4	22.9	19.8	306.4	312.1	311.9	158.8
spambase	2220.9	229.0	557.2	1115.5	1661.7	2002.6	340.7	286.1	292.8	2135.5	2152.4	2139.8	1026
texture	2912.2	300.1	716.6	1458.8	2175.0	2610.9	433.6	333.8	352.5	2759.8	2852.9	2852.8	1240.4
vehicle	1415.3	154.3	380.4	735.3	1075.3	1283	364.1	173.3	193.4	1304.7	1387.6	1380	425.7
waveform	3484.3	354.0	861.5	1749.8	2601.2	3137.6	1355.9	753.1	727.1	3125.9	3408.3	3381.1	828.9
Avg. High	2408.2	247.3	602.1	1212.9	1801.7	2166.1	547.0	342.8	339.2	2222.7	2345.3	2345.3	825.2
Avg. All	2035.2	210.7	511.1	1022.0	1525.5	1829.4	444.1	302.0	298.2	1719.5	1915.2	1926.2	809.9

FRBCS-CM results (VII)

Complexity *benchmarking vs.* *original Bagging FRBCEs*

Dataset	fuzzy MCSs	FRBCS-CM 10%	25%	50%	75%	90%	Greedy FS MV	AVG	DT	Greedy BS MV	AVG	DT	GA Dimil.
Low dim.:													
abalone	3990.9	398.2	995.7	1996.9	2983.6	3578.4	1211.0	1047.6	1037.7	2711.3	3306.9	3398.7	2391.9
breast	435.2	46.1	110.9	217.0	326.2	391	33.0	25.7	24.1	415.9	426.6	427.4	221.1
glass	590.3	57.4	140.6	289.9	434.4	528	88.7	43.6	54.7	560.5	576.4	577.5	173.8
heart	466.0	49.4	120.3	235.3	352.6	421	48.9	35.7	33.4	444.6	455.7	454.7	221.1
magic	3882.1	421.0	968.3	1965.6	2969.9	3475.8	528.2	424.6	417.3	2247.8	3203.6	3319	2123.6
pblocks	1329.4	131.2	328.9	628.1	967.8	1182.2	248.2	108.9	106.1	1259	1288	1297.3	314.1
phoneme	2197.3	241.7	587.8	1132.5	1679.0	2000	493.2	381.1	339.4	1442.8	2046	2049.4	996.9
pima	1050.9	110.9	260.7	530.1	782.4	946	239.3	149.4	118.1	957	1025	1027.7	530
wine	231.4	23.7	57.9	116.4	172.7	208	9.1	6.8	6.2	222.4	226.9	226.9	71.2
yeast	2449.0	260.8	630.9	1198.4	1825.1	2198.4	511.5	389.5	434.9	1901.3	2296.7	2291.9	902.4
Avg. Low	1662.3	174.0	420.2	831.0	1249.4	1492.8	341.1	261.3	257.2	1216.3	1485.2	1507.1	794.6
High dim.:													
ionosphere	367.7	37.8	95.4	211.0	279.8	334	27.0	22.2	24.4	353.3	361.2	360.6	190.3
optdigits	3584.6	359.2	893.5	1787.7	2678.8	3227.2	652.7	428.7	423.7	3398.5	3513.8	3513.1	661.5
pendigits	4395.3	448.8	1098.1	2208.7	3299.9	3964.3	892.1	569.8	470.8	4167.2	4306.4	4307.5	1874.6
sat	4207.2	427.2	1046.9	2107.2	3128.1	3762.8	1214.0	728.7	800.6	3575.2	4006.8	4055	1431.9
segment	1175.3	130.1	290.9	593.4	876.9	1051.4	165.6	109.2	86.7	1100.5	1151.3	1151.4	414.2
sonar	319.3	32.4	80.4	162.0	240.0	288	24.4	22.9	19.8	306.4	312.1	311.9	158.8
spambase	2220.9	229.0	557.2	1115.5	1661.7	2002.6	340.7	286.1	292.8	2135.5	2152.4	2139.8	1026
texture	2912.2	300.1	716.6	1458.8	2175.0	2610.9	433.6	333.8	352.5	2759.8	2852.9	2852.8	1240.4
vehicle	1415.3	154.3	380.4	735.3	1075.3	1283	364.1	173.3	193.4	1304.7	1387.6	1380	425.7
waveform	3484.3	354.0	861.5	1749.8	2601.2	3137.6	1355.9	753.1	727.1	3125.9	3408.3	3381.1	828.9
Avg. High	2408.2	247.3	602.1	1212.9	1801.7	2166.1	547.0	342.8	339.2	2222.7	2345.3	2345.3	825.2
Avg. All	2035.2	210.7	511.1	1022.0	1525.5	1829.4	444.1	302.0	298.2	1719.5	1915.2	1926.2	809.9

FRBCS-CM results (VIII)

Complexity benchmarking vs. low reduction and existing fusion methods

Dataset	fuzzy MCSs	FRBCS-CM 10%	FRBCS-CM 25%	FRBCS-CM 50%	FRBCS-CM 75%	FRBCS-CM 90%	Greedy FS MV	Greedy FS AVG	Greedy FS DT	Greedy BS MV	Greedy BS AVG	Greedy BS DT	GA Dimil.
Low dim.:													
abalone	3990.9	398.2	995.7	1996.9	2983.6	3578.4	1211.0	1047.6	1037.7	2711.3	3306.9	3398.7	2391.9
breast	435.2	46.1	110.9	217.0	326.2	391	33.0	25.7	24.1	415.9	426.6	427.4	221.1
glass	590.3	57.4	140.6	289.9	434.4	528	88.7	43.6	54.7	560.5	576.4	577.5	173.8
heart	466.0	49.4	120.3	235.3	352.6	421	48.9	35.7	33.4	444.6	455.7	454.7	221.1
magic	3882.1	421.0	968.3	1965.6	2969.9	3475.8	528.2	424.6	417.3	2247.8	3203.6	3319	2123.6
pblocks	1329.4	131.2	328.9	628.1	967.8	1182.2	248.2	108.9	106.1	1259	1288	1297.3	314.1
phoneme	2197.3	241.7	587.8	1132.5	1679.0	2000	493.2	381.1	339.4	1442.8	2046	2049.4	996.9
pima	1050.9	110.9	260.7	530.1	782.4	946	239.3	149.4	118.1	957	1025	1027.7	530
wine	231.4	23.7	57.9	116.4	172.7	208	9.1	6.8	6.2	222.4	226.9	226.9	71.2
yeast	2449.0	260.8	630.9	1198.4	1825.1	2198.4	511.5	389.5	434.9	1901.3	2296.7	2291.9	902.4
Avg. Low	1662.3	174.0	420.2	831.0	1249.4	1492.8	341.1	261.3	257.2	1216.3	1485.2	1507.1	794.6
High dim.:													
ionosphere	367.7	37.8	95.4	211.0	279.8	334	27.0	22.2	24.4	353.3	361.2	360.6	190.3
optdigits	3584.6	359.2	893.5	1787.7	2678.8	3227.2	652.7	428.7	423.7	3398.5	3513.8	3513.1	661.5
pendigits	4395.3	448.8	1098.1	2208.7	3299.9	3964.3	892.1	569.8	470.8	4167.2	4306.4	4307.5	1874.6
sat	4207.2	427.2	1046.9	2107.2	3128.1	3762.8	1214.0	728.7	800.6	3575.2	4006.8	4055	1431.9
segment	1175.3	130.1	290.9	593.4	876.9	1051.4	165.6	109.2	86.7	1100.5	1151.3	1151.4	414.2
sonar	319.3	32.4	80.4	162.0	240.0	288	24.4	22.9	19.8	306.4	312.1	311.9	158.8
spambase	2220.9	229.0	557.2	1115.5	1661.7	2002.6	340.7	286.1	292.8	2135.5	2152.4	2139.8	1026
texture	2912.2	300.1	716.6	1458.8	2175.0	2610.9	433.6	333.8	352.5	2759.8	2852.9	2852.8	1240.4
vehicle	1415.3	154.3	380.4	735.3	1075.3	1283	364.1	173.3	193.4	1304.7	1387.6	1380	425.7
waveform	3484.3	354.0	861.5	1749.8	2601.2	3137.6	1355.9	753.1	727.1	3125.9	3408.3	3381.1	828.9
Avg. High	2408.2	247.3	602.1	1212.9	1801.7	2166.1	547.0	342.8	339.2	2222.7	2345.3	2345.3	825.2
Avg. All	2035.2	210.7	511.1	1022.0	1525.5	1829.4	444.1	302.0	298.2	1719.5	1915.2	1926.2	809.9

FRBCS-CM results (IX)

***Complexity** benchmarking vs. <u>high reduction</u> and existing fusion methods*

Dataset	fuzzy MCSs	FRBCS-CM 10%	FRBCS-CM 25%	FRBCS-CM 50%	FRBCS-CM 75%	FRBCS-CM 90%	Greedy FS MV	Greedy FS AVG	Greedy FS DT	Greedy BS MV	Greedy BS AVG	Greedy BS DT	GA Dimil.
Low dim.:													
abalone	3990.9	398.2	995.7	1996.9	2983.6	3578.4	1211.0	1047.6	1037.7	2711.3	3306.9	3398.7	2391.9
breast	435.2	46.1	110.9	217.0	326.2	391	33.0	25.7	24.1	415.9	426.6	427.4	221.1
glass	590.3	57.4	140.6	289.9	434.4	528	88.7	43.6	54.7	560.5	576.4	577.5	173.8
heart	466.0	49.4	120.3	235.3	352.6	421	48.9	35.7	33.4	444.6	455.7	454.7	221.1
magic	3882.1	421.0	968.3	1965.6	2969.9	3475.8	528.2	424.6	417.3	2247.8	3203.6	3319	2123.6
pblocks	1329.4	131.2	328.9	628.1	967.8	1182.2	248.2	108.9	106.1	1259	1288	1297.3	314.1
phoneme	2197.3	241.7	587.8	1132.5	1679.0	2000	493.2	381.1	339.4	1442.8	2046	2049.4	996.9
pima	1050.9	110.9	260.7	530.1	782.4	946	239.3	149.4	118.1	957	1025	1027.7	530
wine	231.4	23.7	57.9	116.4	172.7	208	9.1	6.8	6.2	222.4	226.9	226.9	71.2
yeast	2449.0	260.8	630.9	1198.4	1825.1	2198.4	511.5	389.5	434.9	1901.3	2296.7	2291.9	902.4
Avg. Low	1662.3	174.0	420.2	831.0	1249.4	1492.8	341.1	261.3	257.2	1216.3	1485.2	1507.1	794.6
High dim.:													
ionosphere	367.7	37.8	95.4	211.0	279.8	334	27.0	22.2	24.4	353.3	361.2	360.6	190.3
optdigits	3584.6	359.2	893.5	1787.7	2678.8	3227.2	652.7	428.7	423.7	3398.5	3513.8	3513.1	661.5
pendigits	4395.3	448.8	1098.1	2208.7	3299.9	3964.3	892.1	569.8	470.8	4167.2	4306.4	4307.5	1874.6
sat	4207.2	427.2	1046.9	2107.2	3128.1	3762.8	1214.0	728.7	800.6	3575.2	4006.8	4055	1431.9
segment	1175.3	130.1	290.9	593.4	876.9	1051.4	165.6	109.2	86.7	1100.5	1151.3	1151.4	414.2
sonar	319.3	32.4	80.4	162.0	240.0	288	24.4	22.9	19.8	306.4	312.1	311.9	158.8
spambase	2220.9	229.0	557.2	1115.5	1661.7	2002.6	340.7	286.1	292.8	2135.5	2152.4	2139.8	1026
texture	2912.2	300.1	716.6	1458.8	2175.0	2610.9	433.6	333.8	352.5	2759.8	2852.9	2852.8	1240.4
vehicle	1415.3	154.3	380.4	735.3	1075.3	1283	364.1	173.3	193.4	1304.7	1387.6	1380	425.7
waveform	3484.3	354.0	861.5	1749.8	2601.2	3137.6	1355.9	753.1	727.1	3125.9	3408.3	3381.1	828.9
Avg. High	2408.2	247.3	602.1	1212.9	1801.7	2166.1	547.0	342.8	339.2	2222.7	2345.3	2345.3	825.2
Avg. All	2035.2	210.7	511.1	1022.0	1525.5	1829.4	444.1	302.0	298.2	1719.5	1915.2	1926.2	809.9

FRBCS-CM results (X)

Complexity *benchmarking vs. joint classifier selection (mid red.) and fusion*

Dataset	fuzzy MCSs	FRBCS-CM 10%	25%	50%	75%	90%	Greedy FS MV	AVG	DT	Greedy BS MV	AVG	DT	GA Dimil.
Low dim.:													
abalone	3990.9	398.2	995.7	1996.9	2983.6	3578.4	1211.0	1047.6	1037.7	2711.3	3306.9	3398.7	2391.9
breast	435.2	46.1	110.9	217.0	326.2	391	33.0	25.7	24.1	415.9	426.6	427.4	221.1
glass	590.3	57.4	140.6	289.9	434.4	528	88.7	43.6	54.7	560.5	576.4	577.5	173.8
heart	466.0	49.4	120.3	235.3	352.6	421	48.9	35.7	33.4	444.6	455.7	454.7	221.1
magic	3882.1	421.0	968.3	1965.6	2969.9	3475.8	528.2	424.6	417.3	2247.8	3203.6	3319	2123.6
pblocks	1329.4	131.2	328.9	628.1	967.8	1182.2	248.2	108.9	106.1	1259	1288	1297.3	314.1
phoneme	2197.3	241.7	587.8	1132.5	1679.0	2000	493.2	381.1	339.4	1442.8	2046	2049.4	996.9
pima	1050.9	110.9	260.7	530.1	782.4	946	239.3	149.4	118.1	957	1025	1027.7	530
wine	231.4	23.7	57.9	116.4	172.7	208	9.1	6.8	6.2	222.4	226.9	226.9	71.2
yeast	2449.0	260.8	630.9	1198.4	1825.1	2198.4	511.5	389.5	434.9	1901.3	2296.7	2291.9	902.4
Avg. Low	1662.3	174.0	420.2	831.0	1249.4	1492.8	341.1	261.3	257.2	1216.3	1485.2	1507.1	794.6
High dim.:													
ionosphere	367.7	37.8	95.4	211.0	279.8	334	27.0	22.2	24.4	353.3	361.2	360.6	190.3
optdigits	3584.6	359.2	893.5	1787.7	2678.8	3227.2	652.7	428.7	423.7	3398.5	3513.8	3513.1	661.5
pendigits	4395.3	448.8	1098.1	2208.7	3299.9	3964.3	892.1	569.8	470.8	4167.2	4306.4	4307.5	1874.6
sat	4207.2	427.2	1046.9	2107.2	3128.1	3762.8	1214.0	728.7	800.6	3575.2	4006.8	4055	1431.9
segment	1175.3	130.1	290.9	593.4	876.9	1051.4	165.6	109.2	86.7	1100.5	1151.3	1151.4	414.2
sonar	319.3	32.4	80.4	162.0	240.0	288	24.4	22.9	19.8	306.4	312.1	311.9	158.8
spambase	2220.9	229.0	557.2	1115.5	1661.7	2002.6	340.7	286.1	292.8	2135.5	2152.4	2139.8	1026
texture	2912.2	300.1	716.6	1458.8	2175.0	2610.9	433.6	333.8	352.5	2759.8	2852.9	2852.8	1240.4
vehicle	1415.3	154.3	380.4	735.3	1075.3	1283	364.1	173.3	193.4	1304.7	1387.6	1380	425.7
waveform	3484.3	354.0	861.5	1749.8	2601.2	3137.6	1355.9	753.1	727.1	3125.9	3408.3	3381.1	828.9
Avg. High	2408.2	247.3	602.1	1212.9	1801.7	2166.1	547.0	342.8	339.2	2222.7	2345.3	2345.3	825.2
Avg. All	2035.2	210.7	511.1	1022.0	1525.5	1829.4	444.1	302.0	298.2	1719.5	1915.2	1926.2	809.9

5. Classifier Selection & Fusion via an Interpretable Genetic Fuzzy System
FRBCS-CM results (XI)

Our approach provides interpretability to the FRBCEs to some extent

FRBCS-CM
(sparse matrix obtained by the GFS)

(c,k)	w
(1,4)	0.558
(1,13)	0.72
(1,23)	0.356
(1,24)	0.748
(1,48)	0.044
(2,2)	0.382
(2,3)	0.504
(2,5)	0.586
(2,15)	0.388
(2,17)	0.643
(3,10)	0.703
(3,22)	0.619
(3,32)	0.204
(3,40)	0.221
(3,48)	0.458

FRBCS-CM
(Fuzzy rule base for component classifier fusion)

$\mathbf{R_1}$:
If $\mathbf{member_4}$ ($\mathbf{0.558}$) says that Class is $\mathbf{1}$ or
$\mathbf{member_{13}}$ ($\mathbf{0.72}$) says that Class is $\mathbf{1}$ or
$\mathbf{member_{23}}$ ($\mathbf{0.356}$) says that Class is $\mathbf{1}$ or
$\mathbf{member_{24}}$ ($\mathbf{0.748}$) says that Class is $\mathbf{1}$ or
$\mathbf{member_{48}}$ ($\mathbf{0.044}$) says that Class is $\mathbf{1}$

$\mathbf{R_2}$:
If $\mathbf{member_2}$ ($\mathbf{0.382}$) says that Class is $\mathbf{2}$ or
$\mathbf{member_3}$ ($\mathbf{0.504}$) says that Class is $\mathbf{2}$ or
$\mathbf{member_5}$ ($\mathbf{0.586}$) says that Class is $\mathbf{2}$ or
$\mathbf{member_{15}}$ ($\mathbf{0.388}$) says that Class is $\mathbf{2}$ or
$\mathbf{member_{17}}$ ($\mathbf{0.643}$) says that Class is $\mathbf{2}$

$\mathbf{R_3}$:
If $\mathbf{member_{10}}$ ($\mathbf{0.703}$) says that Class is $\mathbf{3}$ or
$\mathbf{member_{22}}$ ($\mathbf{0.619}$) says that Class is $\mathbf{3}$ or
$\mathbf{member_{32}}$ ($\mathbf{0.204}$) says that Class is $\mathbf{3}$ or
$\mathbf{member_{40}}$ ($\mathbf{0.221}$) says that Class is $\mathbf{3}$ or
$\mathbf{member_{48}}$ ($\mathbf{0.458}$) says that Class is $\mathbf{3}$

FURIA
(selected fuzzy rules of component classifiers)

Class 1:
$\mathbf{R_{1,4}}$ If $\mathbf{x}$ is and $\mathbf{A_7}$ $\mathbf{x}$ is $\mathbf{A_1}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.96}$
$\mathbf{R_{1,13}}$ If $\mathbf{x}$ is $\mathbf{A_{13}}$ and $\mathbf{x}$ is $\mathbf{A_5}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.96}$
$\mathbf{R_{1,23}}$ If $\mathbf{x}$ is $\mathbf{A_{13}}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.94}$
$\mathbf{R_{1,23}}$ If $\mathbf{x}$ is $\mathbf{A_3}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.64}$
$\mathbf{R_{1,24}}$ If $\mathbf{x}$ is $\mathbf{A_{13}}$ and $\mathbf{x}$ is $\mathbf{A_7}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.95}$
$\mathbf{R_{1,48}}$ If $\mathbf{x}$ is $\mathbf{A_{13}}$ and $\mathbf{x}$ is $\mathbf{A_1}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.96}$
$\mathbf{R_{1,48}}$ If $\mathbf{x}$ is $\mathbf{A_5}$ and $\mathbf{x}$ is $\mathbf{A_1}$ then Class is $\mathbf{1}$ with $\mathbf{CF=0.74}$
Class 2:
$\mathbf{R_{2,2}}$ If $\mathbf{x}$ is $\mathbf{A_{10}}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.96}$
$\mathbf{R_{2,2}}$ If $\mathbf{x}$ is $\mathbf{A_2}$ and $\mathbf{x}$ is $\mathbf{A_5}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.94}$
$\mathbf{R_{2,3}}$ If $\mathbf{x}$ is $\mathbf{A_{10}}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.97}$
$\mathbf{R_{2,3}}$ If $\mathbf{x}$ is $\mathbf{A_2}$ and $\mathbf{x}$ is $\mathbf{A_3}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.8}$
$\mathbf{R_{2,5}}$ If $\mathbf{x}$ is $\mathbf{A_1}$ and $\mathbf{x}$ is $\mathbf{A_7}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.97}$
$\mathbf{R_{2,5}}$ If $\mathbf{x}$ is $\mathbf{A_5}$ and $\mathbf{x}$ is $\mathbf{A_1}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.82}$
$\mathbf{R_{2,15}}$ If $\mathbf{x}$ is $\mathbf{A_{10}}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.97}$
$\mathbf{R_{2,15}}$ If $\mathbf{x}$ is $\mathbf{A_2}$ and $\mathbf{x}$ is $\mathbf{A_4}$ then Class is $\mathbf{2}$ with $\mathbf{CF=94}$
$\mathbf{R_{2,17}}$ If $\mathbf{x}$ is $\mathbf{A_{10}}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.96}$
$\mathbf{R_{2,17}}$ If $\mathbf{x}$ is $\mathbf{A_2}$ and $\mathbf{x}$ is $\mathbf{A_3}$ then Class is $\mathbf{2}$ with $\mathbf{CF=0.9}$
Class 3:
$\mathbf{R_{1,10}}$ If $\mathbf{x}$ is $\mathbf{A_{12}}$ and $\mathbf{x}$ is $\mathbf{A_7}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.94}$
$\mathbf{R_{1,10}}$ If $\mathbf{x}$ is $\mathbf{A_{10}}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.91}$
$\mathbf{R_{1,22}}$ If $\mathbf{x}$ is $\mathbf{A_{11}}$ and $\mathbf{x}$ is $\mathbf{A3}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.94}$
$\mathbf{R_{1,22}}$ If $\mathbf{x}$ is $\mathbf{A_7}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.9}$
$\mathbf{R_{1,32}}$ If $\mathbf{x}$ is $\mathbf{A_{11}}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.93}$
$\mathbf{R_{1,40}}$ If $\mathbf{x}$ is $\mathbf{A_{11}}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.95}$
$\mathbf{R_{1,48}}$ If $\mathbf{x}$ is $\mathbf{A_7}$ then Class is $\mathbf{3}$ with $\mathbf{CF=0.95}$

OVERVIEW

1. Introduction
2. Proposed Framework
3. FRBCE Design from Classical ML Approaches
4. EMO Selection of Component Classifiers
5. Classifier Selection and Fusion via an Interpretable GFS
6. **Conclusions**

- An advanced framework to design FRBCEs using classical and recent diversity induction methods and EAs has been presented
- Different specific methods for the two CE design stages have been proposed under the general umbrella
- In particular, the proposal of an interpretable FRBCS-CM performing joint classifier selection and fusion over any weight-based classifier constitute a very novel development
- The obtained FRBCEs have shown to be competitive with state-of-the-art classical Ces
- The framework has been applied to solve a real-world topology-based WiFi indoor localization problem

K. Trawinski et al. A multiclassifier approach for topology-based wifi indoor localization. Soft Computing (2013), to appear. IF 2011: 1.880. Cat: CS, Int. App. O: 24/99. Q1

6. Conclusions
Research team

OVERVIEW

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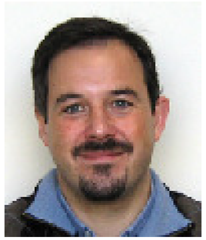

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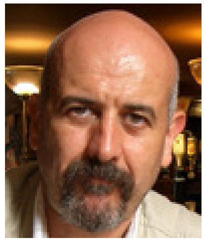
Gradiant Postdoctoral Researcher



Dr.Luciano Sánchez
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Thank you for your attention

Questions?