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# Multi-Objective Evolutionary Fuzzy Systems: An Overview by Problem objectives nature and optimized components

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# Contents

# Introduction to genetic fuzzy systems

Multi-Objective Evolutionary Fuzzy Systems (MOEFSs) are a particular type of Genetic Fuzzy System using Multi-Objective Evolutionary Algorithms (MOEAs)

## An Revision on Genetic Fuzzy Systems (GFSs)

- **Brief Introduction**
- **Taxonomy of Genetic Fuzzy Systems**
- **Why do we use GAs? Considering multiple Objectives**
- **The birth, GFSs roadmap, current state and most cited papers**

# Introduction to genetic fuzzy systems
## Brief Introduction

- The use of genetic/evolutionary algorithms (GAs) to design fuzzy systems constitutes one of the branches of the Soft Computing paradigm: genetic fuzzy systems (GFSs)

- The most known approach is that of genetic fuzzy rule-based systems, where some components of a fuzzy rule-based system (FRBS) are derived (adapted or learnt) using a GA

- Some other approaches include genetic fuzzy neural networks and genetic fuzzy clustering, among others

# Introduction to genetic fuzzy systems
## Brief Introduction

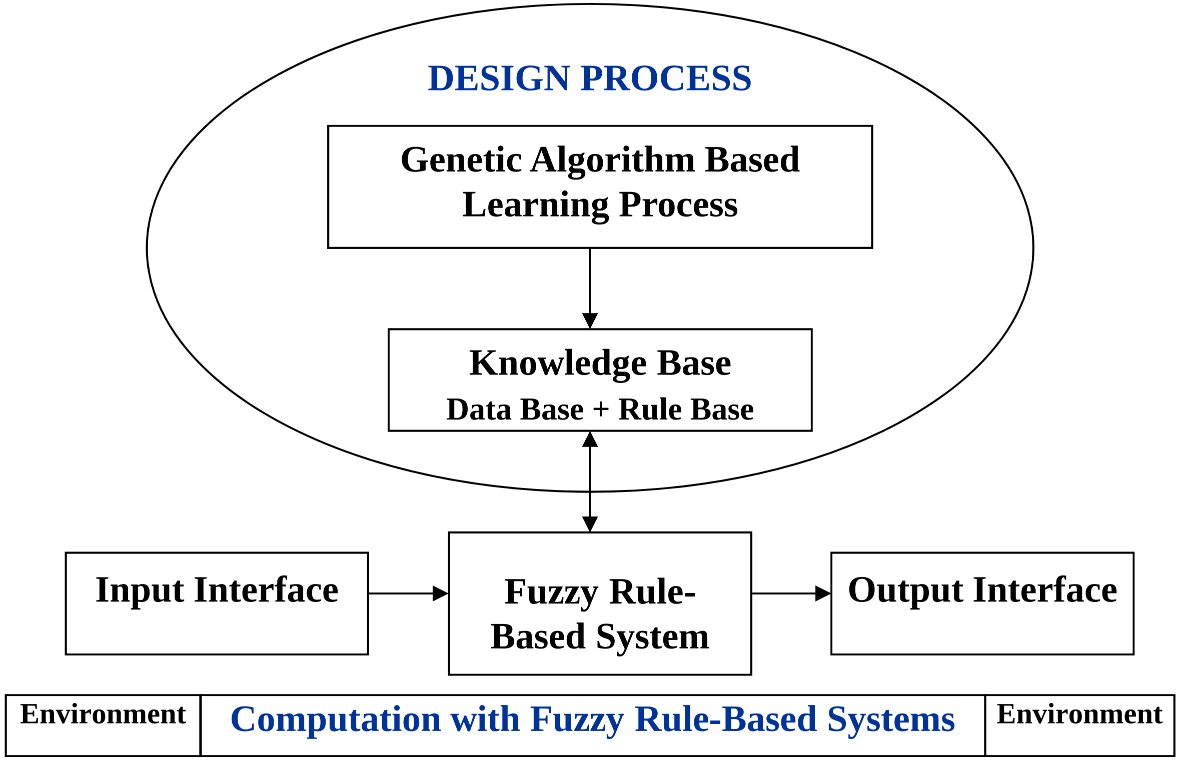
### Evolutionary algorithms and machine learning:

- Evolutionary algorithms were not specifically designed as machine learning techniques, as other approaches like neural networks
- However, it is well known that a learning task can be modelled as an optimization problem, and thus solved through evolution
- Their powerful search in complex, ill-defined problem spaces has permitted applying evolutionary algorithms successfully to a huge variety of machine learning and knowledge discovery tasks
- Their flexibility and capability to incorporate existing knowledge are also very interesting characteristics for the problem solving.

# Introduction to genetic fuzzy systems

## Brief Introduction

### Genetic Fuzzy Rule-Based Systems:



# Introduction to genetic fuzzy systems
## Brief Introduction

### Design of fuzzy rule-based systems:

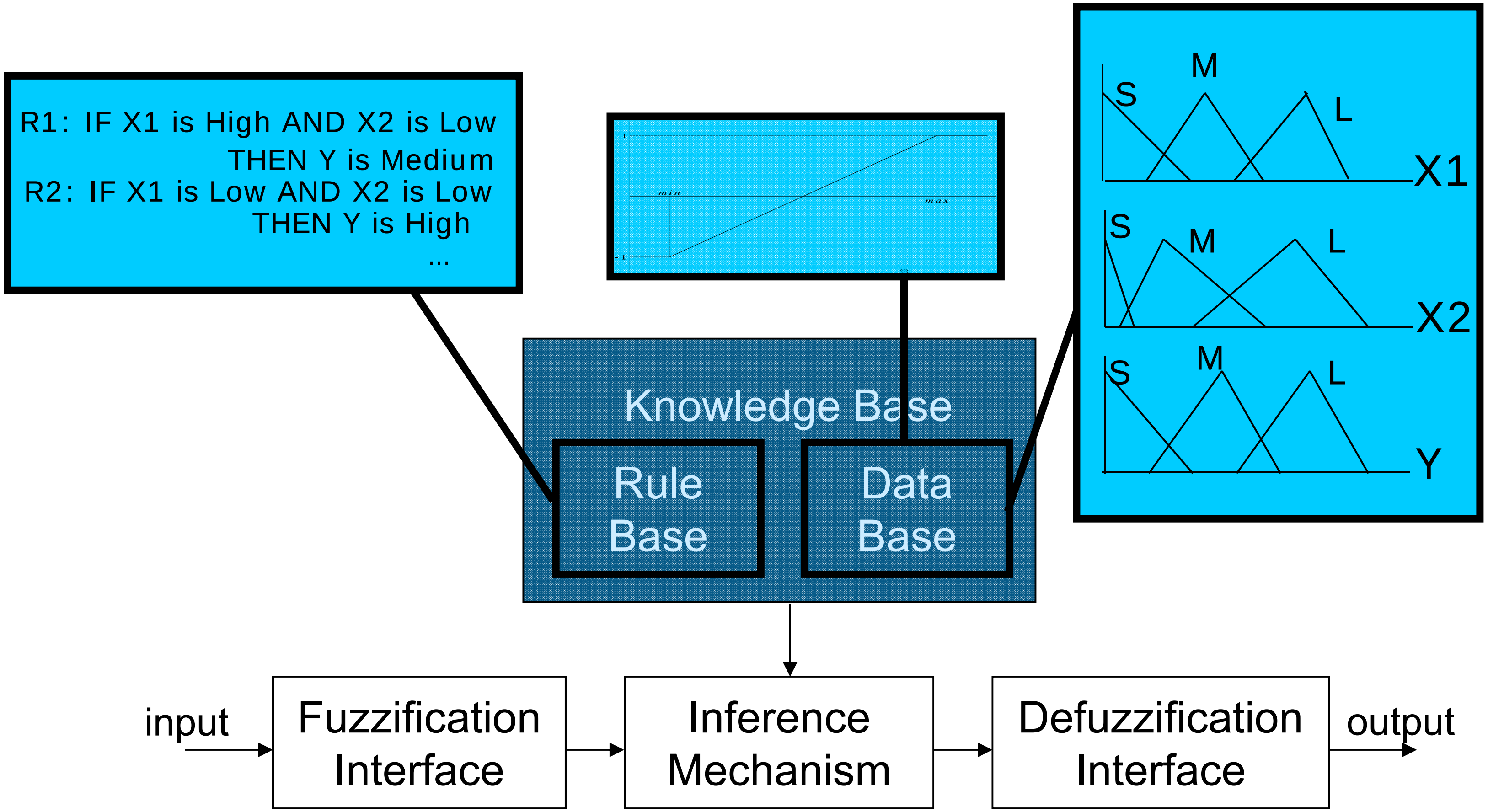
- An FRBS (regardless it is a fuzzy model, a fuzzy logic controller or a fuzzy classifier), is comprised by two main components:
  - The Knowledge Base (KB), storing the available problem knowledge in the form of fuzzy rules
  - The Inference System, applying a fuzzy reasoning method on the inputs and the KB rules to give a system output

- Both must be designed to build an FRBS for a specific application:
  - The KB is obtained from expert knowledge or by machine learning methods
  - The Inference System is set up by choosing the fuzzy operator for each component (conjunction, implication, defuzzifier, etc.)

    **Sometimes, the latter operators are also parametric and can be tuned using automatic methods**

# Introduction to genetic fuzzy systems

## Brief Introduction




**An Example of Fuzzy rule-based system**

# Introduction to genetic fuzzy systems
## Brief Introduction

**The KB design involves two subproblems, related to its two subcomponents:**

- **Definition of the Data Base (DB):**
  - **Variable universes of discourse**
  - **Scaling factors or functions**
  - **Granularity (number of linguistic terms/labels) per variable**
  - **Membership functions associated to the labels**
- **Derivation of the Rule Base (RB): fuzzy rule composition**

# Introduction to genetic fuzzy systems

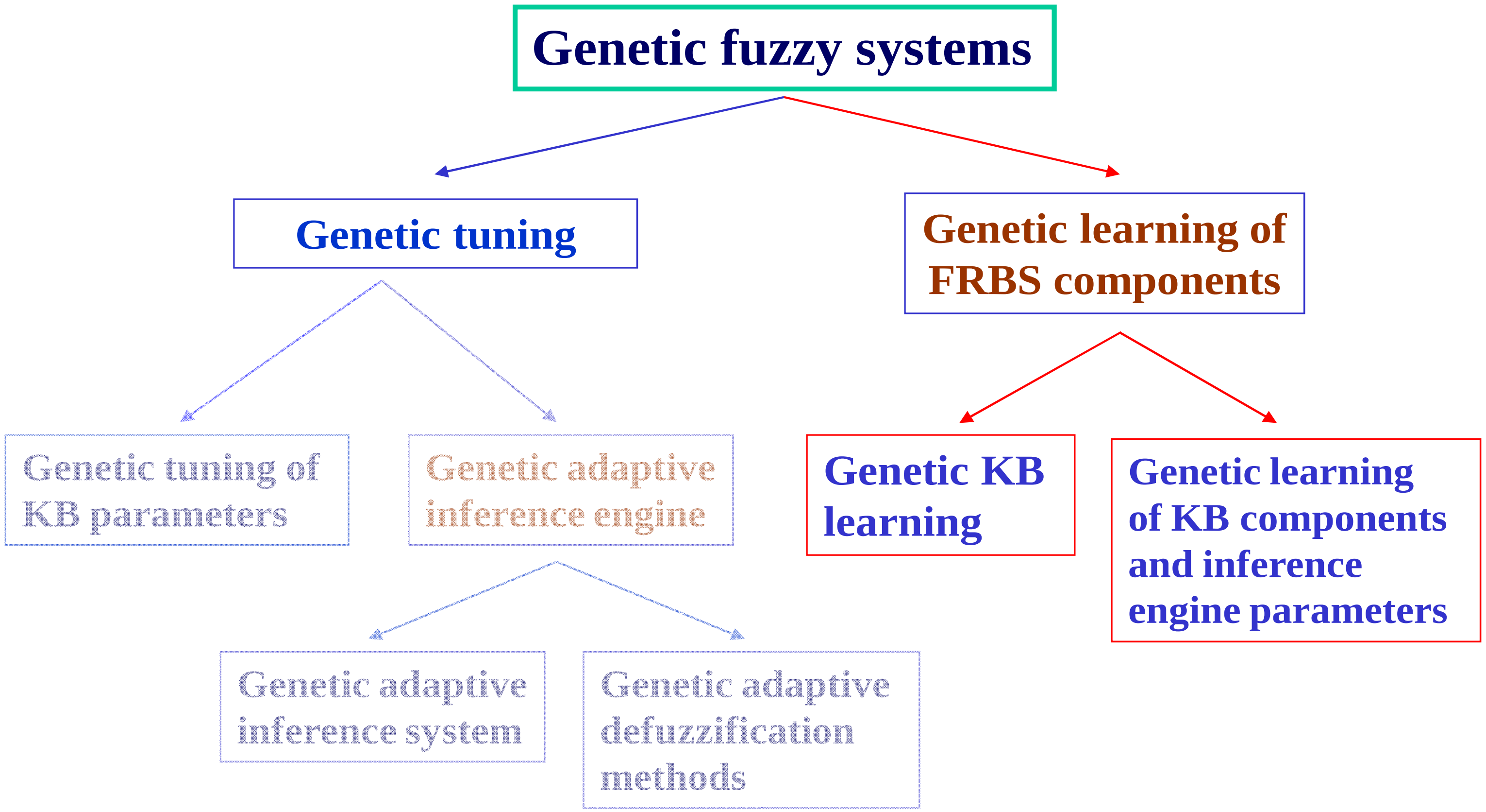
## Brief Introduction

**As said, there are two different ways to design the KB:**

- **From human expert information**
- **By means of machine learning methods guided by the existing numerical information (fuzzy modeling and classification) or by a model of the system being controlled**

# Introduction to genetic fuzzy systems

## Taxonomy of Genetic Fuzzy Systems

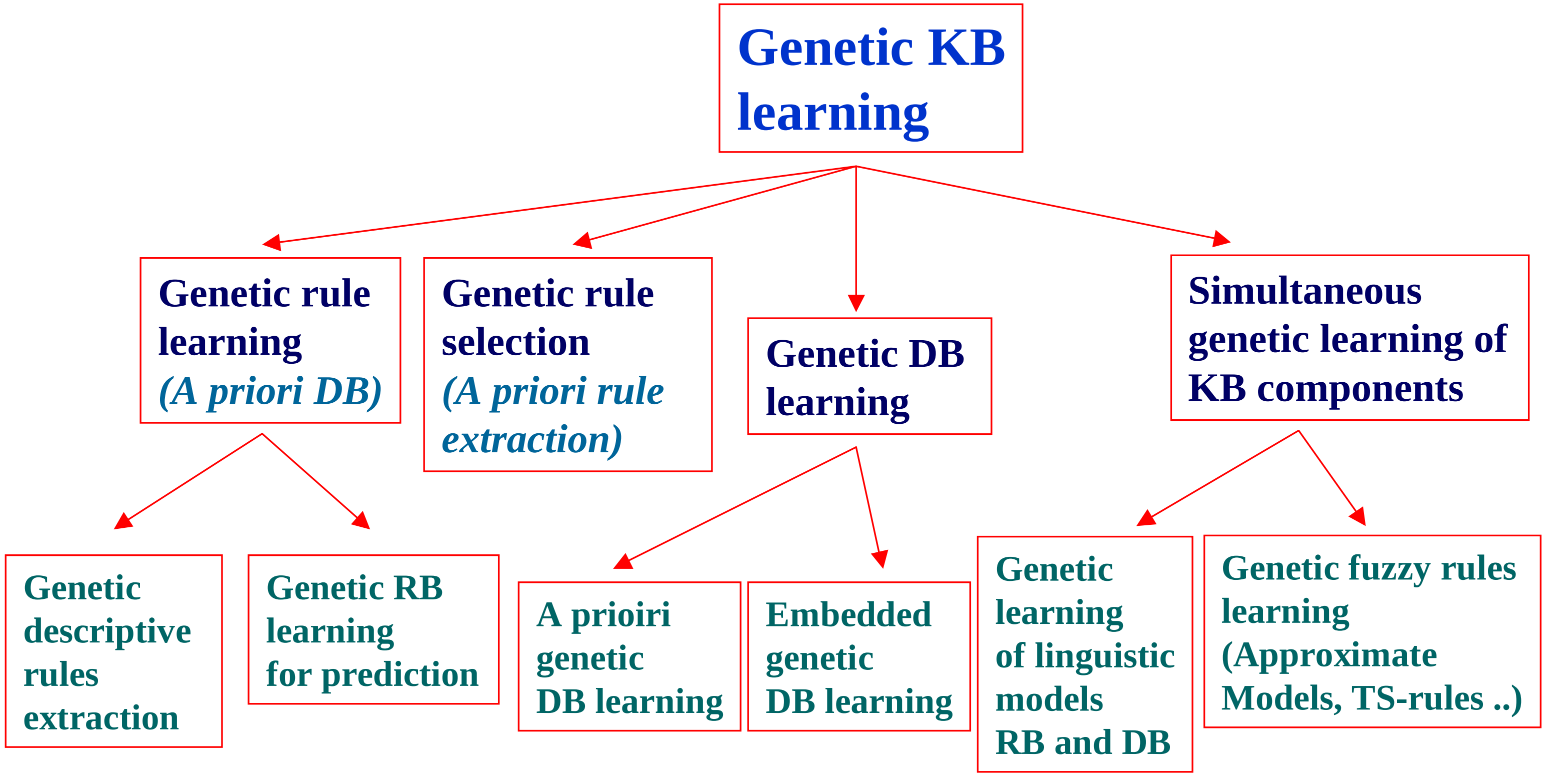



F. Herrera, **Genetic Fuzzy Systems: Taxonomy, Current Research Trends and Prospects**. *Evolutionary Intelligence 1 (2008) 27-46* [doi: 10.1007/s12065-007-0001-5](doi: 10.1007/s12065-007-0001-5)

Associated Website: [http://sci2s.ugr.es/gfs/](http://sci2s.ugr.es/gfs/)

# Introduction to genetic fuzzy systems

## Taxonomy of Genetic Fuzzy Systems

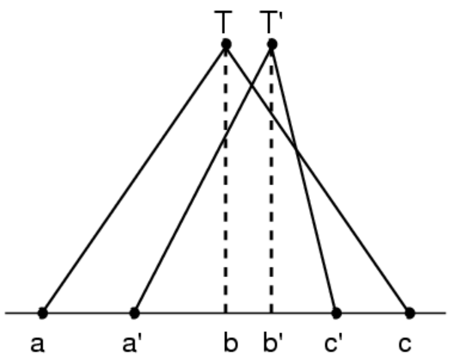


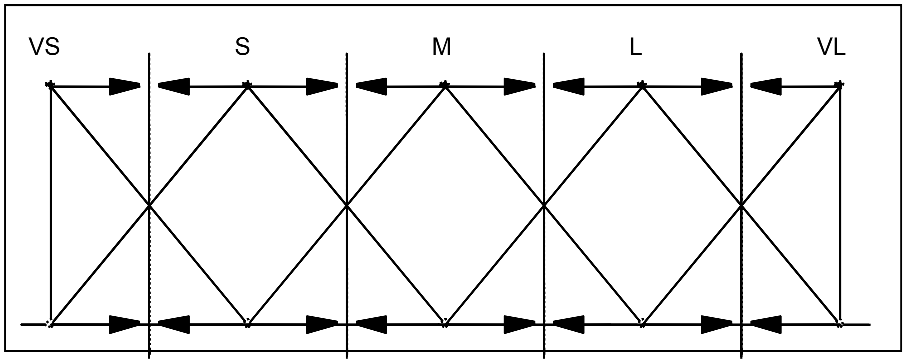
# Introduction to genetic fuzzy systems
## 1. Genetic Tuning

**Classically:**

**– performed on a predefined DB definition**

**– tuning of the membership function shapes by a GA**

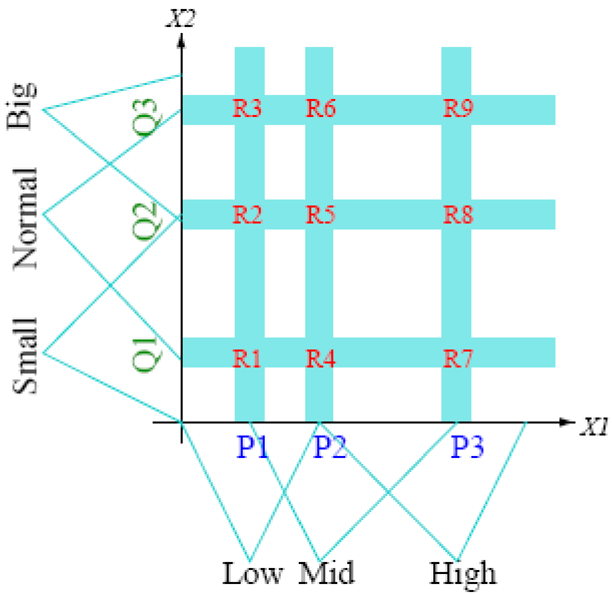


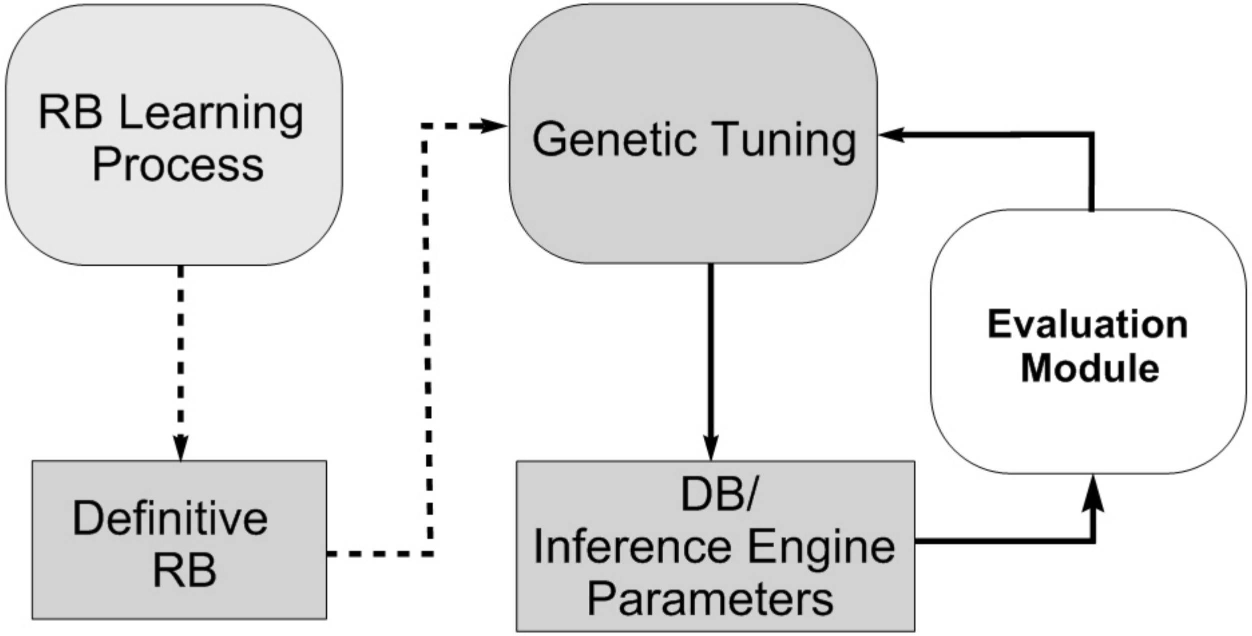




**– tuning of the inference parameters**

# Introduction to genetic fuzzy systems

## 1. Genetic Tuning

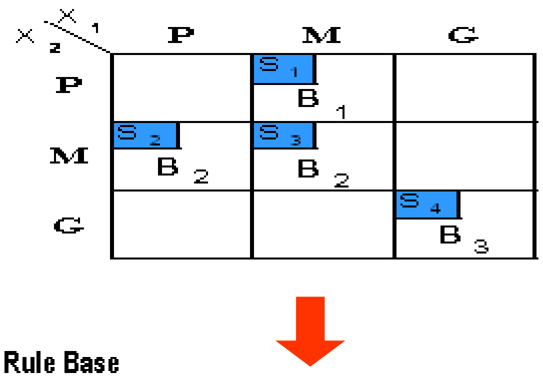


# Introduction to genetic fuzzy systems
## 2. Genetic Rule Learning

**A predefined Data Base definition is assumed**

– **The fuzzy rules (usually Mamdani-type) are derived by a GA**




**Rule Base**

$R_1$ = IF $X_1$ is **M** and $X_2$ is **P** THEN Y is $\mathbf{B_1}$

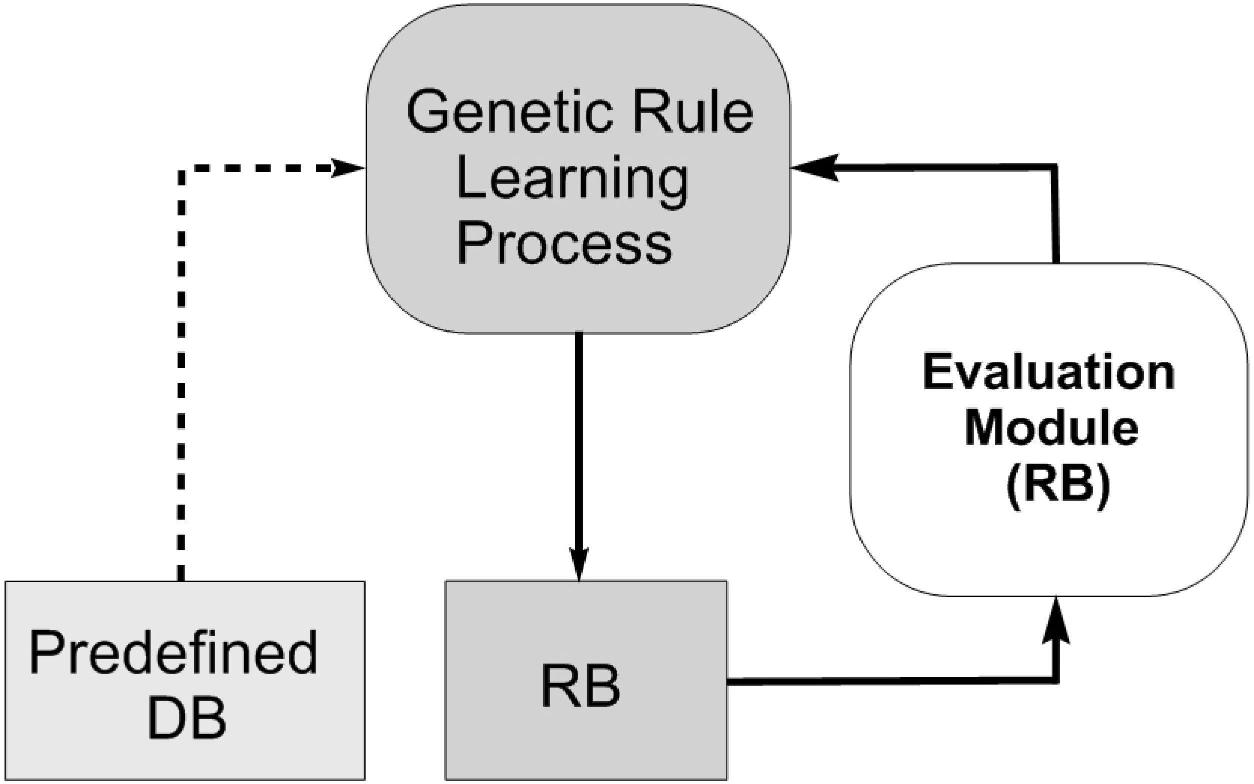
$R_2$ = IF $X_1$ is **P** and $X_2$ is **M** THEN Y is $\mathbf{B_2}$

$R_3$ = IF $X_1$ is **M** and $X_2$ is **M** THEN Y is $\mathbf{B_2}$

$R_4$ = IF $X_1$ is **G** and $X_2$ is **G** THEN Y is $\mathbf{B_3}$

# Introduction to genetic fuzzy systems

## 2. Genetic Rule Learning

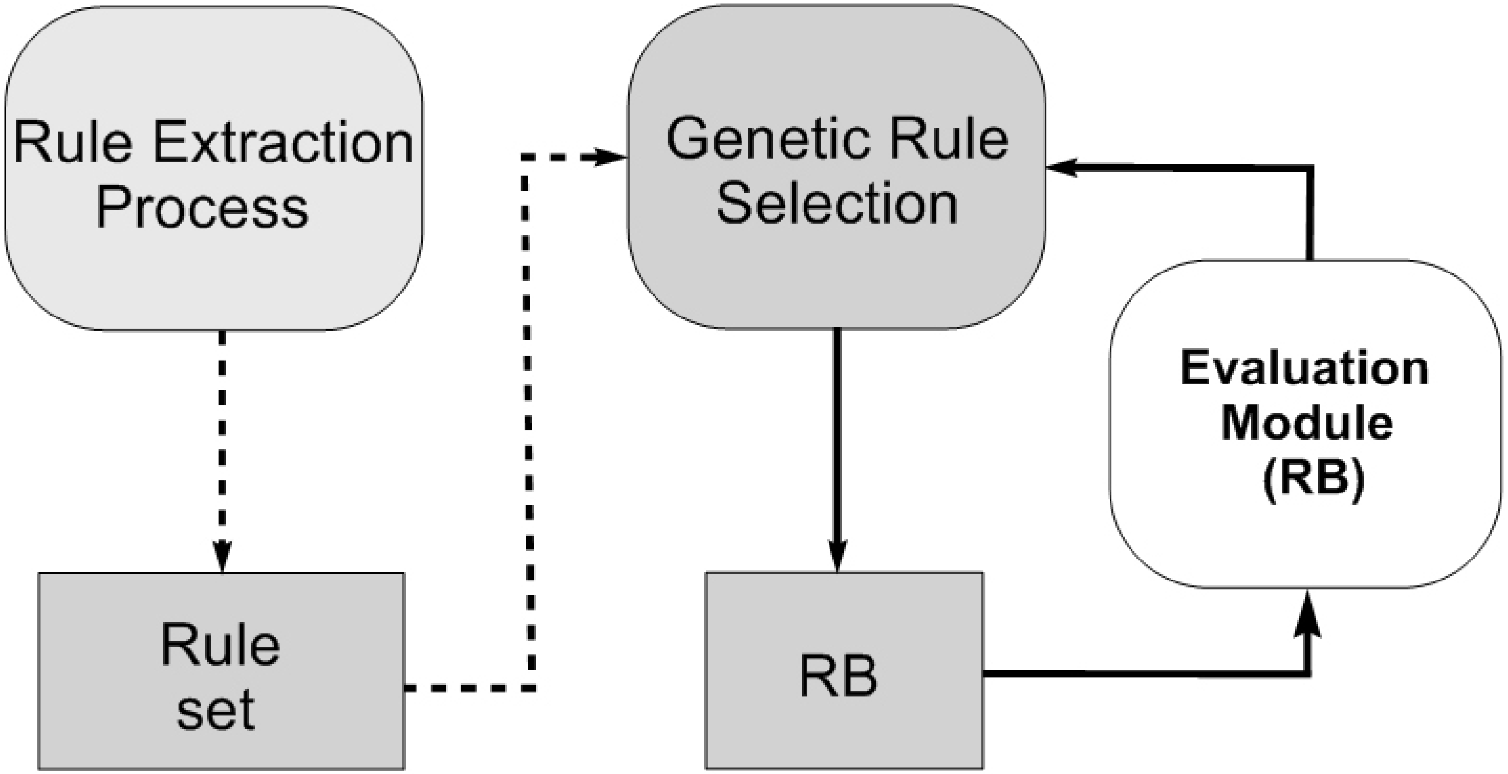


# Introduction to genetic fuzzy systems
## 3. Genetic Rule Selection

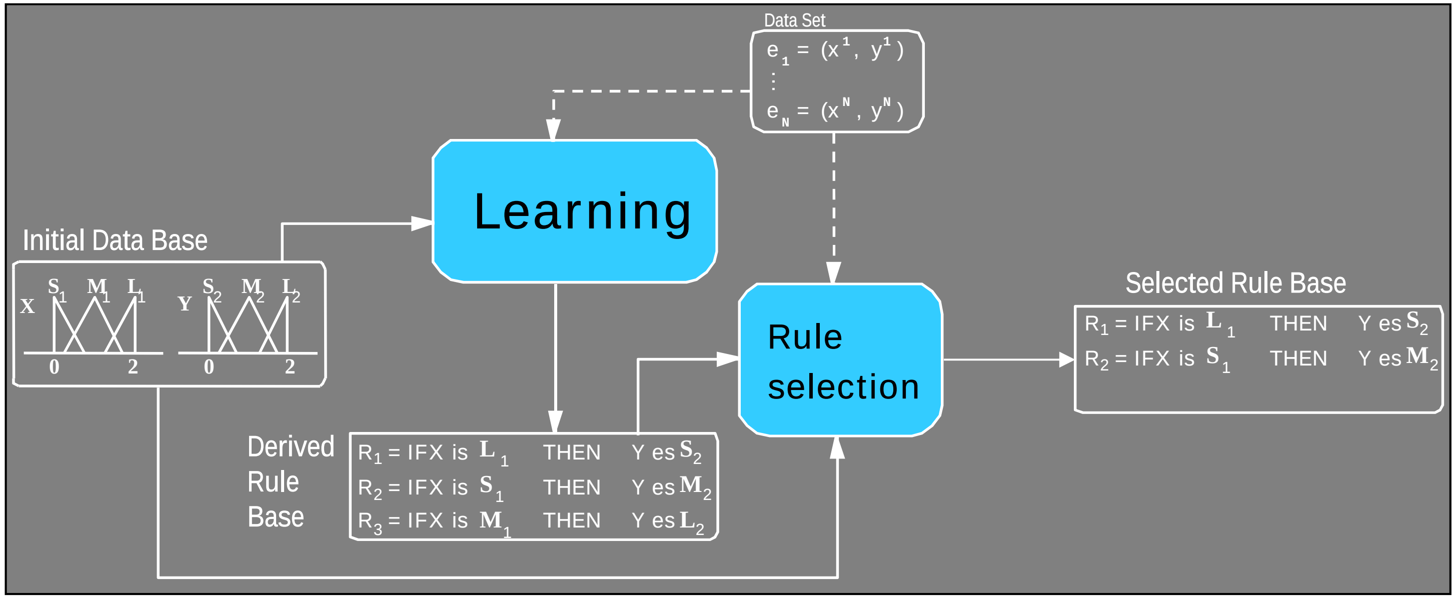
- A predefined set of candidate rules is assumed
- The fuzzy rules are selected by a GA for getting a compact rule base (more interpretable, more precise)

# Introduction to genetic fuzzy systems

## 3. Genetic Rule Selection



# Introduction to genetic fuzzy systems
## 3. Genetic Rule Selection

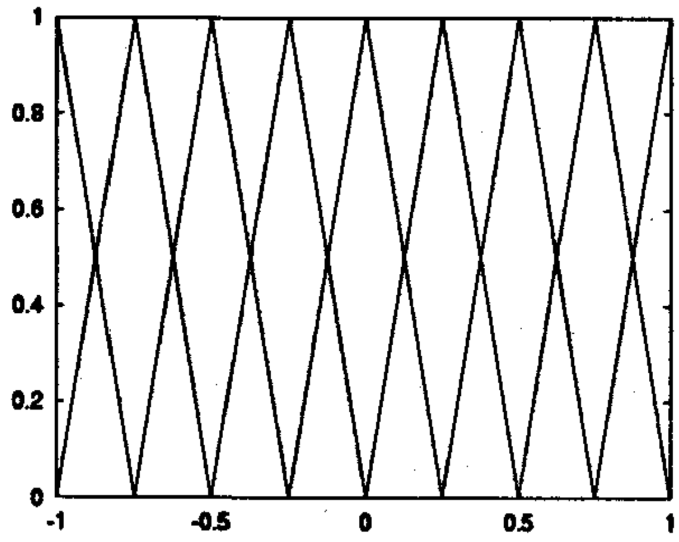


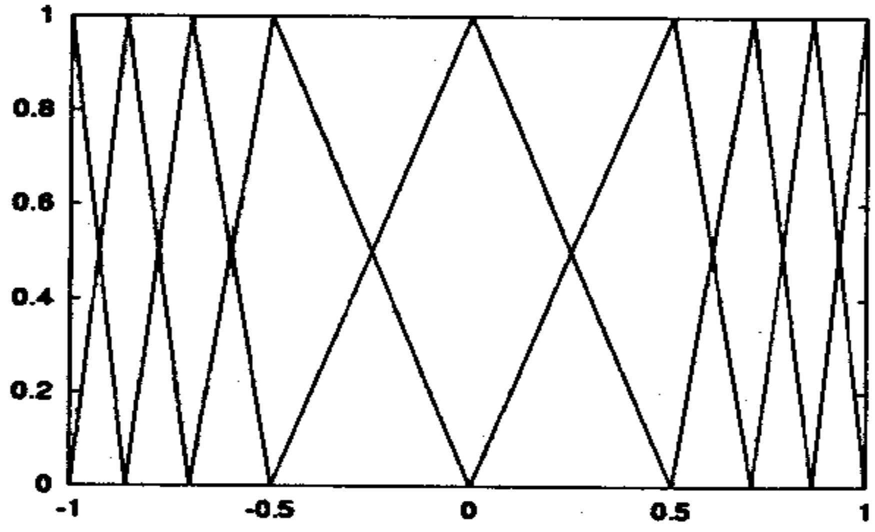

**Example of genetic rule selection**

# Introduction to genetic fuzzy systems

## 4. Genetic DB Learning

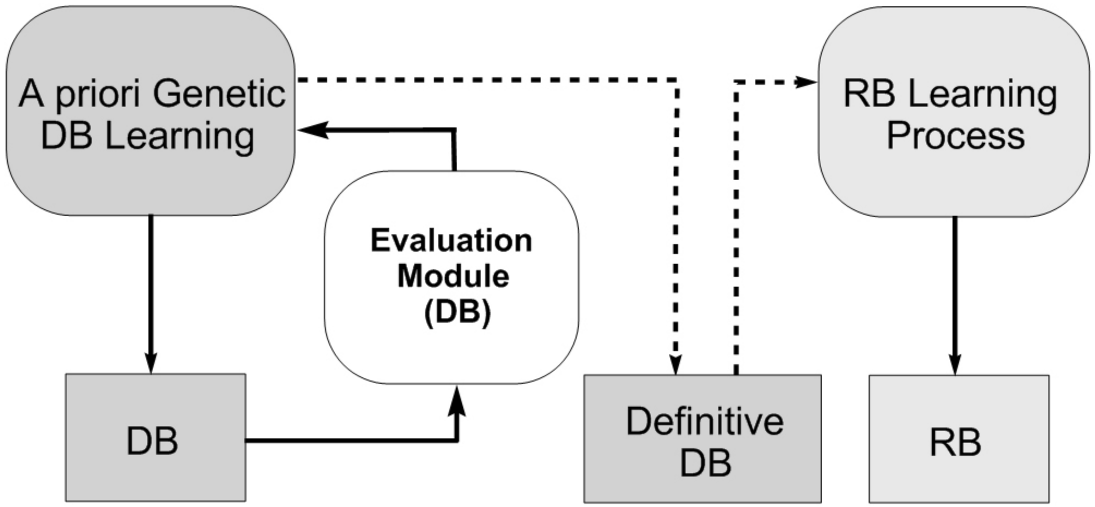
– **Learning of the membership function shapes by a GA**

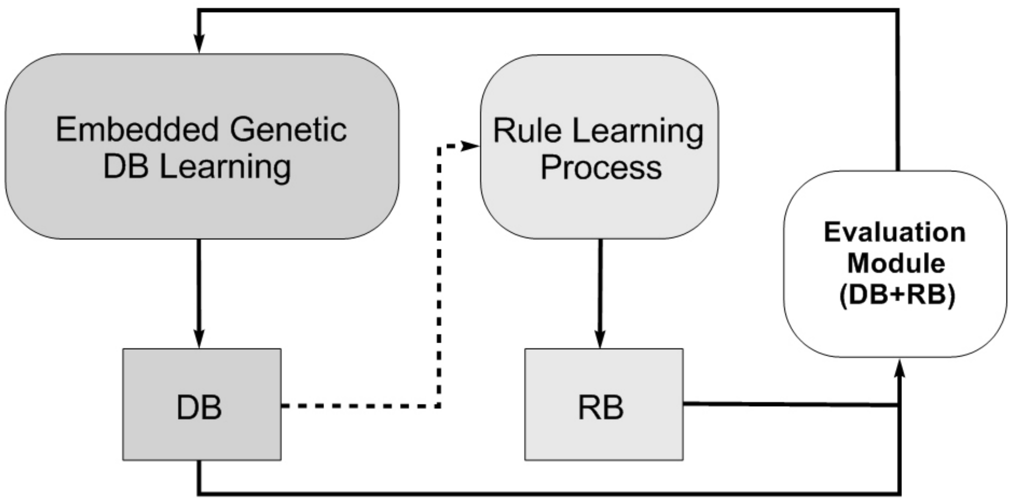


# Introduction to genetic fuzzy systems

## 4. Genetic DB Learning

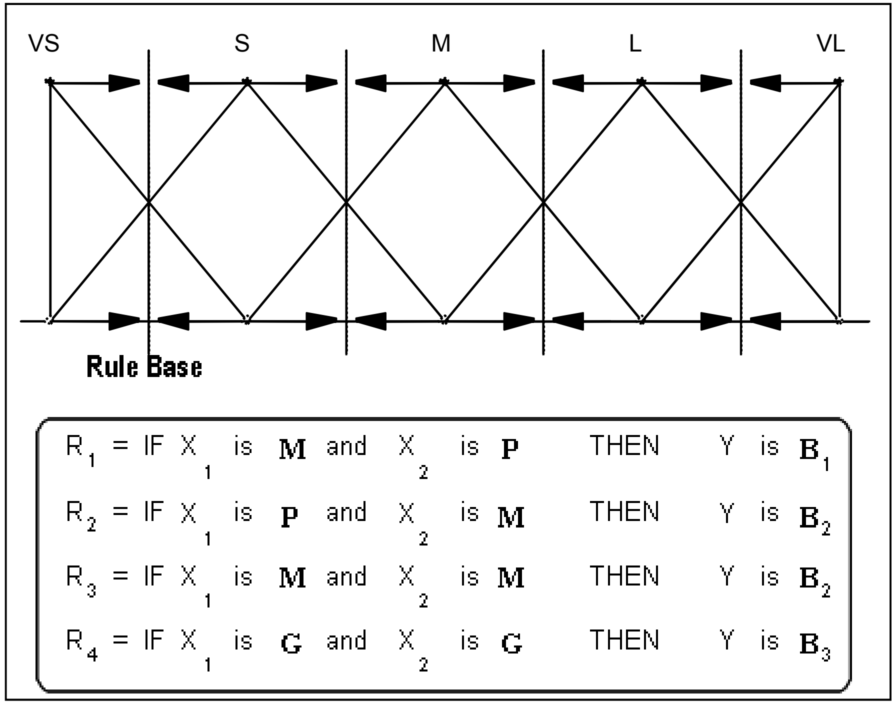


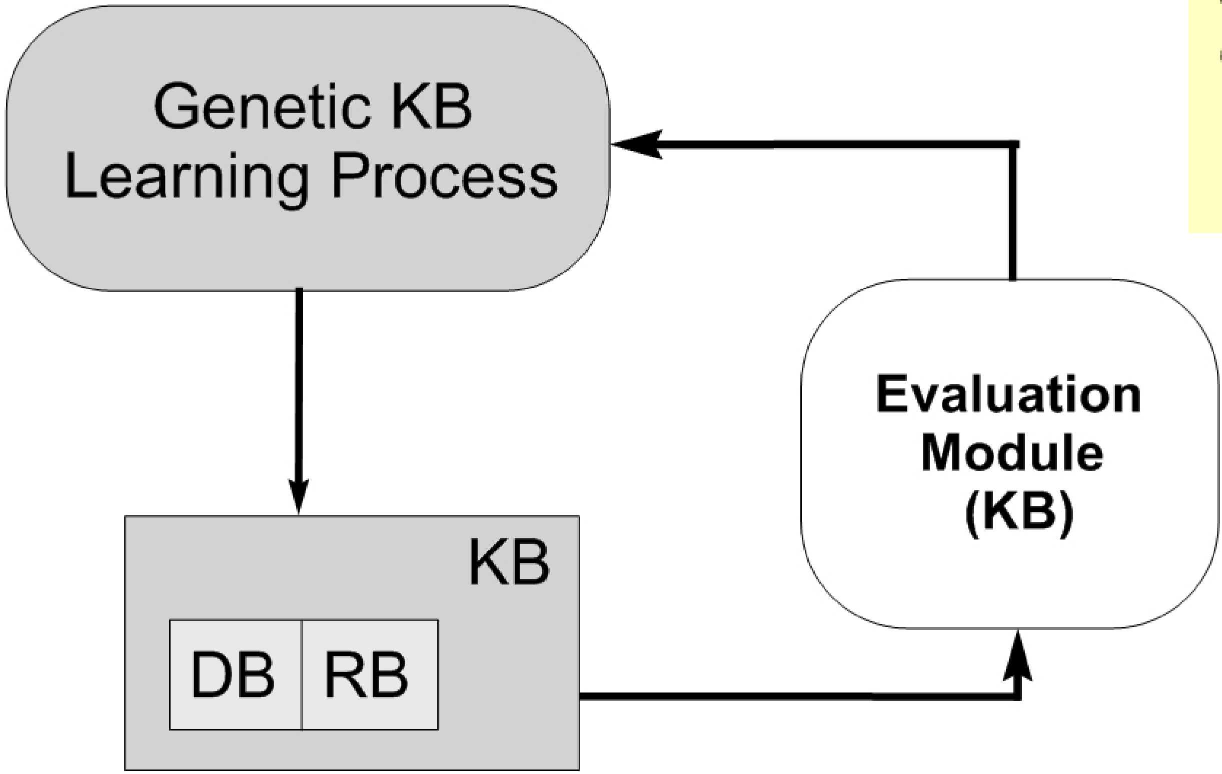
# Introduction to genetic fuzzy systems

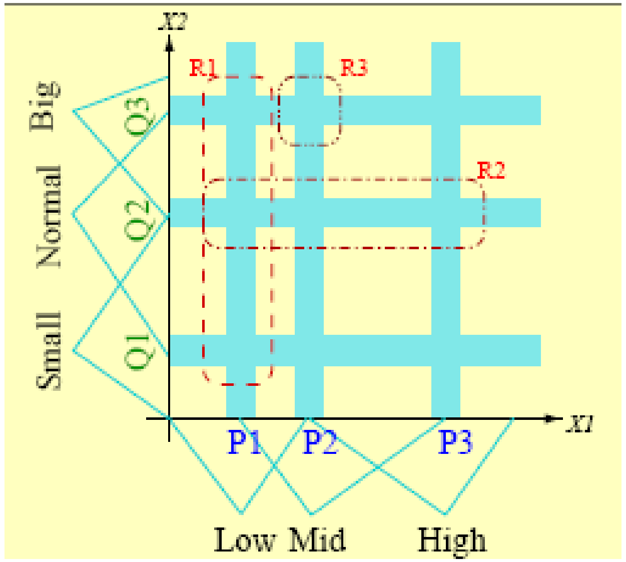
## 5. Simultaneous Genetic Learning of KB Components

**The simultaneous derivation properly addresses the strong dependency existing between the RB and the DB**

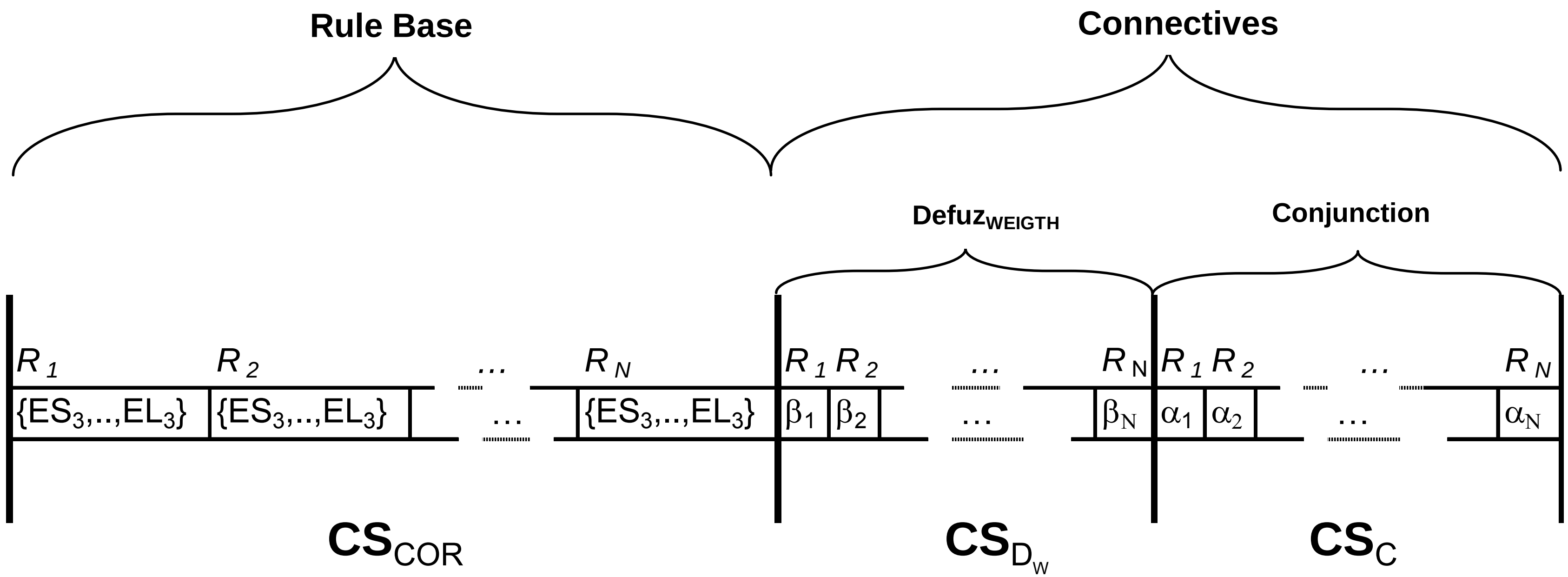


# Introduction to genetic fuzzy systems
## 5. Simultaneous Genetic Learning of KB Components



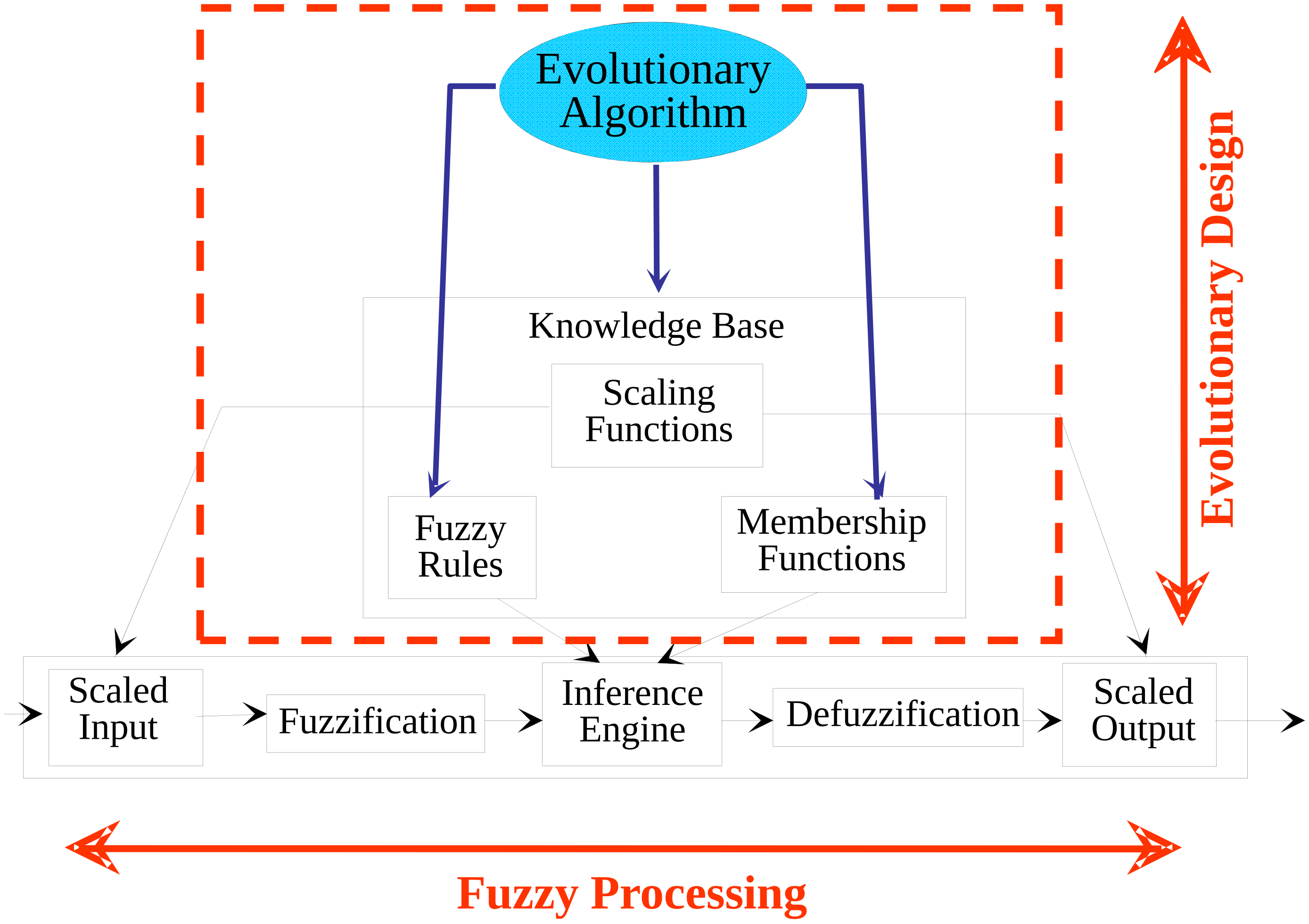



## 6. Genetic Learning of KB Components and Inference Engine Parameters




**Example of the coding scheme for learning an RB and the inference connective parameters**

# Introduction to genetic fuzzy systems



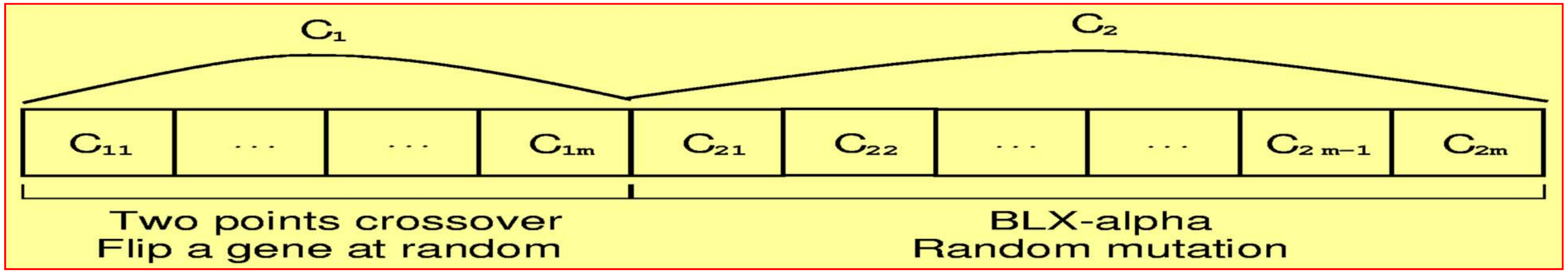
# Introduction to genetic fuzzy systems

## Why do we use GAs?

### Particular Characteristics of the Genetic Fuzzy Systems

- **We can code different FS components in a chromosome:**
  - **Identify relevant inputs**
  - **Scaling factors**
  - **Membership functions, shape functions, optimal shape of membership funct., granularity (number of labels per variable)**
  - **Fuzzy rules, Any inference parameter, ....**

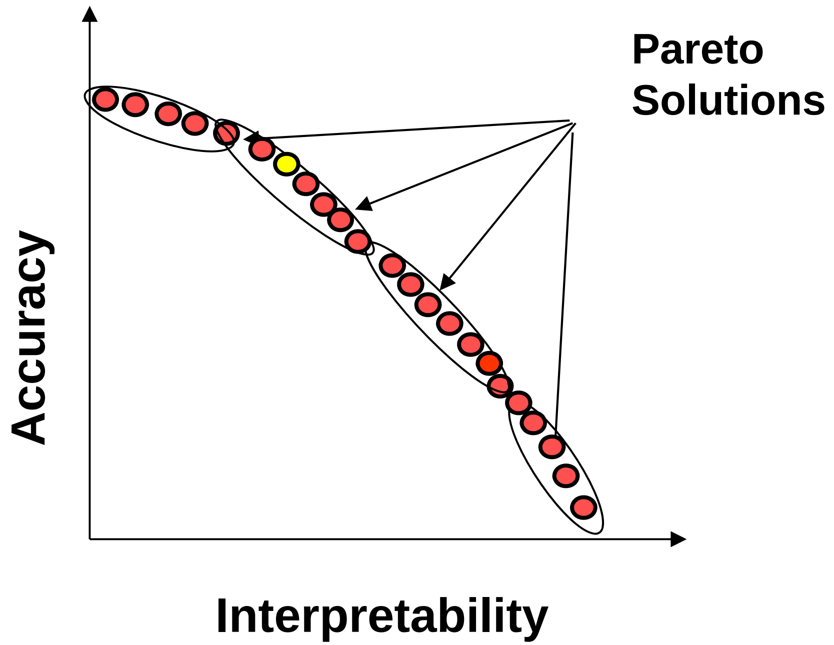
**We can define different mechanism for managing them (combining genetic operators, coevolution,...)**



# Introduction to genetic fuzzy systems
## Considering Multiple Objectives

### Particular Characteristics of the Genetic Fuzzy Systems

- **We can consider multiple objectives in the learning model (interpretability, precision, ....)**



## The birth of GFSs: 1991

- Thrift's ICGA91 paper (Mamdani-type Rule Base Learning. Pittsburgh approach)

  Thrift P (1991) **Fuzzy logic synthesis with genetic algorithms.** *In: Proc. of 4th International Conference on Genetic Algorithms (ICGA'91), pp 509-513*

- Valenzuela-Rendón's PPSN-I paper (Scatter Mamdani-type KB Learning. Michigan approach)

  Valenzuela-Rendon M (1991) **The fuzzy classifier system: A classifier system for continuously varying variables**. *In: Proc. of 4th International Conference on Genetic Algorithms (ICGA'91), pp 346-353*

- Pham and Karaboga's Journal of Systems Engineering paper (Relational matrix-based FRBS learning. Pittsburgh approach)

  Pham DT, Karaboga D (1991) **Optimum design of fuzzy logic controllers using genetic algorithms**. *Journal of Systems Engineering 1:114-118*).

- Karr's AI Expert paper (Mamdani-type Data Base Tuning)

  Karr C (1991) **Genetic algorithms for fuzzy controllers**. *AI Expert 6(2):26-33*.

Almost the whole basis of the area were established in the first year!

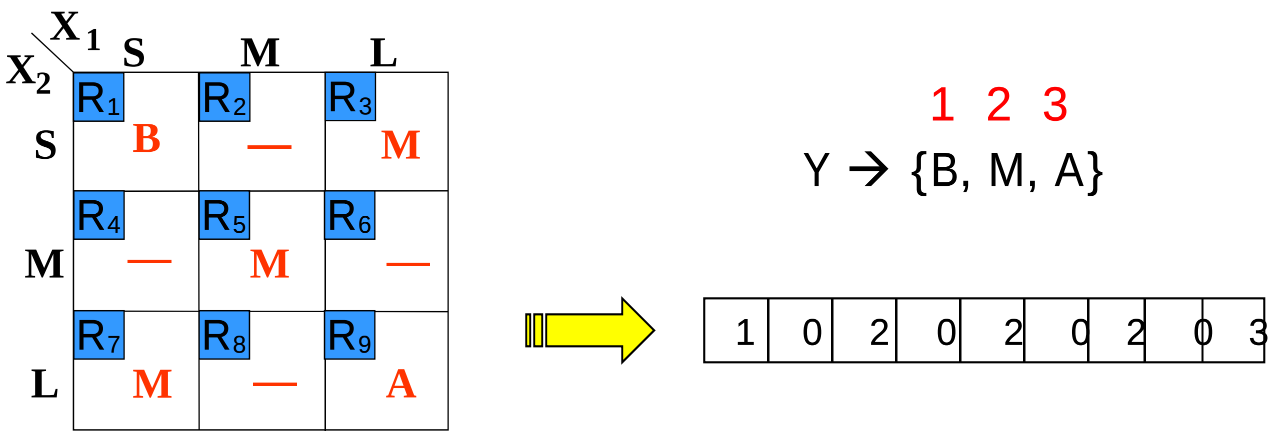
# Introduction to genetic fuzzy systems

The birth, GFSs roadmap, current status and most cited papers

## Thrift's GFS:

**P. Thrift, Fuzzy logic synthesis with genetic algorithms, Proc. Fourth Intl. Conf. on Genetic Algorithms (ICGA'91), San Diego, USA, 1991, pp. 509–513**

- **Classical approach: Pittsburgh – the decision table is encoded in a rule consequent array**
- **The output variable linguistic terms are numbered from 1 to n and comprise the array values. The value 0 represents the rule absence, thus making the GA able to learn the optimal number of rules**
- **The ordered structure allows the GA to use simple genetic operators**



## GFSs roadmap

### 1991-1996/7: INITIAL GFS SETTING: KB LEARNING:

- Establishment of the three classical learning approaches in the GFS field: Michigan, Pittsburgh, and IRL
- Different FRBS types: Mamdani, Mamdani DNF, Scatter Mamdani, TSK
- Generic applications: Classification, Modeling, and Control

### 1995-...: FUZZY SYSTEM TUNING:

- First: Membership function parameter tuning
- Later: other DB components adaptation: scaling factors, context adaptation (scaling functions), linguistic hedges, ...
- Recently: interpretability consideration

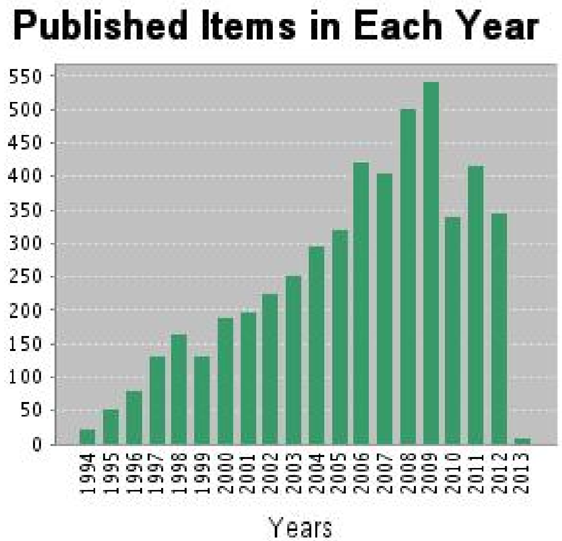
## GFSs roadmap

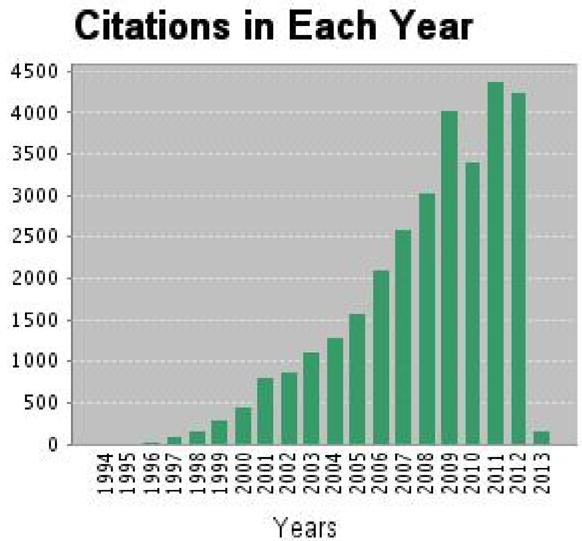
### 1998-...: APPROACHING TO MATURITY? NEW GFS LEARNING APPROACHES:

- New EAs: Bacterial genetics, DNA coding, Virus-EA, genetic local search (memetic algorithms), ...
- Hybrid learning approaches: a priori DB learning, GFNNs, Michigan-Pitt hybrids, ...
- **Multiobjective evolutionary algorithms**
- Interpretability-accuracy trade-off consideration
- Course of dimensionality (handling large data sets and complex problems):
  - Rule selection (1995-...)
  - Feature selection at global level and fuzzy rule level
  - Hierarchical fuzzy modeling
- “Incremental” learning

# Introduction to genetic fuzzy systems
## Current state of the GFS area

Number of papers on GFSs published in JCR journals




Source: **The Thomson Corporation ISI Web of Knowledge**

Query: **(TS = (("GA-" OR "GA based" OR evolutionary OR "genetic algorithm*" OR "genetic programming" OR "evolution strate*" OR "genetic learning" OR "particle swarm" OR "differential evolutio*" OR "ant system*" OR "ant colony" OR "genetic optimi*" OR "estimation of distribution algorithm*") AND ("fuzzy rule*" OR "fuzzy system*" OR "fuzzy neural" OR "neuro-fuzzy" OR "fuzzy control*" OR "fuzzy logic cont*" OR "fuzzy class*" OR "fuzzy if" OR "fuzzy model*" OR "fuzzy association rule*" OR "fuzzy regression"))**



Date of analysis: **January 3th, 2013** Number of papers: **5079**
Number of citations: **30738** Average citations per paper: **6.05**

# Introduction to genetic fuzzy systems

## Current state of the GFS area

### Highly cited papers on GFSs (classic approaches - papers until 2000)

1. Homaifar, A., McCormick, E., Simultaneous Design of Membership Functions and rule sets for fuzzy controllers using genetic algorithms, IEEE TFS 3 (2) (1995) 129-139. Citations: 302
2. Ishibuchi, H., Nozaki, K., Yamamoto, N., Tanaka, H., Selecting fuzzy if-then rules for classification problems using genetic algorithms, IEEE TFS 3 (3) (1995) 260-270. Citations: 284
3. Setnes, M., Roubos, H., GA-fuzzy modeling and classification: complexity and performance, IEEE TFS 8 (5) (2000) 509-522 . Citations: 215
4. Ishibuchi, H., Nakashima, T., Murata, T., Performance evaluation of fuzzy classifier systems for multidimensional pattern classification problems, IEEE TSMC B 29 (5) (1999) 601-618. Citations: 177
5. Shi, Y.H., Eberhart, R., Chen, Y.B., Implementation of evolutionary fuzzy systems, IEEE TFS 7 (2) (1999) 109-119. Citations: 126
6. Park, D., Kandel, A., Langholz, G., Genetic-based new fuzzy reasoning models with application to fuzzy control, IEEE TSMC B 24 (1) (1994) 39-47. Citations: 125
7. Jin, YC (2000) Fuzzy modeling of high-dimensional systems: Complexity reduction and interpretability improvement. IEEE Transactions on Fuzzy Systems 8(2):212-221. Citations: 121
8. Ishibuchi H, Murata T, Turksen IB (1997) Single-objective and **two-objective genetic algorithms** for selecting linguistic rules for pattern classification problems. Fuzzy Sets and Systems 89(2):135-150 Citations: 116
9. Juang, C.F., Lin, J.Y., Lin, C.T., Genetic reinforcement learning through symbiotic evolution for fuzzy controller design, IEEE TSMC B 30 (2) (2000) 290-302. Citations: 109
10. Herrera, F., Lozano, M., Verdegay, J.L., Tuning fuzzy-logic controllers by genetic algorithms, IJAR 12 (3-4) (1995) 299-315. Citations: 108

# Introduction to genetic fuzzy systems

## Current state of the GFS area

### Highly cited papers on GFSs (recent approaches – 2001 to 2010)

1. **Juang, CF (2002) A TSK-type recurrent fuzzy network for dynamic systems processing by neural network and genetic algorithms. IEEE Transactions on Fuzzy Systems 10(2):155-170.** Citations: 144

2. **Cordon O, Gomide F, Herrera F, Hoffmann F, Magdalena L (2004) Ten years of genetic fuzzy systems: current framework and new trends. Fuzzy Sets and Systems 141(1):5-31.** Citations: 142

3. **Roubos H, Setnes M (2001) Compact and transparent fuzzy models and classifiers through iterative complexity reduction. IEEE Transactions on Fuzzy Systems 9(4):516-524.** Citations: 105

4. **Ishibuchi H, Nakashima T, Murata T (2001) Three-objective genetics-based machine learning for linguistic rule extraction. Information Sciences 136(1-4):109-133.** Citations: 97

5. **Ishibuchi H, Yamamoto T (2004) Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining. Fuzzy Sets and Systems 141(1):59-88.** Citations: 96

6. **Cordon O, Herrera F, Villar P (2001) Generating the knowledge base of a fuzzy rule-based system by the genetic learning of the data base. IEEE Transactions on Fuzzy Systems 9(4):667-674.** Citations: 66

7. **Gonzalez J, Rojas I, Ortega J, Pomares H, Fernandez J, Diaz AF (2003) Multiobjective evolutionary optimization of the size, shape, and position parameters of radial basis function networks for function approximation. IEEE Transactions on Neural Networks 14(6):1478-1495.** Citations: 61

8. **Liu BD, Chen CY, Tsao JY (2001) Design of adaptive fuzzy logic controller based on linguistic-hedge concepts and genetic algorithms. IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics 31(1):32-53** Citations: 53

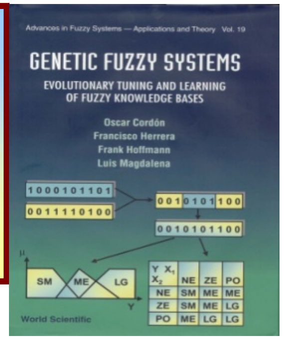
9. **Wang HL, Kwong S, Jin YC, et al. (2005) Multi-objective hierarchical genetic algorithm for interpretable fuzzy rule-based knowledge extraction. Fuzzy Sets And Systems 149(1):149-186.** Citations: 52

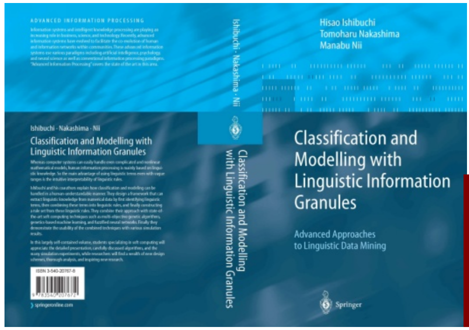
10. **Kuo RJ, Chen CH, Hwang YC (2001) An intelligent stock trading decision support system through integration of genetic algorithm based fuzzy neural network and artificial neural network. Fuzzy Sets and Systems 118(1):21-45.** Citations: 50

# Introduction to genetic fuzzy systems

## Some References

**GENETIC FUZZY SYSTEMS**
**Evolutionary Tuning and Learning of Fuzzy Knowledge Bases.**
O. Cordón, F. Herrera, F. Hoffmann, L. Magdalena
**World Scientific, July 2001**




**H. Ishibuchi, T. Nakashima, M. Nii, Classification and Modeling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer (2005)**

- M. Fazzolari, R. Alcalá, Y. Nojima, H. Ishibuchi, F. Herrera. A review of the application of Multi-Objective Evolutionary Systems: Current status and further directions. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65, doi: 10.1109/TFUZZ.2012.2201338
- F. Herrera, Genetic Fuzzy Systems: Taxonomy, Current Research Trends and Prospects. *Evolutionary Intelligence 1 (2008) 27-46 doi: 10.1007/s12065-007-0001-5,*
- F. Herrera, Genetic Fuzzy Systems: Status, Critical Considerations and Future Directions, International Journal of Computational Intelligence Research 1 (1) (2005) 59-67
- O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena, Ten Years of Genetic Fuzzy Systems: Current Framework and New Trends, FSS 141 (1) (2004) 5-31
- F. Hoffmann, Evolutionary Algorithms for Fuzzy Control System Design, Proceedings of the IEEE 89 (9) (2001) 1318-1333

# Introduction to genetic fuzzy systems

## GFSs and MOEFSs Website

**http://sci2s.ugr.es/gfs/**


**http://sci2s.ugr.es/gfs/biblio.php**

# Contents

1. Basics on MOEFSs
   - Introduction to Genetic Fuzzy Systems (GFSs) and its main types
   - Evolutionary Multiobjective Optimization: Basic concepts and framework

2. Types of MOEFSs by multiobjective nature and optimized components

3. MOEFSs designed for the Interpretability-Accuracy Tradeoff of Fuzzy Systems: *Two contradictory objectives*
   - Interpretability issues in fuzzy systems design
   - Some example approaches

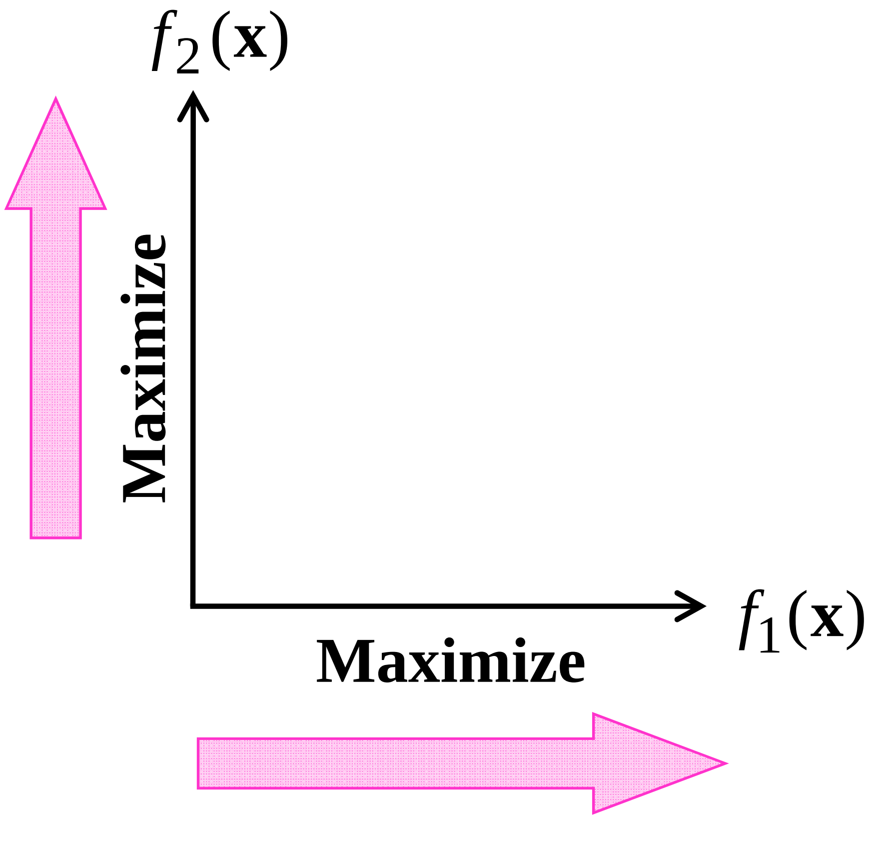
4. Other types of MOEFSs
   - MOEFSs designed for multi-objective control problems
   - MOEFSs designed for fuzzy association rule mining

5. New Research Directions in MOEFSs

# Multiobjective Optimization

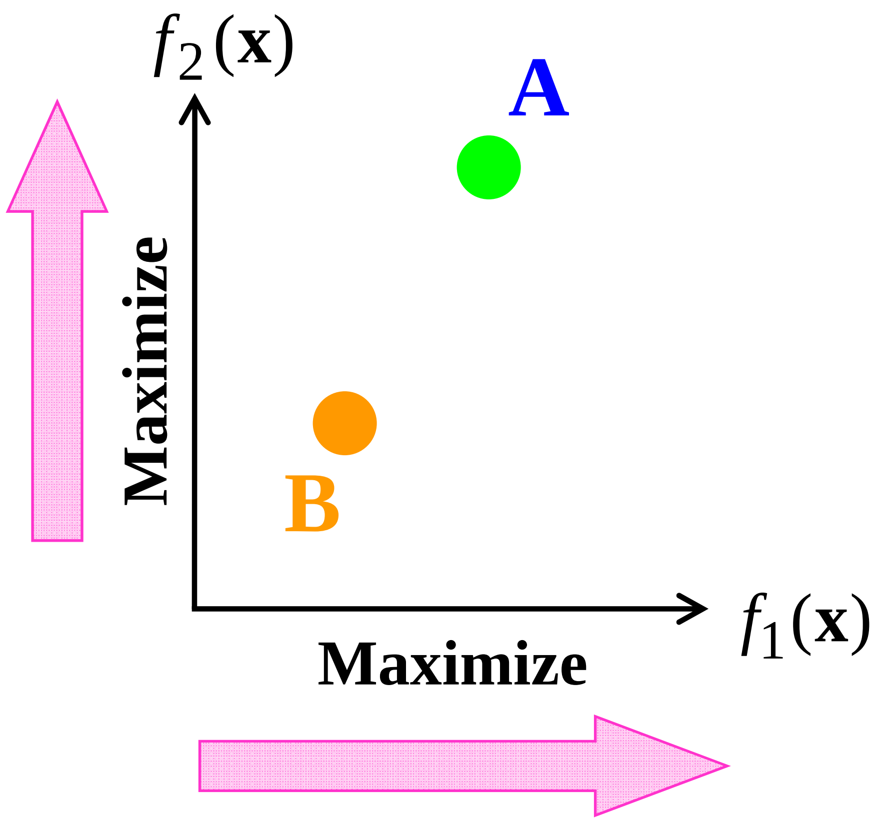
## Two-Objective Maximization Problem:

$$\textbf{Maximize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$$



# Comparison between Two Solutions

**Maximize** $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$



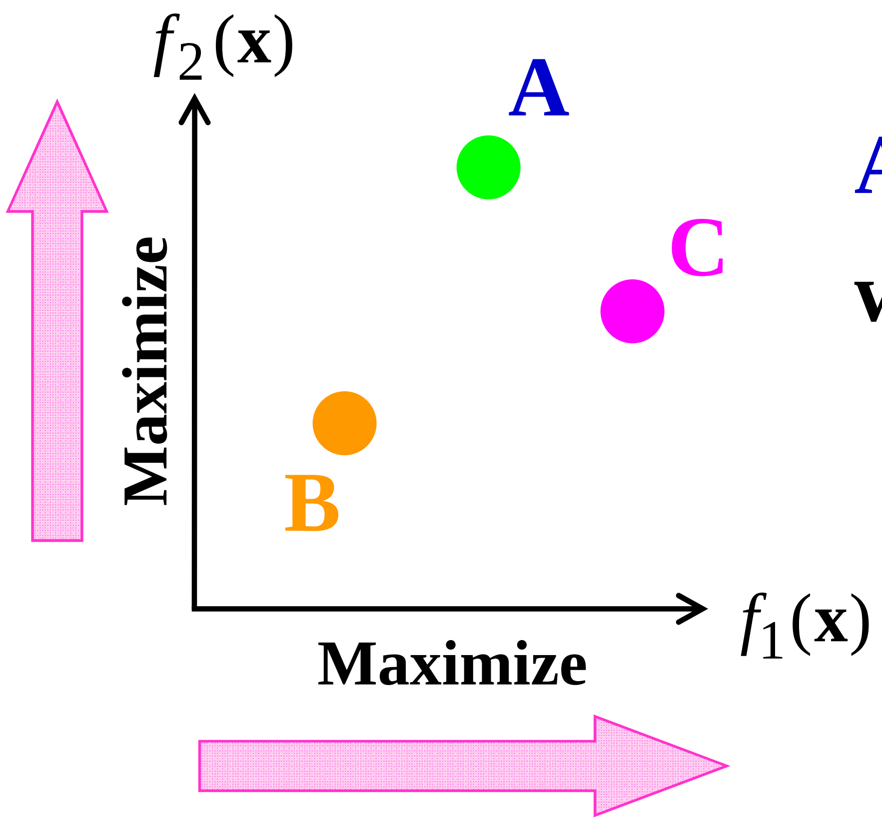

**A dominates B**

**B is dominated by A**

**(A is better than B )**

# Comparison between Two Solutions

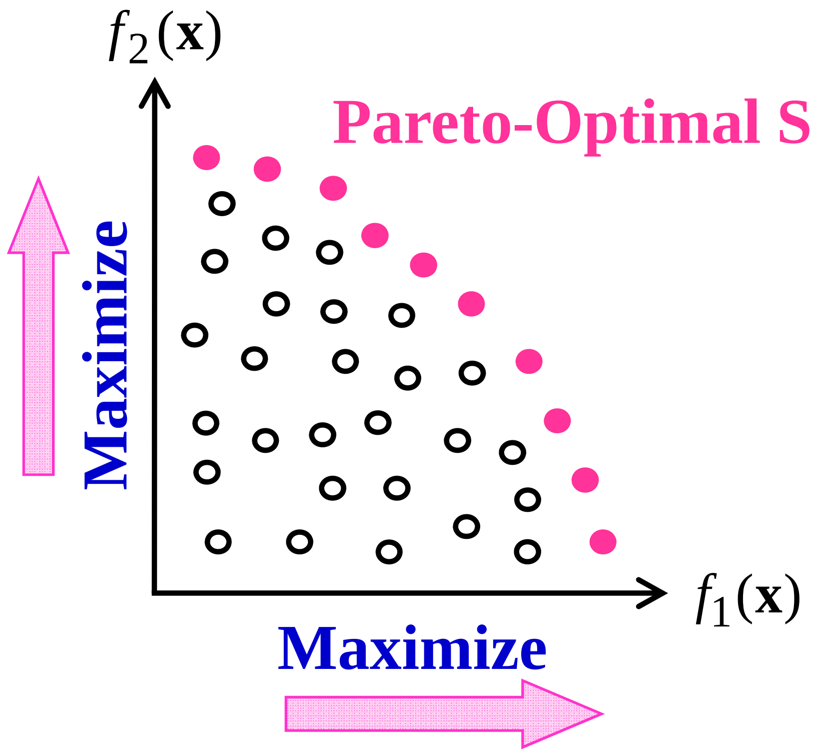
**Maximize** $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$




**A and C are non-dominated with each other.**

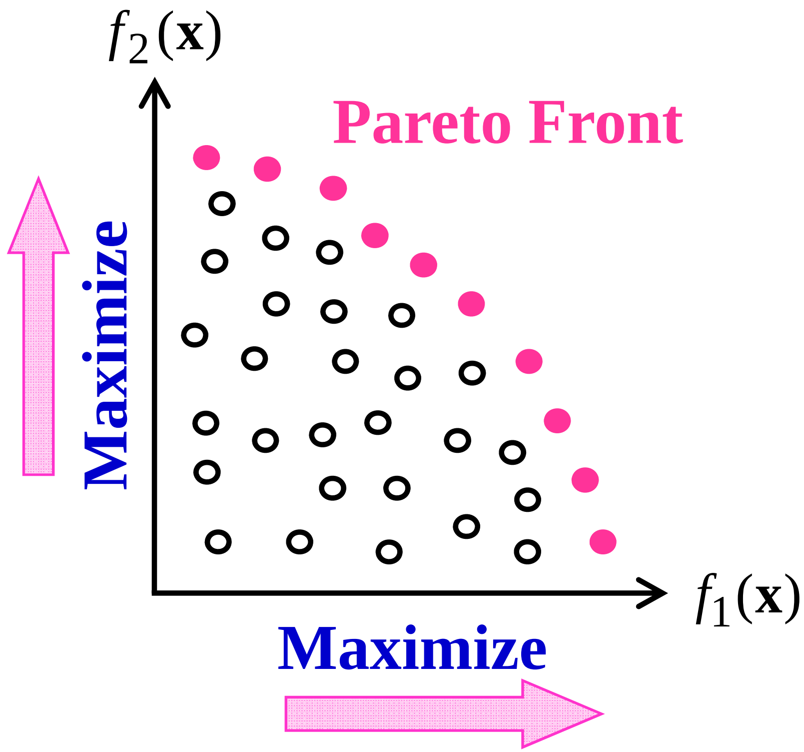
# Pareto-Optimal Solutions

**A Pareto-optimal solution is a solution that is not dominated by any other solutions.**



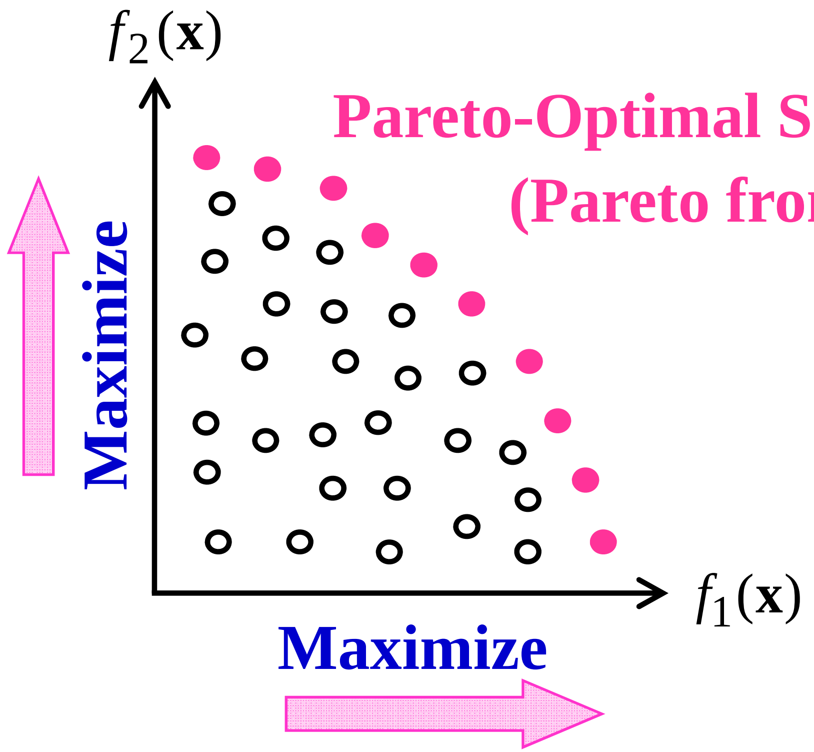
# Pareto Front

**The set of all Pareto-optimal solutions is called the Pareto front of the problem.**



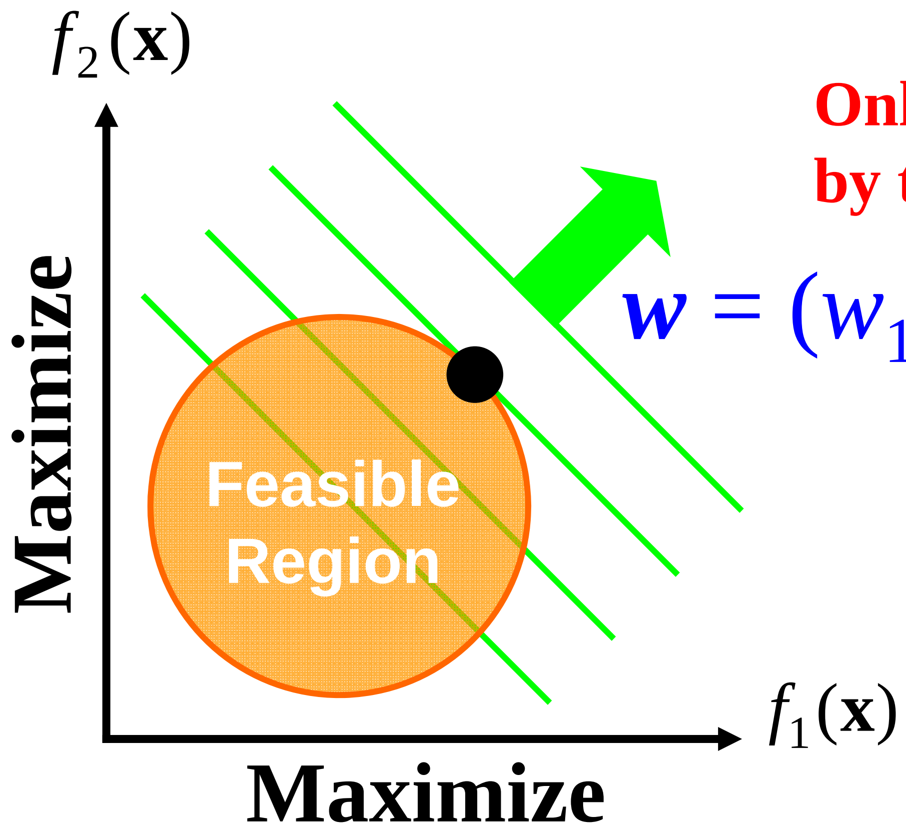
# EMO Algorithms

Evolutionary multiobjective optimization (EMO) algorithms have been designed to search for Pareto-optimal solutions in their single run.



# Comparison: Weighted Sum Approach

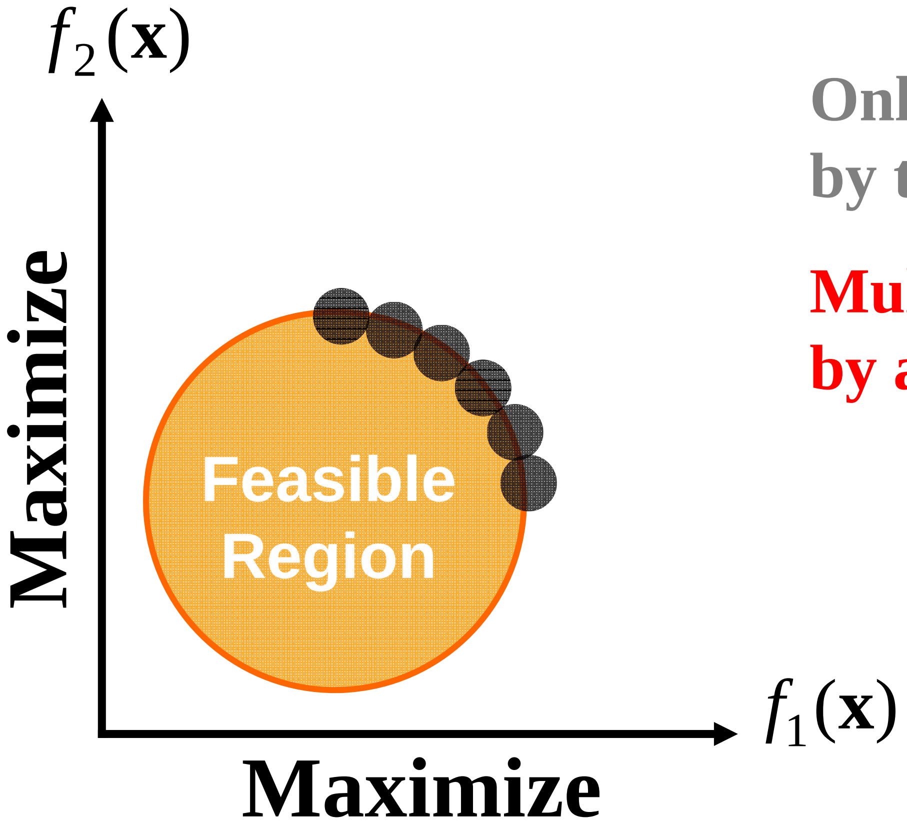
$$\textbf{Maximize} \quad g(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x})$$




Only a single solution is obtained by the weighted sum approach.

# Comparison: EMO Approach

**Maximize** $f_1(\mathbf{x}), \quad f_2(\mathbf{x})$

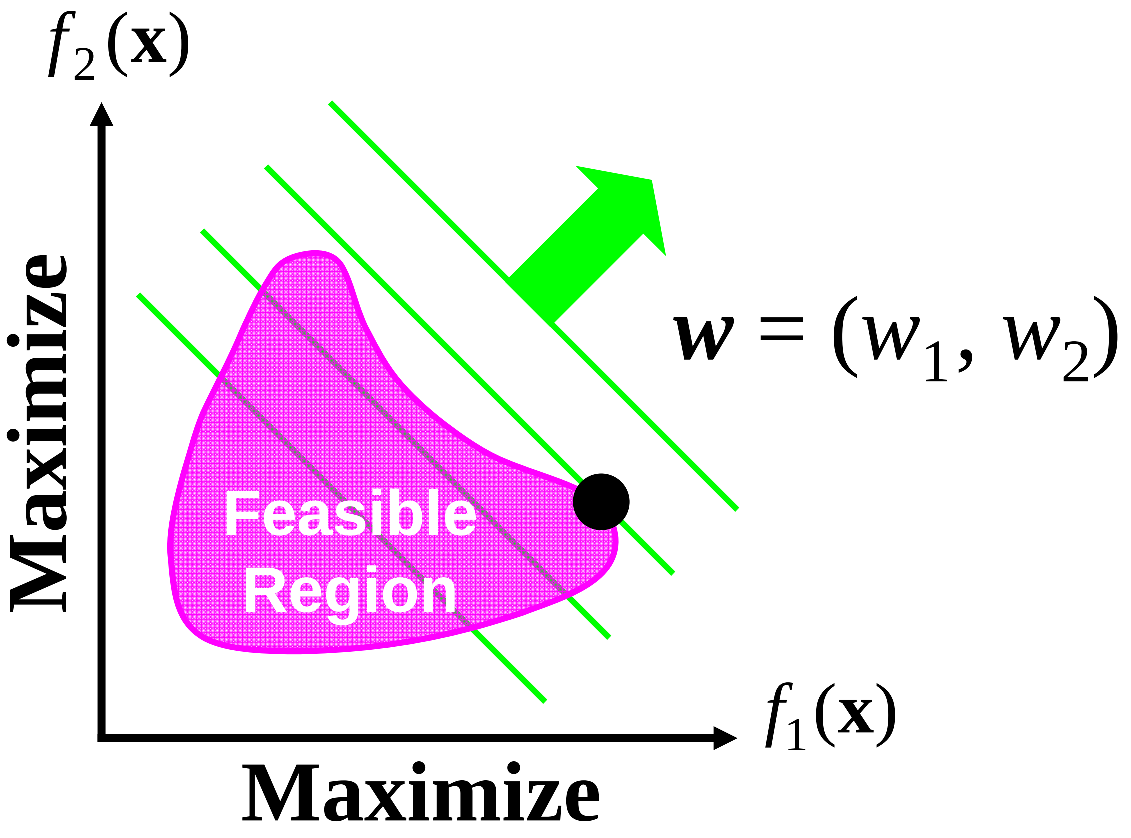



**Only a single solution is obtained by the weighted sum approach.**

**Multiple solutions are obtained by an EMO algorithm.**

# Difficulties in Weighted Sum Approach

- This approach is sensitive to the weight vector specification.
- This approach can not find any Pareto-optimal solutions in a non-convex region of the Pareto front in the objective space.



# Difficulties in Weighted Sum Approach

- This approach is sensitive to the weight vector specification.
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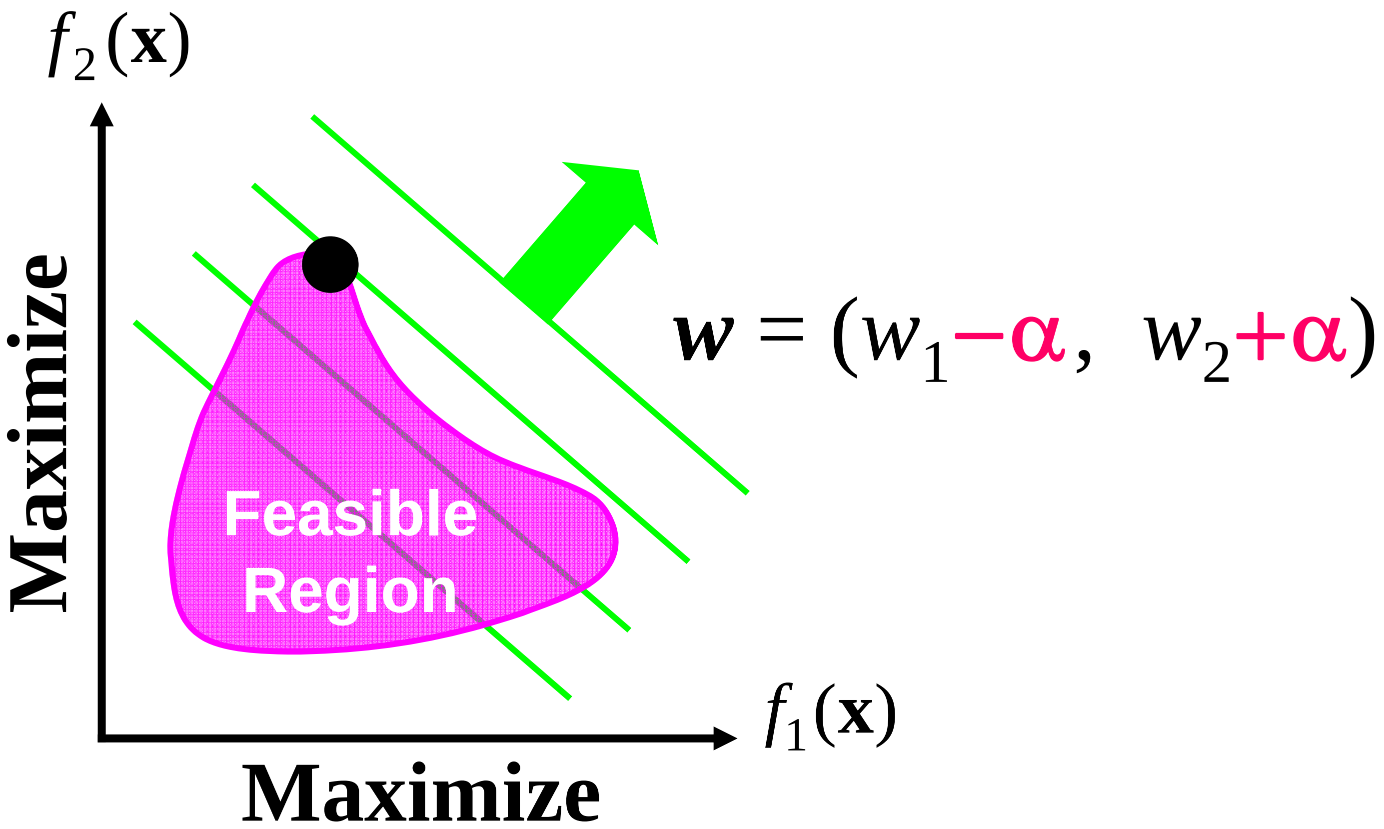
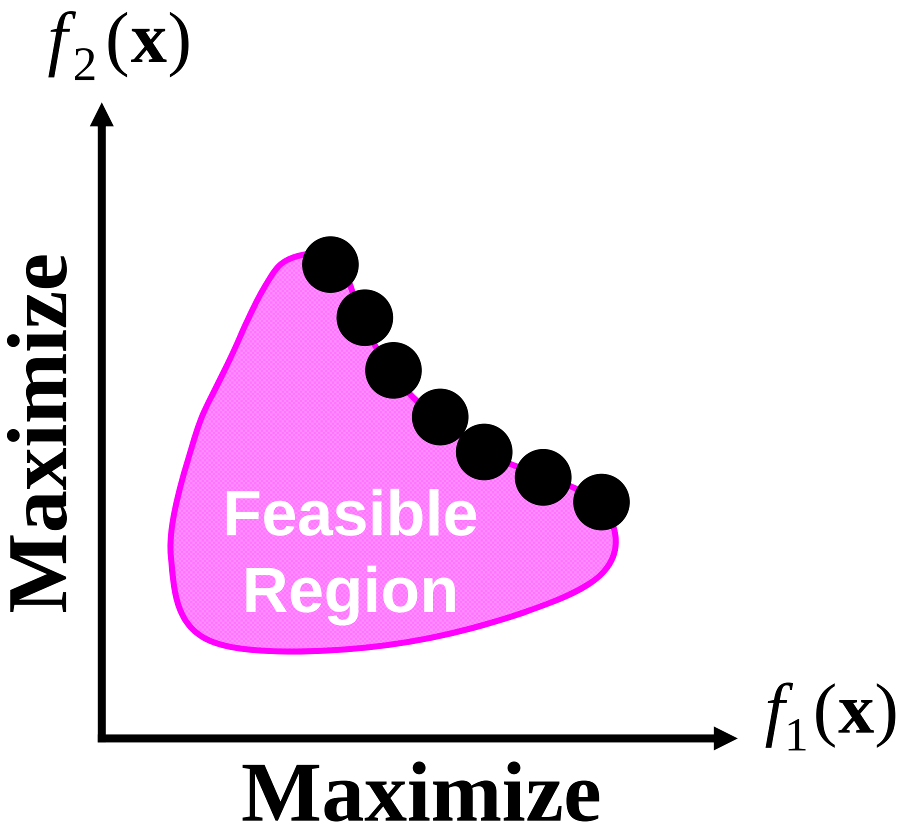


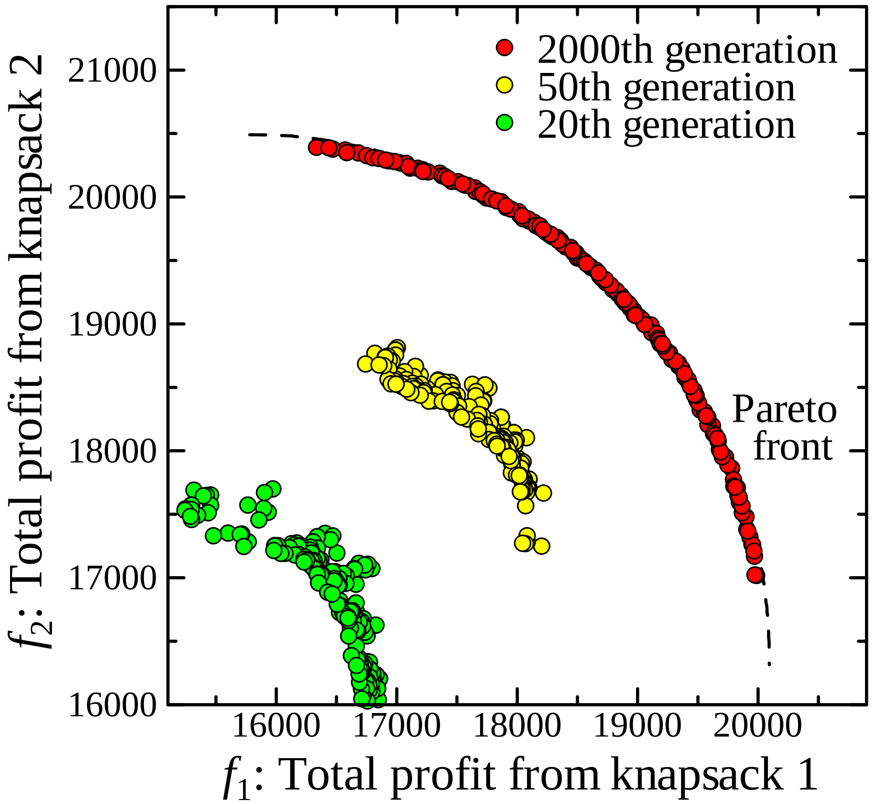
# EMO Approach

- **EMO approach can find Pareto-optimal solutions even in a non-convex region of the Pareto front in the objective space.**

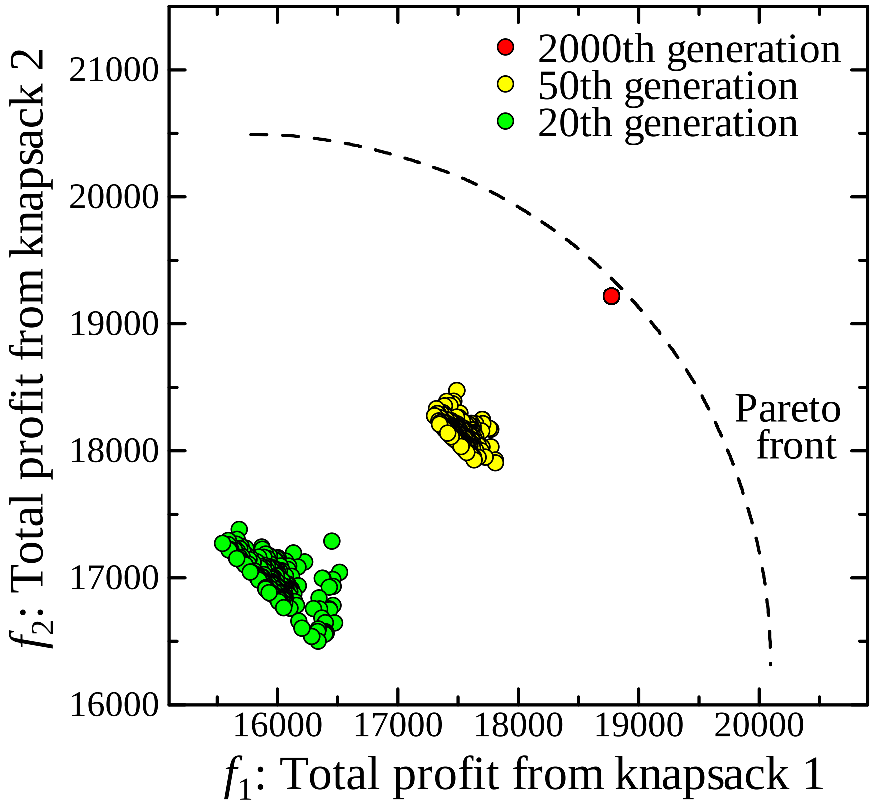


# Comparison of the Two Approaches

## Two-objective maximization problem




**EMO Approach**

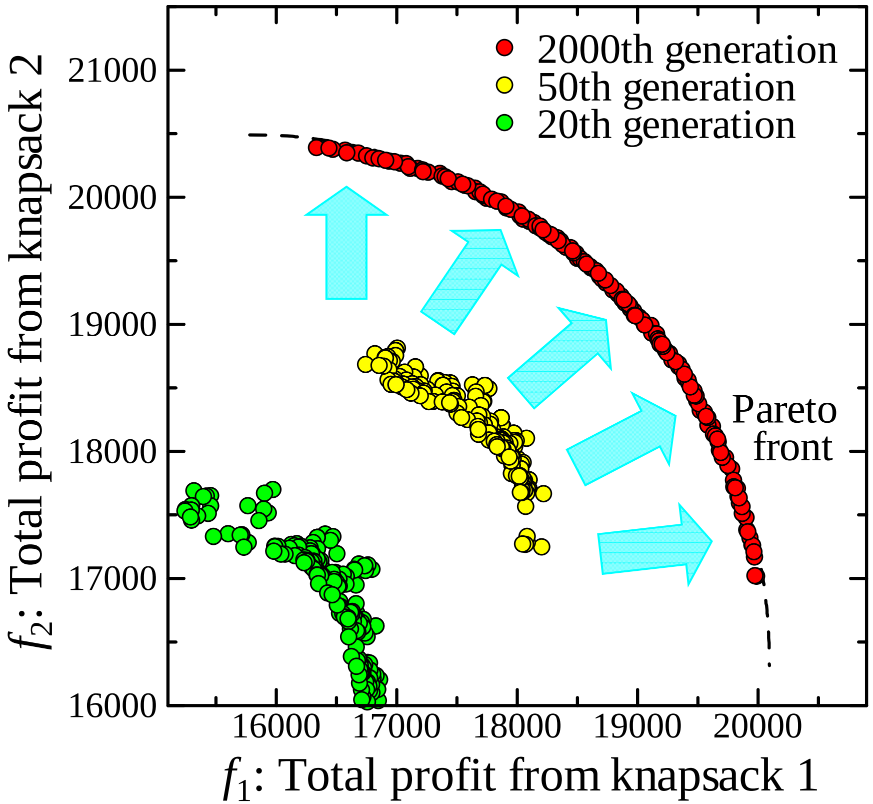



**Weighted Sum Approach**

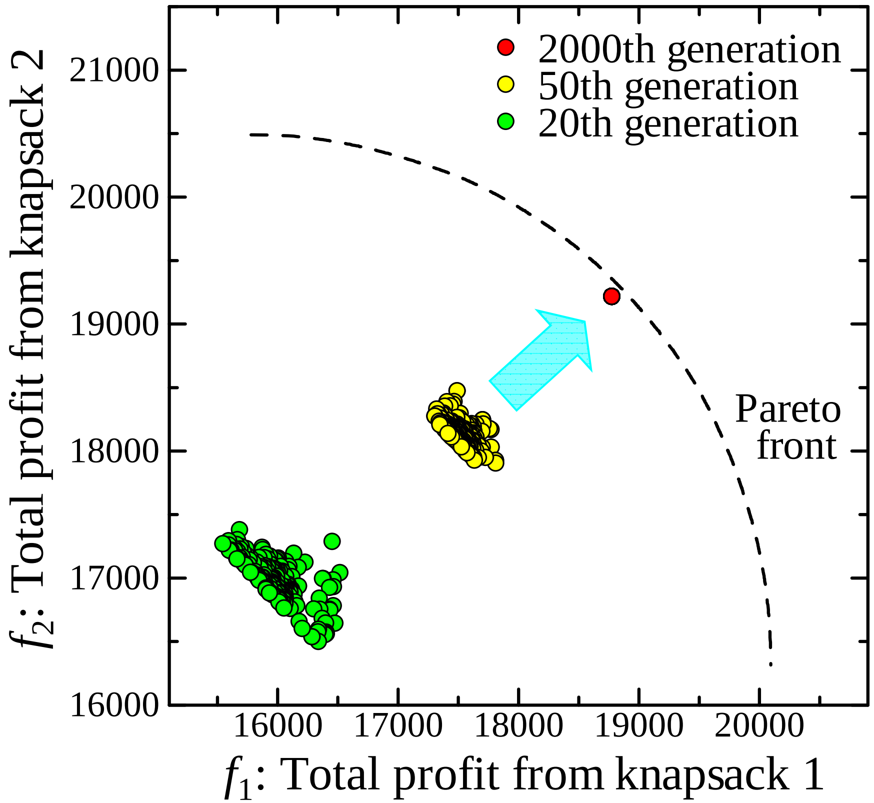
**Experimental results of a single run of each approach**

# Search Direction in Each Approach

## Two-objective maximization problem




**EMO Approach**




**Weighted Sum Approach**

**Both the diversity and the convergence should be improved in EMO.**

# Highly Cited EMO Papers

## Two Dominant Algorithms: NSGA-II and SPEA

1. **Deb K et al. (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE TEC*. NSGA-II**

2. **Zitzler E, Thiele L (1999) Multiobjective evolutionary algorithms: A comparative case study and the Strength Pareto approach. *IEEE TEC*. SPEA (=> SPEA2 in TIK-Report)**

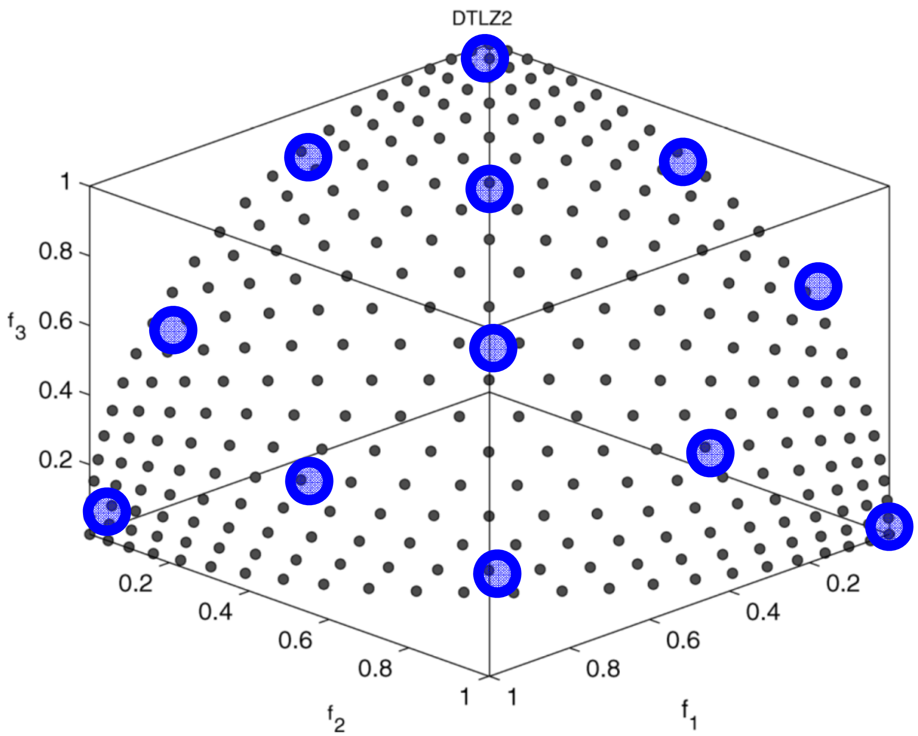
3. **Fonseca CM, Fleming PJ (1998) Multiobjective optimization and multiple constraint handling with evolutionary algorithms (Part I): A unified formulation, *IEEE SMC Part A*.**

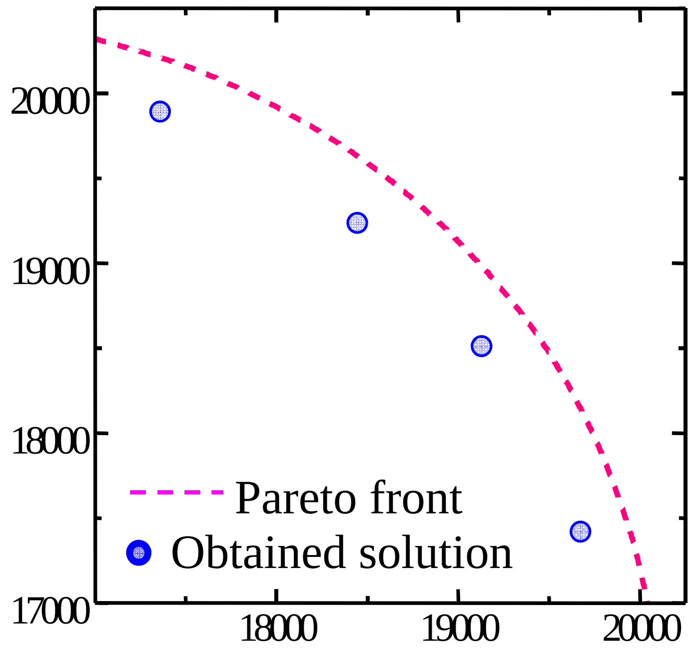
4. **Zitzler E, Thiele L, Laumanns M (2003) Performance assessment of multiobjective optimizers: An analysis and review. *IEEE TEC*.**

5. **Ishibuchi H, Murata T (1998) A multi-objective genetic local search algorithm and its application to flowshop scheduling, *IEEE SMC Part C*.**

# Goal of EMO Algorithms

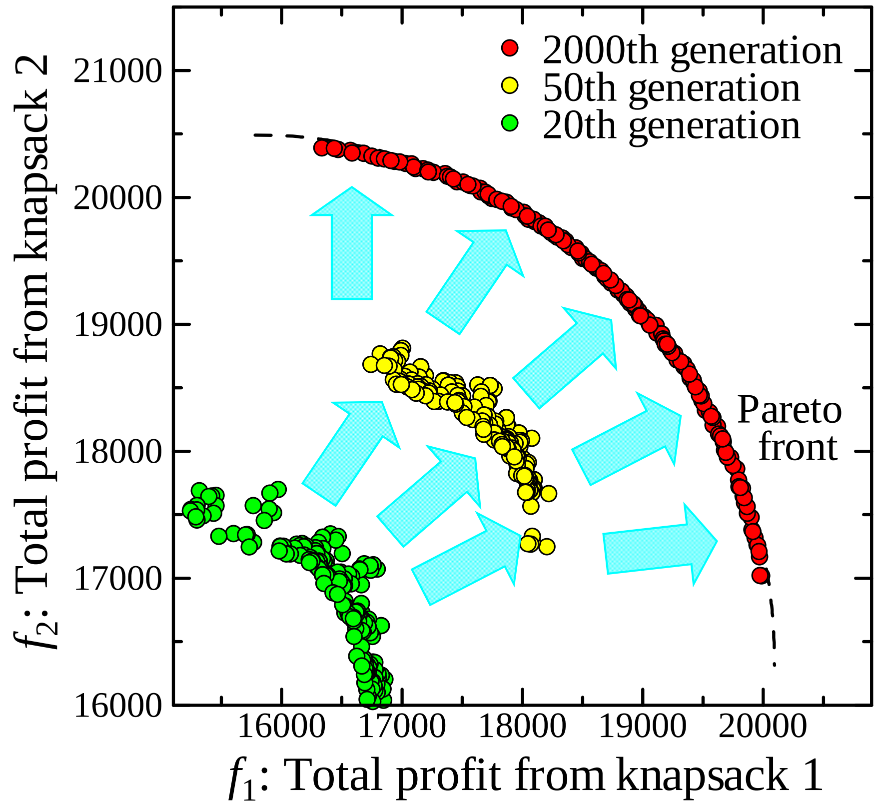
**An EMO algorithm is designed to search for**
**- all Pareto-optimal solutions**
**- uniformly distributed Pareto optimal solutions**
**- a solution set which approximates the Pareto front**
**in their single run.**



# Basic Ideas in EMO Algorithm Design

**Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features.**



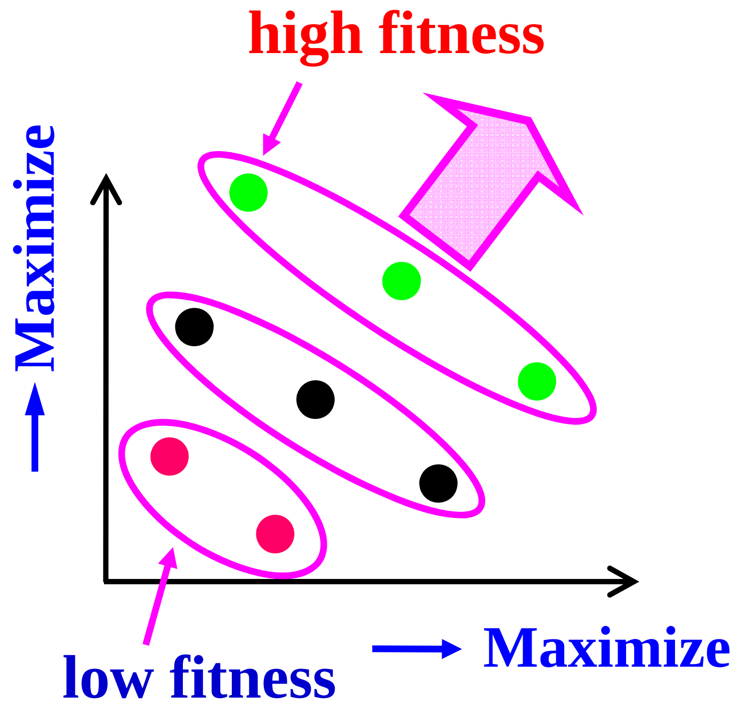

**Desired search behavior of EMO algorithms**

# Basic Ideas in EMO Algorithm Design

**Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features:**

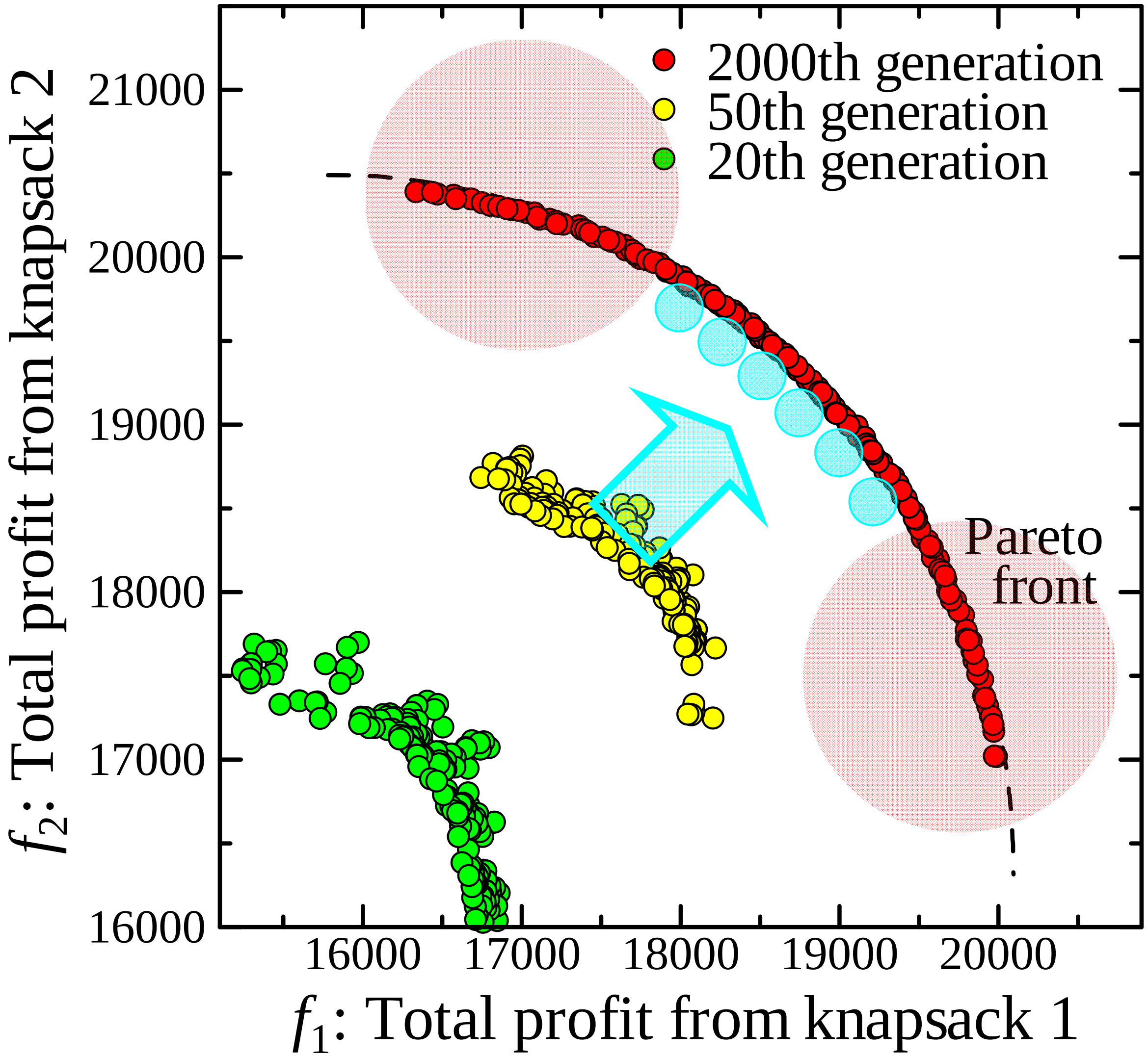
## (1) Pareto Dominance

**Converge to the Pareto front**



# Basic Ideas in Recent EMO Algorithms

**1. Pareto Dominance**



# Basic Ideas in EMO Algorithm Design

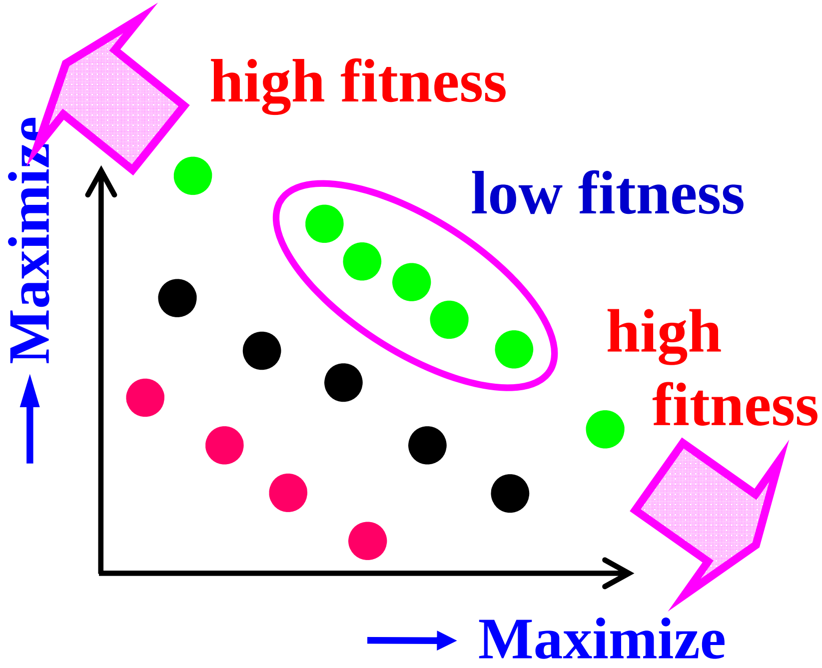
**Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features:**

**(1) Pareto Dominance**

**Converge to the Pareto front**

**(2) Crowding**

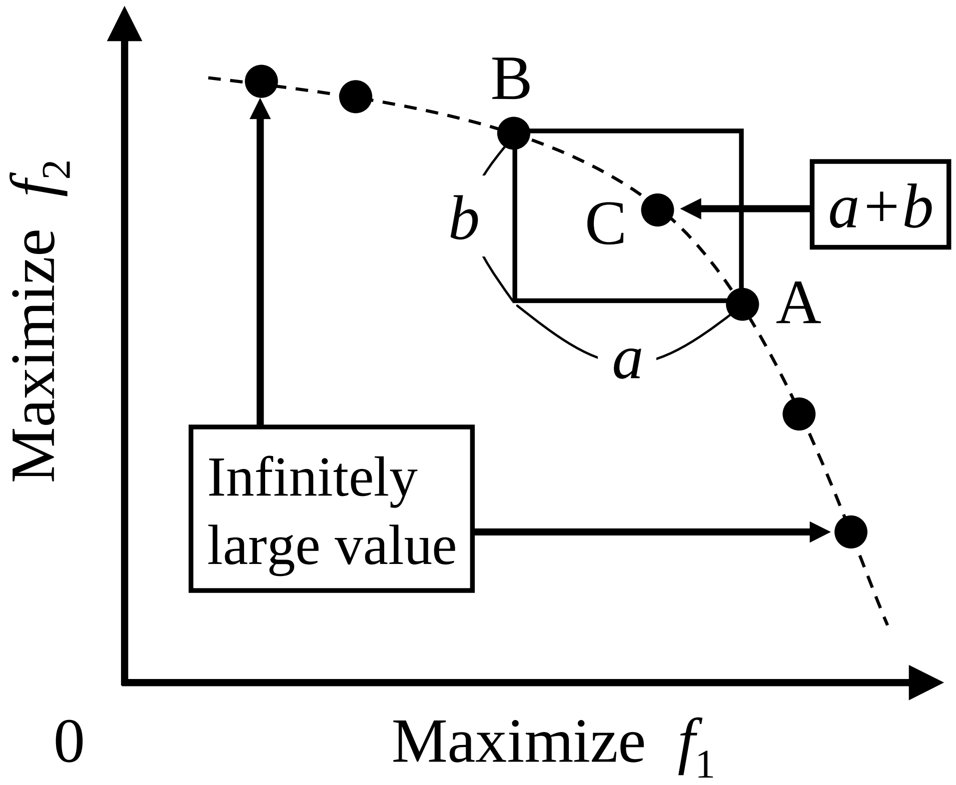
**Diversity maintenance**



# Basic Ideas in EMO Algorithm Design

## Example: Crowding Distance in NSGA-II

**Distance between adjacent individuals**




**Crowding distance of C is (*a* + *b*)**

# Basic Ideas in EMO Algorithm Design

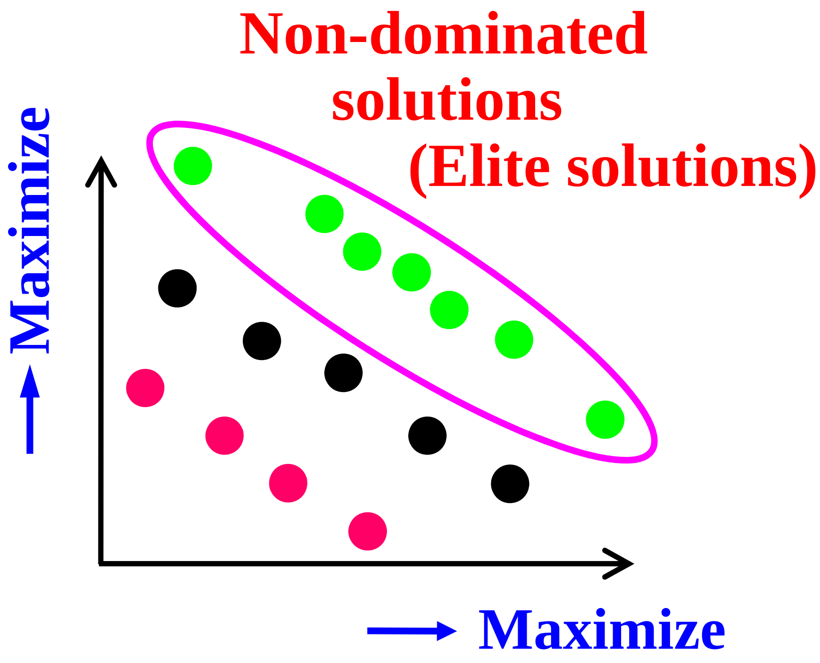
Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features:

## (1) Pareto Dominance

Converge to the Pareto front

## (2) Crowding

Diversity maintenance

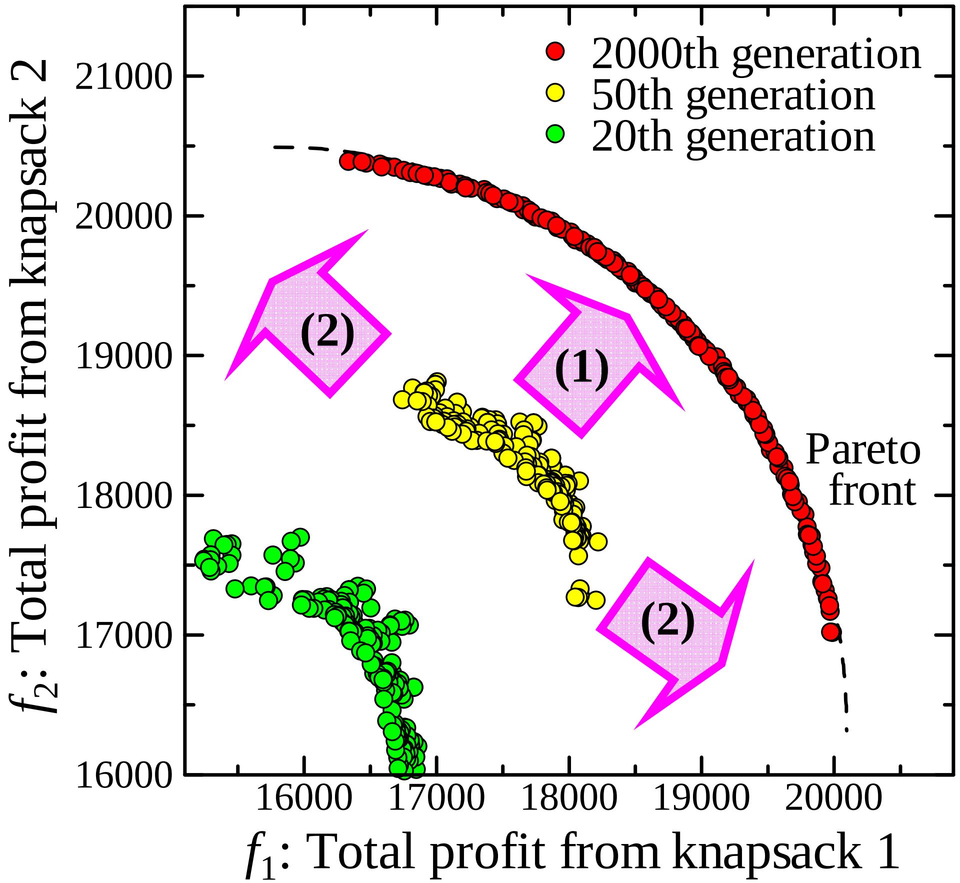



## (3) Elitist Strategy

Non-dominated solutions are handled as elite solutions.

# Basic Ideas in Recent EMO Algorithms

**(1) Pareto Dominance (Convergence to the Pareto front)**
**(2) Crowding (Diversity Maintenance)**
**(3) Elite Strategy (Non-Dominated Solutions)**



# Hot Issues in EMO Research

## Utilization of Decision Maker's Preference

- Preference is incorporated into EMO algorithms.
- Interactive EMO approaches seem to be promising.

## Handling of Many Objectives by EMO Algorithms

- Pareto dominance-based algorithms do not work well.
- More selection pressure is needed.

## Hybridization with Local Search

- Hybridization often improves the performance of EMO.
- Balance between local and genetic search is important.

## Design of New EMO Algorithms (some alternatives to NSGA-II and SPEA2)

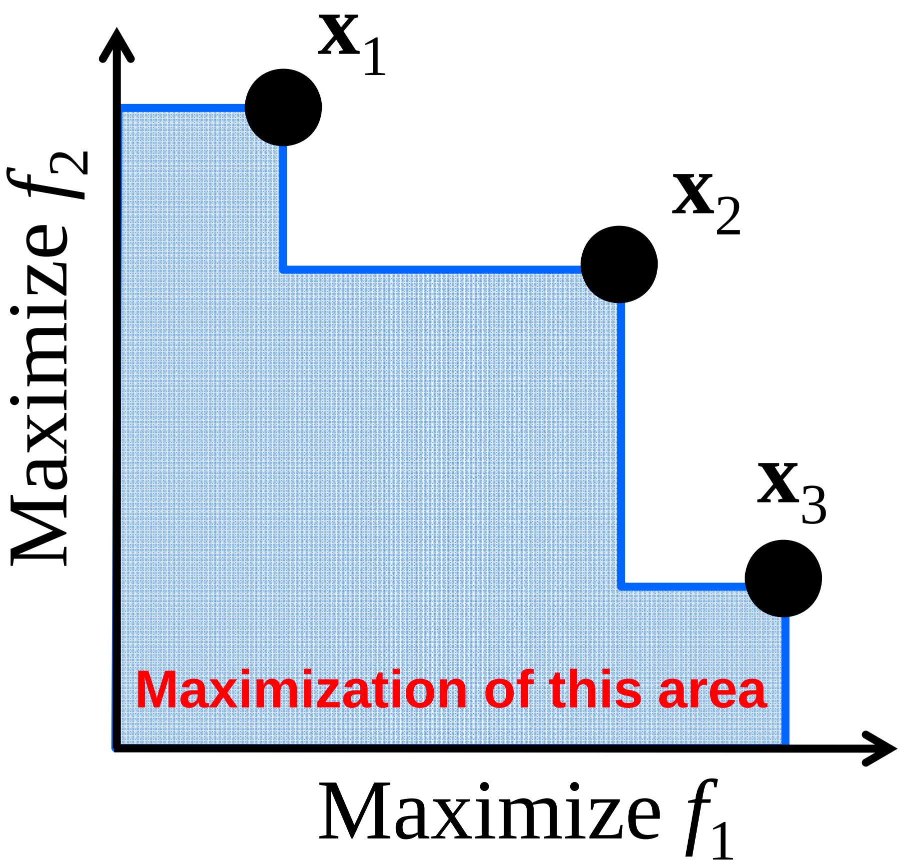
- Indicator-based EMO algorithms
- Scalarizing function-based EMO algorithms
- Use of other search methods such as PSO, ACO and DE.

# New Trend in EMO Algorithm Design

## IBEA: Indicator-Based Evolutionary Algorithm

### Basic Idea

**To maximize a performance indicator of a solution set (not a solution): Hypervolume is often used.**



# New Trend in EMO Algorithm Design
## IBEA: Indicator-Based Evolutionary Algorithm

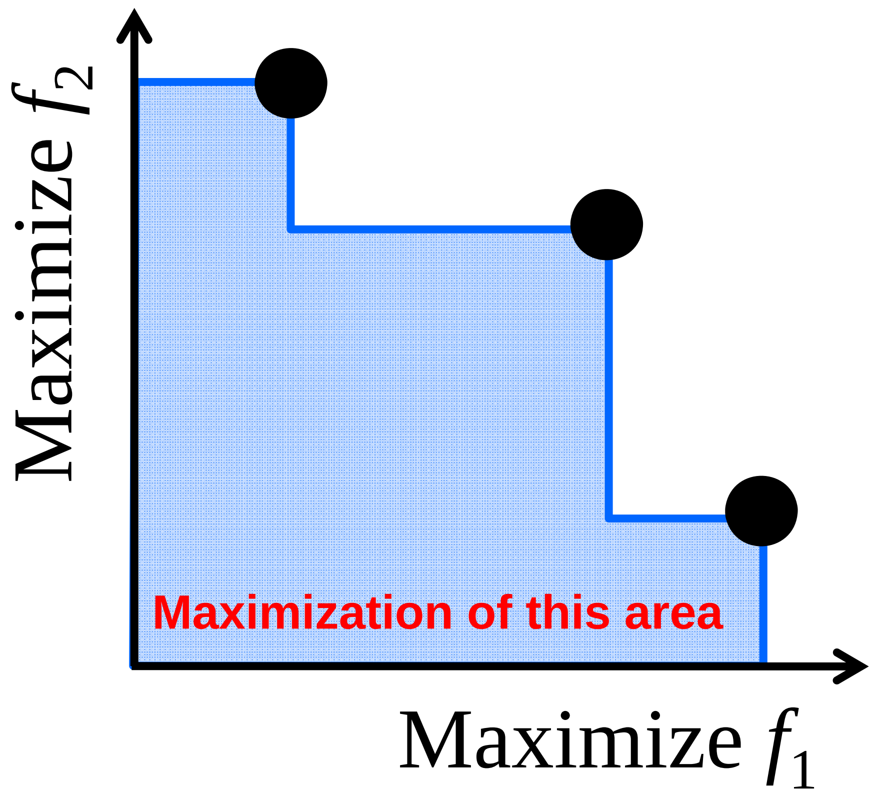
$$\text{Maximize } \boldsymbol{I}(\boldsymbol{S})$$ (Maximization of an Indicator Function)

$$\text{subject to } |\boldsymbol{S}| \leq \boldsymbol{N} \quad \text{where } \boldsymbol{S} \subset \{\mathbf{x} | \mathbf{x} \in \mathbf{X}\}$$

$\boldsymbol{S}$ : A set of solutions

$\boldsymbol{N}$ : A pre-specified number of required solutions

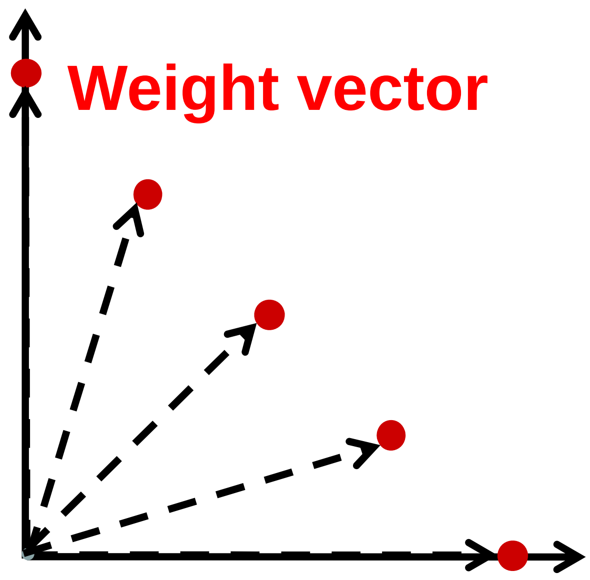
$\mathbf{X}$ : A feasible region



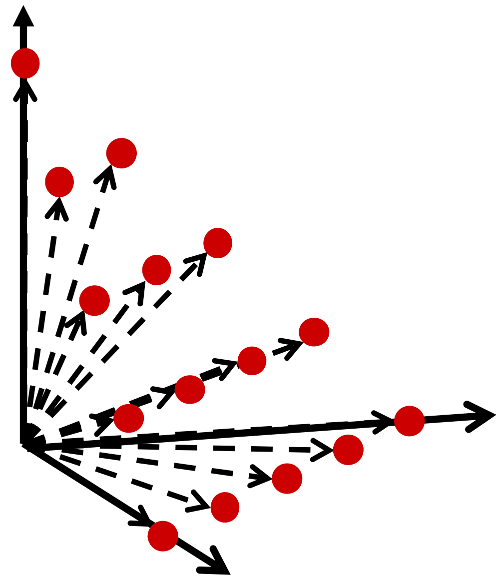
# New Trend in EMO Algorithm Design
## MOEA/D: Use of Scalarizing Functions

**MOEA/D: Multi-objective evolutionary algorithm based on decomposition by Zhang and Li (IEEE TEC 2007)**

**Its Basic Idea (Decomposition): A multi-objective problem is handled as a set of scalarizing function optimization problems with different weight vectors.**

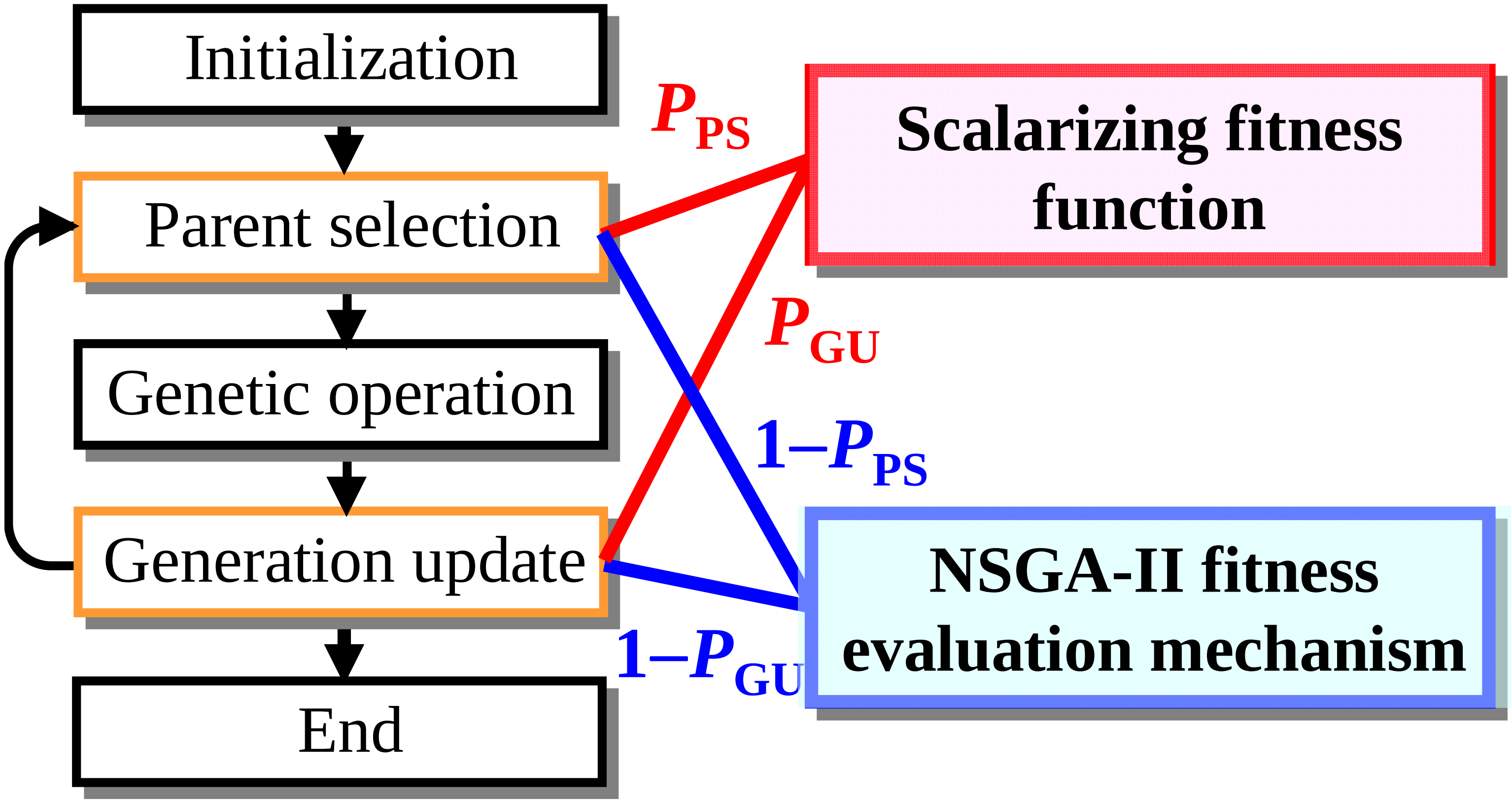



(a) Two-objective case



(b) Three-objective case

# New Trend in EMO Algorithm Design
## Hybrid Method: Use of Scalarizing Functions




**Probability for scalarizing fitness functions:**

**Parent selection: $P_{PS}$ Generation update: $P_{GU}$**

**Ishibuchi et al. (PPSN 2006)**

# New Trend in EMO Algorithm Design
## Use of Other Meta-Heuristics (PSO, ACO, etc.)

### Highly Cited Papers


[1] Coello CAC, Pulido GT, Lechuga MS (2004) Handling Multiple Objectives with Particle Swarm Optimization, IEEE TEC

[2] McMullen PR (2001) An Ant Colony Optimization Approach to Addressing a JIT Sequencing Problem with Multiple Objectives, Artificial Intelligence in Engineering

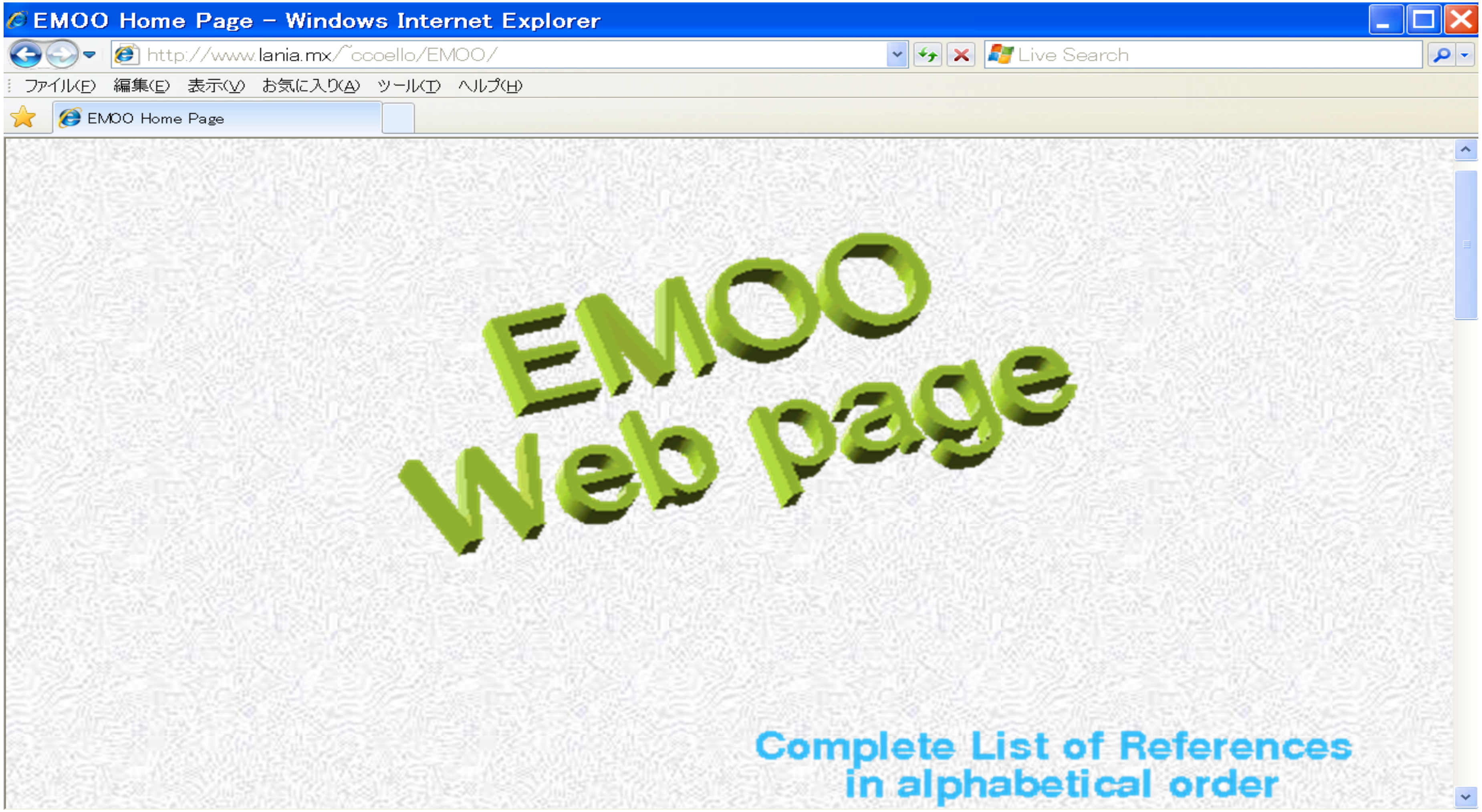
[3] Ray T, Liew KM (2002) A Swarm Metaphor for Multiobjective Design Optimization, Engineering Optimization

[4] Li XD (2003) A Non-Dominated Sorting Particle Swarm Optimizer for Multiobjective Optimization, GECCO 2003.

[5] Ho SL et al. (2005) A Particle Swarm Optimization-Based Method for Multiobjective Design Optimizations, IEEE Trans. on Magnetics

# For More Information

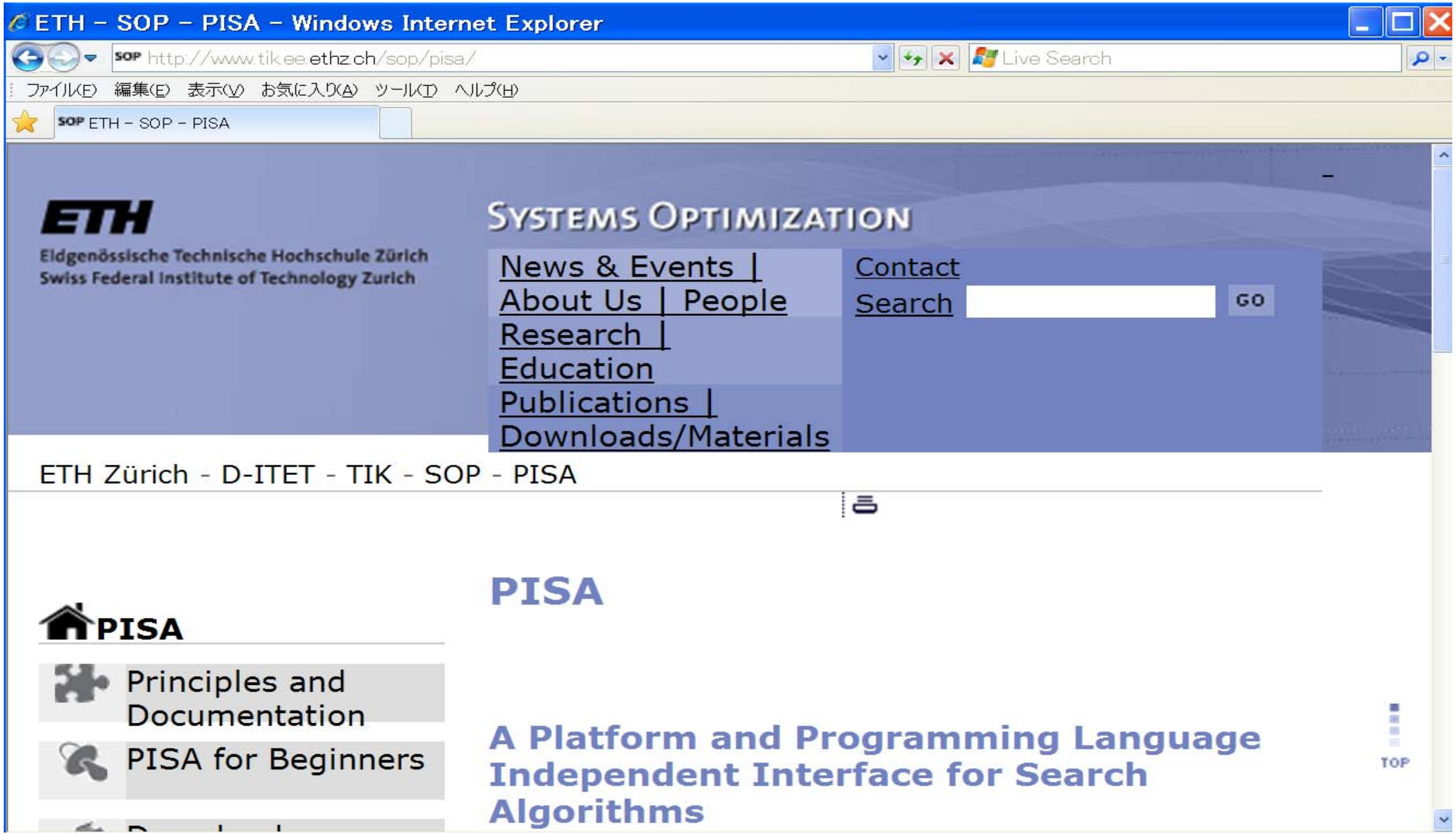
## Webpage for EMO Papers: EMOO




http://www.lania.mx/~ccoello/EMOO/

# For More Information

## Webpage for EMO Algorithms and Problems: PISA




http://www.tik.ee.ethz.ch/sop/pisa/

# Contents

# Types of MOEFSs by Multiobjective Nature and Optimized Components

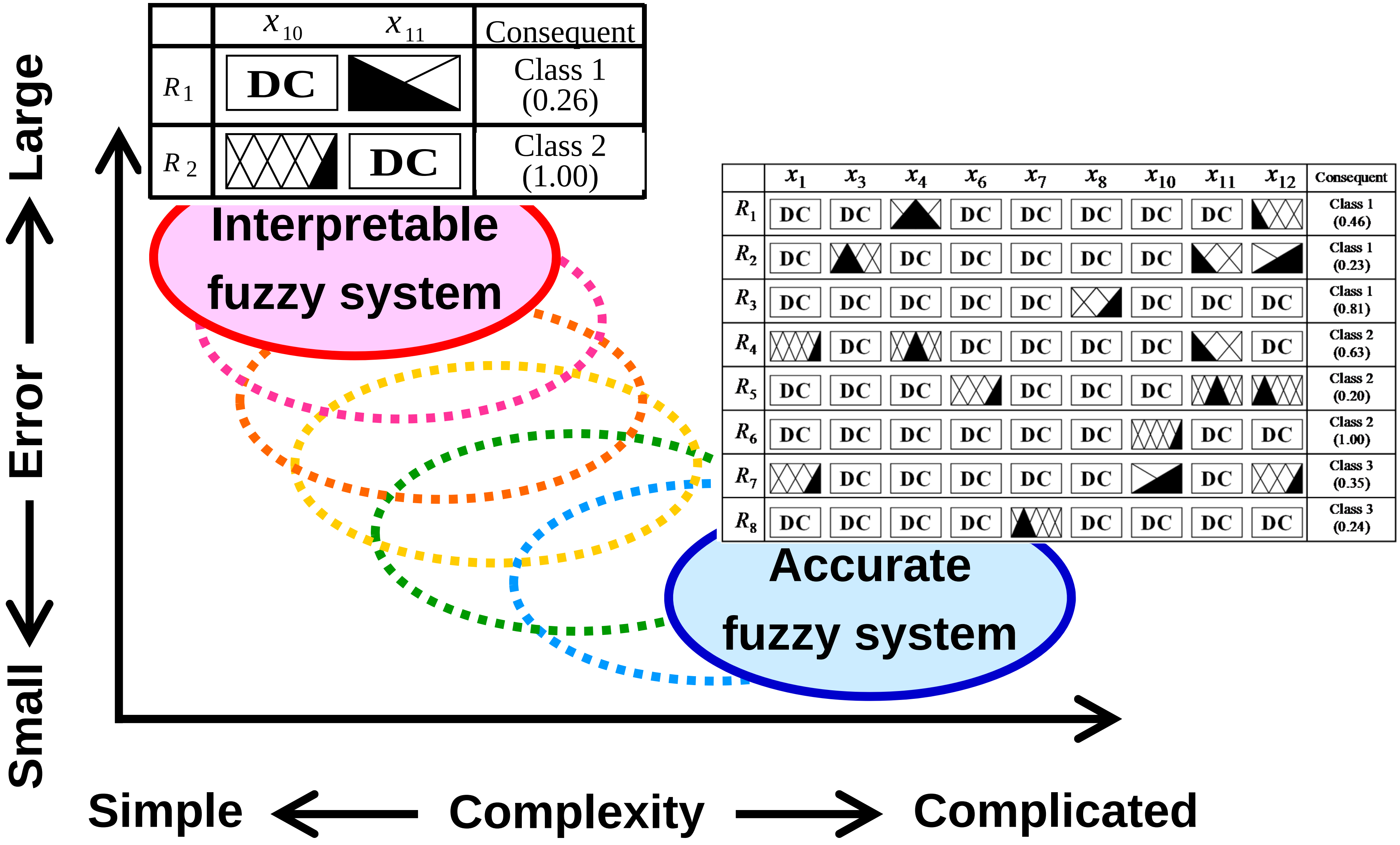
- The flexibility of FRBSs makes them applicable to a wide range of problems.

- From among them, problems with multiple conflicting objectives are of particular interest to researchers, as they are very common and arise wherever optimal decisions need to be taken.

- These problems can be tackled using MOEAs for the design of FRBSs, giving way to the so called MOEFSs.

- MOEFSs are a type of GFS exploiting MOEAs to design sets of FRBSs with different trade-offs among objectives instead of a single one.

# Motivations for MOEFSs at their Origin
## 1) Preventing a Deterioration of Interpretability

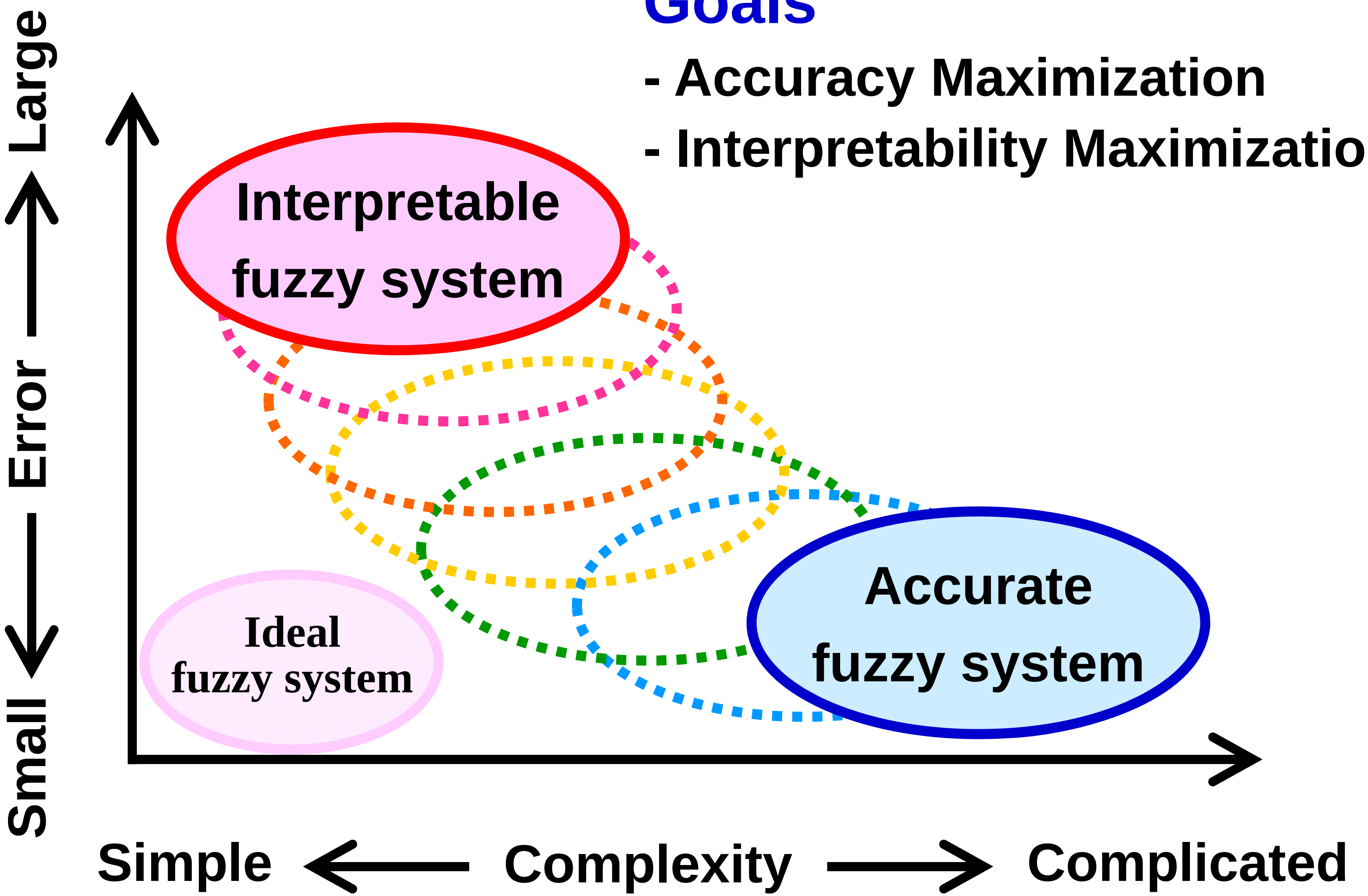


# Motivations for MOEFSs at their Origin

## Multiobjective Fuzzy System Design (Late 1990s - )

### Goals

- Accuracy Maximization
- Interpretability Maximization



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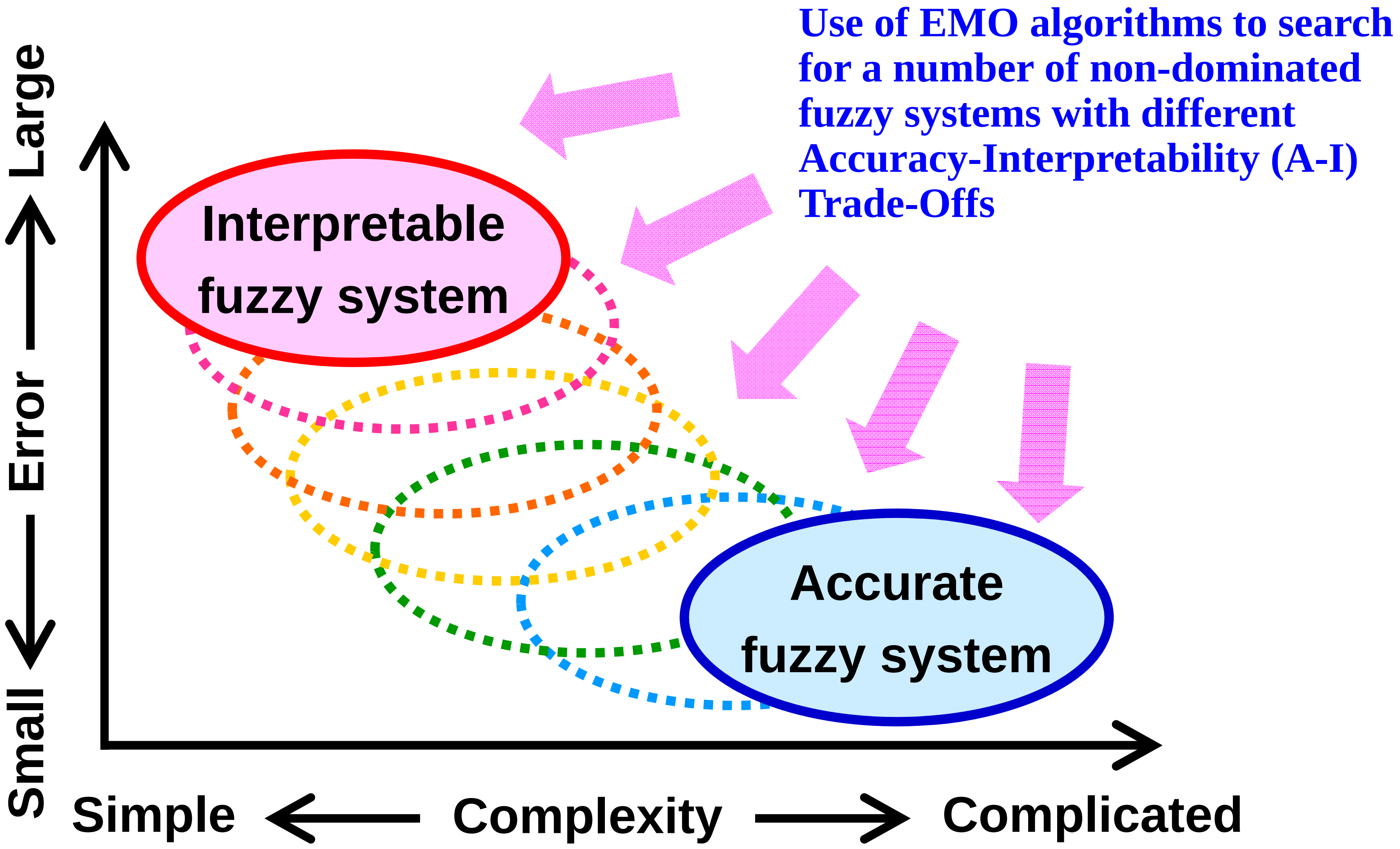
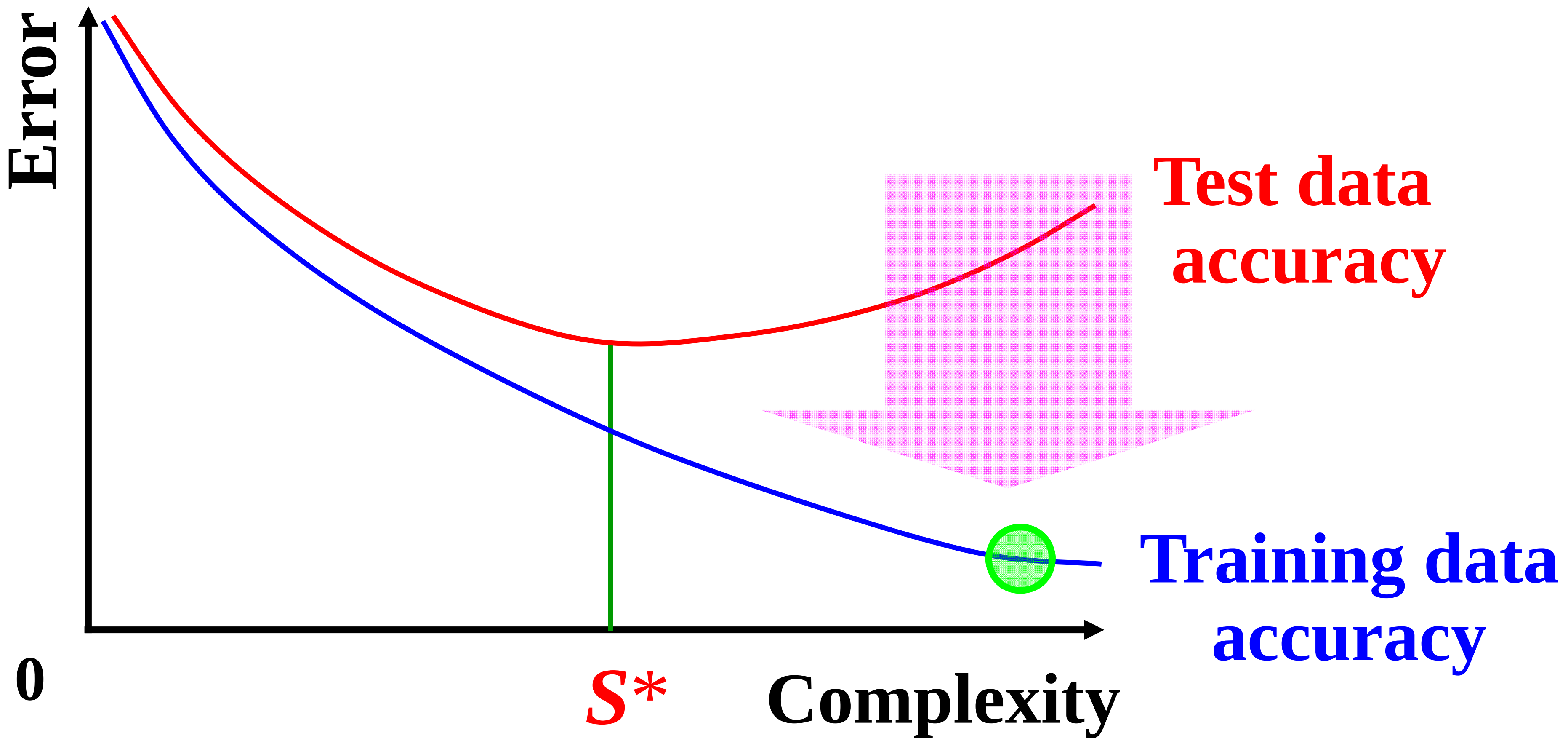


# Motivations for MOEFSs at their Origin

## 2) Avoiding too Complex Models helps to Control Overfitting

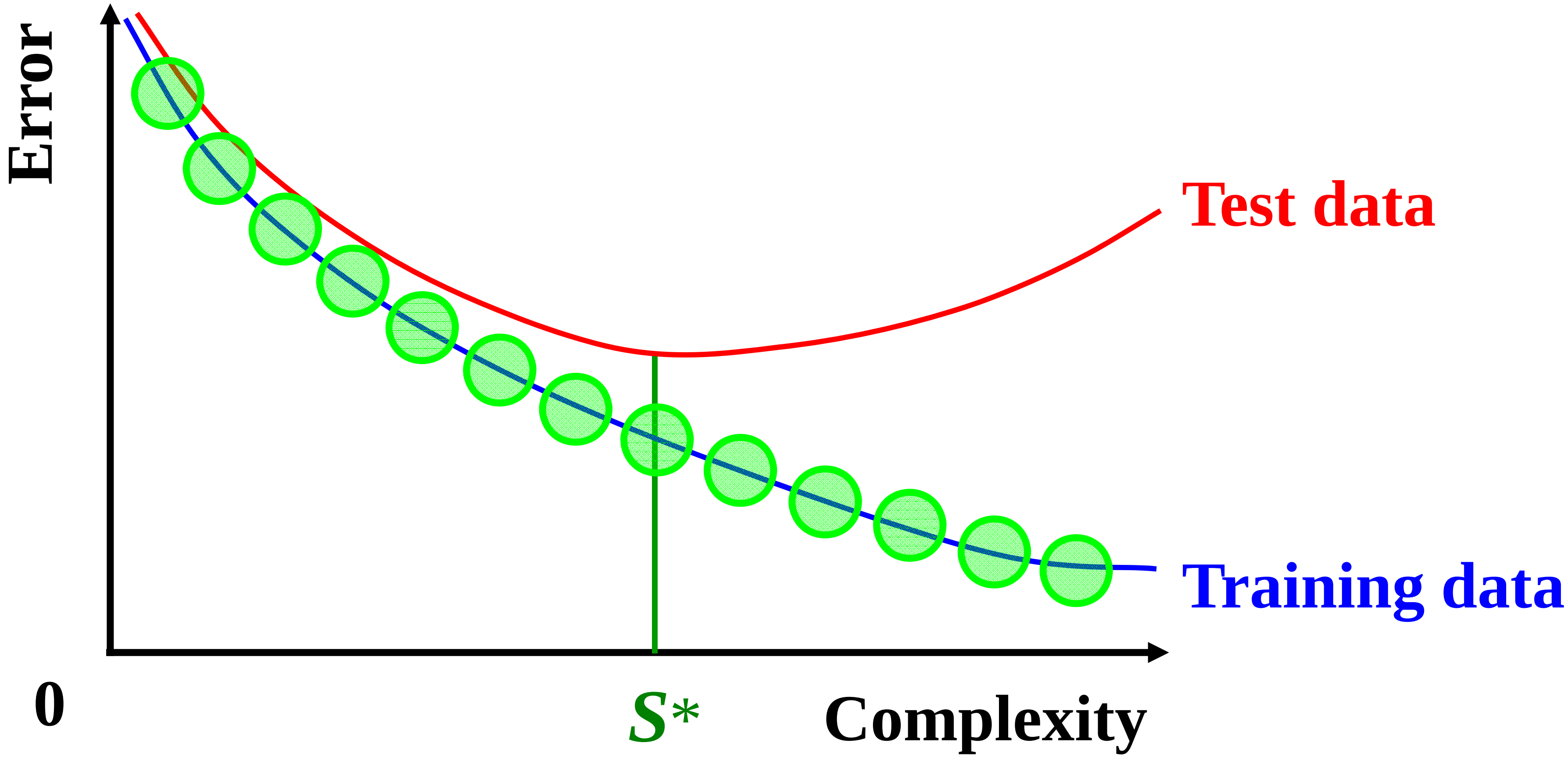
**Accuracy maximization ⇨ Overfitting**



# Motivations for MOEFSs at their Origin

## 2) Avoiding too Complex Models helps to Control Overfitting

**Many non-dominated fuzzy systems can be obtained along the tradeoff surface by a single run of an EMO algorithm.**



# Types of MOEFSs by Multiobjective Nature and Optimized Components

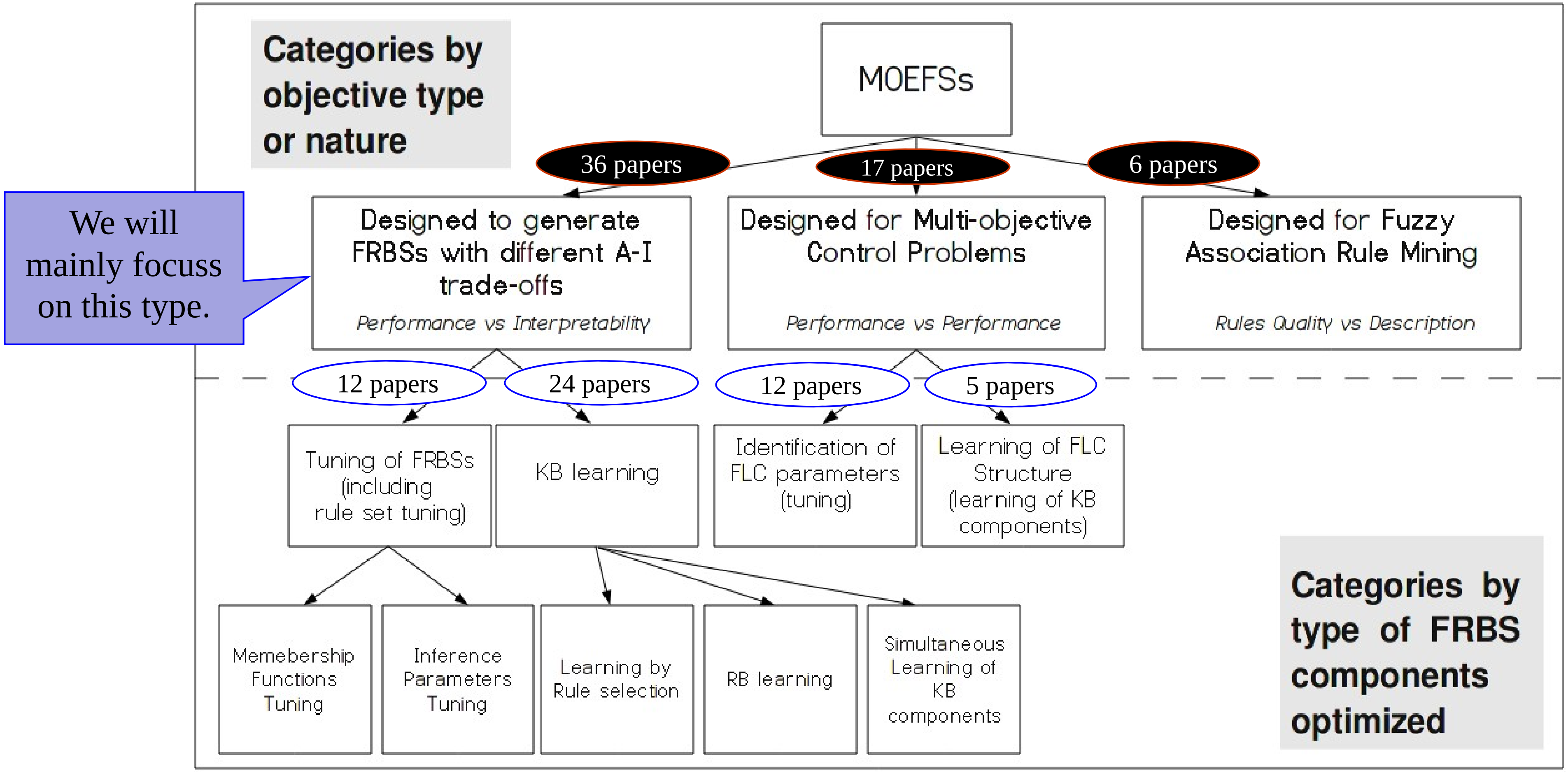
- However, MOEFSs have been also applied to solve multiobjective control problems and for fuzzy association rule mining (where different metrics are considered to describe the quality of the obtained rules).

- The type of objectives used in these three main categories (A-I trade-off, control and mining fuzzy association rules) represent a different multi-objective nature

- Due to this fact, both, the multi-objective nature of the problem faced and type of FRBS components optimized, have been considered recently to propose a two-level taxonomy in,

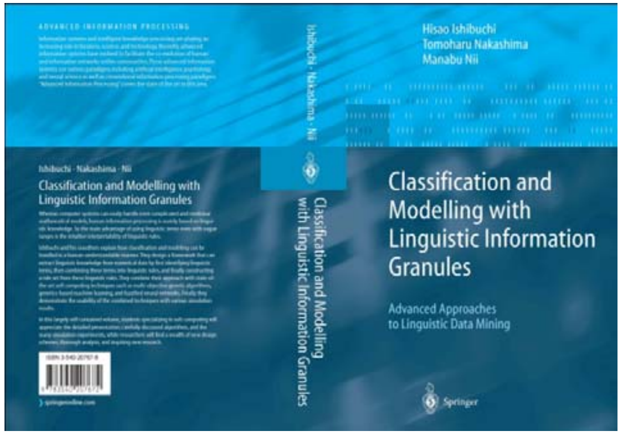
**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****.** **IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65,** doi: 10.1109/TFUZZ.2012.2201338

# Types of MOEFSs by Multiobjective Nature and Optimized Components: A Two-level Taxonomy



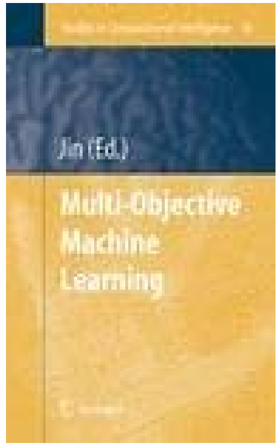

**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*.** **IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65,** doi: 10.1109/TFUZZ.2012.2201338

# Multiobjective Evolutionary Fuzzy Systems
## Bibliography




**H. Ishibuchi, T. Nakashima, M. Hii. Classification and Modelling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer-Verlag, 2004.**

**Jin, Yaochu (Ed.)**
**Multi-Objective Machine Learning**
**Springer-Verlag, 2006**




– M. Fazzolari, R. Alcalá, Y. Nojima, H. Ishibuchi, F. Herrera. A review of the application of Multi-Objective Evolutionary Systems: Current status and further directions. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65, doi: 10.1109/TFUZZ.2012.2201338

**Associated Webpage (http://ssci2s.ugr.es/gfs)**

# Highly Cited MOEFS Papers

[1] Ishibuchi et al. (1997) Single-objective and two-objective genetic algorithms for selecting linguistic rules for pattern classification problems. *Fuzzy Sets & Systems.*

[2] Ishibuchi et al. (2001) Three-objective genetics-based machine learning for linguistic rule extraction. *Information Sciences.*

[3] Ishibuchi & Yamamoto (2004) Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining. *Fuzzy Sets & Systems*.

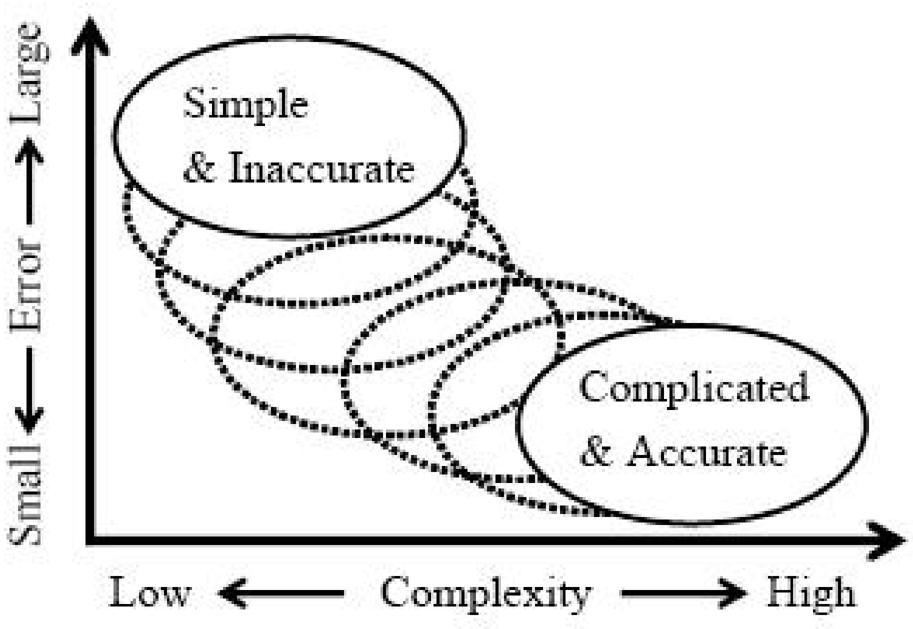
[4] Wang et al. (2005) Multi-objective hierarchical genetic algorithm for interpretable fuzzy rule-based knowledge extraction. *Fuzzy Sets & Systems*.

[5] Johansen & Babuska (2003) Multiobjective identification of Takagi-Sugeno fuzzy models. *IEEE TFS.*

# Contents

# Interpretability Issues in Fuzzy System Design
## Complexity Criteria

- **Highly used criteria: Complexity criteria in the learning of FRBSs.**




**Number of variables, labels, rules, conditions ...**

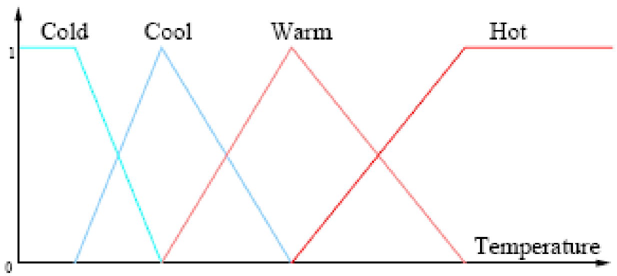
# Interpretability Issues in Fuzzy System Design
## Semantic Criteria

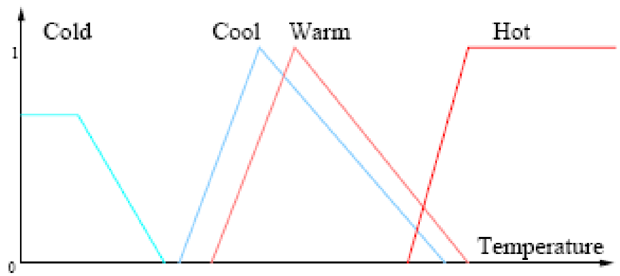
- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

### Interpretability considerations: semantic criteria

Semantics: the study of meanings

- Distinguishability: Each linguistic label has semantic meaning
- Number of elements: Compatible with human capabilities
- Coverage: Any element belongs to at least one fuzzy set
- Normalization: At least one element has unitary membership
- Complementarity: For each element, the sum of memberships is one



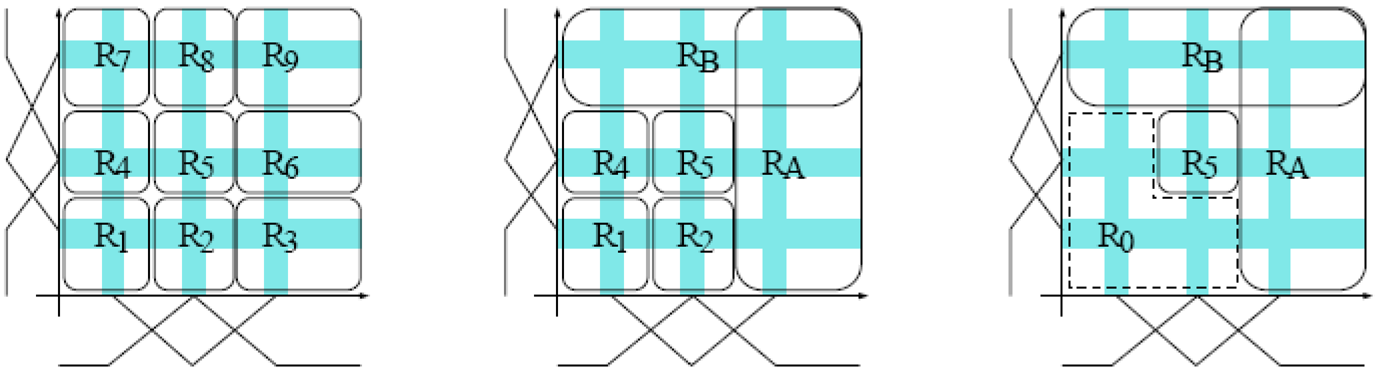
# Interpretability Issues in Fuzzy System Design
## Syntactic Criteria

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

### Interpretability considerations: syntactic criteria

Syntax: the way in which linguistic elements are put together

- Completeness: for any input, at least one rule must fire
- Rule–base simplicity: Set of rules as small as possible
- Rule readability: small number of conditions in rule antecedents
- Consistency: rules firing simultaneously must have similar consequents

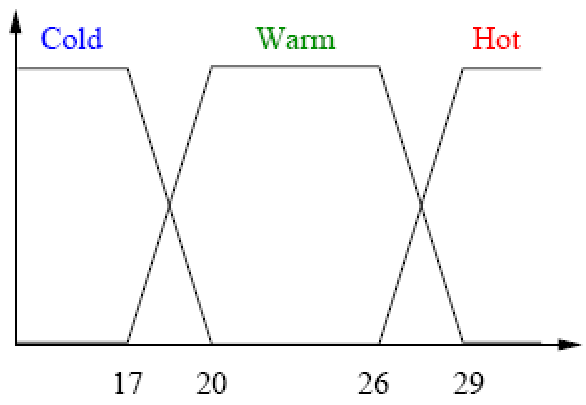


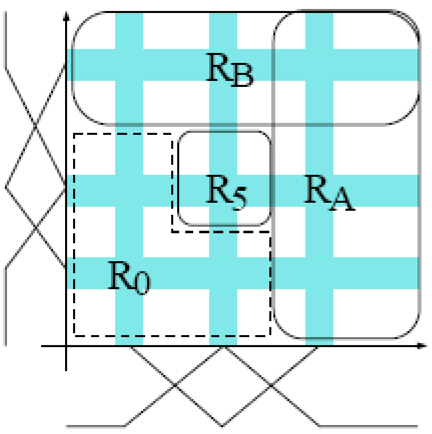
# Interpretability Issues in Fuzzy System Design
## Strategies to Satisfy Interpretability

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

**Strategies to satisfy interpretability criteria**

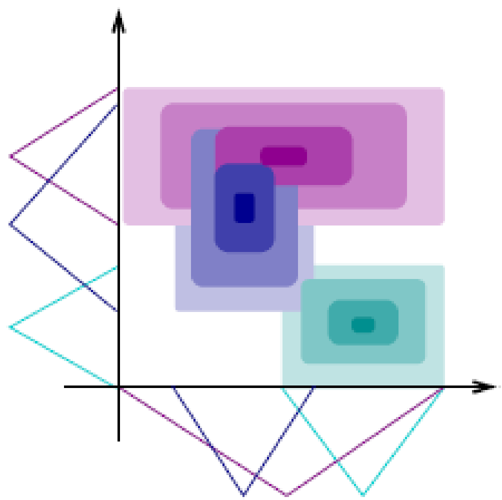
- Linguistic labels shared by all rules
- Normal, orthogonal membership functions
- Don't care conditions

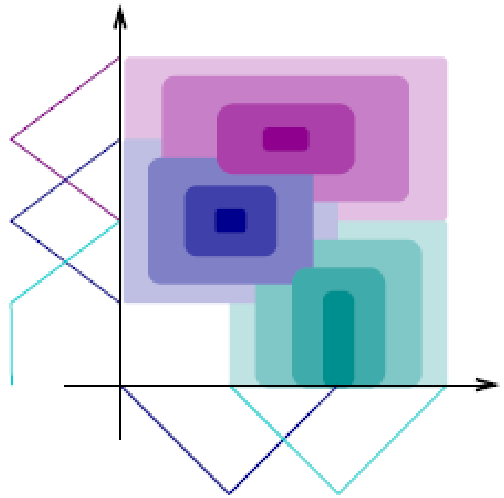


# Interpretability Issues in Fuzzy System Design
## Still not Clear Concepts

- **Interpretability quality:**





What is the most interpretable rule base?

# A Taxonomy on the Existent Interpretability Measures for Linguistic FRBSs

Most works in C1 and C2 are applied to classification problems. They are the classic measures.

| | Rule Base Level | Fuzzy Partition Level |
|---|---|---|
| Complexity-based Interpretability | $C_1$<br>Number of rules<br>Number of conditions | $C_2$<br>Number of membership functions<br>Number of features |
| Semantic-based Interpretability | $C_3$<br>Consistency of rules<br>Rules fired at the same time<br>Transparency of rule structure (rule weights, etc.)<br>Cointension | $C_4$<br>Absolute Measures:<br>Completeness or coverage, normalization, distinguishability, complementarity<br>Relative Measures |

There are few works in C3
Still an open problem

Most works in C4 impose absolute measures or restrictions. Relativity could be a new possibility.
Still an open problem.

# A Taxonomy on the Existent Interpretability Measures for Linguistic FRBSs (2)

- Interpretability of FRBSs is still an open problem since there is no single (or global) comprehensive measure to quantify the interpretability of linguistic models

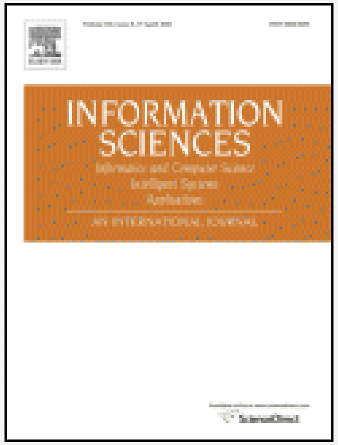
- To get a good global measure it would be necessary to consider appropriate measures from all of the quadrants, in order to take into account the different interpretability properties required for these kinds of systems together.

M.J. Gacto, R. Alcalá, F. Herrera

**Interpretability of Linguistic Fuzzy Rule-Based Systems: An Overview on Interpretability Measures,** *Information Sciences 181:20 (2011) 4340–4360, doi: 10.1016/j.ins.2011.02.021*

A thematic website has been developed to maintain this study at:

***http://sci2s.ugr.es/fuzzy-interpretability/***



# Applicability of MOEFSs to the I-A problem

❑ The different measures from each quadrant could be optimized as different objectives within a multi-objective framework.

❑ They are contradictory to some degree. Not only accuracy is contradictory to interpretability. The different measures represent different properties and requirements.

❑ Together with accuracy, many interpretability objectives should be optimized at the same. Two different solutions:

- Development of new EMO algorithms for many objective problems *(incoming for future)*
- By grouping complexity measures and semantic measures into two respective indexes. *(it would represent the present)*

❑ With respect to the objectives nature, while accuracy is hard to improve, interpretability is easy to obtain, since interpretable models can even be provided by hand.

❑ These differences between both types of objectives influence the optimization process, by which the applied MOEAs are usually modified or extended.

# Contents

# A-I Trade-Off: Some Example Approaches

## Bibliography on this category

| | | A-I trade-off | | | FRBS approach | | Objectives | | MOEA | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | Authors | Ref. | Year | Rules | Type | #Obj. | Type | Name | Gen. | Type | Problem type |
| DB TUNING (INCLUDING RULE SET LEARNING) | MEMBERSHIP FUNCTIONS TUNING | Wang et al. | [79] | 2005 | TSK | LING. ∗ | 5 | A+C+C+S+S | MOHGA | 1st | I ∘ | REG. |
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I.P.=Inference Parameters, MAM.=Mamdani, TSK=Takagi-Sugeno-Kang, LING.=Linguistic, SCAT.=Scatter, ∗In the antecedent;
A=Accuracy, C=Complexity, S=Semantic aspects;
NON.=No name, N=Novel algorithm, I=Improved version, G=General use;
CLAS.=Classification, REG.=Regression, TS.=Time Series, IMB.=Imbalanced;
†NSGA-II based, ⋆PAES based, ∘MOGA based, •SPEA2 based.

- Most of them are based on 2nd gen. MOEAs
- Usually no more than 3 objectives
- Complexity at the beginning; Semantic aspects in the last years
- Most of them are Linguistic and Mamdani type based approaches
- KB learning in the last years *(granularity as important factor)*
- Most of them are improved versions of the most known MOEAs *(particularly in the case of KB learning)*

**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65.**

# A-I Trade-Off: Some Example Approaches

## Some Example Cases

| | | A-I trade-off | | | FRBS approach | | Objectives | | MOEA | | | |
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| | | Authors | Ref. | Year | Rules | Type | #Obj. | Type | Name | Gen. | Type | Problem type |
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†NSGA-II based, ⋆PAES based, ∘MOGA based, •SPEA2 based.

In the following we will see a representative example for each type:

- FIRST TYPE: RB Learning
- SECOND TYPE: DB Tuning + Rule Sel.
- THIRD TYPE: KB Learning

**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65.**

# A-I Trade-Off: Some Example Approaches
## MODEL 1: Multiobjective Rule Selection

### FIRST TYPE: RULE BASE LEARNING - CLASSIFICATION

**H. Ishibuchi, T. Yamamoto, Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining,** ***Fuzzy Sets and Systems, Vol. 141, pp. 59-88 (2004)***

# A-I Trade-Off: Some Example Approaches

## MODEL 1: Multiobjective Rule Selection

### Two-Stage Approach for Rule Base Learning

1. Heuristic Rule Extraction

   **A pre-specified number of candidate fuzzy rules are extracted from numerical data using a heuristic rule evaluation criterion (data mining).**

2. Multiobjective Genetic Fuzzy Rule Selection

   **A small number of fuzzy rules are selected from the extracted candidate rules using a multi-objective genetic algorithm (evolutionary optimization).**

H. Ishibuchi and T. Yamamoto, "Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining," ***Fuzzy Sets and Systems***, Vol. 141, pp. 59-88 (2004).

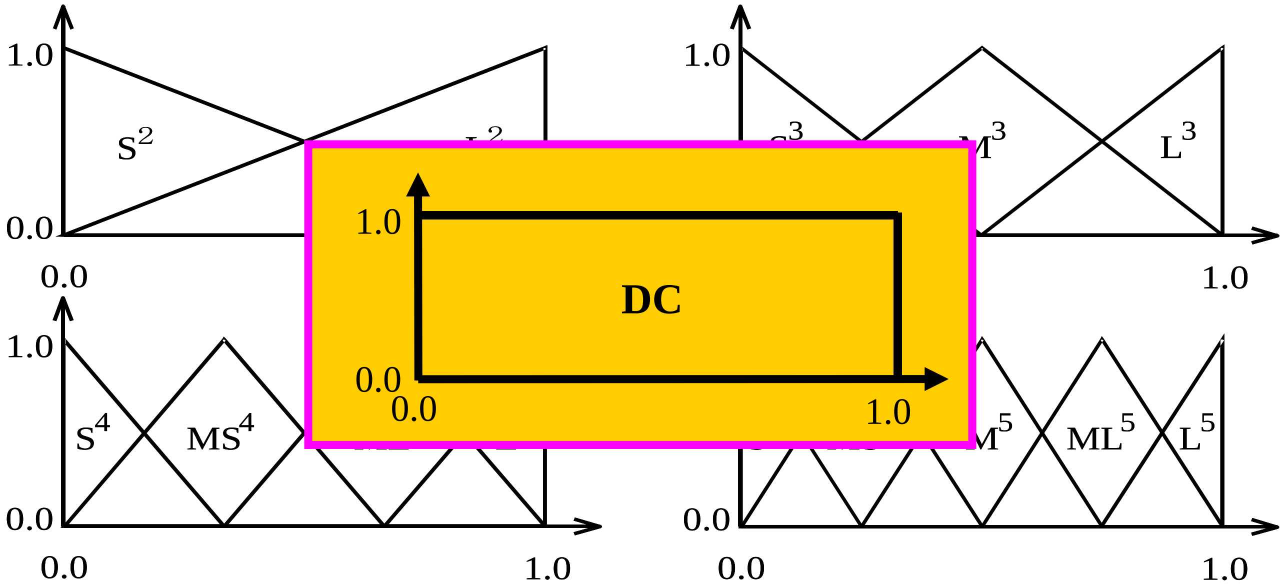
## A-I Trade-Off: Some Example Approaches
## MODEL 1: Multiobjective Rule Selection

### Fuzzy Rules for *n*-dimensional Problems

If $x_1$ is $A_1$ and … and $x_n$ is $A_n$ then Class $C$ with $CF$

| | |
|---|---|
| $A_i$ : | Antecedent fuzzy set |
| Class $C$ : | Consequent class |
| $CF$ : | Rule weight (Certainty factor) |

### Antecedent Fuzzy Sets (Multiple Partitions)

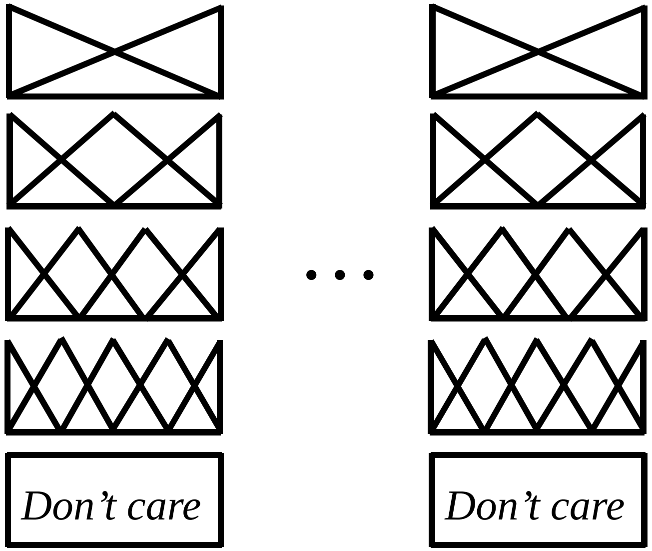



Usually we do not know an appropriate fuzzy partition for each input variable.

# A-I Trade-Off: Some Example Approaches
## MODEL 1: Multiobjective Rule Selection

### Possible Fuzzy Rules

Total number of possible fuzzy rules




$$x_1 \qquad\qquad x_n$$

$$(14+1)\times \cdots \times(14+1)=(15)^n$$

### Examined Fuzzy Rules

They only examine short fuzzy rules with only a few antecedent conditions.

If $x_1$ is *small* and $x_{48}$ is *large* then Class 1 with 0.58

# A-I Trade-Off: Some Example Approaches

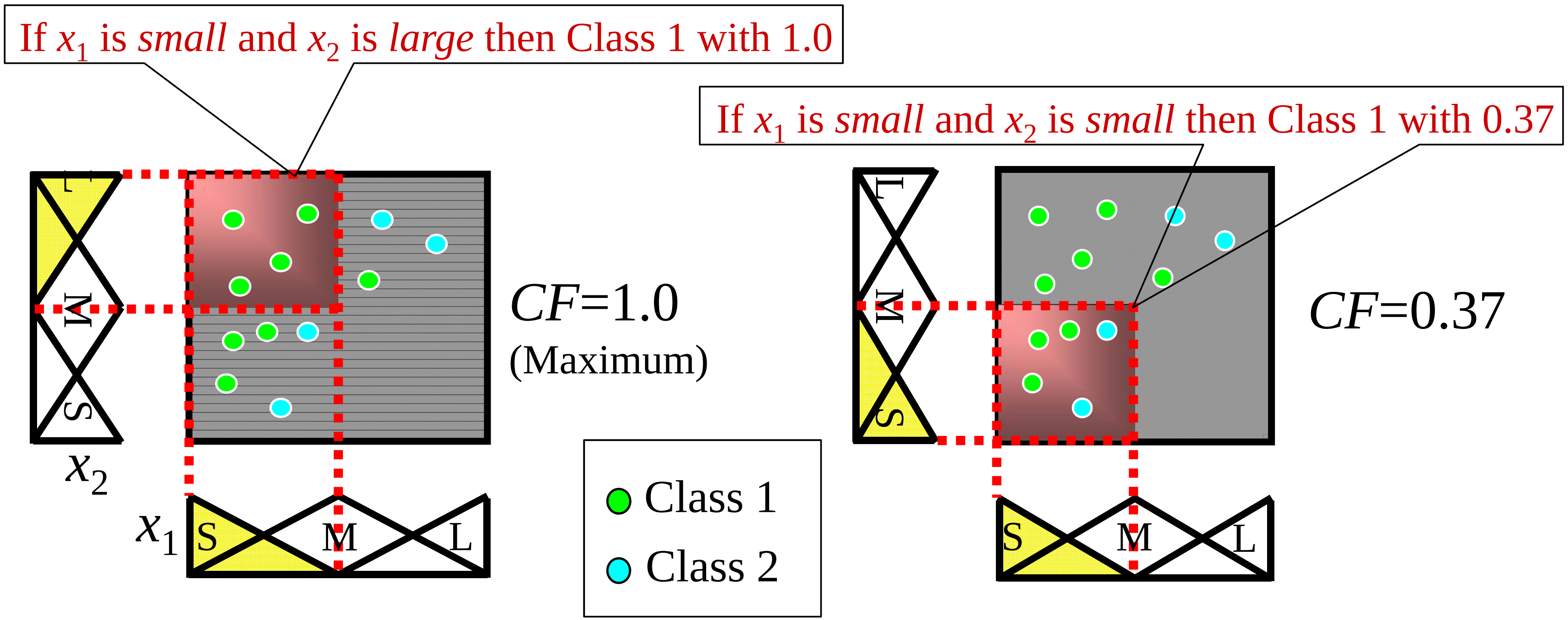
## MODEL 1: Multiobjective Rule Selection

### Consequent Class

The consequent class of each fuzzy rule is determined by compatible training patterns (i.e., the dominant class in the corresponding fuzzy subspace).

### Rule Weight **(Certainty Factor)**

The rule weight *CF* of each fuzzy rule is calculated from compatible training patterns.



# A-I Trade-Off: Some Example Approaches
## MODEL 1: Multiobjective Rule Selection

### 1. Heuristic Rule Extraction

They extract a pre-specified number of the best fuzzy rules with respect to a pre-specified heuristic rule evaluation criterion.

Possible fuzzy rules: $(15)^n$ rules

↓

Restriction on the rule length :

Only short fuzzy rules

↓

Rule evaluation criterion:

The best rules for each class

**300 fuzzy rules for each class**

# A-I Trade-Off: Some Example Approaches
# MODEL 1: Multiobjective Rule Selection

## 2. Multiobjective Genetic Fuzzy Rule Selection

**Algorithm:** Multi-objective Genetic Local Search (MOGLS)

- **Selection based on a weighted fitness function (**Number of correctly classified training patterns and number of rules**)**
- Tentative set of **non-dominated solutions** preserved externally
- **Elitist strategy**: $N_{elite}$ individuals of the population are randomly replaced with $N_{elite}$ individuals randomly extracted from the tentative set of non-dominated solutions

Coding: $$S = s_1 s_2 \cdots s_N$$

$N$: Total number of candidate rules

$s_j$={0, 1}: Inclusion or exclusion of the $j$-th rule

Objectives: $f_1(S),\ f_2(S),\ f_3(S)$

$f_1(S)$ : Number of correctly classified patterns by $S$

$f_2(S)$ : Number of selected rules in $S$

$f_3(S)$ : Total number of antecedent conditions in $S$

# A-I Trade-Off: Some Example Approaches

## MODEL 1: Multiobjective Rule Selection

### Comparison of Four Approaches

(1) Two-objective approach

Maximize $f_1(S)$ and minimize $f_2(S)$

(2) Weighted sum of the two objectives

Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S)$

(3) Three-objective approach

Maximize $f_1(S)$ and minimize $f_2(S)$, $f_3(S)$

(4) Weighted sum of the three objectives

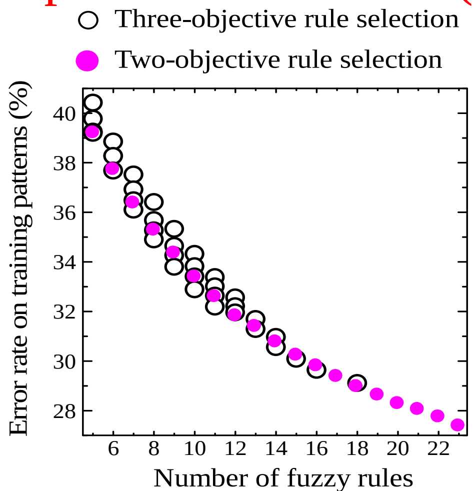
Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S) - w_3 \cdot f_3(S)$

### Data Sets

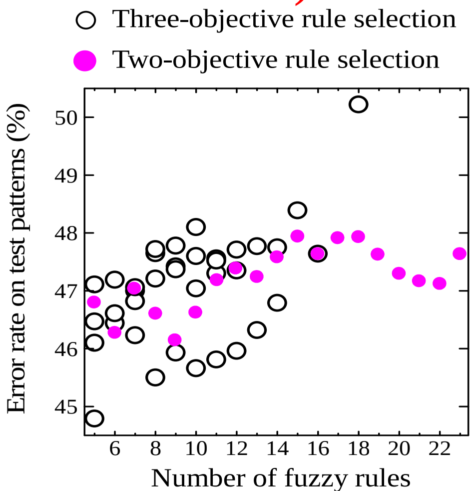
| Data set | Attributes | Patterns | Classes | Length |
|---|---|---|---|---|
| Breast W | 9 | 683* | 2 | 3 |
| Diabetes | 8 | 768 | 2 | 3 |
| Glass | 9 | 214 | 6 | 3 |
| Heart C | 13 | 297* | 5 | 3 |
| Iris | 4 | 150 | 3 | 3 |
| Sonar | 60 | 208 | 2 | 2 |
| Wine | 13 | 178 | 3 | 3 |

# A-I Trade-Off: Some Example Approaches
## MODEL 1: Multiobjective Rule Selection

### Experimental Results (Cleveland Heart)



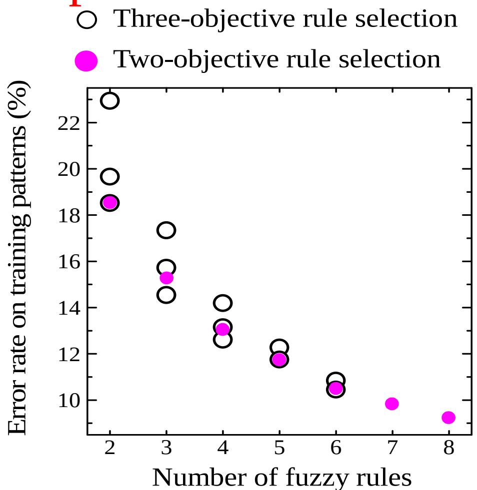
(a) Error rates on **training data**



(b) Error rates on **test data**

**We can observe the overfitting due to the increase in the number of fuzzy rules.**

### Experimental Results (Sonar)



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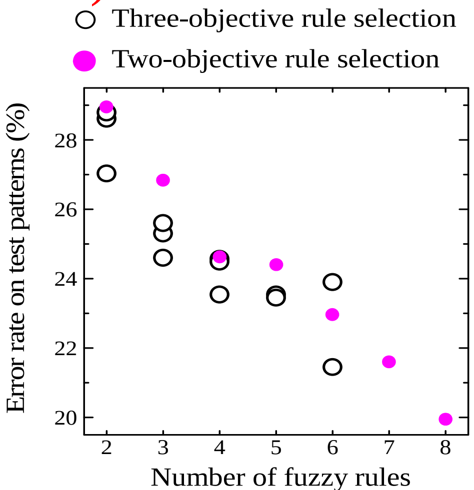


(b) Error rates on **test data**

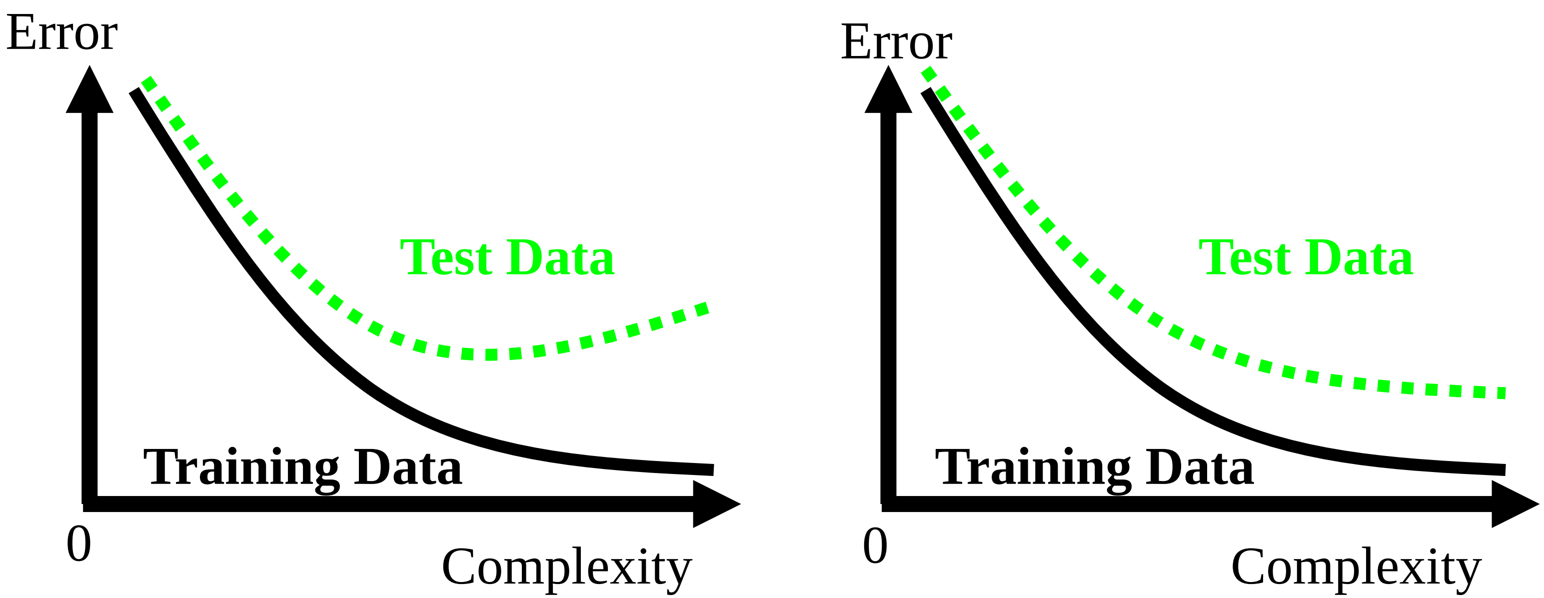
**The generalization ability is increased by increasing the number of fuzzy rules (i.e., the overfitting is not observed).**

# A-I Trade-Off: Some Example Approaches
## MODEL 1: Multiobjective Rule Selection

### Observation

(1) Experimental results showed that each test problem has a different tradeoff structure.

(2) Knowledge on the tradeoff structure is useful in the design of fuzzy rule-based classification systems.



# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

### SECOND TYPE: DATA BASE TUNING (+ RULE SELECT.) - REGRESSION

**R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15:5 (2007) 539–557***

**M.J. Gacto, R. Alcalá, F. Herrera, Adaptation and Application of Multi-Objective Evolutionary Algorithms for Rule Reduction and Parameter Tuning of Fuzzy Rule-Based Systems. *Soft Computing 13:5 (2009) 419-436***

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

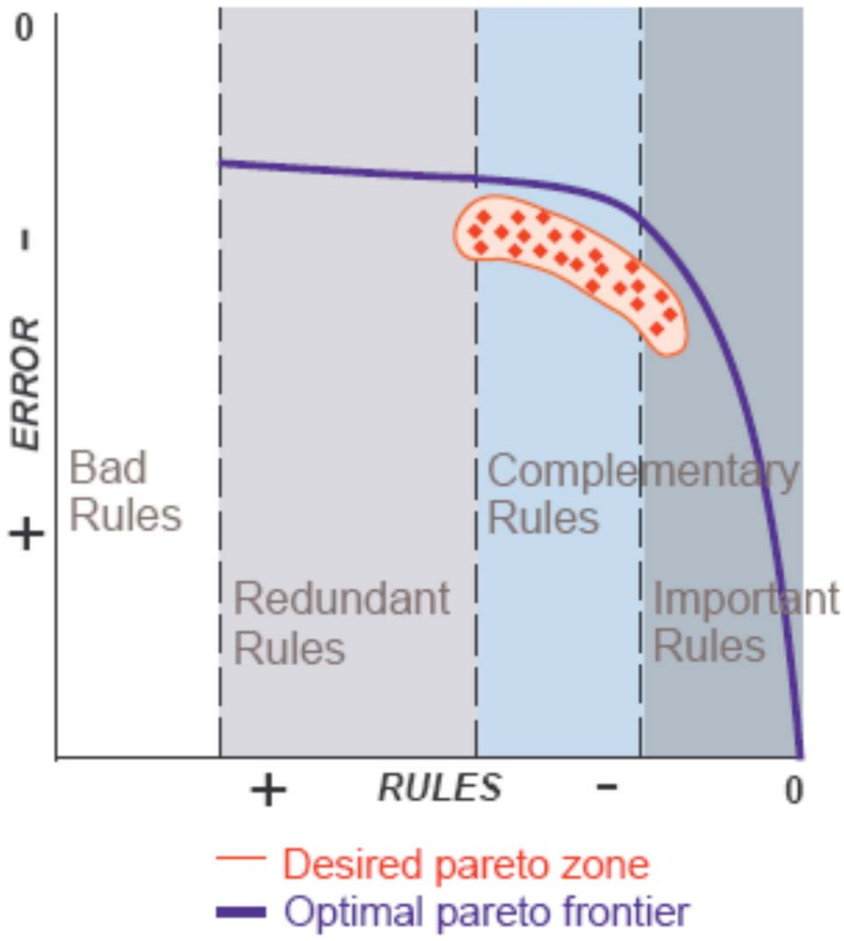
R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15:5 (2007) 539–557*,

**Multi-objective EAs are powerful tools to generate GFSs but they are based on getting a large, well distributed and spread off, Pareto set of solutions**

- **The two criteria to optimize in GFSs are accuracy and interpretability. The former is more important than the latter, so many solutions in the Pareto set are not useful**
- **Solution: Inject knowledge through the MOEA run to bias the algorithm to generate the desired Pareto front part**

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

Pareto front classification in an interpretability-accuracy GFSs:




- Bad rules zone: solutions with bad performance rules. Removing them improves the accuracy, so no Pareto solutions are located here
- Redundant rules zone: solutions with irrelevant rules. Removing them does not affect the accuracy and improves the interpretability
- Complementary rules zone: solutions with neither bad nor irrelevant rules. Removing them slightly decreases the accuracy
- Important rules zone: solutions with essential rules. Removing them significantly decreases the accuracy

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

Accuracy-oriented modifications performed:

- **Restart the genetic population at the middle of the run time, keeping the individual with the highest accuracy as the only one in the external population and generating all the new individuals with the same number of rules it has**

- **In each MOGA step, the number of chromosomes in the external population considered for the binary tournament is decreased, focusing the selection on the higher accuracy individuals**

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

**Obtained results for the medium voltage line problem:**

Multi-objective genetic tuning + rule selection method:

| Method | #R | $MSE_{tra}$ | $\sigma_{tra}$ | t-test | $MSE_{tst}$ | $\sigma_{tst}$ | t-test |
|---|---|---|---|---|---|---|---|
| WM | 65 | 57605 | 2841 | + | 57934 | 4733 | + |
| WM+T | 65 | 18602 | 1211 | + | 22666 | 3386 | + |
| WM+S | 40.8 | 41086 | 1322 | + | 59942 | 4931 | + |
| WM+TS | 41.9 | 14987 | 391 | + | 18973 | 3772 | + |
| NSGAII | 41.0 | 14488 | 965 | + | 18419 | 3054 | + |
| $NSGAII_{ACC}$ | 48.1 | 16321 | 1636 | + | 20423 | 3138 | + |
| SPEA2 | $\boxed{\textbf{33}}$ | 13272 | 1265 | + | 17533 | 3226 | + |
| $SPEA2_{ACC}$ | 34.5 | $\boxed{\textbf{11081}}$ | 1186 | * | $\boxed{\textbf{14161}}$ | 2191 | * |

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

**STUDY ON SEVERAL ALTERNATIVE APPROACHES AND IMPROVEMENTS:**
ADAPTATION AND APPLICATION OF MOEAs

M.J. Gacto, R. Alcalá, F. Herrera,
Adaptation and Application of Multi-Objective Evolutionary Algorithms for Rule Reduction and Parameter Tuning of Fuzzy Rule-Based Systems, *Soft Computing 13:5 (2009) 419-436*,

- **To perform the study we have applied six different approaches based on the two most known and successful MOEAs:**
  - **Application of *SPEA2* and *NSGA-II***
  - **Two versions of NSGA-II for finding knees, $NSGA\text{-}II_A$ and $NSGA\text{-}II_U$**
  - **Two extensions for specific application, $SPEA2_{Acc}$ and $SPEA2_{Acc2}$**

| Method | Description |
|---|---|
| WM | Wang & Mendel algorithm |
| T | Tuning of Parameters |
| S | Rule Selection |
| TS | Tuning & Selection |
| Application of standard MOEAs for general use | |
| TS-SPEA2 | Tuning & Selection by SPEA2 |
| TS-NSGA-II | Tuning & Selection by NSGA-II |
| TS-NSGA-II$_A$ | Tuning & Selection by NSGA-II$_{angle}$ |
| TS-NSGA-II$_U$ | Tuning & Selection by NSGA-II$_{utility}$ |
| Extended MOEAs for specific application | |
| TS-SPEA2$_{Acc}$ | Accuracy-Oriented SPEA2 |
| TS-SPEA2$_{Acc^2}$ | Extension of SPEA2$_{Acc}$ |

- **Two objectives are considered: MSE and Number of Rules**

- **Proper operators have to be selected.**

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

### NSGA-II FOR FINDING KNEES

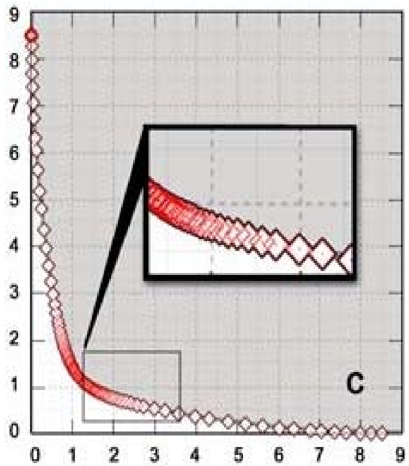
J. Branke, K. Deb, H. Dierolf, and M. Osswald, “Finding Knees in Multi-objective Optimization,” Proc. Parallel Problem Solving from Nature Conf. - PPSN VIII, LNCS 3242, (Birmingham, UK, 2004) 722–731.

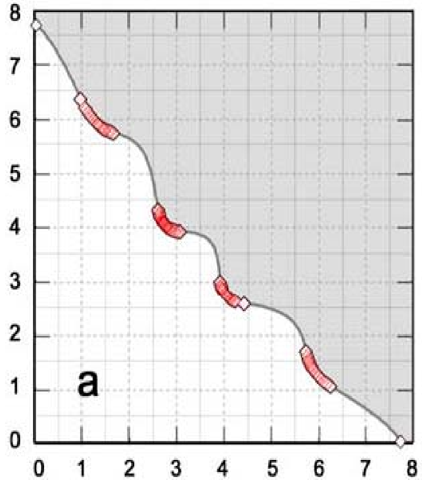
- **A variation of NSGAII in order to find knees in the Pareto front by replacing the crowding measure by either an angle-based measure or an utility-based measure**

**Two different approaches**

**Angle Based Approach**

**Utility Based Approach**




- **In our case, a knee could represent the best compromise between accuracy and number of rules.**

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

### Extension of $SPEA2_{Acc}$ ($SPEA2_{Acc2}$)

**A New Crossover Operator for the Rule Part**

- **Objective:** to improve the search with a more intelligent operator replacing the HUX crossover in $SPEA2_{ACC}$
- **Once BLX is applied a normalized euclidean distance is calculated between the centric point of the MFs used by each rule of the offpring and each parent**
- **The closer parent determines if this rule is selected or not for this offpring**
- **Whit this crossover operator, mutation can be particularly used to remove rules**

# A-I Trade-Off: Some Example Approaches

## MODEL 2: Multiobjective Tuning and Rule Selection

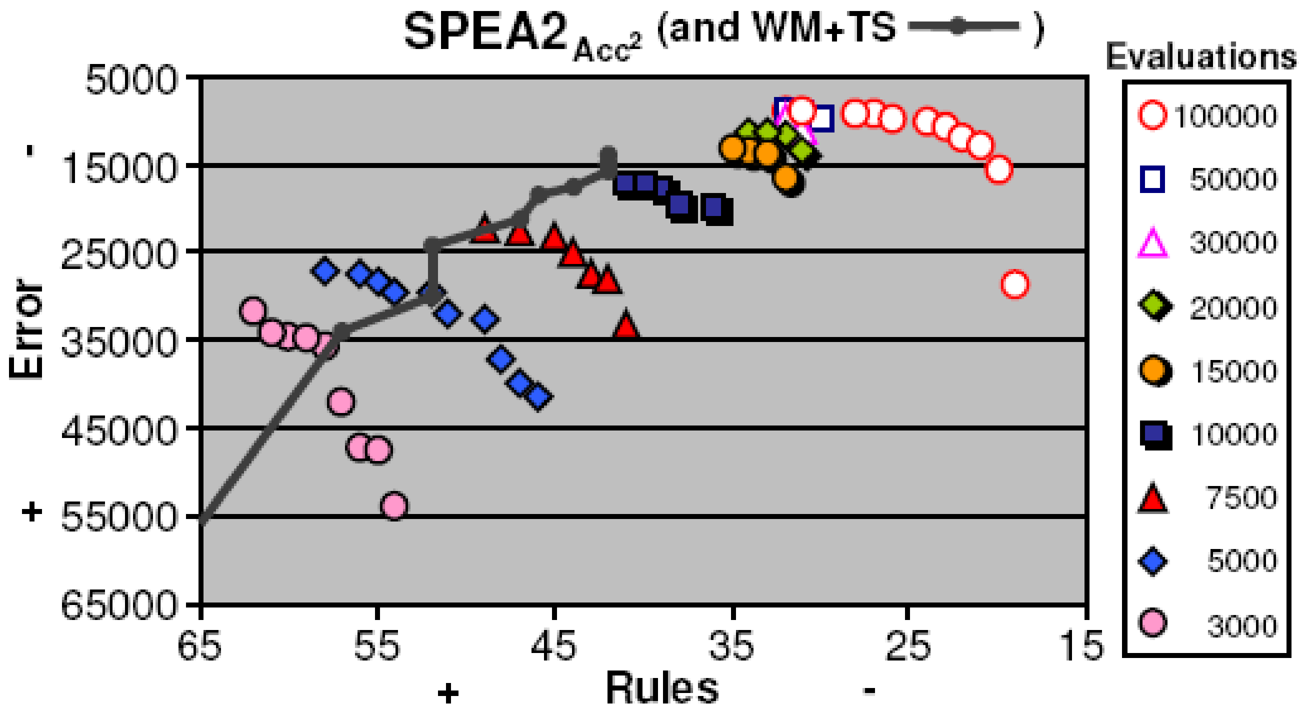
**Obtained results for the medium voltage line problem:**

| Method | #R | $MSE_{tra}$ | $\sigma_{tra}$ | t | $MSE_{tst}$ | $\sigma_{tst}$ | t |
|---|---|---|---|---|---|---|---|
| | | | 100,000 evaluations | | | | |
| WM | **65.0** | **57605** | **2841** | + | **57934** | **4733** | **+** |
| T | **65.0** | **17020** | **1893** | **+** | **21027** | **4225** | **+** |
| S | **40.9** | **41158** | **1167** | **+** | **42988** | **4441** | **+** |
| TS | **41.3** | **13387** | **1153** | **+** | **17784** | **3344** | **+** |
| TS-SPEA2 | **28.9** | **11630** | **1283** | + | **15387** | **3108** | **+** |
| TS-NSGA-II | **31.4** | **11826** | **1354** | **+** | **16047** | **4070** | **+** |
| $TS\text{-}NSGA\text{-}II_A$ | **29.7** | **11798** | **1615** | **+** | **16156** | **4091** | **+** |
| $TS\text{-}NSGA\text{-}II_U$ | **30.7** | **11954** | **1768** | **+** | **15879** | **4866** | **+** |
| $TS\text{-}SPEA2_{Acc}$ | **32.3** | **10714** | **1392** | = | **14252** | **3181** | = |
| $TS\text{-}SPEA2_{Acc2}$ | **29.8** | **10325** | **1121** | * | **13935** | **2759** | * |

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

# A-I Trade-Off: Some Example Approaches
## MODEL 2: Multiobjective Tuning and Rule Selection

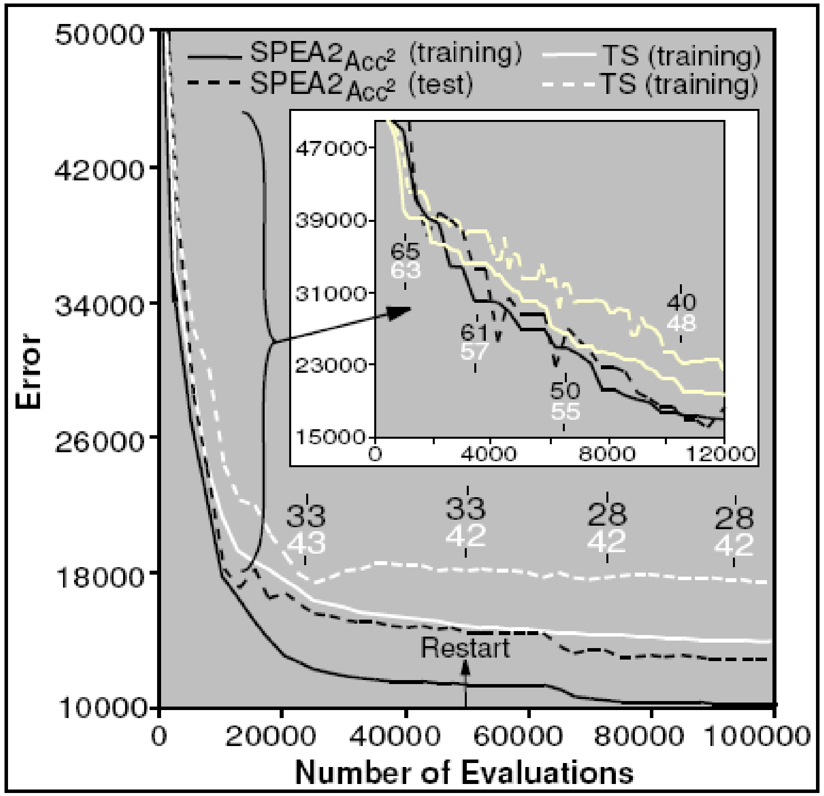
**Comparison of the SPEA2acc² and classical GA for for the medium voltage line problem:**

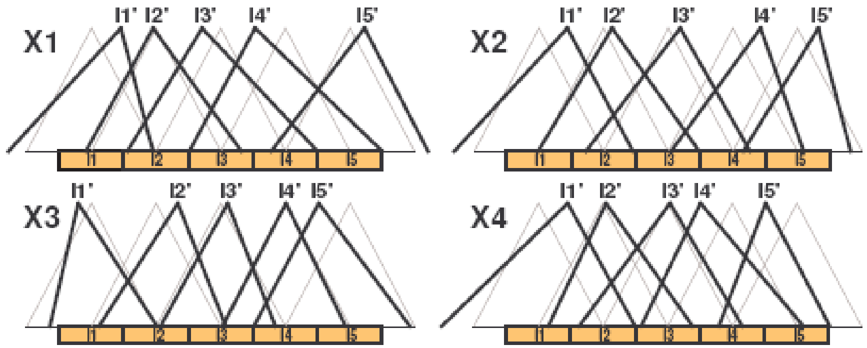


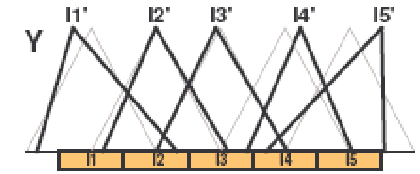
# A-I Trade-Off: Some Example Approaches

## MODEL 2: Multiobjective Tuning and Rule Selection

**Convergence and an example model**




Labelling the final MFs:

l1' = Very Small
l2' = Small
l3' = Medium
l4' = Large
l5' = Very Large

#R: 28 *MSE-tra: 8232* *MSE-tst: 14670*

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| l1' | l1' | l1' | l1' | l1' |
| l1' | l1' | l1' | l2' | l2' |
| l2' | l1' | l1' | l1' | l1' |
| l2' | l1' | l1' | l2' | l2' |
| l2' | l1' | l2' | l2' | l2' |
| l2' | l2' | l2' | l1' | l2' |
| l2' | l3' | l3' | l1' | l3' |
| l3' | l2' | l1' | l1' | l1' |
| l3' | l2' | l1' | l2' | l2' |

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| l3' | l2' | l1' | l3' | l2' |
| l3' | l2' | l2' | l3' | l3' |
| l3' | l3' | l2' | l2' | l2' |
| l3' | l3' | l3' | l2' | l3' |
| l3' | l4' | l3' | l3' | l3' |
| l4' | l2' | l2' | l2' | l2' |
| l4' | l3' | l2' | l1' | l2' |
| l4' | l3' | l2' | l3' | l3' |
| l4' | l3' | l2' | l4' | l3' |

| X1 | X2 | X3 | X4 | Y |
|---|---|---|---|---|
| l4' | l3' | l3' | l2' | l3' |
| l4' | l3' | l3' | l3' | l4' |
| l4' | l4' | l3' | l2' | l3' |
| l4' | l4' | l4' | l2' | l4' |
| l4' | l4' | l4' | l4' | l5' |
| l4' | l5' | l4' | l2' | l3' |
| l4' | l5' | l5' | l3' | l5' |
| l5' | l2' | l2' | l5' | l4' |
| l5' | l2' | l3' | l2' | l3' |
| l5' | l4' | l3' | l5' | l5' |

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

**THIRD TYPE: KNOWLEDGE BASE LEARNING - REGRESSION**

**R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, F. Marcelloni, A Multi-Objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems, *IEEE Transactions on Fuzzy Systems* 17:5 (2009) 1106-1122, doi:10.1109/TFUZZ.2009.2023113**

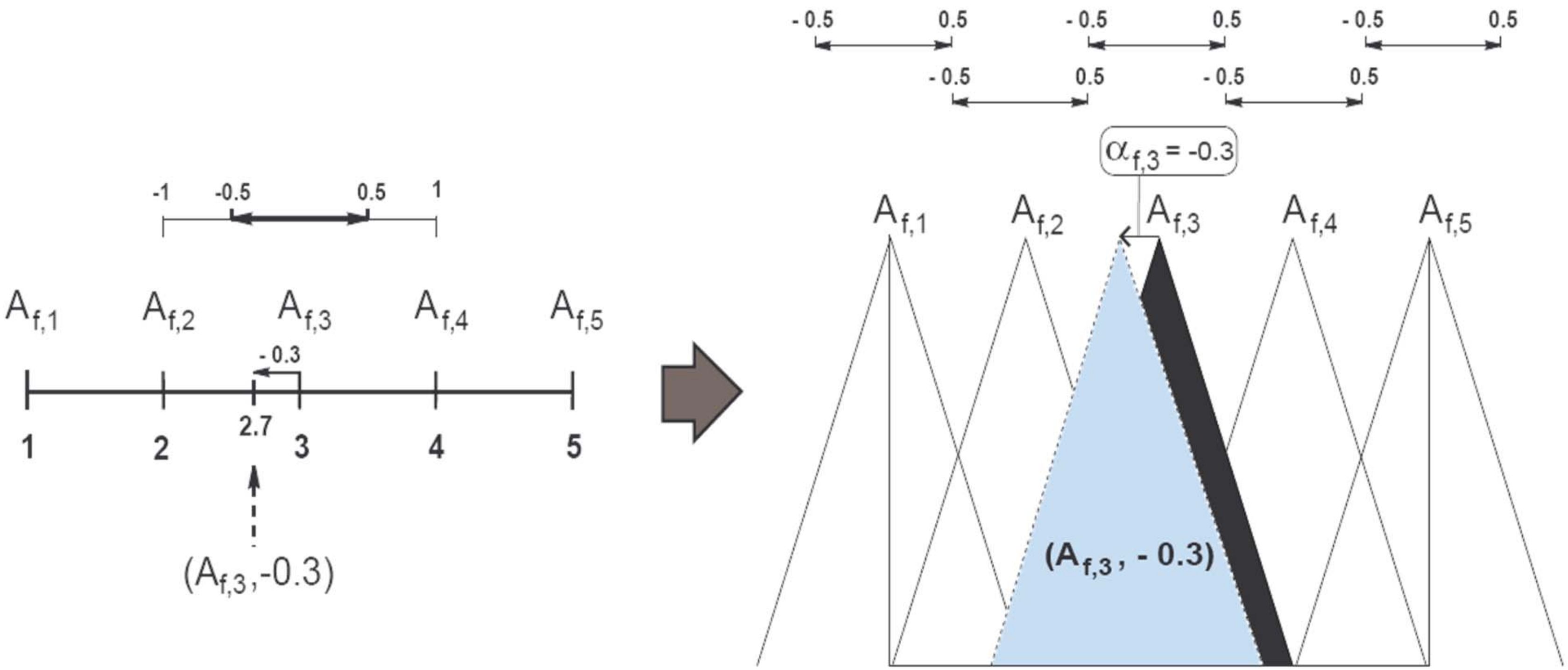
# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, F. Marcelloni, A Multi-Objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems 17:5 (2009) 1106-1122, *IEEE Transactions on Fuzzy Systems, doi:10.1109/TFUZZ.2009.2023113*,

- **Rule bases and parameters of the membership functions of the associated linguistic labels are learnt concurrently.**
- **Accuracy and interpretability are measured in terms of approximation error (MSE) and rule base complexity (#Conditions), respectively.**
- **To manage the size of the search space, the linguistic 2-tuple representation model, which allows the symbolic translation of a label by only considering one parameter, has been exploited**

# A-I Trade-Off: Some Example Approaches

## MODEL 3: Multiobjective Learning of DB and RB




a) Simbolic Translation of a label

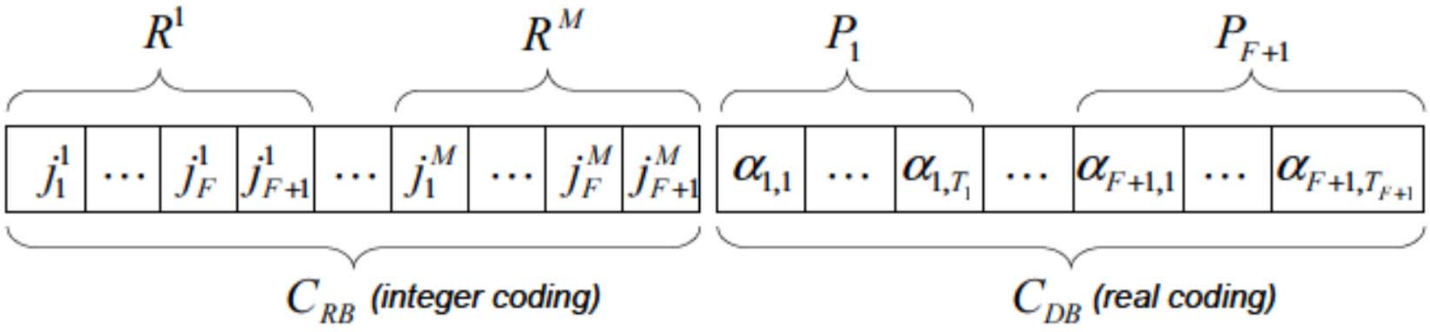
b) Lateral Displacement of a Membership function

This proposal decreases the tuning complexity, since the 3 parameters per label of the classical tuning are reduced to only 1 translation parameter (the tuning is applied to the level of linguistic partitions)

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

### Coding Scheme and Operators

- **A double coding scheme (C = $C_{RB}$ + $C_{DB}$)**




- **Crossover operator: one point + BLX-$\alpha$ crossovers (2 offsprings)**

- **Mutation operators:**
  - **Rule Adding: It adds $\gamma$ random rules to the RB, where $\gamma$ is randomly chosen in [1, $\gamma_{max}$]**

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

### Operators and Selection Schemes

- **Modify RB:** It randomly changes $\delta$ elements of the RB part. The number $\delta$ is randomly generated in $[1, \delta_{max}]$

- **Modify DB:** It changes a gene value at random in the DB part

**PAES**, **NSGA-II** and **SOGA** were applied using this representation and crossover

```
[p1, p2] = selection(archive/population);

if (rand() < Pcross)
        [s1, s2] = crossover(p1,p2);
        PmRB = 0.01;
else
        s1 = p1;
        s2 = p2;
        PmRB = 1;
endif
Loop i=1,2
        if (rand() < PmRB)
                if (rand <PmAdd)
                        s1 = add_rule();
                else
                        s1 = modify_rule_base();
                endif
        endif
        if (rand() < PmDB)
                s1 = mutate_DB();
        endif
endLoop
```

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

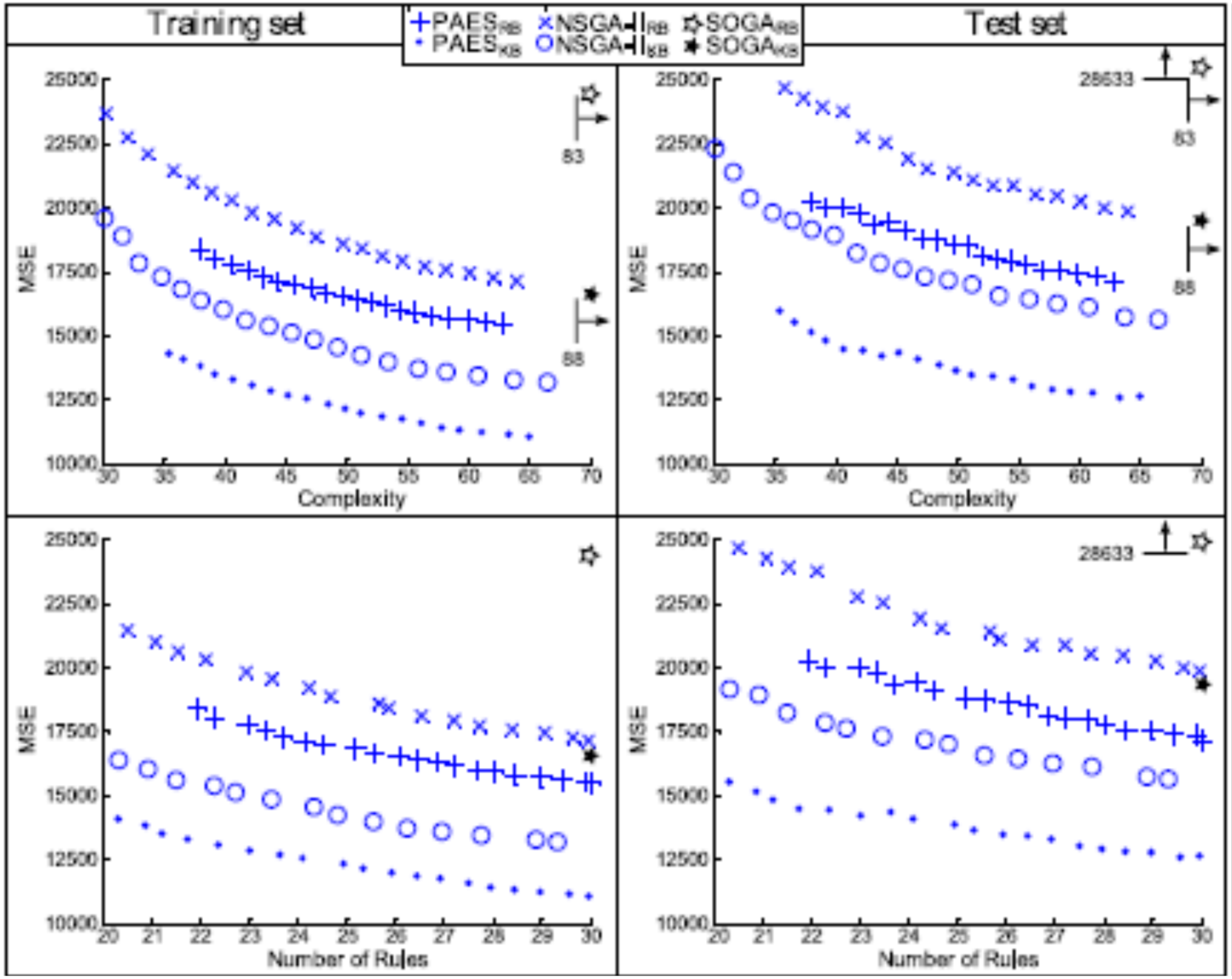
### Analysed Methods

| Method | Description | Pop. size |
|---|---|---|
| $SOGA_{RB}$ | **Rule Base learning with SOGA** | **64** |
| $NSGA\text{-}II_{RB}$ | **Rule Base learning with NSGA-II** | **64** |
| $PAES_{RB}$ | **Rule Base learning with PAES** | **64** |
| $SOGA_{KB}$ | **(Rule Base + Data Base) learning with SOGA** | **64** |
| $NSGA\text{-}II_{KB}$ | **(Rule Base + Data Base) learning with NSGA-II** | **64** |
| $PAES_{KB}$ | **(Rule Base + Data Base) learning with PAES** | **64** |

- **Different population sizes were probed for these MOEAs showing better results when the population used for parent selection has similar sizes than those considered by single objective oriented algorithms.**
- **300,000 evaluations to allow complete convergence in all the algorithms**

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

Average Pareto Fronts and average solution by SOGA
(medium voltage lines problem)




5 Data partitions 80% - 20%
6 Runs per partition
A total of 30 Runs
Test t-student α = 0.05

1. **Most accurate solution is selected from each Pareto**
2. **Average values are computed and represented**
3. **These solutions are no more used**
4. **Repeat to extract the desired avarage Pareto**

**Only the first 20 solutions are considered**

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

Statistical Analysis

**Statistical comparison among MOEAs**

| Method | Using the Pareto most accurate solution (First) | | | | | | | Using the Pareto median solution (Median) | | | | | | | Using the Pareto simplest solution (Last) | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | # R/C | $E_{tra}$ | $\sigma_{tra}$ | t-t | $E_{tst}$ | $\sigma_{tst}$ | t-t | # R/C | $E_{tra}$ | $\sigma_{tra}$ | t-t | $E_{tst}$ | $\sigma_{tst}$ | t-t | # R/C | $E_{tra}$ | $\sigma_{tra}$ | t-t | $E_{tst}$ | $\sigma_{tst}$ | t-t |
| NSGA-II$_{RB}$ | 30/64 | 17116 | 4283 | + | 19834 | 4996 | + | 25/48 | 18853 | 4672 | + | 21533 | 5149 | + | 18/30 | 23649 | 5852 | + | 26660 | 6342 | + |
| PAES$_{RB}$ | 30/63 | 15454 | 3882 | + | 17135 | 4234 | + | 27/51 | 16378 | 4112 | + | 18472 | 4740 | + | 22/38 | 18352 | 4631 | + | 20238 | 5419 | + |
| NSGA-II$_{KB}$ | 29/67 | 13137 | 3378 | + | 15587 | 4806 | + | 23/46 | 15073 | 4126 | + | 17581 | 5853 | + | 17/29 | 21629 | 12156 | + | 25716 | 14722 | + |
| PAES$_{KB}$ | 30/65 | **11044** | **2771** | * | **12607** | **3106** | * | 25/50 | **12133** | **3380** | * | **13622** | **3353** | * | 20/35 | **14297** | **4449** | * | **15951** | **4405** | * |

**Statistical comparison of the best MOEA with SOGA**

| Method | # R/C | $E_{tra}$ | $\sigma_{tra}$ | t-t | $E_{tst}$ | $\sigma_{tst}$ | t-t |
|---|---|---|---|---|---|---|---|
| SOGA$_{RB}$ | 30/83 | 24340 | 8450 | + | 28633 | 11861 | + |
| SOGA$_{KB}$ | 30/88 | 16502 | 5136 | ⋄ | 19112 | 6273 | ⋄ |
| PAES$_{KB}$ (First) | 30/65 | **11044** | **2771** | - | **12607** | **3106** | - |
| PAES$_{KB}$ (Median) | 25/50 | 12133 | 3380 | - | 13622 | 3353 | - |
| PAES$_{KB}$ (Last) | 20/35 | 14297 | 4449 | =‡ | 15951 | 4405 | - |

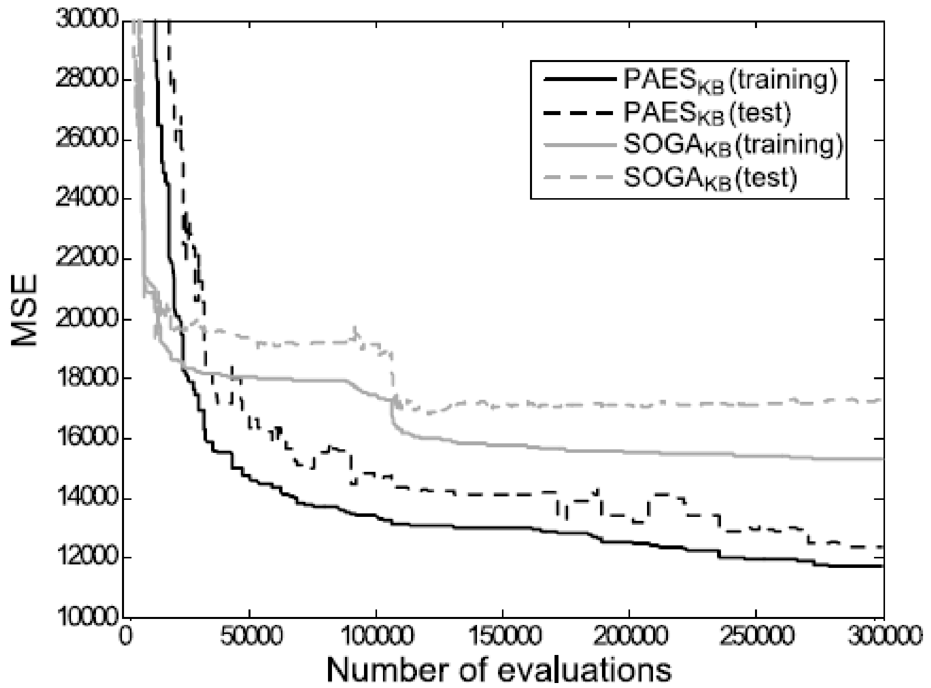
‡ It is (-) with 91% confidence

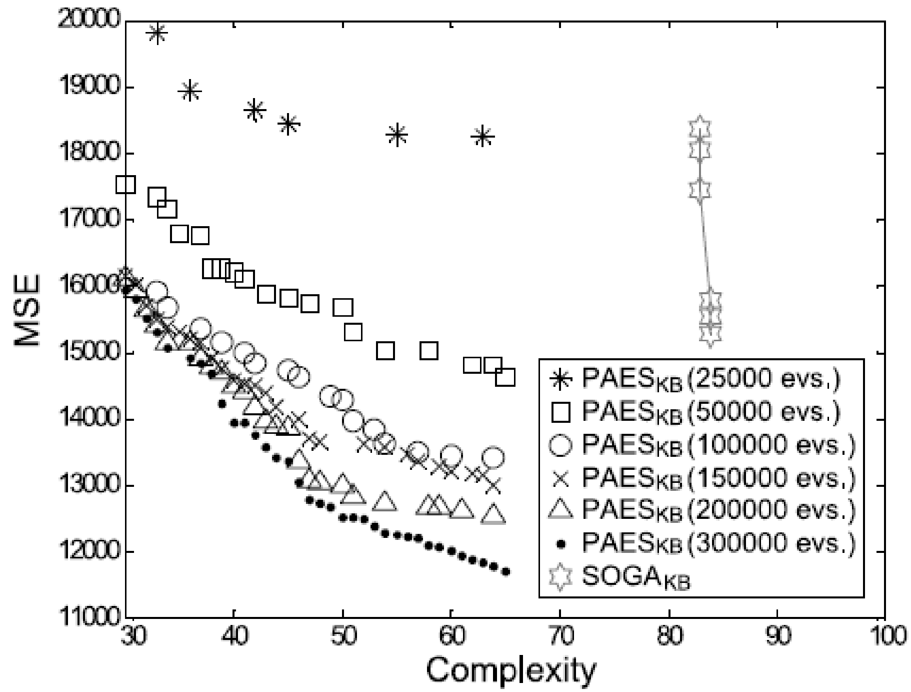
REMINDER
5 Data partitions 80% - 20%
6 Runs per partition
A total of 30 Runs
Test t-student α = 0.05

# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

Convergence

| Evolution of SOGA and PAES$_{KB}$ (FIRST) | Pareto evolution of PAES$_{KB}$ and SOGA evolution |
| --- | --- |



# A-I Trade-Off: Some Example Approaches
## MODEL 3: Multiobjective Learning of DB and RB

- **The models obtained by these new approaches presented a better trade-off than those obtained by only considering performance measures.**

- **Between both multi-objective experimented, namely a modified (2+2)PAES and the classical NSGA-II, the modified (2+2)PAES has shown a better behavior than NSGA-II.**

- **Finally, the linguistic 2-tuples representation presented has shown a good positive synergy.**

# Contents

# MOEFSs for Multiobjective Control Problems

## Bibliography on this category

| | Fuzzy Control | | | FRBS approach | | | MOEA | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Authors | Ref. | Year | Rules | Type | #Obj. | Name | Gen. | Type | Application Framework |
| Controller parameters' identification | Ahlawat et al. | [96] | 2001 | Mam. | Ling. | 2 | NoN. | 1st | I ⋆ | Building vibration |
| | Ahlawat et al. | [97], [99] | 2002,2004 | Mam. | Ling. | 2 | NoN. | 1st | I ⋆ | Building vibration |
| | Ahlawat et al. | [98] | 2002 | Mam. | Ling. | 3 | NoN. | 1st | I ⋆ | Building vibration |
| | Chipperfield et al. | [101] | 2002 | Mam. | Ling. | 9 | NoN. | 1st | N | Gas turbine engine |
| | Ahlawat et al. | [100] | 2004 | Mam. | Ling. | 2 | NoN. | 1st | I ⋆ | Building vibration |
| | Jurado et al. | [102] | 2005 | Mam. | Ling. | 16 | NoN. | 1st | I ∘ | Solid oxide fuel cell |
| | Kim et al. | [103] | 2006 | Mam. | Scat. | 2 | NSGA-II | 2nd | G | Base-isolation system |
| | Kim et al. | [104] | 2007 | Mam. | Scat. | 4 | NSGA-II | 2nd | G | Base-isolation system |
| | Shook et al. | [105] | 2008 | Mam. | Ling. | 4 | NSGA-II CE | 2nd | I † | Seismic loads mitigation |
| | Muñoz et al. | [106] | 2008 | Mam. | Ling. | 2 | various | 2nd | G | Fuzzy visual system for robots |
| | Daum et al. | [108] | 2010 | TSK | Scat. | 2 | NSGA-II | 2nd | G | HVAC systems |
| | Ebner et al. | [109] | 2010 | ‡ | ‡ | 3 | NoN. | 2nd | I • | Water treatment |
| | Gacto et al. | [93] | 2012 | Mam. | Ling. | 2 | $SPEA2_{E/E}$ | 2nd | I • | HVAC systems |
| Learning FLC struct. | Blumel et al. | [110] | 2001 | Mam. | Ling. | 4 | NSGA | 1st | N | Missile autopilot |
| | Chen et al. | [113] | 2002 | TSK | Ling. ∗ | 2 | NoN. | 1st | N | Incineration process |
| | Stewart et al. | [111] | 2004 | Mam. | Ling. | 3 | NoN. | 1st | N | DC motor motion ctrl. |
| | Serra et al. | [114] | 2006 | Mam. | Ling. | 3 | NoN. | 2nd | N | Nonlinear plants |
| | Fazendeiro et al. | [112] | 2007 | Mam. | Ling. | 2 | NoN. | 2nd | I • | Drug dosage for surgeries |

Mam.=Mamdani, TSK=Takagi-Sugeno-Kang, Ling.=Linguistic, Scat.=Scatter, ∗In the antecedent, ‡Patented FLC, not available information;
A=Accuracy, C=Complexity, S=Semantic aspects;
NoN.=No name, N=Novel algorithm, I=Improved version, G=General use;
⋆2-branch tournament GA, ∘MOGA based, †NSGA-II based, •SPEA2 based.

The multiobjective nature is specific to each problem

- Most of them deal with the post-processing of FLC parameters (simplest with reduced search space)
- Earlier works consider 1st-gen. algorithms and only recently the 2nd-gen. have been applied (2006)
- Almost all of them are Linguistic and Mamdani-type based approaches

**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65.**

# MOEFSs for Multiobjective Control Problems

## An example for the control of HVAC Systems

| | Fuzzy Control | | | FRBS approach | | | MOEA | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Authors | Ref. | Year | Rules | Type | #Obj. | Name | Gen. | Type | Application Framework |
| Controller parameters' identification | Ahlawat et al. | [96] | 2001 | Mam. | Ling. | 2 | NoN. | 1st | I ⋆ | Building vibration |
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| | Ahlawat et al. | [98] | 2002 | Mam. | Ling. | 3 | NoN. | 1st | I ⋆ | Building vibration |
| | Chipperfield et al. | [101] | 2002 | Mam. | Ling. | 9 | NoN. | 1st | N | Gas turbine engine |
| | Ahlawat et al. | [100] | 2004 | Mam. | Ling. | 2 | NoN. | 1st | I ⋆ | Building vibration |
| | Jurado et al. | [102] | 2005 | Mam. | Ling. | 16 | NoN. | 1st | I ◦ | Solid oxide fuel cell |
| | Kim et al. | [103] | 2006 | Mam. | Scat. | 2 | NSGA-II | 2nd | G | Base-isolation system |
| | Kim et al. | [104] | 2007 | Mam. | Scat. | 4 | NSGA-II | 2nd | G | Base-isolation system |
| | Shook et al. | [105] | 2008 | Mam. | Ling. | 4 | NSGA-II CE | 2nd | I † | Seismic loads mitigation |
| | Muñoz et al. | [106] | 2008 | Mam. | Ling. | 2 | various | 2nd | G | Fuzzy visual system for robots |
| | Daum et al. | [108] | 2010 | TSK | Scat. | 2 | NSGA-II | 2nd | G | HVAC systems |
| | Ebner et al. | [109] | 2010 | ‡ | ‡ | 3 | NoN. | 2nd | I • | Water treatment |
| | Gacto et al. | [93] | 2012 | Mam. | Ling. | 2 | $SPEA2_{E/E}$ | 2nd | I • | HVAC systems |
| Learning FLC struct. | Blumel et al. | [110] | 2001 | Mam. | Ling. | 4 | NSGA | 1st | N | Missile autopilot |
| | Chen et al. | [113] | 2002 | TSK | Ling. ∗ | 2 | NoN. | 1st | N | Incineration process |
| | Stewart et al. | [111] | 2004 | Mam. | Ling. | 3 | NoN. | 1st | N | DC motor motion ctrl. |
| | Serra et al. | [114] | 2006 | Mam. | Ling. | 3 | NoN. | 2nd | N | Nonlinear plants |
| | Fazendeiro et al. | [112] | 2007 | Mam. | Ling. | 2 | NoN. | 2nd | I • | Drug dosage for surgeries |

Mam.=Mamdani, TSK=Takagi-Sugeno-Kang, Ling.=Linguistic, Scat.=Scatter, ∗In the antecedent, ‡Patented FLC, not available information;
A=Accuracy, C=Complexity, S=Semantic aspects;
NoN.=No name, N=Novel algorithm, I=Improved version, G=General use;
⋆2-branch tournament GA, ◦MOGA based, †NSGA-II based, •SPEA2 based.

In the following we will see a representatibe example for the control HVAC Systems

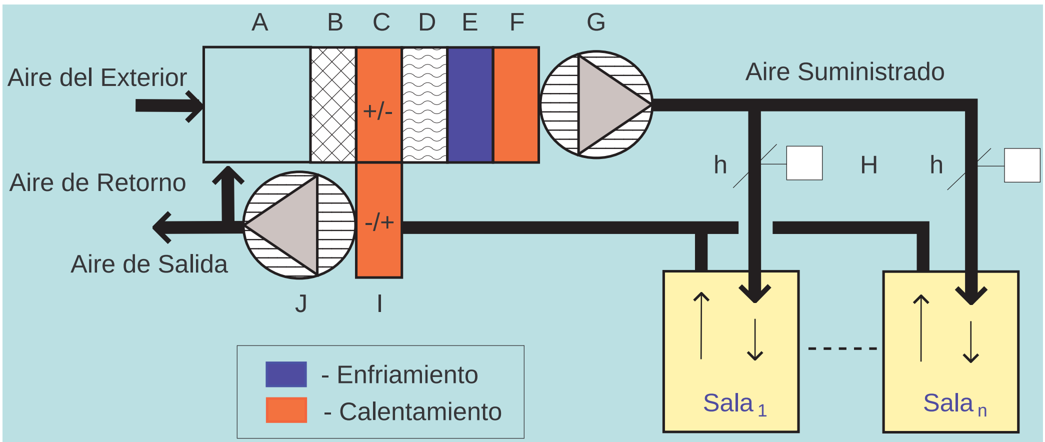
**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65.**

# MOEFSs: APPLICATION TO A HVAC CONTROL PROBLEM

## Heating Ventilating and Air Conditioning Systems

GENESYS

**JOULE-THERMIE JOE-CT98-0090**



# Models for Fuzzy Control of HVAC Systems

**Single Objective Previous Approaches**


**R. Alcalá, J.M. Benítez, J. Casillas, O. Cordón, R. Pérez, Fuzzy Control of HVAC Systems Optimized by Genetic Algorithms. *Applied Intelligence 18:2 (2003) 155-177*.**

**R. Alcalá, J. Casillas, O. Cordón, A. González, F. Herrera, A Genetic Rule Weighting and Selection Process for Fuzzy Control of Heating, Ventilating and Air Conditioning Systems. *Engineering Applications of Artificial Intelligence 18:3 (2005) 279-296*.**

**R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Improving Fuzzy Logic Controllers Obtained by Experts: A Case Study in HVAC Systems. *Applied Intelligence 31:1 (2009) 10-35*.**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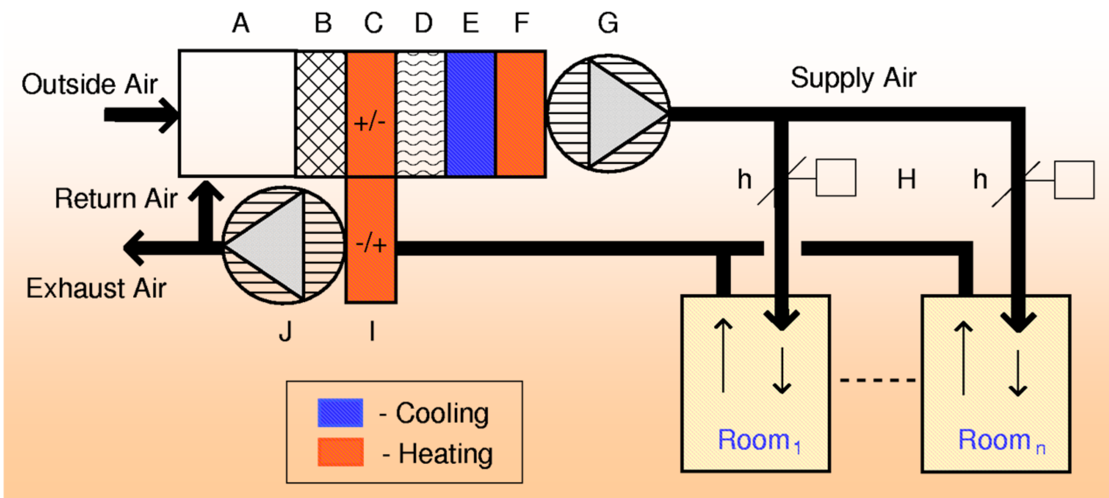
**A new MOEFS to Solve the Problem**


M.J. Gacto, **R. Alcalá**, **F. Herrera**, **A Multi-Objective Evolutionary Algorithm for an Effective Tuning of Fuzzy Logic Controllers in Heating, Ventilating and Air Conditioning Systems*. Applied Intelligence 36:2 (2012) 330-347***

# Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

- Energy consumption in buildings is the 40% of the total and more than a half is for indoor climate conditions
- The use of appropriate automatic control strategies could result in energy savings ranging 15-85 %
- Moreover, in current systems, several criteria are considered and optimized independently without a global strategy

**Generic Structure of an Office Building HVAC System**

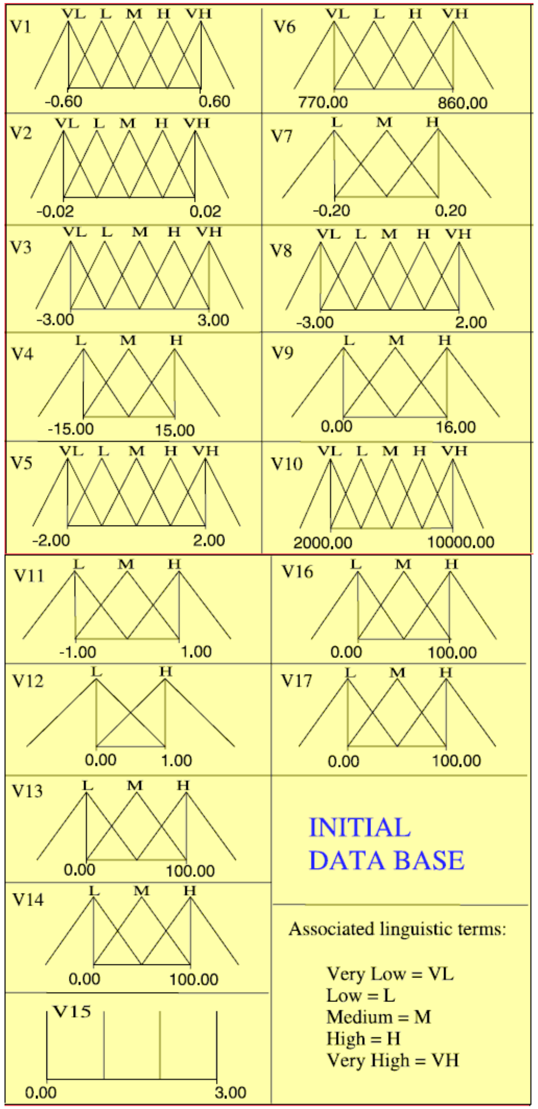



- It maintain a good thermal quality in summer and winter
- It dilutes and removes emissions from people, equipment and activities and supplies clean air

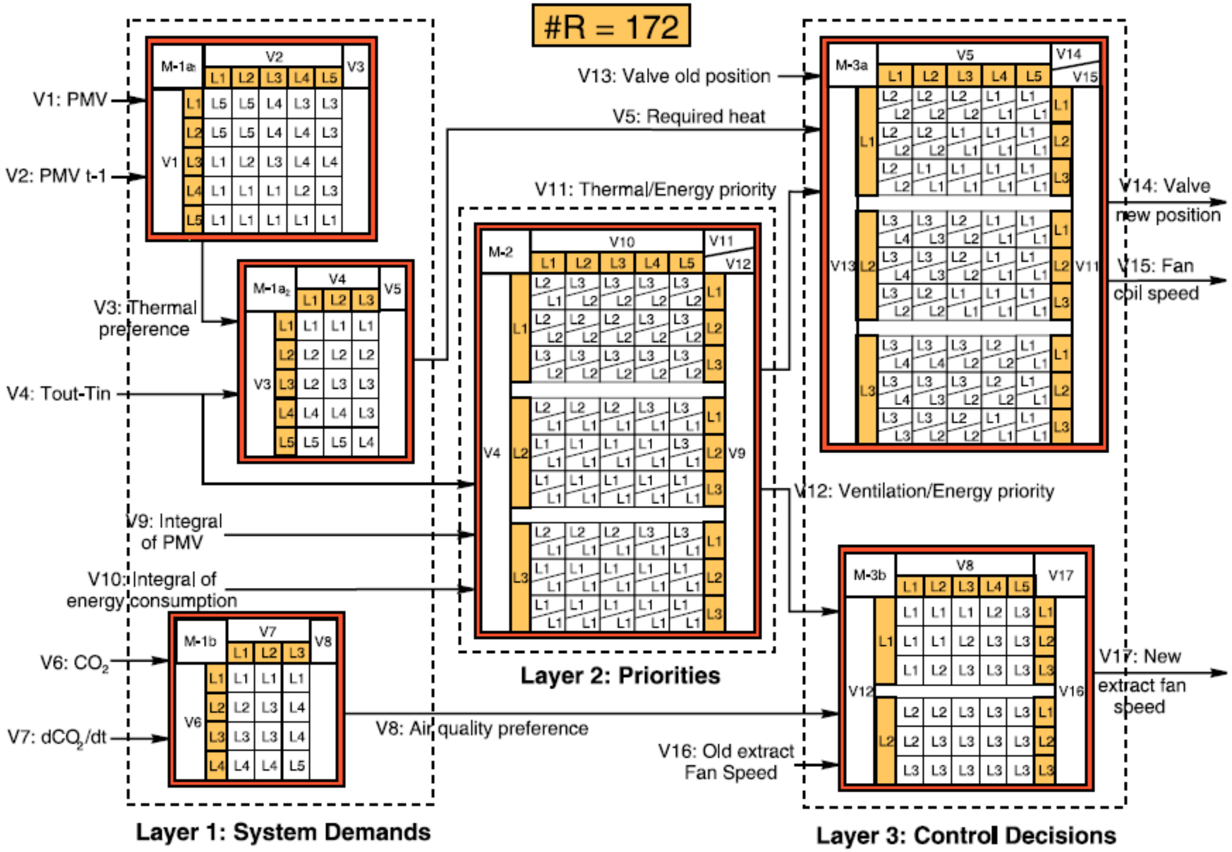
# Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

## Initial Data Base

**17 Variables**




## Initial Rule Base and FLC Structure

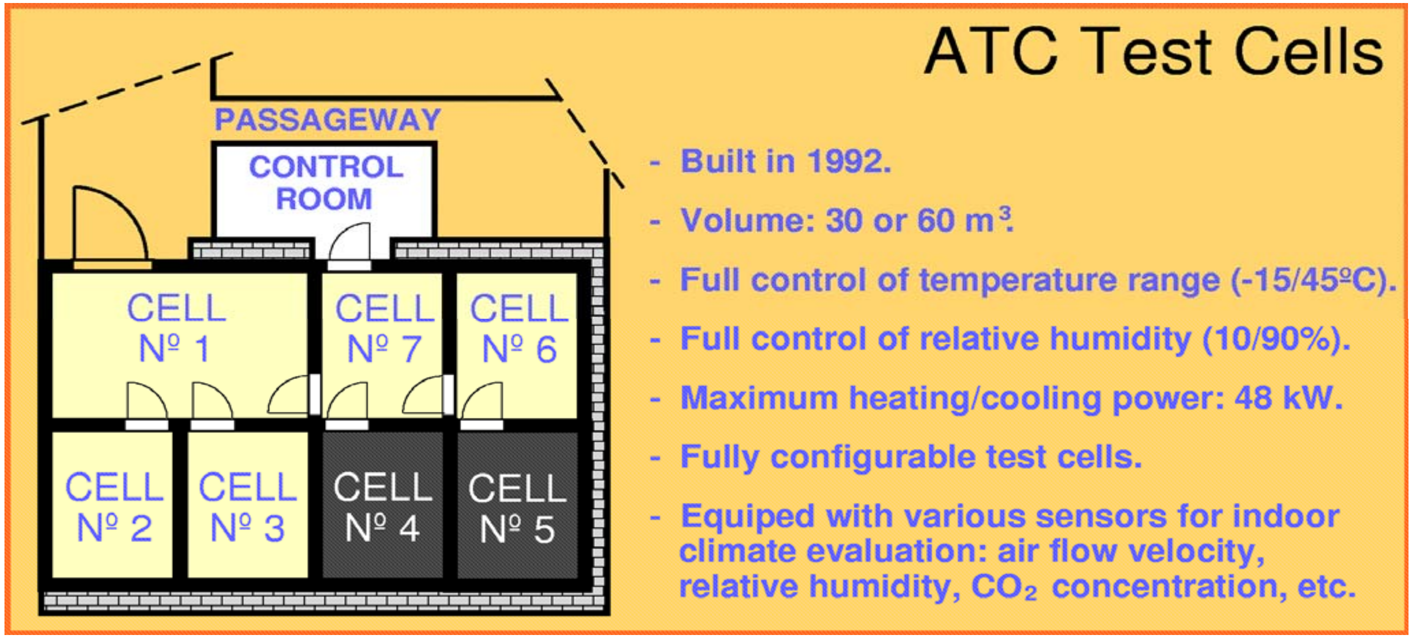



Module 1a₁: Thermal Demands
Module 1a₂: Thermal Preference
Module 1b: Air Quality Demands

Module 2: Energy Priorities
Module 3a: Required HVAC System Status
Module 3b: Required Ventilation System Status

# Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

## Representation of the Test Cells




- **Two adjacent twin cells were available**

- **A calibrated and validated model of this site was developed to evaluate each FLC**

# Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

- **Goal: multi-criteria optimization of an expert FLC for an HVAC system: reduction of the energy consumption but maintaining the required indoor comfort levels**

$O_1$ Upper thermal comfort limit [3]: $if\ PMV > 0.5, O_1 = O_1 + (PMV - 0.5)$.

$O_2$ Lower thermal comfort limit: $if\ PMV < -0.5, O_2 = O_2 + (-PMV - 0.5)$.

$O_3$ IAQ requirement: $if\ CO_2\ conc. > 800ppm, O_3 = O_3 + (CO_2 - 800)$.

$O_4$ Energy consumption: $O_4 = O_4 +$ Power at time $t$.

$O_5$ System stability: $C_5 = C_5 +$ System change from time $t$ to $(t-1)$.

- **INITIAL RESULTS**

| MODELS | #R | PMV>0.5 $0_1$ | | PMV<-0.5 $0_2$ | | $C0_2$ $0_3$ | | ENERGY $0_4$ | % | STABILITY $0_5$ | % |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ON-OFF | - | 0,0 | | 0 | | 0 | | 3206400 | - | 1136 | - |
| FLC | 172 | 0,0 | | 0 | | 0 | | 2901686 | 9,50 | 1505 | -32,48 |

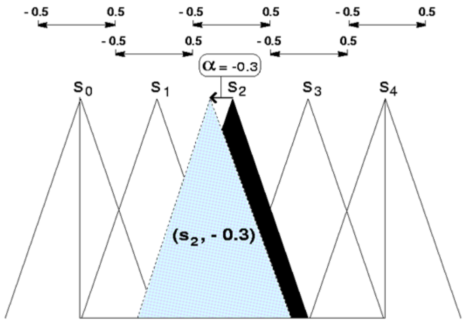
# MOEFSs for Fuzzy Control of HVAC Systems: Problem Restrictions and Tuning Approach

- **The controller accuracy is assessed by means of simulations which approximately take 3-4 minutes**
  - Necessity of efficient tuning methodologies:
    - Efficient adjustment of the MF parameters
    - Steady-State Genetic Algorithms were applied in the previous aproaches: quick convergence

      2000 evaluations $\Rightarrow$ 1 run took approximately 4 days
    - Considering a small population (31 individuals)

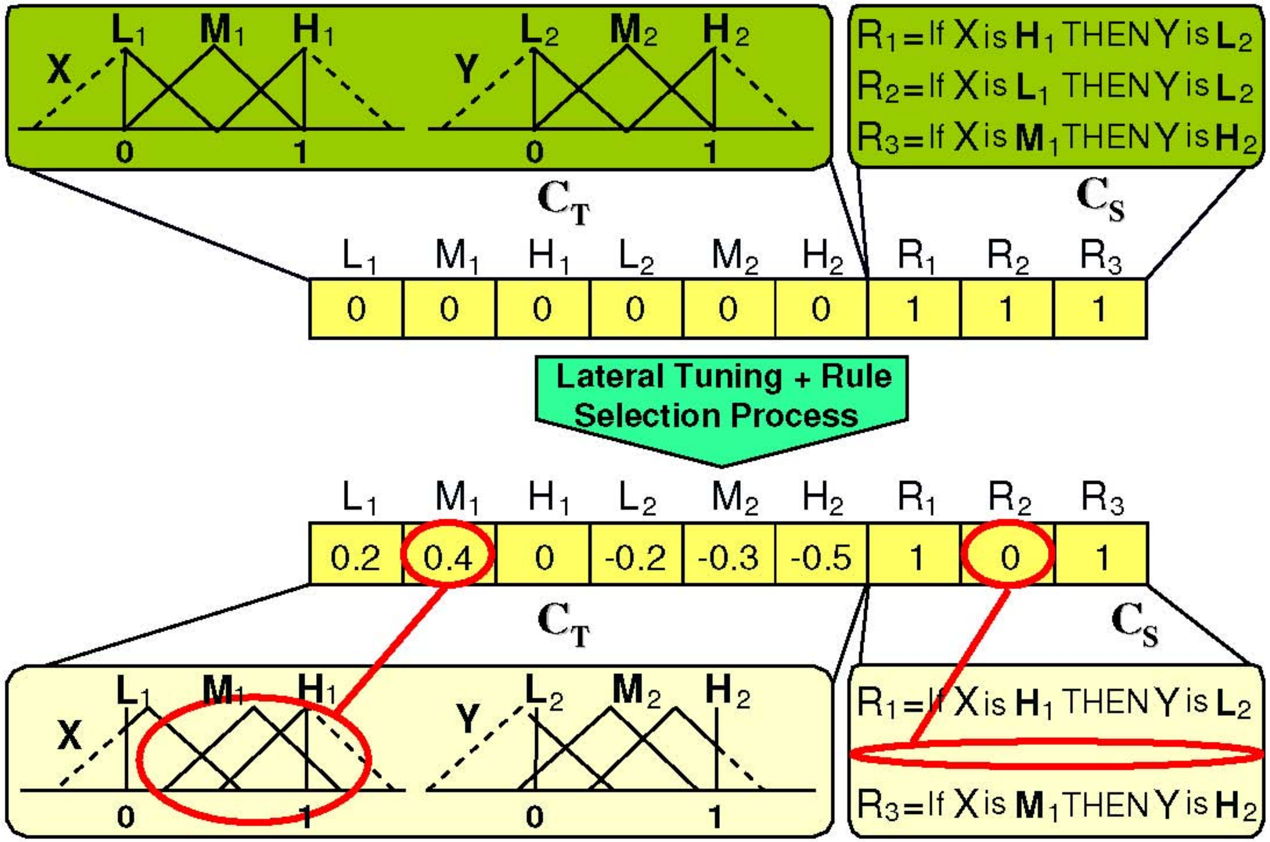
- **The Lateral Tuning is combined with a Rule Selection**
  - A doble coding scheme is considered with the joining of the selection binary values and the lateral parameters

$$\mathbf{C = C_S\ C_T}$$



b) Lateral Displacement of a Membership function

# MOEFSs for Fuzzy Control of HVAC Systems:
# Lateral Tuning + Rule Selection




**Example of genetic lateral tuning and rule selection**

# MOEFSs for Fuzzy Control of HVAC Systems: An Improved MOEA: SPEA2$_{E/E}$

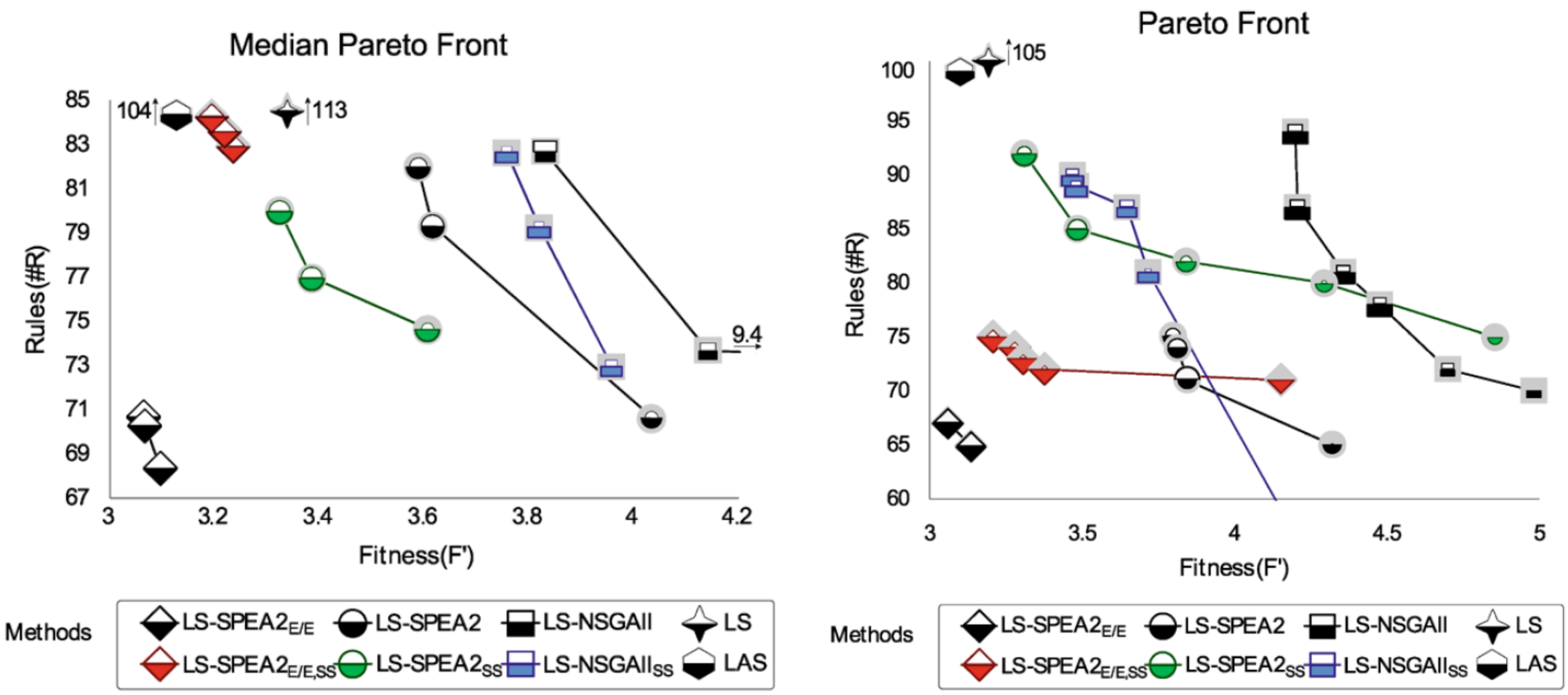
- Since the experts were able to provide trusted weights, performance criteria have been combined into a single function F. Thus the objectives are:
  - Minimization of F (to improve the performance)
  - Minimization of the number of rules (to favour the tuning efficiency)

- The following mechanisms or operators have been integrated into the well-known SPEA2 algorithm to improve the Exploration/Exploitation trade-off
  - An incest prevention mechanism as the well-known CHC algorithm
  - Automatic restarting aplication to avoid local optima
  - Progressive concentration on the most accurate solutions for parent selection
  - An intelligent crossover operator

# MOEFSs for Fuzzy Control of HVAC Systems: RESULTS

| Method | #R | $F'$ | PMV $F$ | PMV $M_1$ | PMV $M_2$ | $CO_2$ $M_3$ | Energy $M_4$ | Energy % | Stability $M_5$ | Stability % |
|---|---|---|---|---|---|---|---|---|---|---|
| Initial controllers | | | | | | | | | | |
| ON-OFF | – | 6.58 | 6.58 | 0.0 | 0 | 0 | 3206400 | – | 1136 | – |
| Initial controller | 172 | 5.69 | 6.32 | 0.0 | 0 | 0 | 2901686 | 9.5 | 1505 | −32.5 |
| Mono-objective steady-state genetic algorithms | | | | | | | | | | |
| S | 160 | 5.91 | 6.15 | 0.1 | 0 | 0 | 2886422 | 10.0 | 1312 | −15.5 |
| T | 172 | 4.55 | 5.71 | 0.0 | 0 | 0 | 2586717 | 19.3 | 1081 | 4.8 |
| TS | 109 | 4.36 | 5.66 | 0.1 | 0 | 0 | 2536849 | 20.9 | 1057 | 7.0 |
| W | 172 | 5.37 | 5.88 | 0.1 | 0 | 1 | 2783010 | 13.2 | 1202 | 5.8 |
| WS | 109 | 4.95 | 5.64 | 0.6 | 0 | 0 | 2755851 | 14.1 | 949 | 16.5 |
| L | 172 | 3.75 | 4.97 | 0.9 | 0 | 0 | 2325093 | 27.5 | 1072 | 5.7 |
| LS | 113 | 3.35 | 4.69 | 0.7 | 0 | 0 | 2287993 | 28.6 | 800 | 29.6 |
| LA | 172 | 3.23 | 4.61 | 0.9 | 0 | 0 | 2245812 | 30.0 | 797 | 29.8 |
| LAS | 104 | 3.14 | 4.50 | 0.8 | 0 | 0 | 2253996 | 29.7 | 634 | 44.2 |
| Multi-objective evolutionary algorithms | | | | | | | | | | |
| LS-NSGA-II | 82.7 | 3.830 | 4.909 | 0.5 | 0 | 1.3 | 2480182 | 22.6 | 636 | 44.0 |
| LS-NSGA-II$_A$ | **69.3** | 3.964 | 5.003 | 0.7 | 0 | 0 | 2502374 | 21.9 | 706 | 37.8 |
| LS-NSGA-II$_U$ | 71.3 | 4.304 | 5.264 | 0.6 | 0 | 0 | 2562149 | 20.1 | 909 | 19.9 |
| LS-SPEA2 | 82 | 3.587 | 4.830 | 0.8 | 0 | 0 | 2373620 | 26.0 | 780 | 31.3 |
| LS-SPEA2$_{ACC}$ | 96.3 | 3.383 | 4.708 | 1.0 | 0 | 0 | 2264251 | 29.4 | 874 | 23.0 |
| LS-SPEA2$_{E/E}$ | 70.7 | **3.064** | **4.412** | 0.9 | 0 | 0 | 2231310 | **30.4** | 564 | **50.3** |

# MOEFSs for Fuzzy Control of HVAC Systems: Pareto Fronts Obtained




The obtained fronts are not so wide but they dominate the remaining wider ones

# Contents

# MOEFSs for Fuzzy Association Rule Mining
## Fuzzy Association Rule Mining

- **Predictive induction: Induces rule sets acting as classifiers for solving classification and prediction tasks**
- **Descriptive induction: Discovers individual rules describing interesting regularities in the data**

**Therefore: Different goals, different heuristics, different evaluation criteria**

- One way to represent knowledge extracted with data mining techniques is by means of association rules, whose basic concept is to represent associations (simultaneity and not causality) between different pairs of sets of attribute values

**The use of fuzzy sets to describe associations between data:**

- **extends the types of relationships that may be represented,**
- **facilitates the interpretation of rules in linguistic terms, and**
- **avoids unnatural boundaries in the partitioning of the attribute domains**

# MOEFSs for Fuzzy Association Rule Mining

## Bibliography on this category

| Fuzzy association rule mining | | | Objectives | | MOEA | | |
|---|---|---|---|---|---|---|---|
| Authors | Ref. | Year | #Obj. | Description | Name | Gen. | Type |
| Kaya et al. | [115] | 2006 | 3 | ↑Sup. + ↑Con. + ↓Att. | NoN. | 2nd | N |
| Alhajj et al. | [116] | 2008 | 2 | ↑LI + ↓Tim. | NoN. | 2nd | I • |
| Chen et al. | [117] | 2008 | 2 | ↑L1I + ↑Sui. | NoN. | 1st | I ∘ |
| Thilgam et al. | [118] | 2008 | 2 | ↑Sup. + ↑Con. | MOGA | 1st | G |
| Casillas et al. | [119] | 2009 | 3 | ↓Err. + ↓DNF-FR + ↓MAM-FR | NoN. | 2nd | I † |
| Carmona et al. | [120] ∗ | 2010 | 3 | ↑Sup. + ↑FCon. + ↑Unu. | NMEEF-SD | 2nd | I † |

∗Applied for Subgroup Discovery;
Con.=Confidence, Sup.=Support, Tim.=Time, Err.=Error, LI=#Large itemsets, L1I=#Large 1-itemsets, Att.=#Attributes, Sui.=Suitability, DNF-FR=#DNF-type Fuzzy Rules, MAM-FR=#Equivalent Mamdani-type Fuzzy Rules, Unu.=Unusualness, FCon.=Fuzzy confidence, ↑Maximize, ↓Minimize, NoN.=No name, N=Novel algorithm, I=Improved version, G=General use; †NSGA-II based, ⋆PAES based, ∘MOGA based, •SPEA based.

With respect to the multiobjective nature in this category, the aim of the optimization process is not only to improve the general trade-off between the usual metrics of the data mining for the whole set of rules, but also to obtain a large number of different rules, each of them satisfying the objectives to different degrees.

- In most cases, the classical measures of data mining, support and confidence, are used as objectives
- The application of MOEAs to extract fuzzy association rules is quite recent, beginning in 2006
- Therefore, the majority of works exploit a 2nd-generation MOEA

**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65.**

# MOEFSs for Fuzzy Association Rule Mining

## An example on Subgroup Discovery

| Fuzzy association rule mining | | | Objectives | | MOEA | | |
|---|---|---|---|---|---|---|---|
| Authors | Ref. | Year | #Obj. | Description | Name | Gen. | Type |
| Kaya et al. | [115] | 2006 | 3 | ↑Sup. + ↑Con. + ↓Att. | NoN. | 2nd | N |
| Alhajj et al. | [116] | 2008 | 2 | ↑LI + ↓Tim. | NoN. | 2nd | I • |
| Chen et al. | [117] | 2008 | 2 | ↑L1I + ↑Sui. | NoN. | 1st | I ∘ |
| Thilgam et al. | [118] | 2008 | 2 | ↑Sup. + ↑Con. | MOGA | 1st | G |
| Casillas et al. | [119] | 2009 | 3 | ↓Err. + ↓DNF-FR + ↓MAM-FR | NoN. | 2nd | I † |
| Carmona et al. | [120] ∗ | 2010 | 3 | ↑Sup. + ↑FCon. + ↑Unu. | NMEEF-SD | 2nd | I † |

∗Applied for Subgroup Discovery;
Con.=Confidence, Sup.=Support, Tim.=Time, Err.=Error, LI=#Large itemsets, L1I=#Large 1-itemsets, Att.=#Attributes, Sui.=Suitability, DNF-FR=#DNF-type Fuzzy Rules, MAM-FR=#Equivalent Mamdani-type Fuzzy Rules, Unu.=Unusualness, FCon.=Fuzzy confidence, ↑Maximize, ↓Minimize, NoN.=No name, N=Novel algorithm, I=Improved version, G=General use; †NSGA-II based, ⋆PAES based, ∘MOGA based, •SPEA based.

In the following we will see a representatibe example for Subgroup Discovery on Databases

**Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera.** ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****. IEEE Transactions on Fuzzy Systems 21(1) (2013) 45-65.**

# MOEFSs for Subgroup Discovery
## How does subgroup discovery work?

Subgroup discovery is a process to identify relations between a dependent variable (target variable) and usually many explaining, independent variables.

For example, consider the subgroup described by

”smoker=true AND family history=positive”

for the target variable coronary heart disease=true.

Subgroup discovery does not necessarily focus on finding complete relations; instead partial relations, i.e., (small) subgroups with ”interesting” characteristics can be sufficient.

# MOEFSs for Subgroup Discovery
## NMEEF-SD

- **Non-dominated Multi-objective Evolutionary algorithm based on Fuzzy rules extraction for Subgroup Discovery (NMEEF-SD)**

C. J. Carmona, P. González, M. J. del Jesus, and F. Herrera,

"NMEEF-SD: Non-dominated Multiobjective Evolutionary Algorithm for Extracting Fuzzy Rules in Subgroup Discovery",

IEEE Transactions on Fuzzy Systems, vol. 18, no. 5, pp. 958–970, 2010

## MOEFSs for Subgroup Discovery
### NMEEF-SD

- **Each candidate solution is codified according to the “Chromosome = Rule“ approach, where only the antecedent is represented**

- **NMEF-SD is able to work with crisp or fuzzy rules**

- **The fuzzy logic:**
  - **Is used in continuous variables**
  - **Linguistic labels are defined by means of the corresponding membership functions**
  - **Defines uniform partitions with triangular membership functions**

# MOEFSs for Subgroup Discovery
## NMEEF-SD

- **NMEEF-SD can extract canonical or DNF rules.**
  - **For the canonical rules, only the antecedent is represented through a conjunction of value-variable pairs.**

**IF $X_1$ = Value$_3$ AND $X_4$ = $LL_4^2$ THEN *Class 2***

| $X_1$ | $X_1$ | $X_1$ | $X_1$ |
|---|---|---|---|
| 3 | 0 | 0 | 2 |

  - **For the DNF rules extension, a fixed-length binary representation is used**

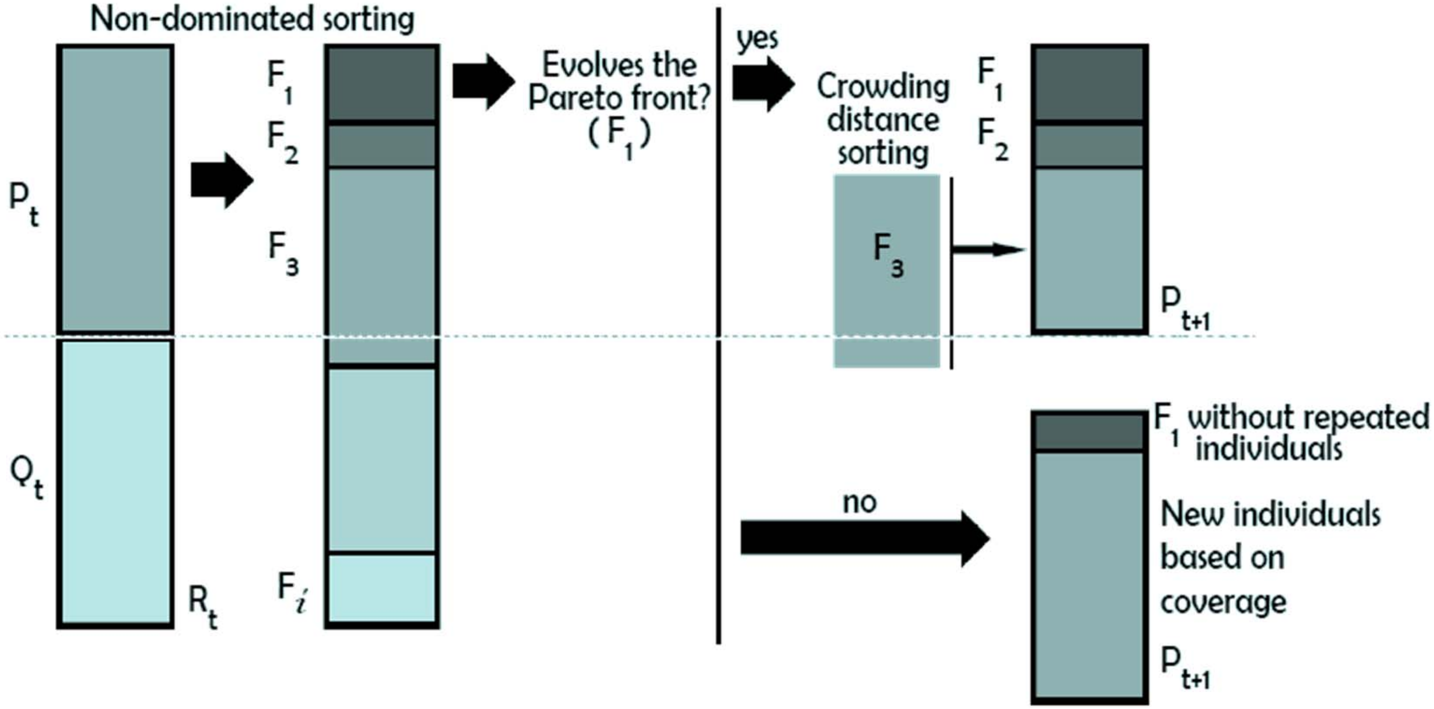
**IF $X_1$ = (Value$_1$ OR Value$_3$) AND $X_3$ = $LL_3^1$ THEN *Class 2***

| $X_1$ | | | | | $X_2$ | | | $X_3$ | | | | | $X_4$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

# MOEFSs for Subgroup Discovery
## NMEEF-SD

- **This algorithm is based on NSGA-II approach.**
- **The quality measures selected as objectives:**
  - **Support ($Sup_cN$)**
  - **Unusualness ($WRA_{cc}$)**

### Operation diagram of NMEEF-SD



# MOEFSs for Subgroup Discovery
## NMEEF-SD

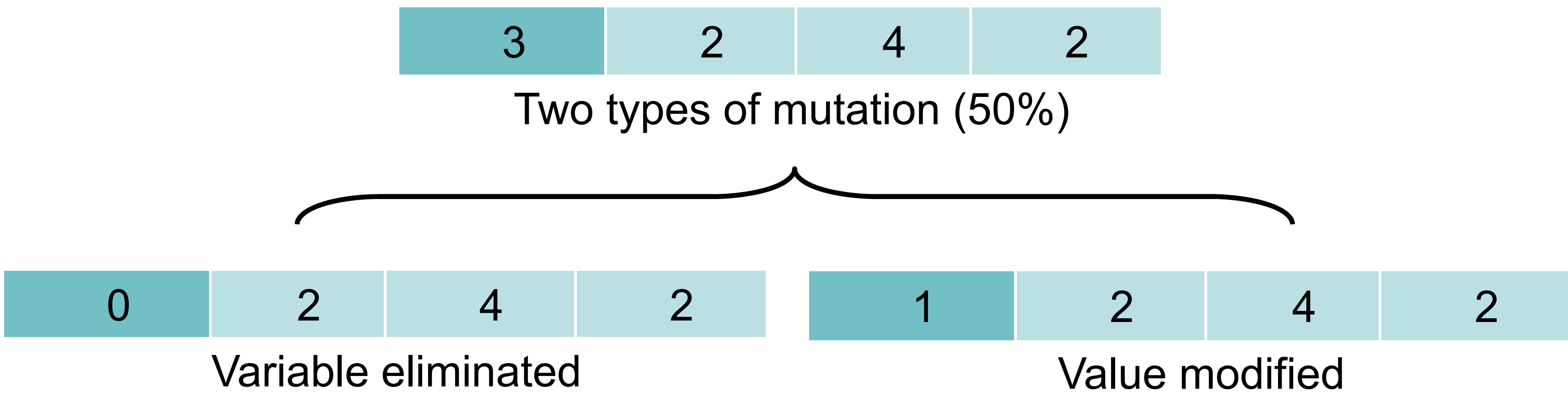
### Biased initialisation

- **To create an initial population whose size is prefixed by an external parameter.**
- **A part of the population (75%) using only a maximum percentage of the variables (25% of the rule) which form part of the rule.**
- **The rest of variables and rules of the population are randomly generated.**
- **This operator obtains a set of rules with a high generality.**

# MOEFSs for Subgroup Discovery
## NMEEF-SD

### Genetic operators

- **The algorithm uses different operators:**
  - **Tournament Selection**
  - **Multi-point Crossover**
  - **Biased Mutation**



# MOEFSs for Subgroup Discovery
## NMEEF-SD

### Fast non-dominated sort

- **The algorithm joins two populations in only one:**
  - **Initial population**
  - **Offspring population**

- **The algorithm applies the fast non-dominated sort over the population obtained previously.**

- **The individuals of the population are classified in fronts of dominance.**

- **The first front is the Pareto front.**

- **The algorithm obtains diversity with the operator of crowding distance.**

# MOEFSs for Subgroup Discovery
## NMEEF-SD

### Re-Initialisation based on coverage

- **When the algorithm obtains the fronts of dominance checks the evolution of the Pareto front.**

**If the Pareto front evolves during more than five percent of the evolutive process**

DOES NOT EVOLVE

1. Eliminates the individuals repeated in the Pareto front.
2. Replaces these individuals with new individuals generated based on coverage.

EVOLVES

1. Introduce the fronts in the next population.
2. If the front has more individuals than can enter in the population, the inviduals are introduced by greater crowding distance.

# MOEFSs for Subgroup Discovery

## NMEEF-SD

### Stop condition

- **The evolutionary process ends when the number of evaluations is reached.**
- **The algorithm returns the rules in the Pareto front which reach a predefined fuzzy confidence value threshold.**
- **The fuzzy confidence is defined in:**

M.J. Del Jesus, P. González, F. Herrera, M. Mesonero

Evolutionary fuzzy rule induction process for subgroup discovery: a case study in Marketing

IEEE Transactions on Fuzzy Systems, Vol. 15 (4), 2007, pp. 578-592.

# MOEFSs for Subgroup Discovery
## NMEEF-SD

### Experimentation

- **Different data sets available in UCI repository has been carried out: Australian, Balance, Echo and Vote.**
  **http://www.ics.uci.edu/~mlearn/MLRepository.html**

- **Ten fold cross validation**

- **Algorithms compared:**
  - **Evolutionary algorithms SDIGA and MESDIF.**
  - **Classical methods CN2-SD and Apriori SD.**

- **Parameters for NMEF-SD:**
  - **Population size: 25**
  - **Maximum number of evaluations: 5000**
  - **Crossover probability 0.60 and mutation probability 0.01**

# MOEFSs for Subgroup Discovery

## NMEEF-SD

## Experimentation

| Database | Algorithm | Rul | Var | COV | SIGN | $WRA_{cc}$ | $SUP_cN$ | FCNF |
|---|---|---|---|---|---|---|---|---|
| Australian | **NMEF-SD** | 3.58 | 2.92 | **0.454** | **23.178** | **0.171** | **0.783** | **0.930** |
| | MESDIF | 10.00 | 3.52 | 0.311 | 7.594 | 0.060 | 0.577 | 0.807 |
| | SDIGA | **2.68** | 3.28 | 0.310 | 16.348 | 0.120 | 0.803 | 0.591 |
| | CN2-SD | 30.50 | 4.58 | 0.400 | 15.350 | 0.055 | 0.649 | 0.830 |
| | AprioriSD | 10.00 | **2.02** | 0.377 | 16.998 | 0.074 | 0.654 | 0.863 |
| Balance | **NMEF-SD** | **2.30** | 2.00 | **0.362** | 5.326 | **0.070** | **0.530** | **0.698** |
| | MESDIF | 28.10 | 3.08 | 0.163 | 3.516 | 0.022 | 0.318 | 0.557 |
| | SDIGA | 7.40 | 2.39 | 0.291 | 5.331 | 0.049 | 0.487 | 0.664 |
| | CN2-SD | 15.60 | 2.23 | 0.336 | **8.397** | 0.063 | 0.512 | 0.583 |
| | AprioriSD | 10.00 | **1.20** | 0.333 | 5.444 | 0.058 | 0.480 | 0.649 |
| Echo | **NMEF-SD** | 3.62 | 2.35 | **0.428** | **1.293** | **0.043** | **0.628** | **0.757** |
| | MESDIF | 19.74 | 3.30 | 0.164 | 0.877 | 0.017 | 0.355 | 0.591 |
| | SDIGA | **2.32** | 2.27 | 0.394 | 1.165 | 0.013 | 0.566 | 0.590 |
| | CN2-SD | 17.30 | 3.23 | 0.400 | 1.181 | 0.019 | 0.490 | 0.667 |
| | AprioriSD | 9.80 | **1.70** | 0.194 | 0.901 | 0.034 | 0.226 | 0.510 |
| Vote | **NMEF-SD** | **1.10** | 2.05 | **0.577** | **21.974** | **0.217** | **0.946** | **0.979** |
| | MESDIF | 7.86 | 3.44 | 0.429 | 19.937 | 0.187 | 0.827 | 0.957 |
| | SDIGA | 3.06 | 3.19 | 0.422 | 18.243 | 0.180 | 0.802 | 0.891 |
| | CN2-SD | 8.00 | 1.79 | 0.438 | 18.830 | 0.176 | 0.858 | 0.932 |
| | AprioriSD | 10.00 | **1.44** | 0.428 | 17.060 | 0.147 | 0.800 | 0.930 |

# MOEFSs for Subgroup Discovery
## NMEEF-SD

### Experimentation

- **When analysing the results is important to take into account:**
  - **The relation between Support and Confidence.**
  - **Good results in the quality measures of Subgroup Discovery: Unusualness and Significance.**
  - **A good interpretability of the results.**

- **NMEF-SD obtains:**
  - **The best results for the quality measures in the data sets selected.**
  - **Better results in generality and precision than others.**
  - **The subgroups are good, useful and representative.**

# Contents

# Current and Future Research Directions in MOEFSs

## 1) Development of New MGFS Methods with Improved Algorithms

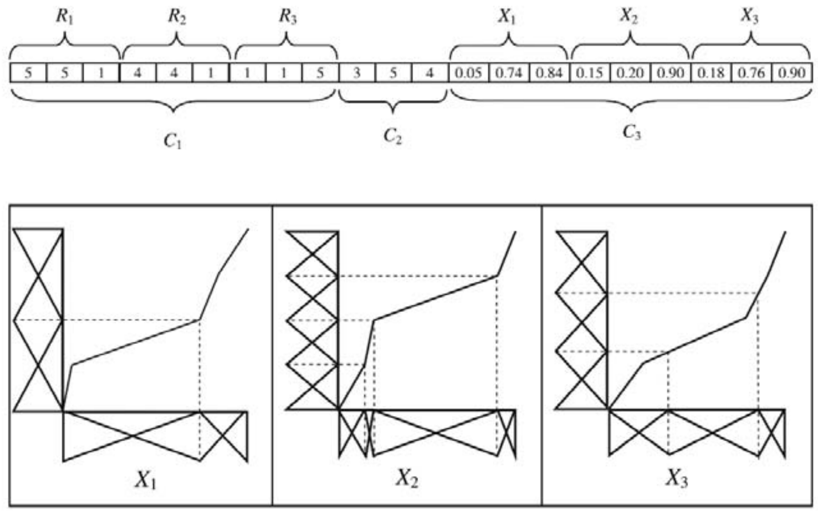
- **Particular algorithms for multiobjective input selection**
- **Particular algorithms for multiobjective fuzzy partition learning**
- **. . .**

**An example for learning granularities and selecting conditions can be found in:**

*M. Antonelli, P. Ducange, B. Lazzerini, and F. Marcelloni, "Learning concurrently partition granularities and rule bases of Mamdani fuzzy systems in a multi-objective evolutionary framework," Int. J. Approx. Reason., vol. 50, n. 7, pp. 1066–1080, 2009.*

*M. Antonelli, P. Ducange, B. Lazzerini, and F. Marcelloni, "Multi-objective evolutionary learning of granularity, membership function parameters and rules of Mamdani fuzzy systems," Evolutionary Intelligence, vol. 2, n. 1-2, pp. 21–37, 2009.*

Exploiting the concept of virtual partitions with modified PAES



# Current and Future Research Directions in MOEFSs (2)

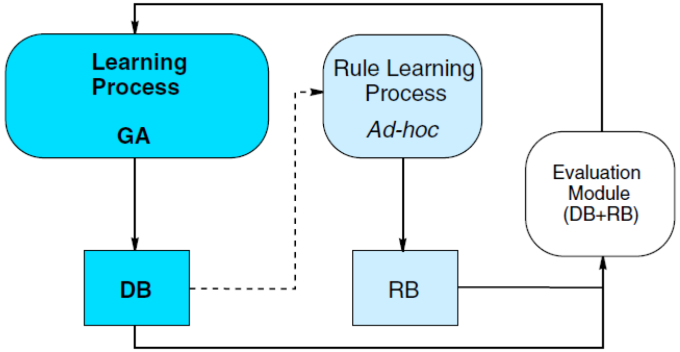
## 1) Development of New MGFS Methods with Improved Algorithms (2)

**An example for learning granularities and for selecting variables can be found in:**

*R. Alcala, M. J. Gacto, and F. Herrera, "A Fast and Scalable Multi-Objective Genetic Fuzzy System for Linguistic Fuzzy Modeling in High-Dimensional Regression Problems," IEEE Transactions on Fuzzy Systems 19:4 (2011) 666-681, doi: 10.1109/TFUZZ.2011.2131657*

Exploiting the embedded learning of the DB with improved SPEA2

$C_1 = (L^1, \ldots, L^N) \quad C_2 = (\alpha^1, \ldots, \alpha^N)$




## 2) Performance evaluation of MOGFSs

- **Visualization of Pareto-Optimal Fuzzy Systems**
- **How to compare MGFSs**
  - A statistical Analysis is needed
  - Use of non-parametric statistical tests

Evaluation indexes in the EMO framework evaluate the exploration and exploitation capabilities of the MOEA. But we are also interested in generalization capabilities of the FRBSs

# Current and Future Research Directions in MOEFSs (3)

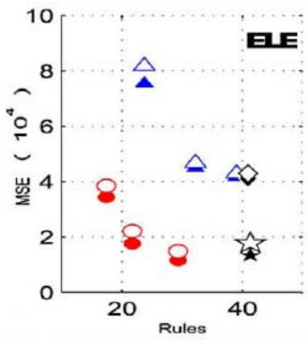
## 2) Performance evaluation of MOGFSs

- **How to compare MGFSs**

**A recent possibility to apply non-parametric statistical tests:**

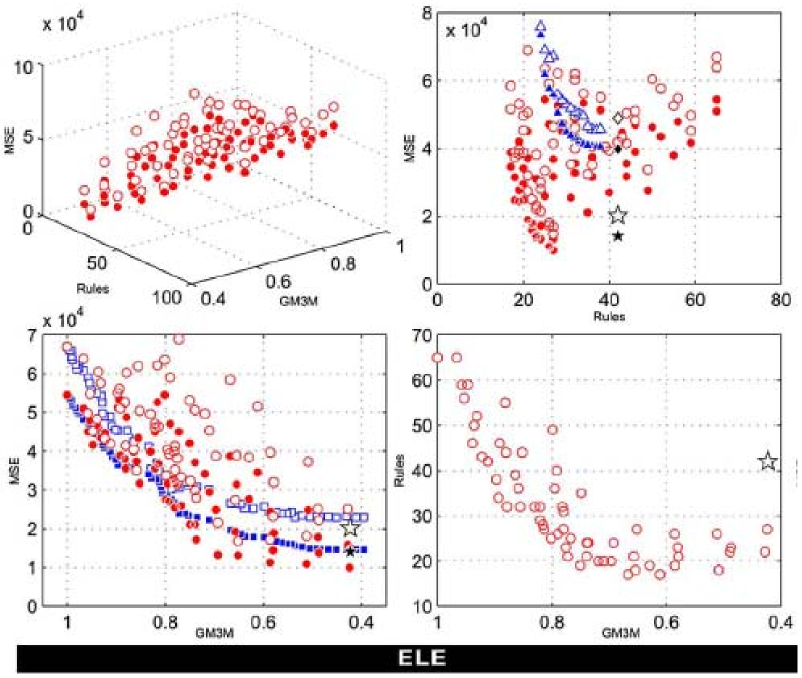
*R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, and F. Marcelloni, "A Multi-objective evolutionary approach to concurrently learn rule and data bases of linguistic fuzzy rule-based systems," IEEE Trans. Fuzzy. Syst., vol. 17, n. 5, pp. 1106–1122, 2009.*

Analyzing the averages on three representative points by non-parametric statistical tests for bi-objective problems (FIRST, MEDIAN, LAST)




**An extension for the case of more than two objectives:**

*M. J. Gacto, R. Alcala, and F. Herrera, "Integration of an index to preserve the semantic interpretability in the multi-objective evolutionary rule selection and tuning of linguistic fuzzy systems," IEEE Trans. Fuzzy. Syst. vol. 18, n.3, pp. 515-531, 2010.*




Projections on bi-objective planes.Then, representative points can be obtained in the new non-dominated solutions

# Current and Future Research Directions in MOEFSs (4)

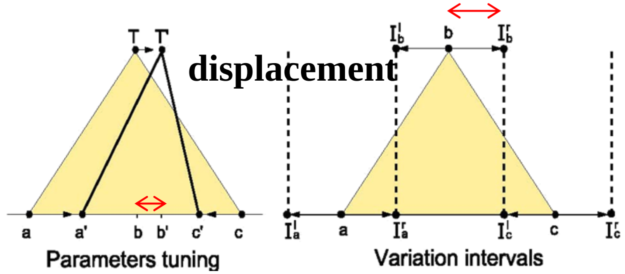
## 3) Reliable Interpretability Measures *(Formulations of the Interpretability)*

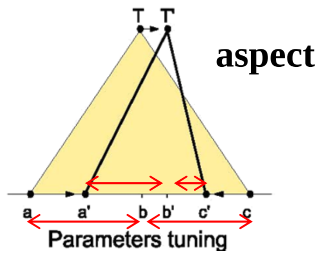
- **We need well established and accepted measures**
- **Use of new ones for C3 (semantic-RB) as cointension or number of fired rules**

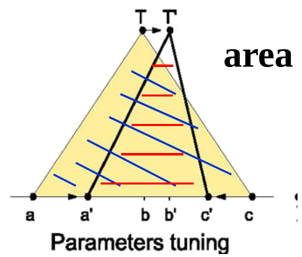
**The use of relative measures for C4 (semantic-DB) could be promising. First proposal in:**

*M. J. Gacto, R. Alcala, and F. Herrera, "Integration of an index to preserve the semantic interpretability in the multi-objective evolutionary rule selection and tuning of linguistic fuzzy systems," IEEE Trans. Fuzzy. Syst. vol. 18, n.3, pp. 515-531, 2010.*

Measuring the differences to a given linguistic partition (obtained from experts or automatically by using absolute measures): GM3M index based on three metrics




**Some recent approaches are also using this kind of measures:**

*M. Antonelli, P. Ducange, B. Lazzerini, and F. Marcelloni, "Learning knowledge bases of multi-objective evolutionary fuzzy systems by simultaneously optimizing accuracy, complexity and partition integrity, Soft Computing vol. 15, n.12, pp. 2335–2354, 2011.*

# Current and Future Research Directions in MOEFSs (5)

## 4) Objective dimensionality

- New EMO algorithms
- Aggregation or selection of a reasonable set of significant measures

## 5) Scalability issues

- High Dimensinality (handling the length of the rules)
- Large scale problems (using a reduced subset of examples)

- **Some approaches dealing with large scale problems:**

• *M.A. de Vega, J.M Bardallo, F.A. Marquez, A. Peregrin, "Parallel distributed two-level evolutionary multiobjective methodology for granularity learning and membership functions tuning in linguistic fuzzy systems," in Proc. of ISDA 2009, pp. 134–139.* → ***Parallelization***

• *M. Cococcioni, B. Lazzerini, F. Marcelloni, "On reducing computational overhead in multi-objective genetic Takagi–Sugeno fuzzy systems," Appl. Soft Computing 11:1 (2011), 675-688.* → ***Fitness estimation***

• *M. Antonelli, P. Ducange, F. Marcelloni, "Exploiting a coevolutionary approach to concurrently select training instances and learn rule bases of Mamdani fuzzy dystems," in Proc. of WCCI 2010, 1366–1372.* → ***Instance Selection***

# Current and Future Research Directions in MOEFSs (6)

## 5) Scalability issues (2)

- **Some approaches dealing with high dimensional problems:**

• *H. Ishibuchi, and T. Yamamoto, "Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining," Fuzzy Sets and Systems, vol. 141, pp. 59–88, 2004.*

• *M. Antonelli, P. Ducange, B. Lazzerini, F. Marcelloni, "Multi-objective Evolutionary Generation of Mamdani Fuzzy Rule-Based Systems based on Rule and Condition Selection," in Proc. of GEFS 2011.*

***Imposing a maximum rule lenght***

***Condition selection by specific approach***

- **An approach dealing with both high dimensional and large scale problems:**

• *R. Alcala, M. J. Gacto, F. Herrera, "A Fast and Scalable Multi-Objective Genetic Fuzzy System for Linguistic Fuzzy Modeling in High-Dimensional Regression Problems," IEEE Trans. on Fuzzy Systems 19:4 (2011) 666-681.*

***Using a specific approach for variable selection and fitness stimation by using a short subset of the examples***

# Current and Future Research Directions in MOEFSs (7)

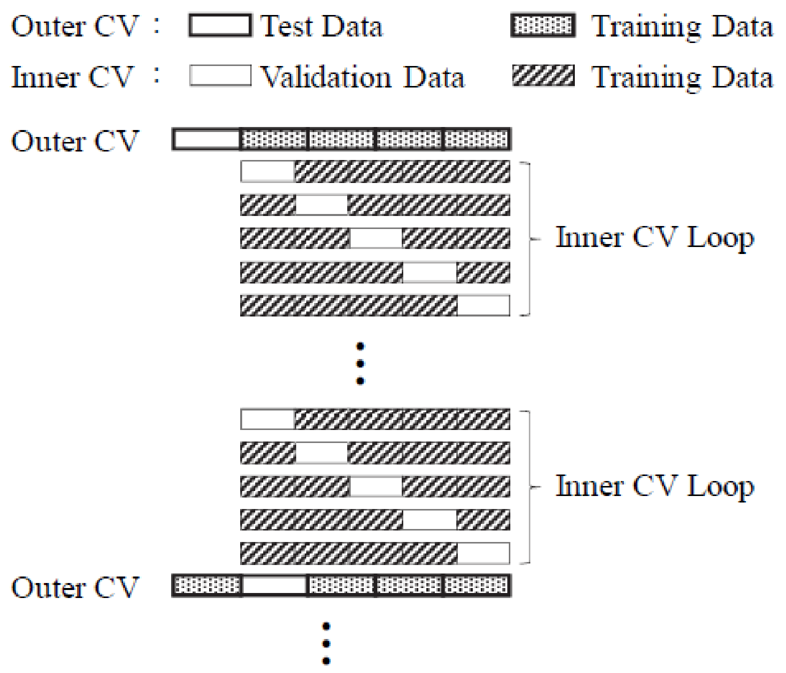
## 6) Automatic selection of the most suitable solution

- Determining those solutions with the best generalization ability
- Only training data can be took into account

■ **A recent approach on this topic:**

• *Ishibuchi H, Nakashima Y, Nojima Y, Double cross-validation for performance evaluation of multi-objective genetic fuzzy systems. In GEFS 2011, pp 31-38.*

***Using a double cross-validation with two cross-validation loops. The inner loop uses the training data to determine the complexity of the systems with the best validation measure, which is used to select the solutions used for the outer loop.***



FUZZ-IEEE 2013 Tutorial, Hyderabad, India
Afternoon Session: 14:00-17:00, July 7, 2013

# Multi-Objective Evolutionary Fuzzy Systems: An Overview by Problem objectives nature and optimized components

Thank you very much for your attention !!!
Questions?

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