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Evolutionary Multi-Objective Design of Fuzzy Rule-Based Systems

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Introduction to genetic fuzzy systems

- **Brief Introduction**
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- **Why do we use GAs?**
- **The birth, GFSs roadmap, current state and most cited papers**

Introduction to genetic fuzzy systems
Brief Introduction

- The use of genetic/evolutionary algorithms (GAs) to design fuzzy systems constitutes one of the branches of the Soft Computing paradigm: genetic fuzzy systems (GFSs)

- The most known approach is that of genetic fuzzy rule-based systems, where some components of a fuzzy rule-based system (FRBS) are derived (adapted or learnt) using a GA

- Some other approaches include genetic fuzzy neural networks and genetic fuzzy clustering, among others

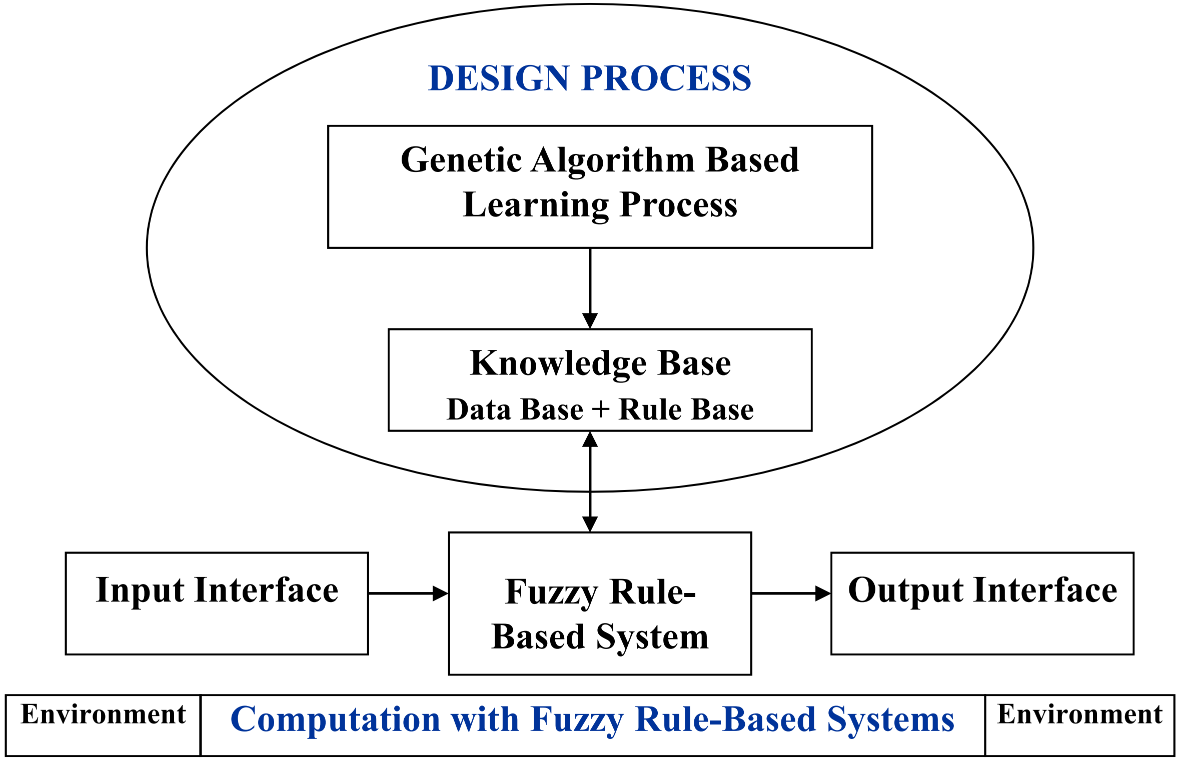
Introduction to genetic fuzzy systems
Brief Introduction

Evolutionary algorithms and machine learning:

- Evolutionary algorithms were not specifically designed as machine learning techniques, like other approaches like neural networks
- However, it is well known that a learning task can be modelled as an optimization problem, and thus solved through evolution
- Their powerful search in complex, ill-defined problem spaces has permitted applying evolutionary algorithms successfully to a huge variety of machine learning and knowledge discovery tasks
- Their flexibility and capability to incorporate existing knowledge are also very interesting characteristics for the problem solving.

Introduction to genetic fuzzy systems
Brief Introduction

Genetic Fuzzy Rule-Based Systems:



Introduction to genetic fuzzy systems
Brief Introduction

Design of fuzzy rule-based systems:

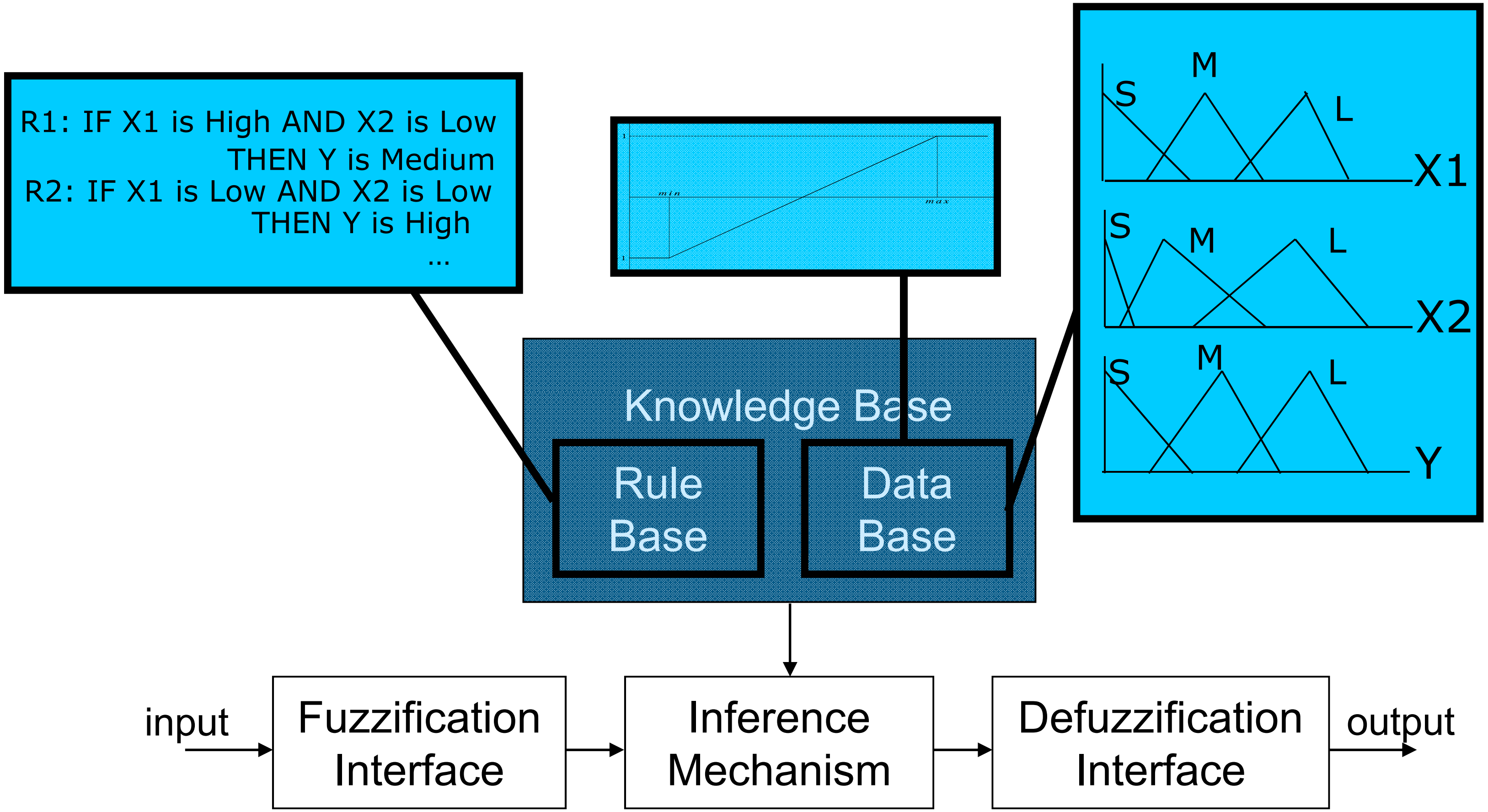
- An FRBS (regardless it is a fuzzy model, a fuzzy logic controller or a fuzzy classifier), is comprised by two main components:
  - The Knowledge Base (KB), storing the available problem knowledge in the form of fuzzy rules
  - The Inference System, applying a fuzzy reasoning method on the inputs and the KB rules to give a system output

- Both must be designed to build an FRBS for a specific application:
  - The KB is obtained from expert knowledge or by machine learning methods
  - The Inference System is set up by choosing the fuzzy operator for each component (conjunction, implication, defuzzifier, etc.)

    Sometimes, the latter operators are also parametric and can be tuned using automatic methods

Introduction to genetic fuzzy systems

Brief Introduction



An Example of Fuzzy rule-based system

Introduction to genetic fuzzy systems

Brief Introduction

The KB design involves two subproblems, related to its two subcomponents:

- **Definition of the Data Base (DB):**
  - **Variable universes of discourse**
  - **Scaling factors or functions**
  - **Granularity (number of linguistic terms/labels) per variable**
  - **Membership functions associated to the labels**
- **Derivation of the Rule Base (RB): fuzzy rule composition**

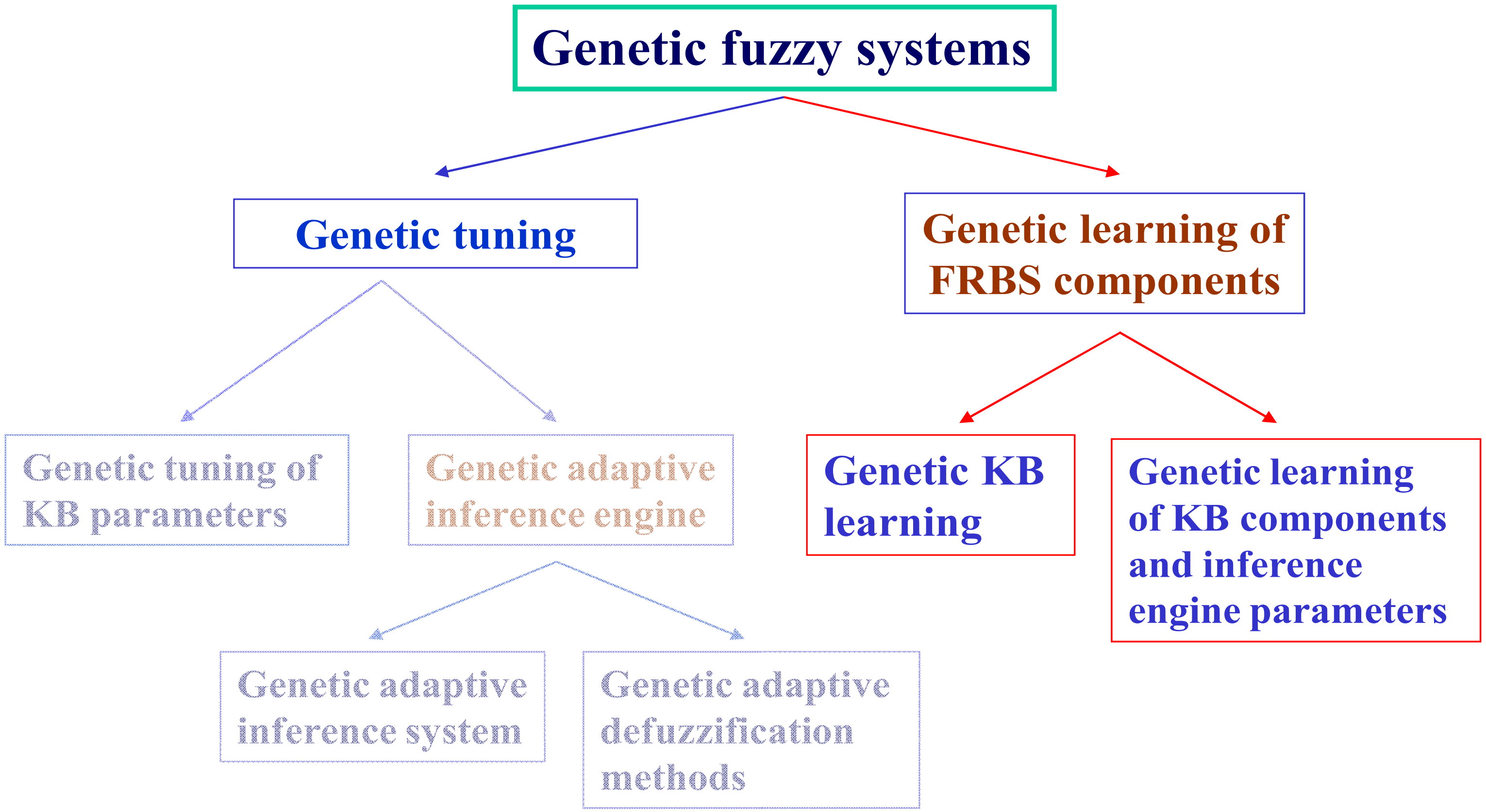
Introduction to genetic fuzzy systems
Brief Introduction

As said, there are two different ways to design the KB:

- **From human expert information**
- **By means of machine learning methods guided by the existing numerical information (fuzzy modeling and classification) or by a model of the system being controlled**

Introduction to genetic fuzzy systems

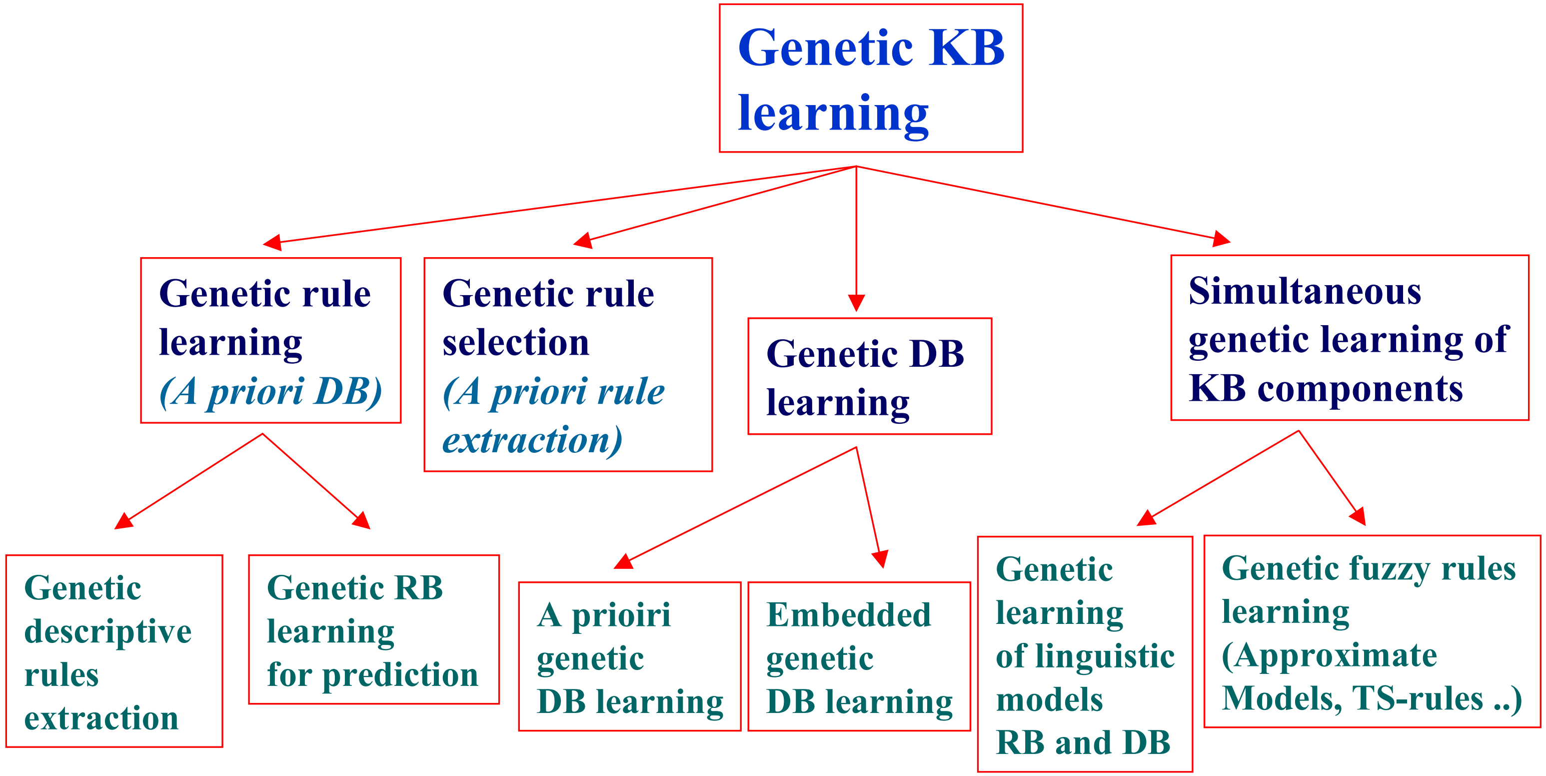
Taxonomy of Genetic Fuzzy Systems



F. Herrera, **Genetic Fuzzy Systems: Taxonomy, Current Research Trends and Prospects**. *Evolutionary Intelligence 1 (2008) 27-46* [doi: 10.1007/s12065-007-0001-5](doi: 10.1007/s12065-007-0001-5)

Associated Website: http://sci2s.ugr.es/gfs/

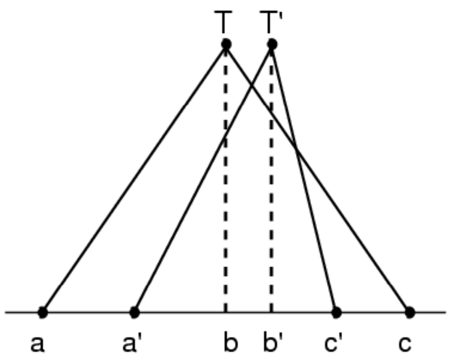
Introduction to genetic fuzzy systems
Taxonomy of Genetic Fuzzy Systems

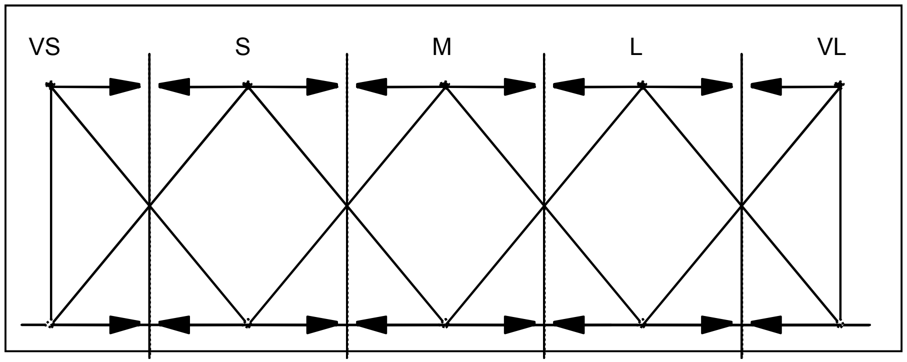


Introduction to genetic fuzzy systems
1. Genetic Tuning

Classically:

- **performed on a predefined DB definition**
- **tuning of the membership function shapes by a GA**

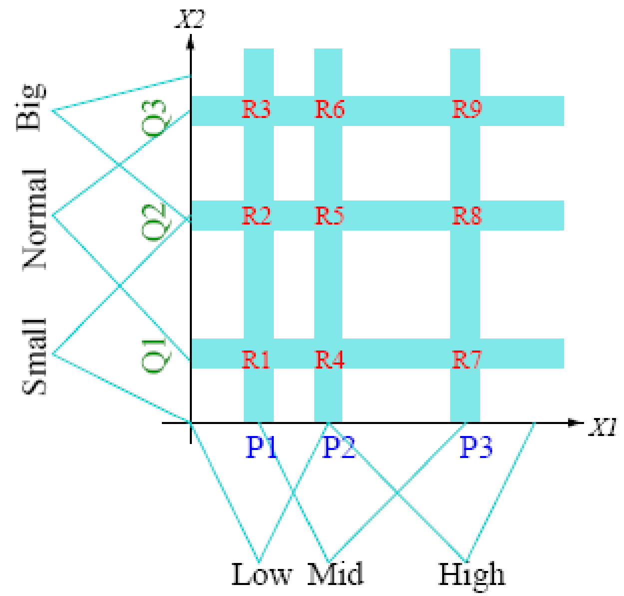


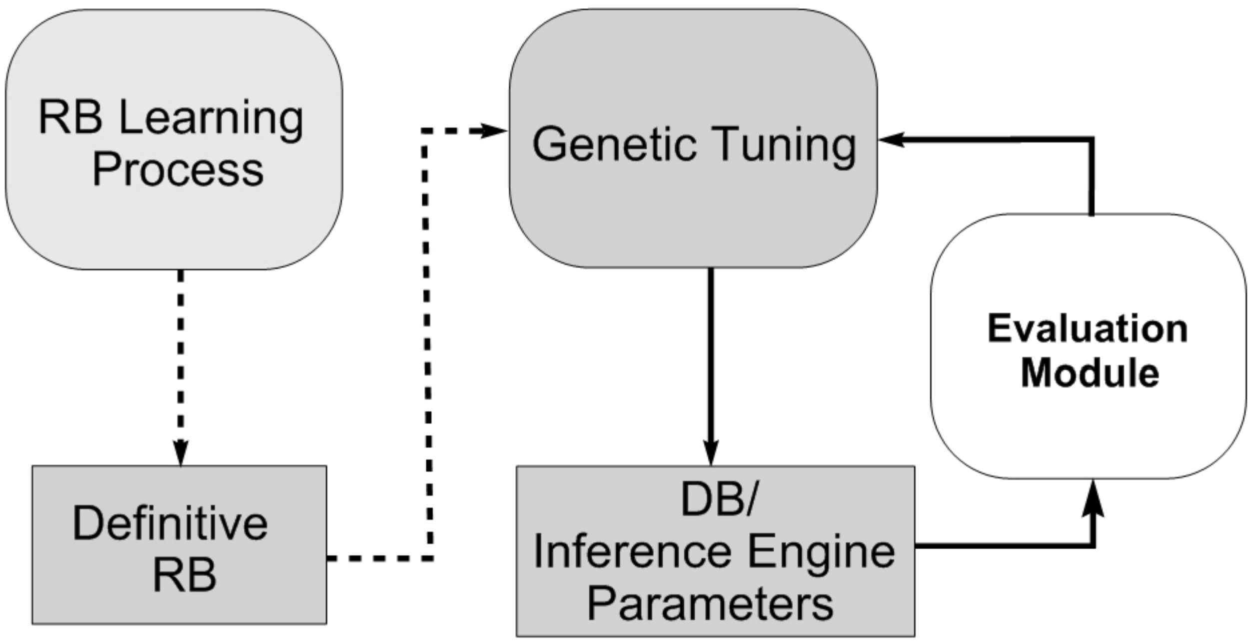


- **tuning of the inference parameters**

Introduction to genetic fuzzy systems

1. Genetic Tuning



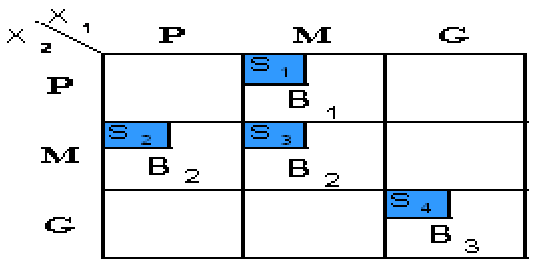


Introduction to genetic fuzzy systems

2. Genetic Rule Learning

A predefined Data Base definition is assumed

– **The fuzzy rules** **(usually Mamdani-type)** **are derived by a GA**



Rule Base

R_1 = IF X_1 is **M** and X_2 is **P** THEN Y is $\mathbf{B}_1$

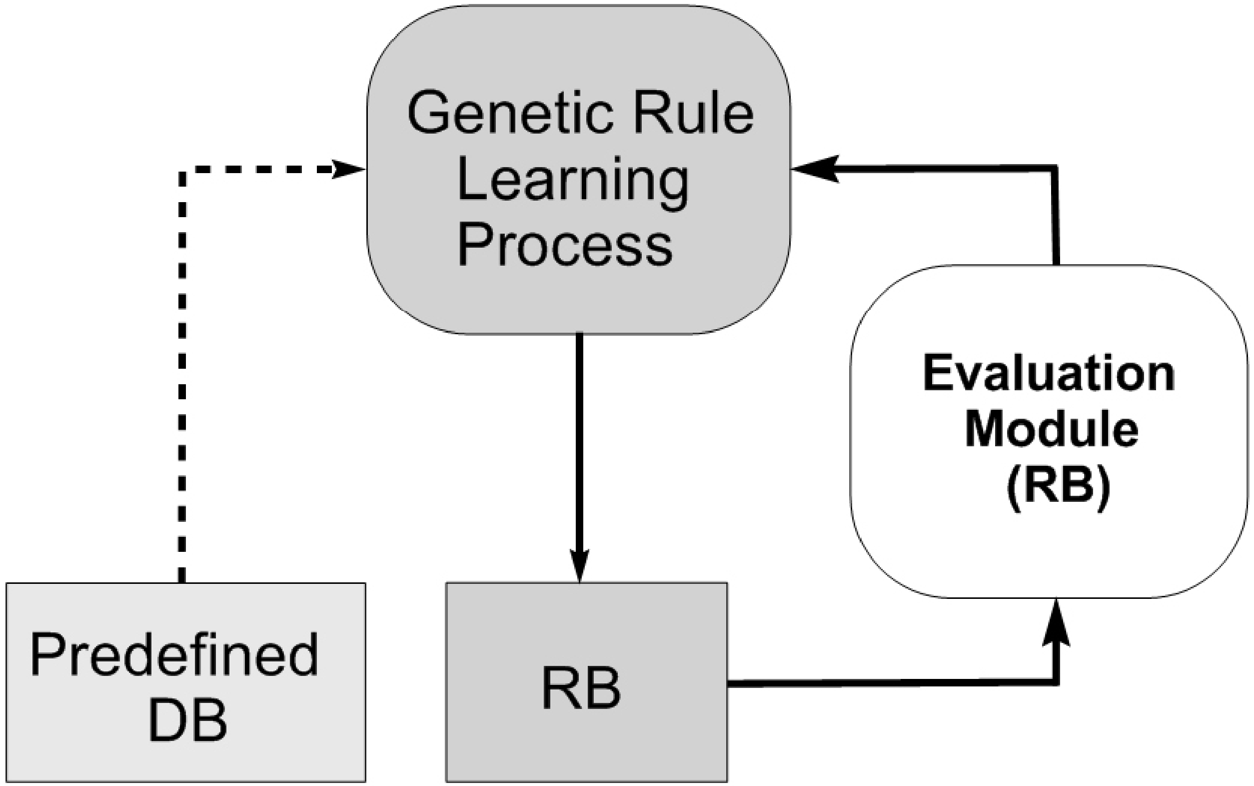
R_2 = IF X_1 is **P** and X_2 is **M** THEN Y is $\mathbf{B}_2$

R_3 = IF X_1 is **M** and X_2 is **M** THEN Y is $\mathbf{B}_2$

R_4 = IF X_1 is **G** and X_2 is **G** THEN Y is $\mathbf{B}_3$

Introduction to genetic fuzzy systems

2. Genetic Rule Learning



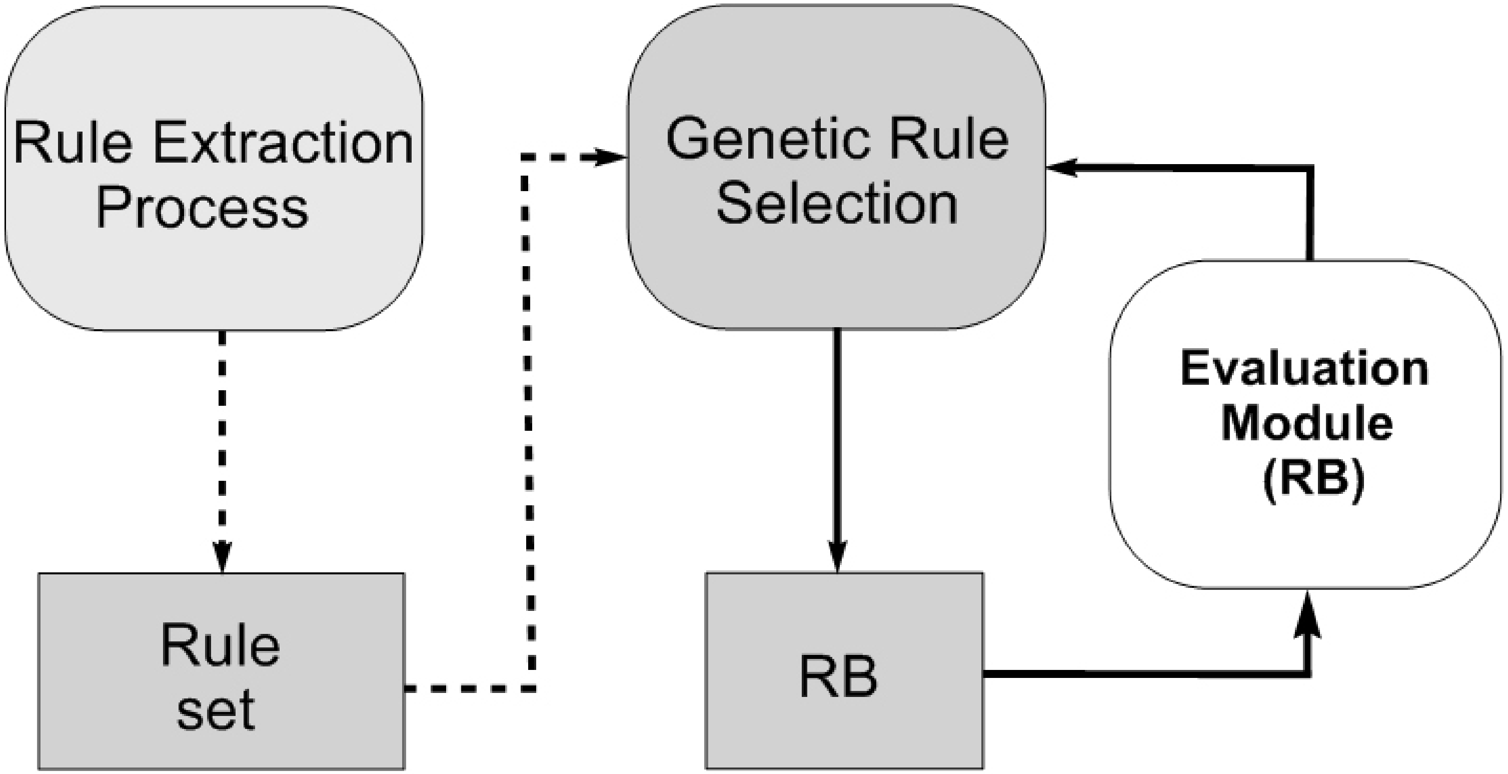
Introduction to genetic fuzzy systems

3. Genetic Rule Selection

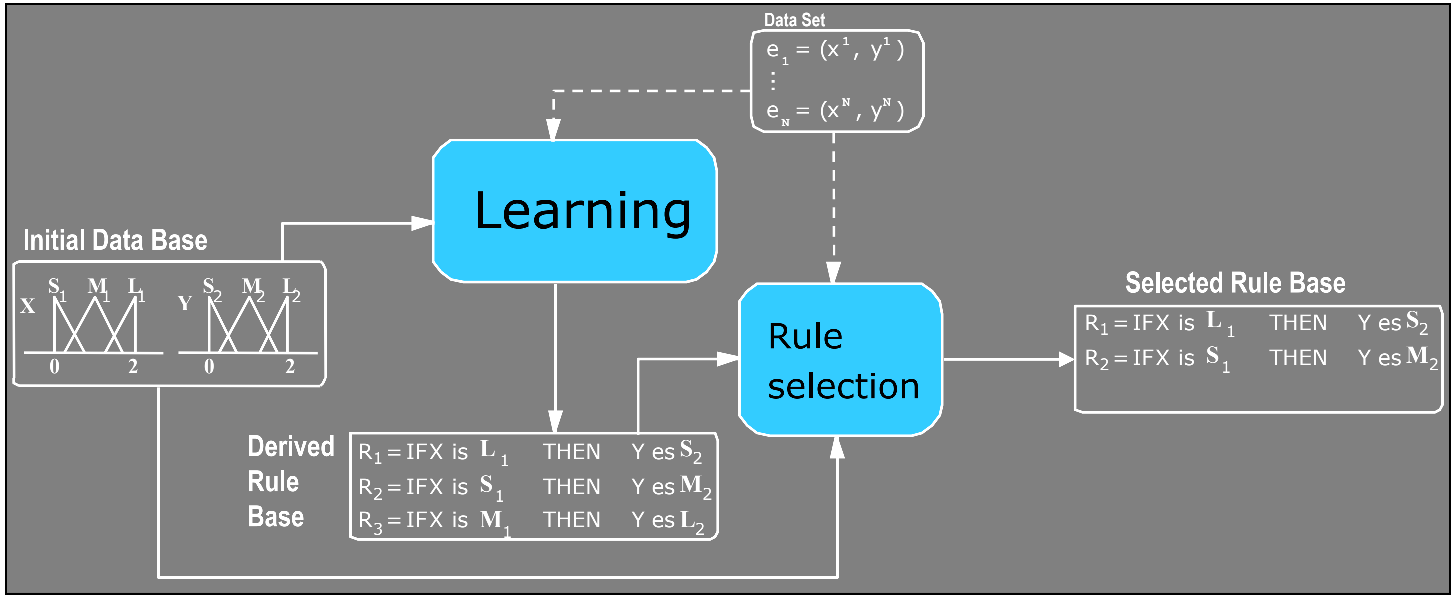
- **A predefined Rule Bases definition is assumed**
- **The fuzzy rules are selection by a GA for getting a compact rule base (more interpretable, more precise)**

Introduction to genetic fuzzy systems

3. Genetic Rule Selection



Introduction to genetic fuzzy systems
3. Genetic Rule Selection

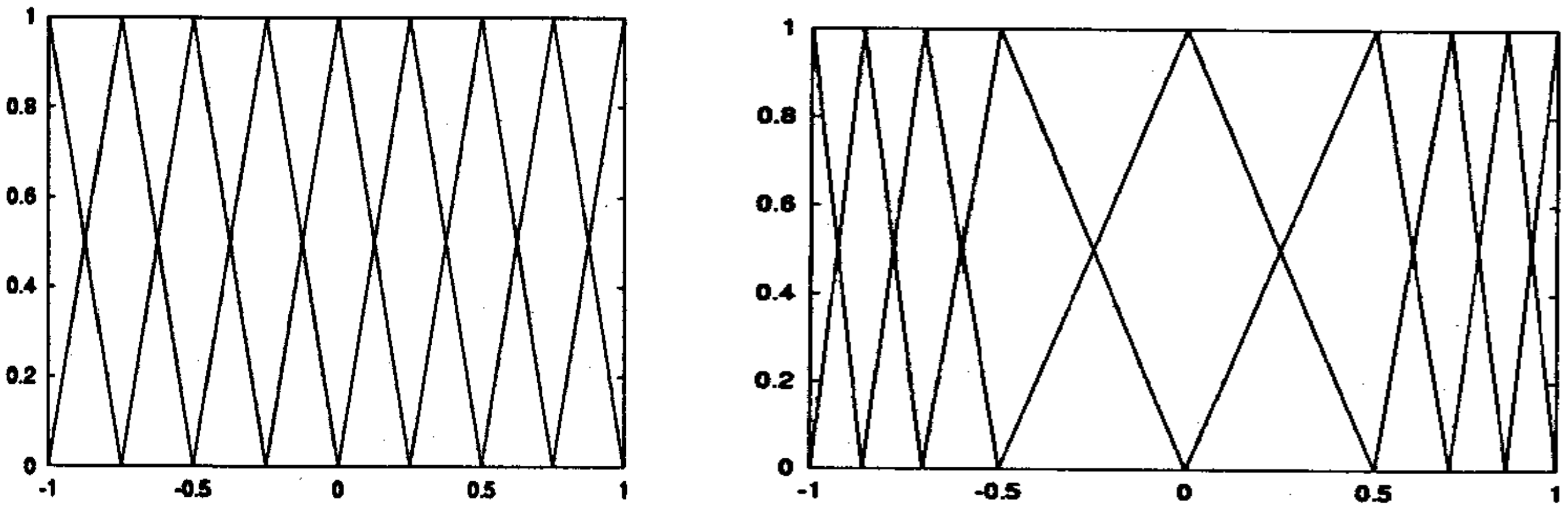


Example of genetic rule selection

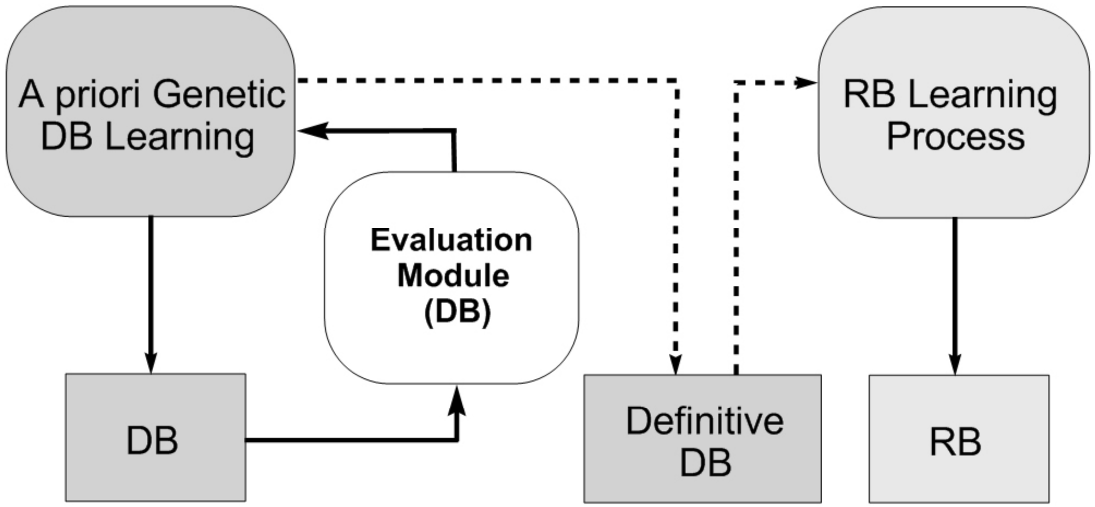
Introduction to genetic fuzzy systems

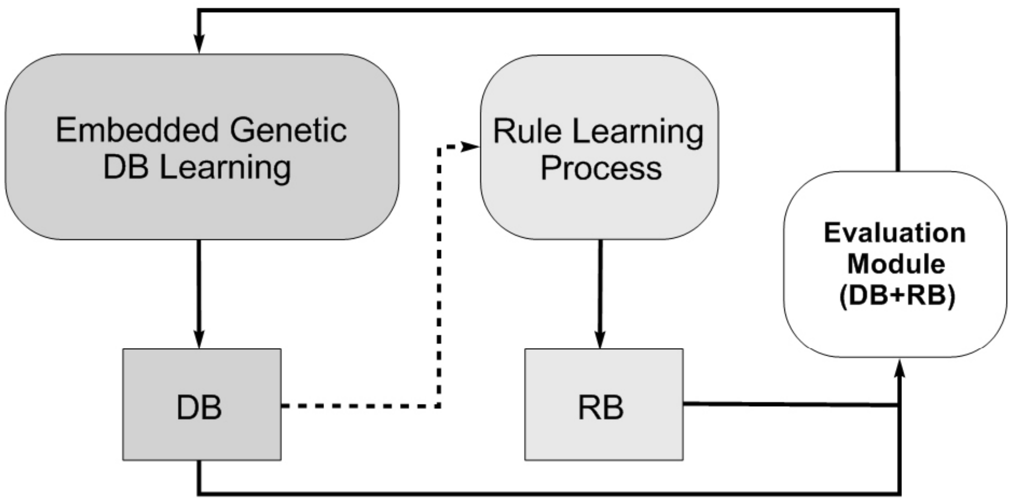
4. Genetic DB Learning

– **Learning of the membership function shapes by a GA**



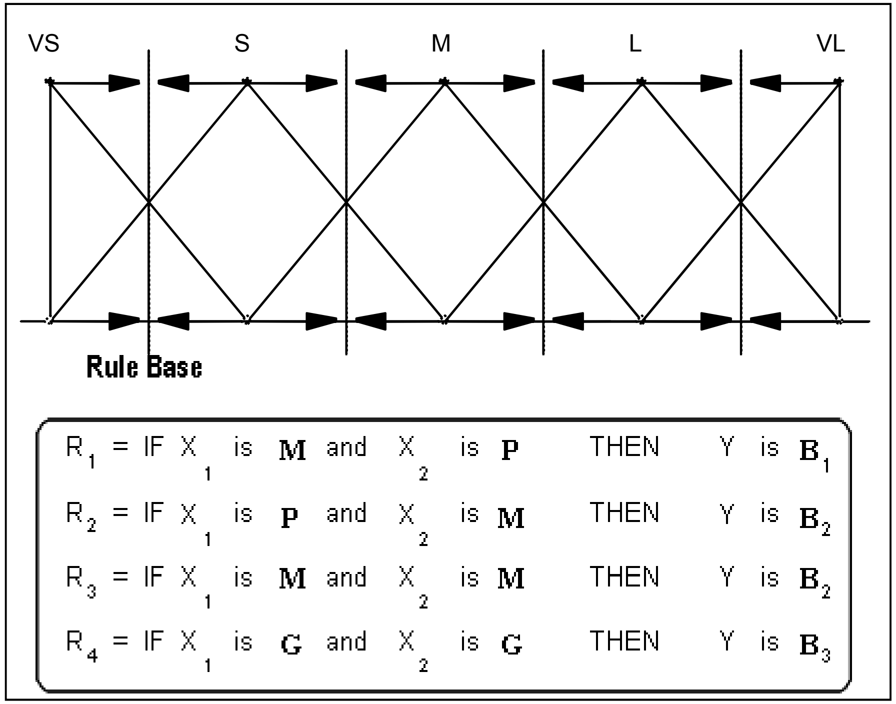
Introduction to genetic fuzzy systems
4. Genetic DB Learning





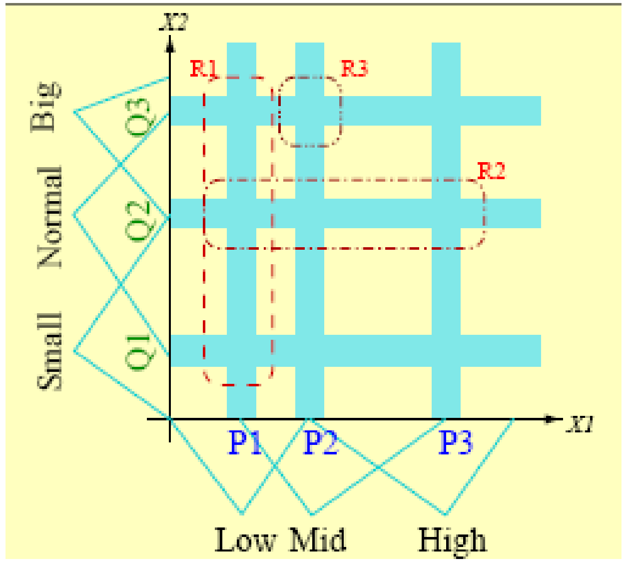
Introduction to genetic fuzzy systems
5. Simultaneous Genetic Learning of KB Components

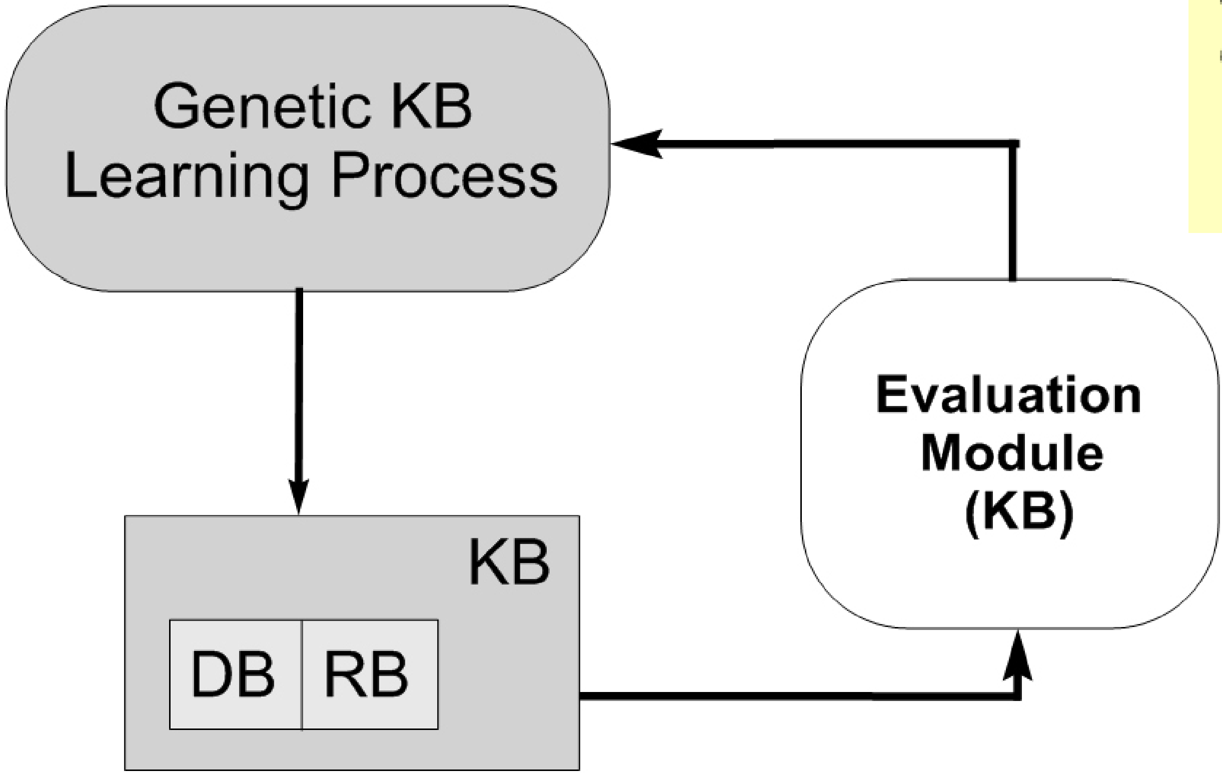
The simultaneous derivation properly addresses the strong dependency existing between the RB and the DB



Introduction to genetic fuzzy systems

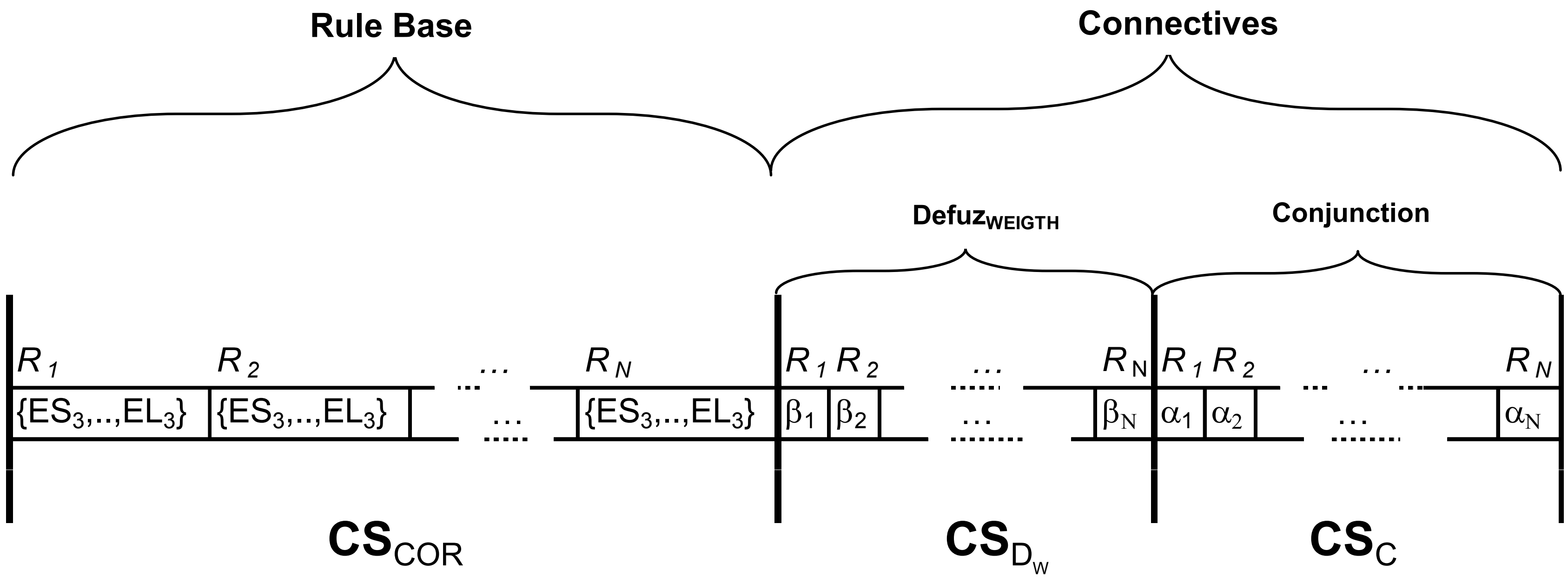
5. Simultaneous Genetic Learning of KB Components





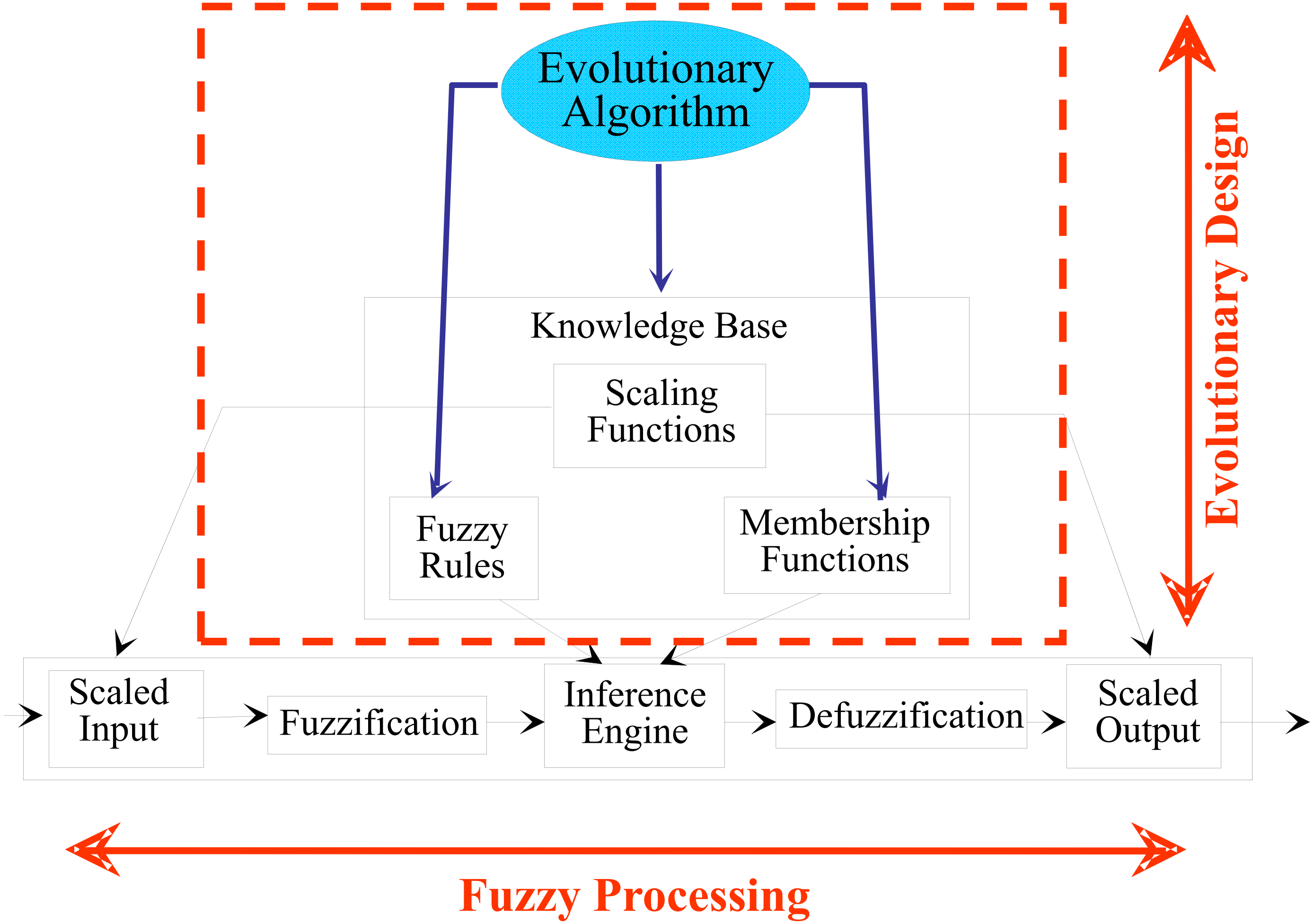
Introduction to genetic fuzzy systems

6. Genetic Learning of KB Components and Inference Engine Parameters



Example of the coding scheme for learning an RB and the inference connective parameters

Introduction to genetic fuzzy systems



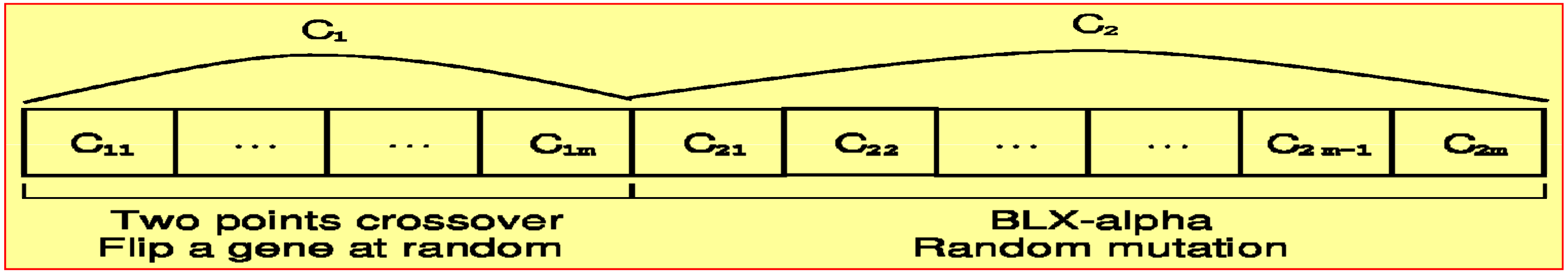
Introduction to genetic fuzzy systems

Why do we use GAs?

Advantages of the Genetic Fuzzy Systems

- **We can code different FS components in a chromosome:**
  - **Identify relevant inputs**
  - **Scaling factors**
  - **Membership functions, shape functions, optimal shape of membership funct., granularity (number of labels per variable)**
  - **Fuzzy rules, Any inference parameter, ...**

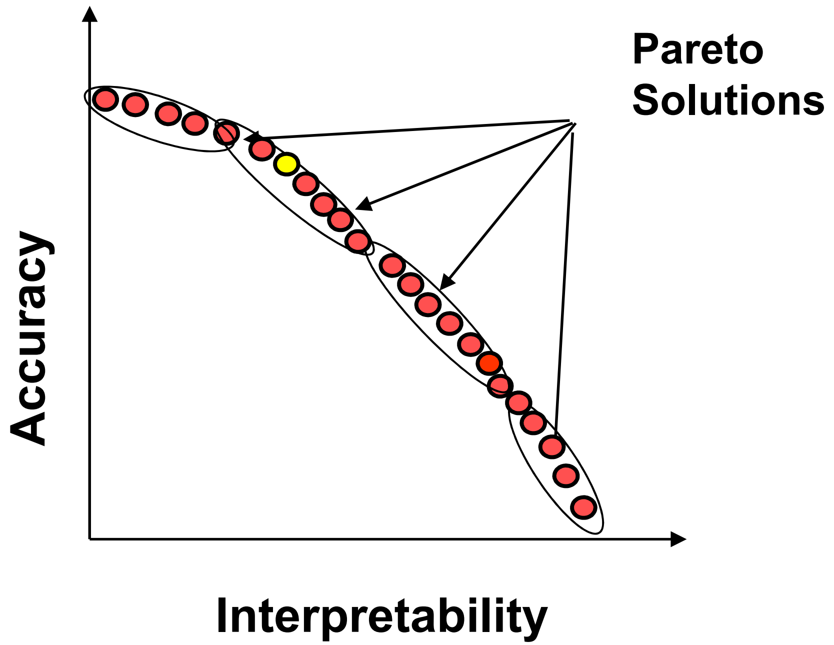
We can define different mechanism for managing them (combining genetic operators, coevolution,...)



Introduction to genetic fuzzy systems
Why do we use GAs?

Advantages of the Genetic Fuzzy Systems

- **We can consider multiple objectives in the learning model (interpretability, precision, ...)**



Introduction to genetic fuzzy systems

The birth, GFSs roadmap, current status and most cited papers

The birth of GFSs: 1991

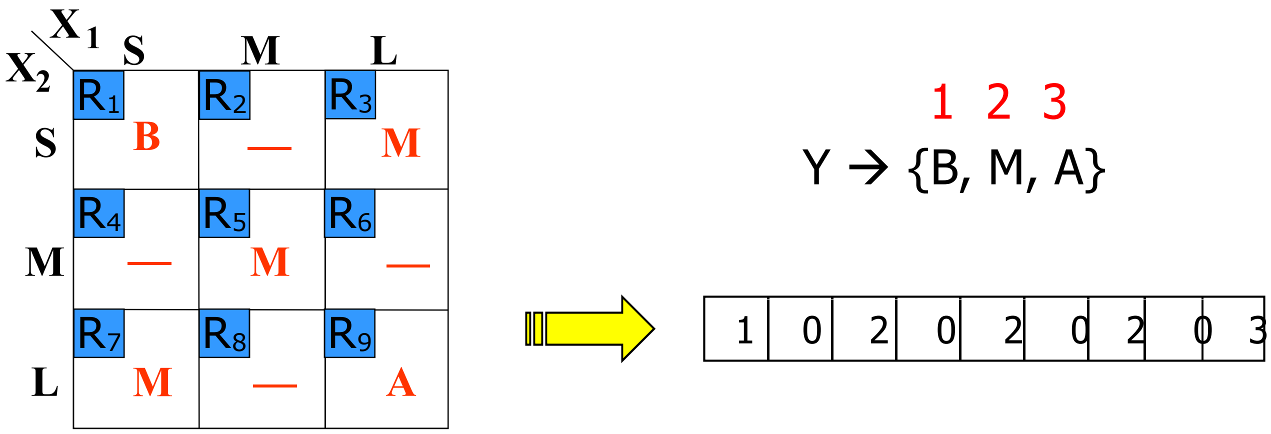
- Thrift's ICGA91 paper (Mamdani-type Rule Base Learning. Pittsburgh approach)
- Thrift P (1991) **Fuzzy logic synthesis with genetic algorithms.** *In: Proc. of 4th International Conference on Genetic Algorithms (ICGA'91), pp 509-513*
- Valenzuela-Rendón's PPSN-I paper (Scatter Mamdani-type KB Learning. Michigan approach)
- Valenzuela-Rendon M (1991) **The fuzzy classifier system: A classifier system for continuously varying variables**. *In: Proc. of 4th International Conference on Genetic Algorithms (ICGA'91), pp 346-353*
- Pham and Karaboga's Journal of Systems Engineering paper (Relational matrix-based FRBS learning. Pittsburgh approach)
- Pham DT, Karaboga D (1991) **Optimum design of fuzzy logic controllers using genetic algorithms**. *Journal of Systems Engineering 1:114-118*).
- Karr's AI Expert paper (Mamdani-type Data Base Tuning)
- Karr C (1991) **Genetic algorithms for fuzzy controllers**. *AI Expert 6(2):26-33*.

Almost the whole basis of the area were established in the first year!

Thrift's GFS:

P. Thrift, Fuzzy logic synthesis with genetic algorithms, Proc. Fourth Intl. Conf. on Genetic Algorithms (ICGA'91), San Diego, USA, 1991, pp. 509–513

- **Classical approach: Pittsburgh – the decision table is encoded in a rule consequent array**
- **The output variable linguistic terms are numbered from 1 to n and comprise the array values. The value 0 represents the rule absence, thus making the GA able to learn the optimal number of rules**
- **The ordered structure allows the GA to use simple genetic operators**



GFSs roadmap

1991-1996/7: INITIAL GFS SETTING: KB LEARNING:

- Establishment of the three classical learning approaches in the GFS field: Michigan, Pittsburgh, and IRL
- Different FRBS types: Mamdani, Mamdani DNF, Scatter Mamdani, TSK
- Generic applications: Classification, Modeling, and Control

1995-...: FUZZY SYSTEM TUNING:

- First: Membership function parameter tuning
- Later: other DB components adaptation: scaling factors, context adaptation (scaling functions), linguistic hedges, …
- Recently: interpretability consideration

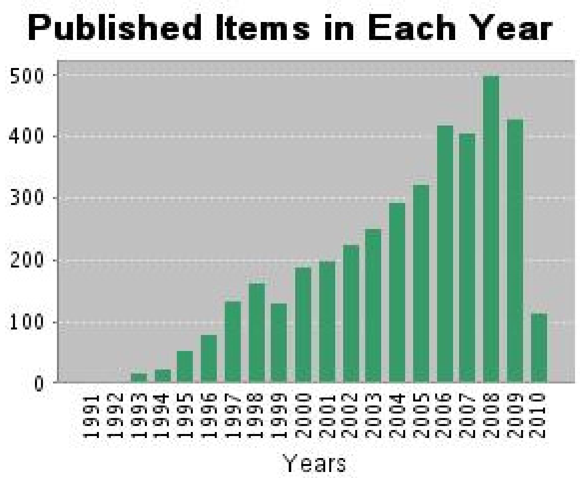
GFSs roadmap

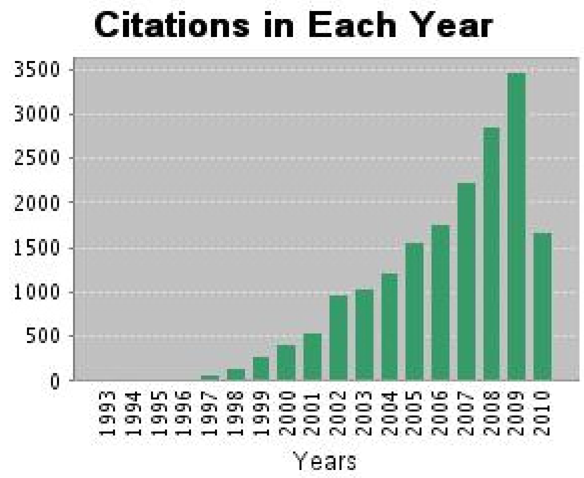
1998-…: APPROACHING TO MATURITY?
NEW GFS LEARNING APPROACHES:

- New EAs: Bacterial genetics, DNA coding, Virus-EA, genetic local search (memetic algorithms), …
- Hybrid learning approaches: a priori DB learning, GFNNs, Michigan-Pitt hybrids, …
- Multiobjective evolutionary algorithms
- Interpretability-accuracy trade-off consideration
- Course of dimensionality (handling large data sets and complex problems):
    - Rule selection (1995-…)
    - Feature selection at global level and fuzzy rule level
    - Hierarchical fuzzy modeling
- "Incremental" learning

Introduction to genetic fuzzy systems
Current state of the GFS area

Number of papers on GFSs published in JCR journals





Source: **The Thomson Corporation ISI Web of Knowledge**

Query: **(TS = (("GA-" OR "GA based" OR evolutionary OR "genetic algorithm*" OR "genetic programming" OR "evolution strate*" OR "genetic learning" OR "particle swarm" OR "differential evolutio*" OR "ant system*" OR "ant colony" OR "genetic optimi*" OR "estimation of distribution algorithm*") AND ("fuzzy rule*" OR "fuzzy system*" OR "fuzzy neural" OR "neuro-fuzzy" OR "fuzzy control*" OR "fuzzy logic cont*" OR "fuzzy class*" OR "fuzzy if" OR "fuzzy model*" OR "fuzzy association rule*" OR "fuzzy regression"))**

Date of analysis: **July 6th, 2010** Number of papers: **3962**

Number of citations: **18298** Average citations per paper: **4.62**

Introduction to genetic fuzzy systems

Current state of the GFS area

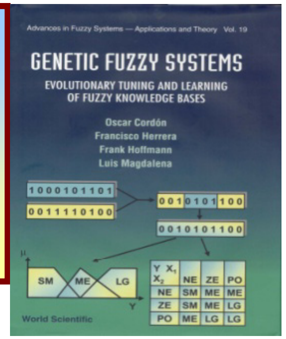
Most cited papers on GFSs (classic approaches - papers until 2000)

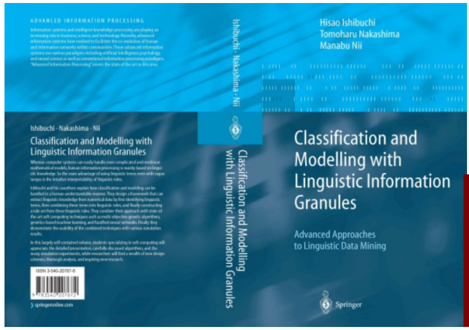
1. **Homaifar, A., McCormick, E., Simultaneous Design of Membership Functions and rule sets for fuzzy controllers using genetic algorithms, IEEE TFS 3 (2) (1995) 129-139.** Citations: 302
2. **Ishibuchi, H., Nozaki, K., Yamamoto, N., Tanaka, H., Selecting fuzzy if-then rules for classification problems using genetic algorithms, IEEE TFS 3 (3) (1995) 260-270.** Citations: 284
3. **Setnes, M., Roubos, H., GA-fuzzy modeling and classification: complexity and performance, IEEE TFS 8 (5) (2000) 509-522 .** Citations: 215
4. **Ishibuchi, H., Nakashima, T., Murata, T., Performance evaluation of fuzzy classifier systems for multidimensional pattern classification problems, IEEE TSMC B 29 (5) (1999) 601-618.** Citations: 177
5. **Shi, Y.H., Eberhart, R., Chen, Y.B., Implementation of evolutionary fuzzy systems, IEEE TFS 7 (2) (1999) 109-119.** Citations: 126
6. **Park, D., Kandel, A., Langholz, G., Genetic-based new fuzzy reasoning models with application to fuzzy control, IEEE TSMC B 24 (1) (1994) 39-47.** Citations: 125
7. **Jin, YC (2000) Fuzzy modeling of high-dimensional systems: Complexity reduction and interpretability improvement. IEEE Transactions on Fuzzy Systems 8(2):212-221.** Citations: 121
8. **Ishibuchi H, Murata T, Turksen IB (1997) Single-objective and two-objective genetic algorithms for selecting linguistic rules for pattern classification problems. Fuzzy Sets and Systems 89(2):135-150** Citations: 116
9. **Juang, C.F., Lin, J.Y., Lin, C.T., Genetic reinforcement learning through symbiotic evolution for fuzzy controller design, IEEE TSMC B 30 (2) (2000) 290-302.** Citations: 109
10. **Herrera, F., Lozano, M., Verdegay, J.L., Tuning fuzzy-logic controllers by genetic algorithms, IJAR 12 (3-4) (1995) 299-315.** Citations: 108

Introduction to genetic fuzzy systems

Some References

GENETIC FUZZY SYSTEMS
Evolutionary Tuning and Learning of Fuzzy Knowledge Bases.
O. Cordón, F. Herrera, F. Hoffmann, L. Magdalena
World Scientific, July 2001





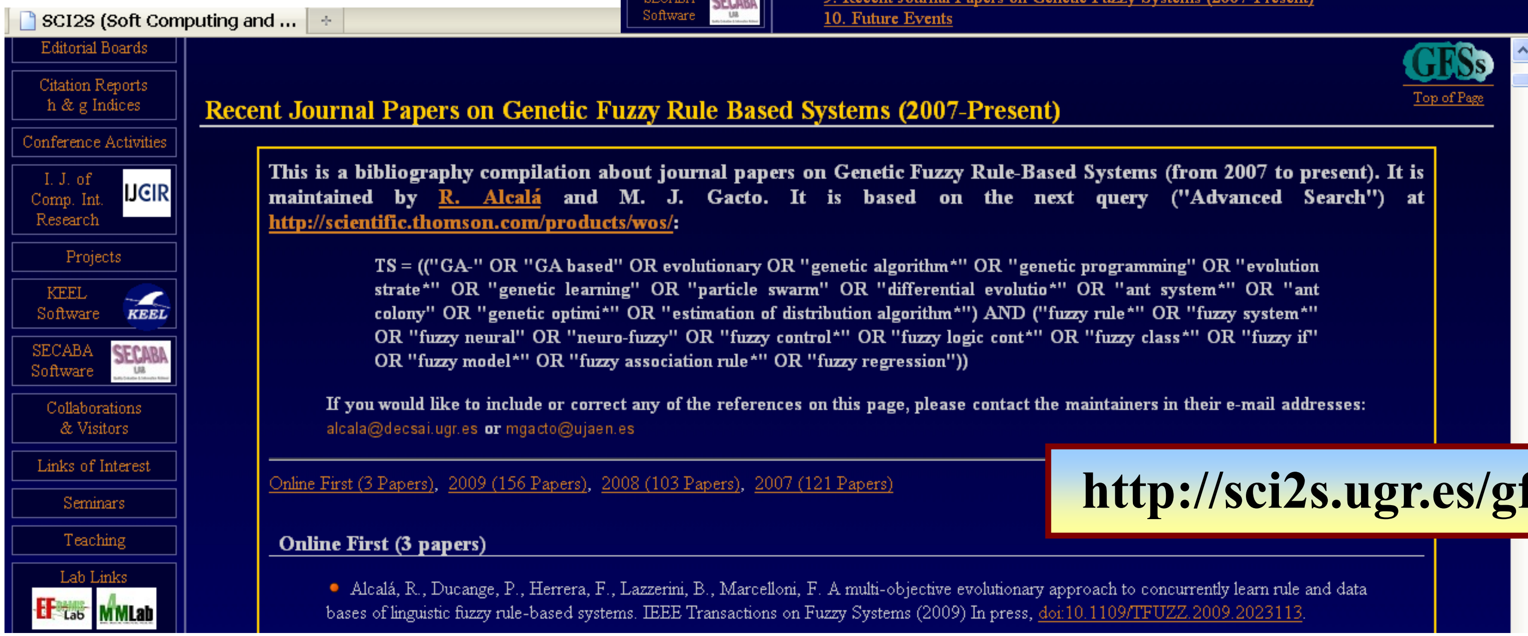
H. Ishibuchi, T. Nakashima, M. Nii, Classification and Modeling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer (2005)

- F. Herrera, Genetic Fuzzy Systems: Taxonomy, Current Research Trends and Prospects. *Evolutionary Intelligence 1 (2008) 27-46 doi: 10.1007/s12065-007-0001-5,*
- F. Herrera, Genetic Fuzzy Systems: Status, Critical Considerations and Future Directions, International Journal of Computational Intelligence Research 1 (1) (2005) 59-67
- O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena, Ten Years of Genetic Fuzzy Systems: Current Framework and New Trends, FSS 141 (1) (2004) 5-31
- F. Hoffmann, Evolutionary Algorithms for Fuzzy Control System Design, Proceedings of the IEEE 89 (9) (2001) 1318-1333

Introduction to genetic fuzzy systems

GFSs Website

http://sci2s.ugr.es/gfs/



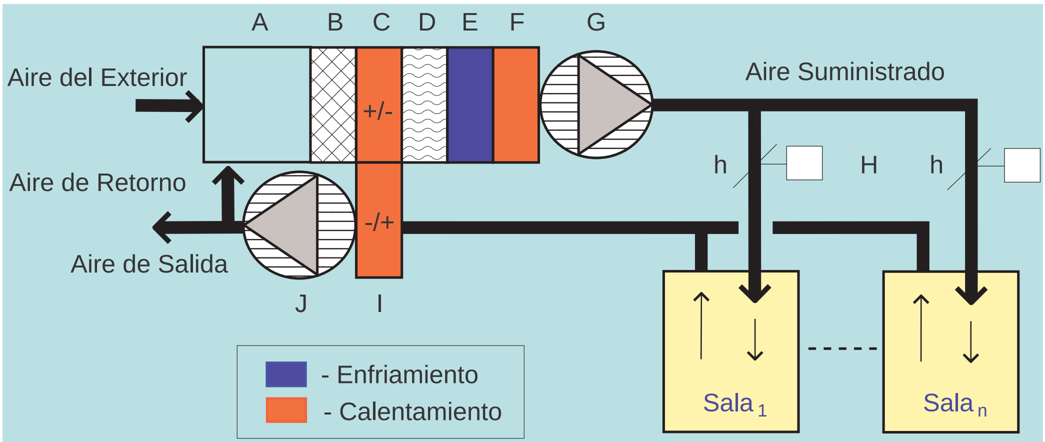
http://sci2s.ugr.es/gfs/biblio.php

GENETIC FUZZY SYSTEMS: APPLICATION TO A HVAC PROBLEM

Heating Ventilating and Air Conditioning Systems: Problem

GENESYS

JOULE-THERMIE JOE-CT98-0090



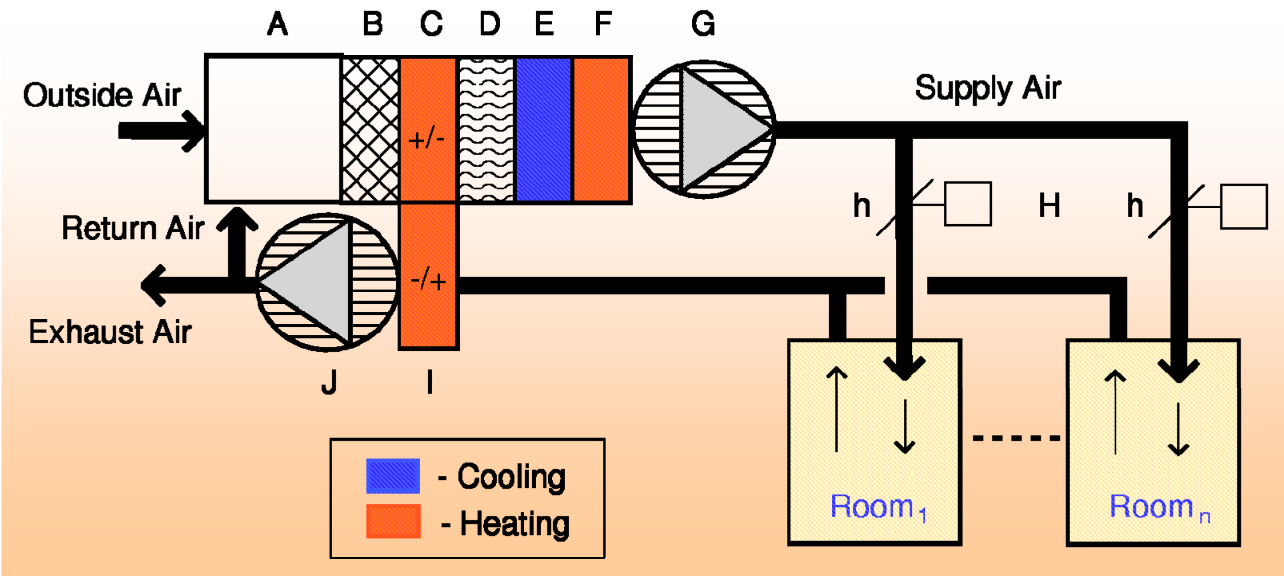
Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

Heating Ventilating and Air Conditioning Systems: Problem

- Energy consumption in buildings is the 40% of the total and more than a half is for indoor climate conditions
- The use of specific technologies can save up to a 20% of the energy consumption
- The use of appropriate automatic control strategies could result in energy savings ranging 15-85 %
- Moreover, in current systems, several criteria are considered and optimized independently without a global strategy

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

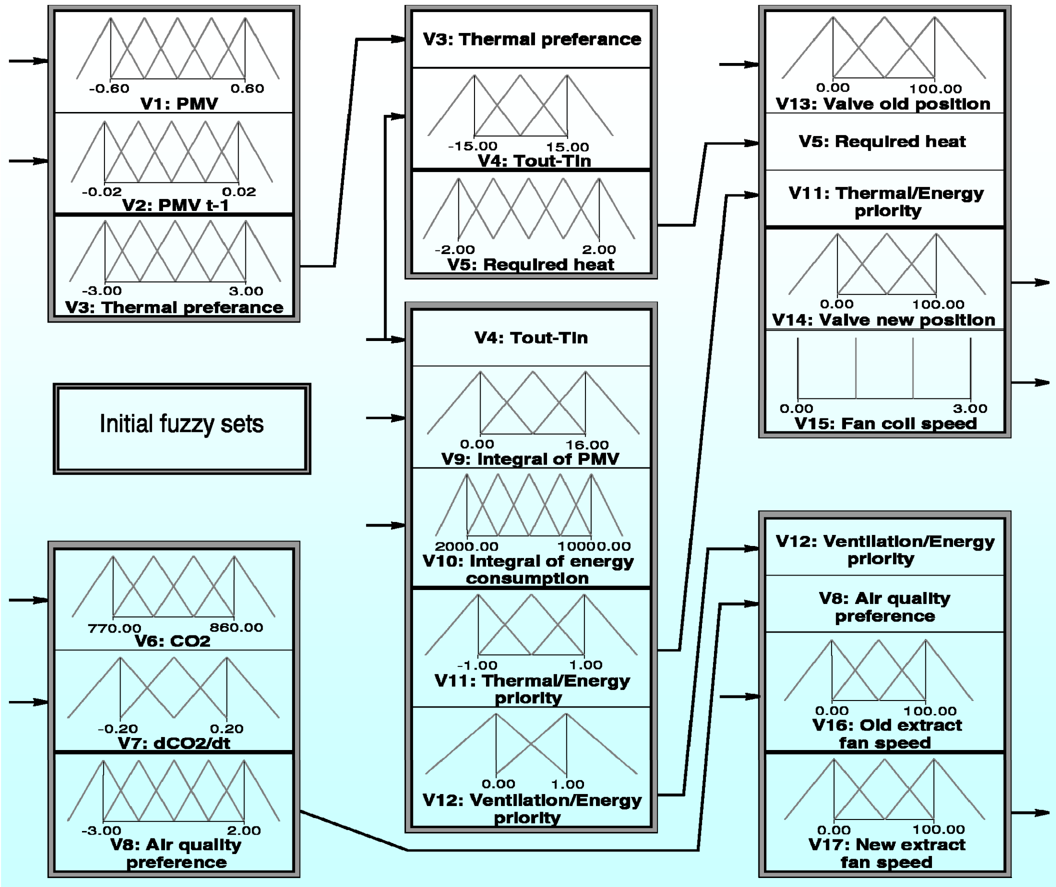
Generic Structure of an Office Building HVAC System



- It maintain a good thermal quality in summer and winter
- It dilutes and removes emissions from people, equipment and activities and supplies clean air

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

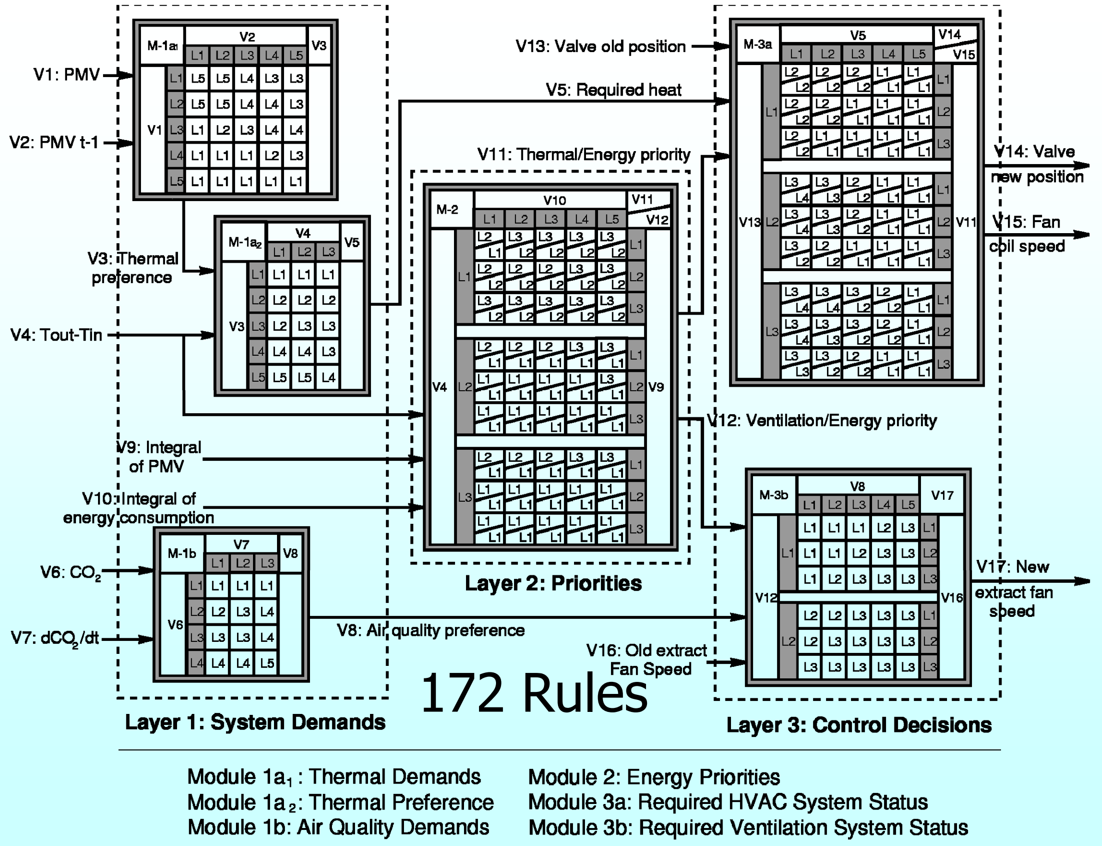
Initial Data Base



17 Variables

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

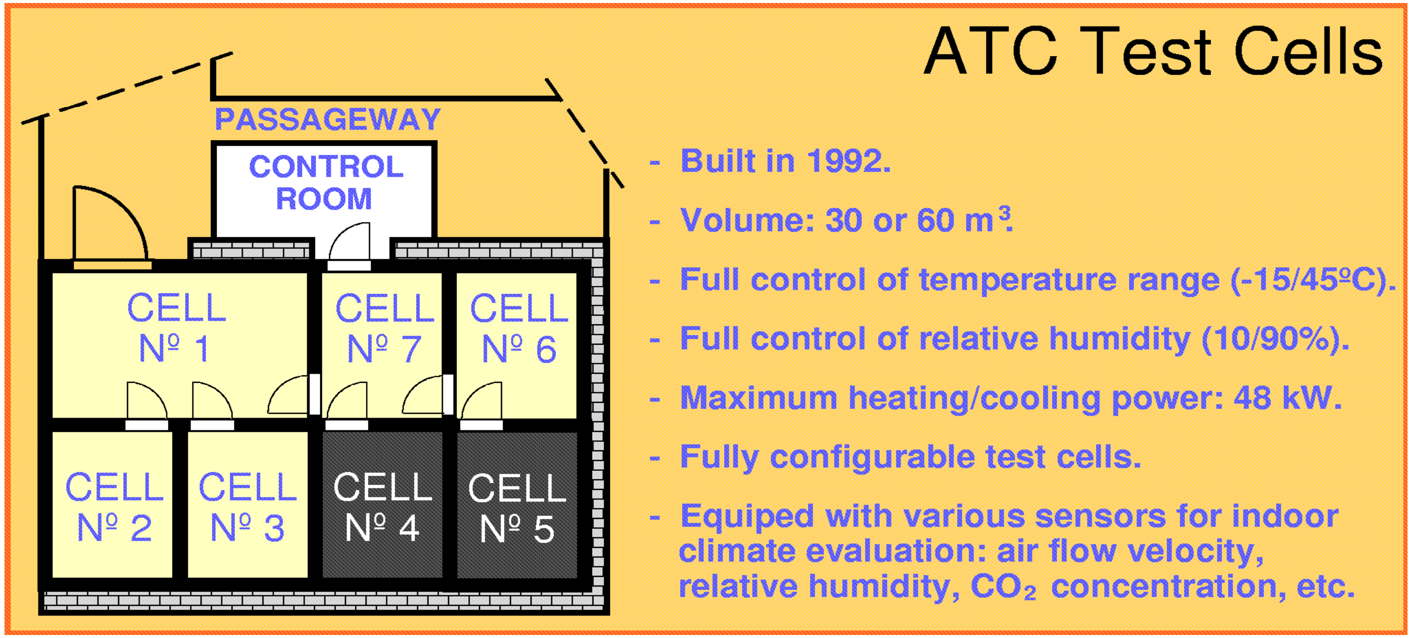
Initial Rule Base and FLC Structure



172 Rules

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

Representation of the Test Cells



- **Two adjacent twin cells were available**
- **A calibrated and validated model of this site was developed to evaluate each FLC**

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

- **Goal: multi-criteria optimization of an expert FLC for an HVAC system: reduction of the energy consumption but maintaining the required indoor comfort levels**

O_1 Upper thermal comfort limit [3]: $if\ PMV > 0.5, O_1 = O_1 + (PMV - 0.5)$.

O_2 Lower thermal comfort limit: $if\ PMV < -0.5, O_2 = O_2 + (-PMV - 0.5)$.

O_3 IAQ requirement: $if\ CO_2\ conc. > 800ppm, O_3 = O_3 + (CO_2 - 800)$.

O_4 Energy consumption: $O_4 = O_4 +$ Power at time t.

O_5 System stability: $C_5 = C_5 +$ System change from time t to $(t-1)$.

- **INITIAL RESULTS**

MODELS	#R	PMV>0.5 0_1		PMV<-0.5 0_2		$C0_2$ 0_3		ENERGY 0_4	%	STABILITY 0_5	%
ON-OFF	-	0,0		0		0		3206400	-	1136	-
FLC	172	0,0		0		0		2901686	9,50	1505	-32,48

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Problem Restrictions

- **The controller accuracy is assessed by means of simulations which approximately take 3-4 minutes**

  - Efficient tuning methodologies:

    - Local adjustment of each tuned parameter
    - Steady-State Genetic Algorithms: quick convergence

      2000 evaluations ⇒ 1 run takes approximately 4 days

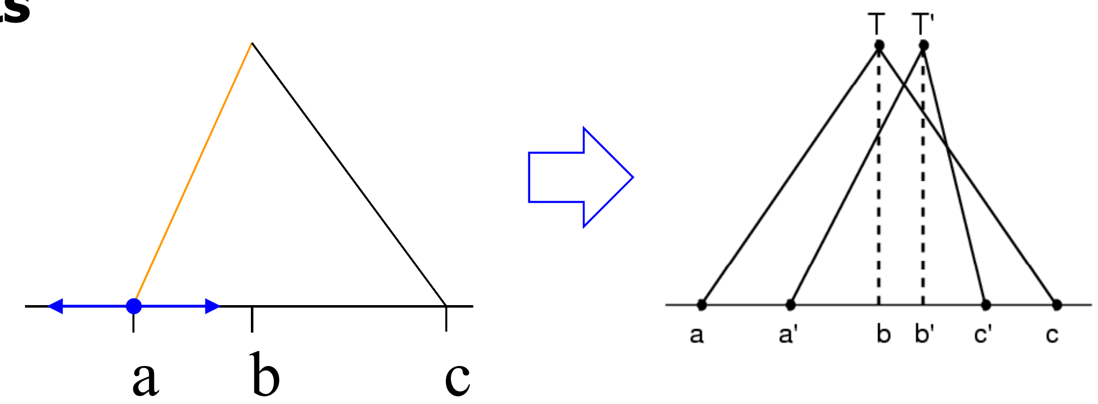
    - Considering a small population (31 individuals)

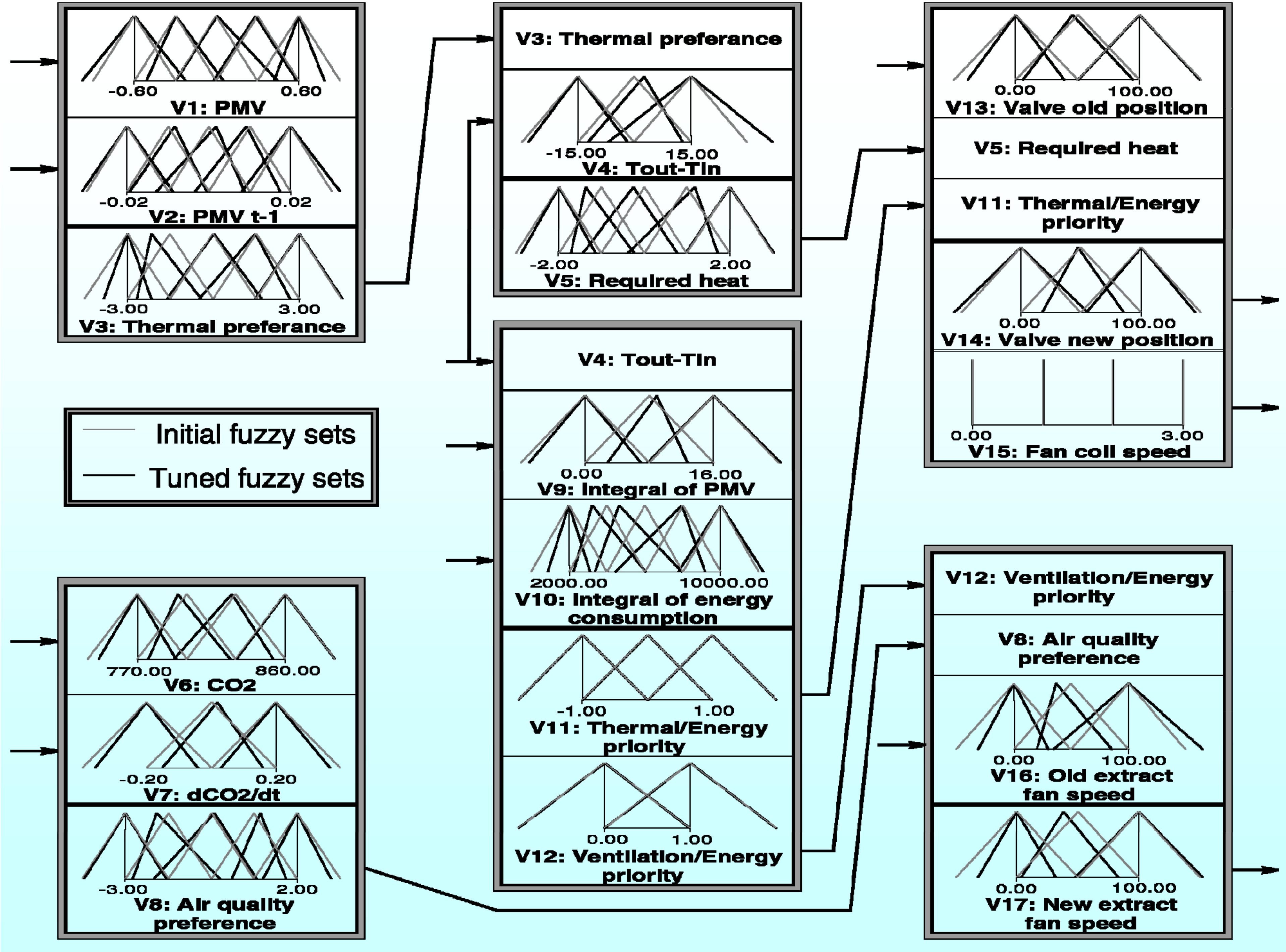
GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Improving the FLC Performance

The main objective was the reduction of the energy consumption (10%), improving the stability of the controller, maintaining the required indoor comfort levels

- **Classic genetic tuning of the Data Base**
  - **Local modification of the membership function definition points**





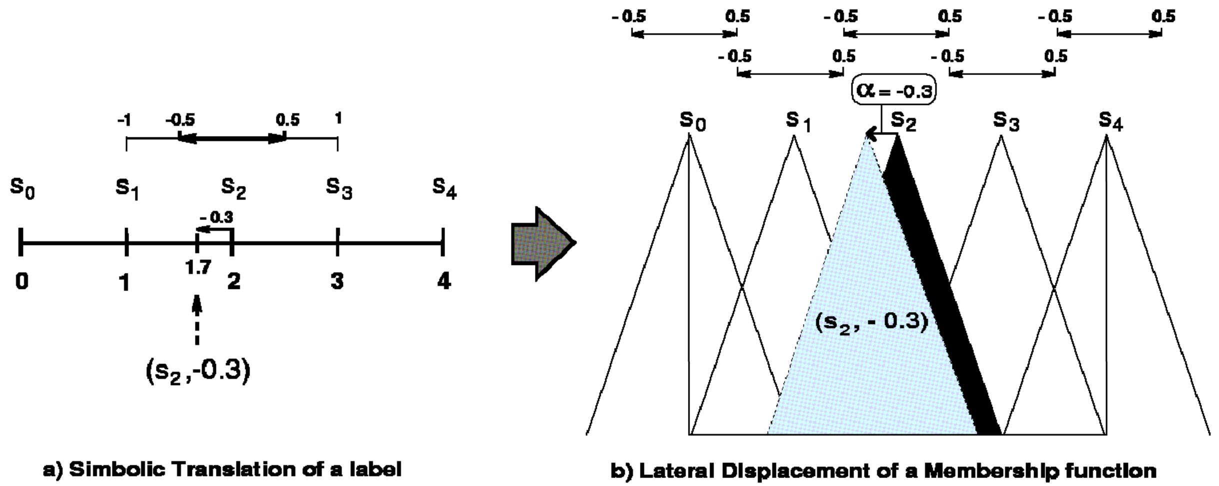
-0.60
0.60
V1: PMV
-0.02
0.02
V2: PMV t-1
-3.00
3.00
V3: Thermal preferance
V3: Thermal preferance
-15.00
15.00
V4: Tout-Tin
-2.00
2.00
V5: Required heat
0.00
100.00
V13: Valve old position
V5: Required heat
V11: Thermal/Energy priority
0.00
100.00
V14: Valve new position
0.00
3.00
V15: Fan coil speed
Initial fuzzy sets
Tuned fuzzy sets
V4: Tout-Tin
0.00
16.00
V9: Integral of PMV
2000.00
10000.00
V10: Integral of energy consumption
-1.00
1.00
V11: Thermal/Energy priority
0.00
1.00
V12: Ventilation/Energy priority
770.00
860.00
V6: CO2
-0.20
0.20
V7: dCO2/dt
-3.00
2.00
V8: Air quality preference
V12: Ventilation/Energy priority
V8: Air quality preference
0.00
100.00
V16: Old extract fan speed
0.00
100.00
V17: New extract fan speed

GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning + Rule Selection

New coding schemes: 2- and 3-tuples:

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Improving Fuzzy Logic Controllers Obtained by Experts: A Case Study in HVAC Systems. *Applied Intelligence*, *doi:10.1007/s10489-007-0107-6, 31:1 (2009) 10-35.*

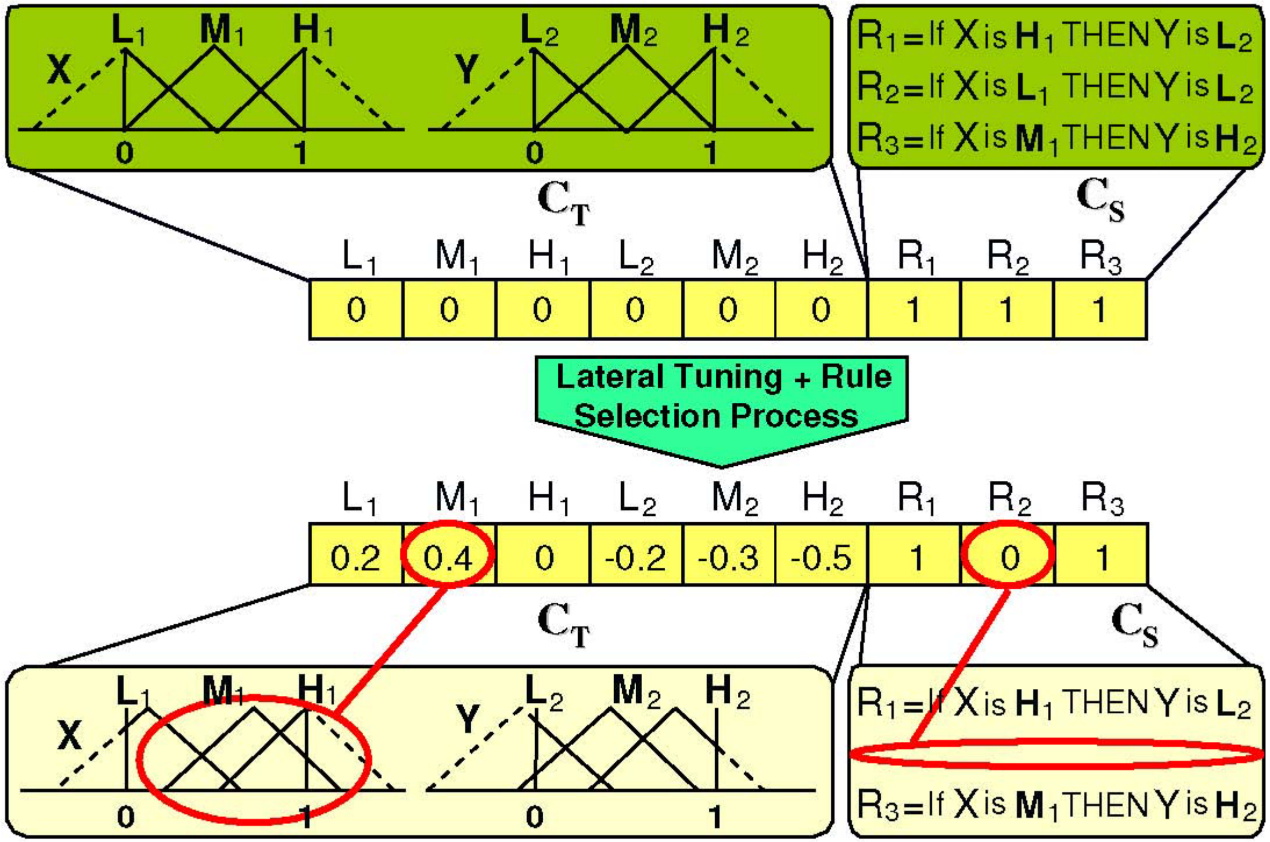
– 2-tuples: **label id. i and a displacement parameter $\alpha_i \in [-0.5,0.5]$**



– **New rule structure:**

IF X_1 IS $(S^1{}_i, \alpha_1)$ AND … AND X_n IS $(S^n{}_i, \alpha_n)$ THEN Y IS $(S^y{}_i, \alpha_y)$

GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning + Rule Selection (2)



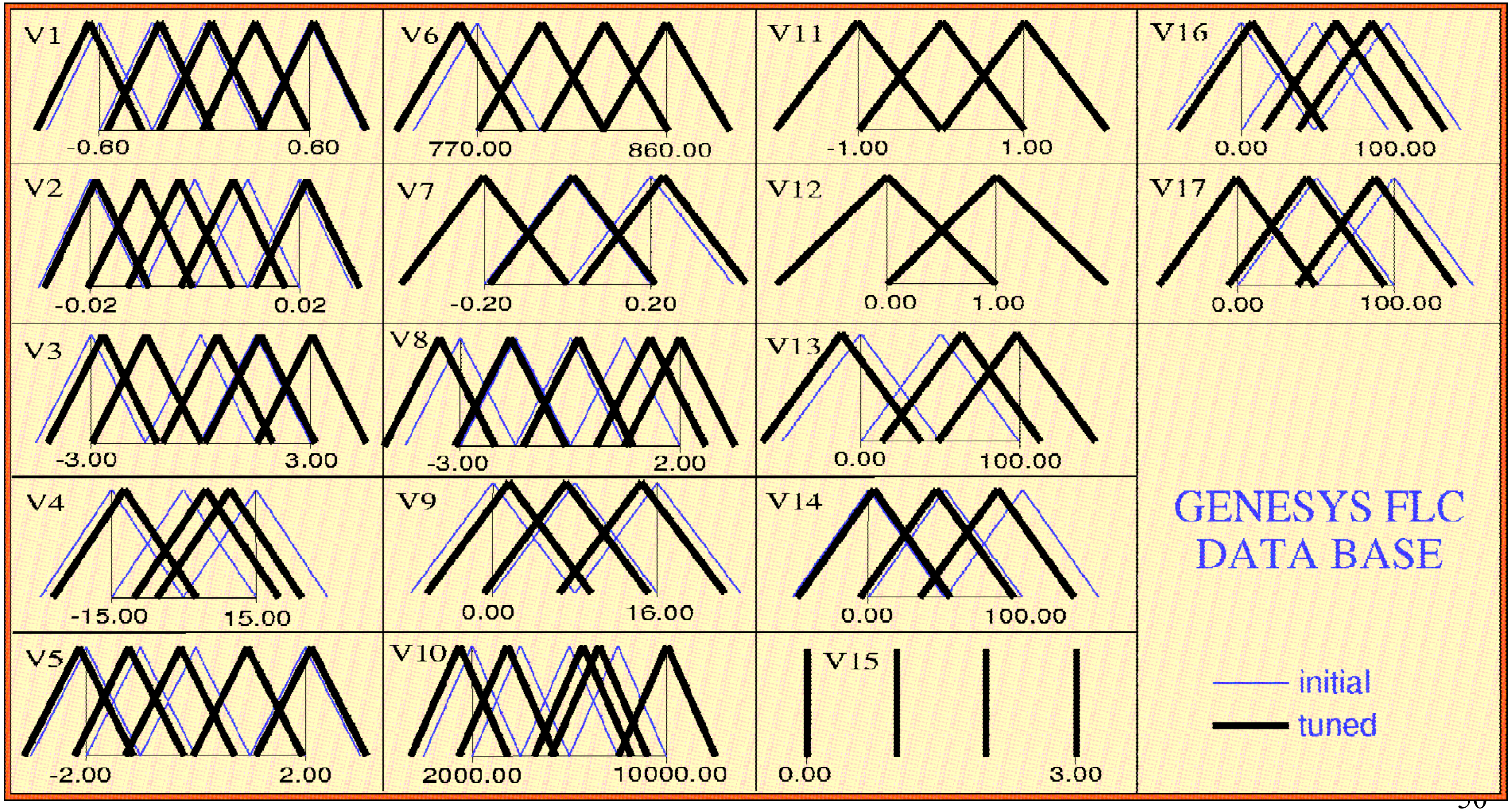
Example of genetic lateral tuning and rule selection

GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

MODELS	#R	PMV>0.5 0_1	PMV<-0.5 0_2	$C0_2$ 0_3	ENERGY 0_4	ENERGY %	ESTABILITY 0_5	ESTABILITY %
ON-OFF	-	0,0	0	0	3206400	-	1136	-
FLC	172	0,0	0	0	2901686	9,50	1505	-32,48
TUNING	172	0,0	0	0	2596875	19,01	1051	7,48
SELECTION	147	0,2	0	0	2867692	10,56	991	12,76
SEL + TUNING	109	0,1	0	0	2492462	22,27	989	12,94
SEL + WEIGHTS	102	0,7	0	0	2731798	14,80	942	17,08
GL 2	172	0,9	0	0	2268689	29,25	1080	4,93
LL 1	172	0,9	0	0	2386033	25,59	896	21,13
GL - S 1	105	1,0	0	0	2218598	30,81	710	37,50
GL - S 2	115	0,4	0	0	2358405	26,45	818	27,99
GL - S 3	118	0,8	0	0	2286976	28,68	872	23,24
LL – S 1	133	0,5	0	0	2311986	27,90	788	30,63
LL – S 2	104	0,6	0	0	2388470	25,51	595	47,62
LL – S 3	93	0,5	0	0	2277807	28,96	1028	9,51

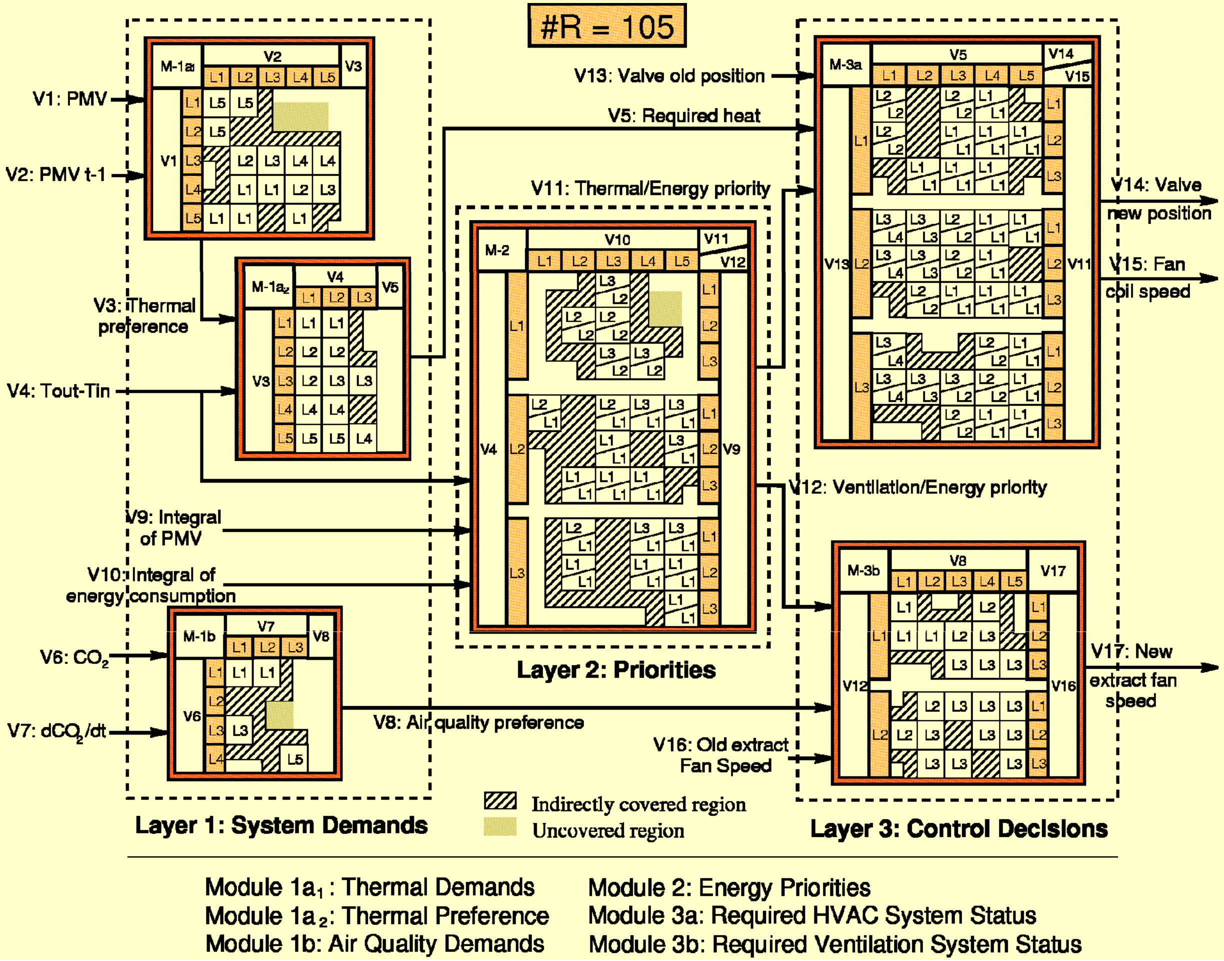
GFS Models for Fuzzy Control of HVAC Systems:
Genetic Lateral Tuning and Rule Selection

Tuned Data Base (GL-S_1):



GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

Selected Rule Base (GL-S$_1$):



GFS Models for Fuzzy Control of HVAC Systems

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R. Alcalá, J.M. Benítez, J. Casillas, O. Cordón, R. Pérez, Fuzzy Control of HVAC Systems Optimized by Genetic Algorithms. *Applied Intelligence 18:2 (2003) 155-177*.

R. Alcalá, J. Casillas, O. Cordón, A. González, F. Herrera, A Genetic Rule Weighting and Selection Process for Fuzzy Control of Heating, Ventilating and Air Conditioning Systems. *Engineering Applications of Artificial Intelligence 18:3 (2005) 279-296*.

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Improving Fuzzy Logic Controllers Obtained by Experts: A Case Study in HVAC Systems. *Applied Intelligence 31:1 (2009) 10-35*.

M.J. Gacto, R. Alcalá, F. Herrera, **A Multi-Objective Evolutionary Algorithm for an Effective Tuning of Fuzzy Logic Controllers in Heating, Ventilating and Air Conditioning Systems**. *Applied Intelligence, doi: 10.1007/s10489-010-0264-x, **in press (2011)***

An Example on the usefulness of MOGFSs: Improved results with more than 30% in Energy and more than 50% in Stability using an improved MOEA

Contents

1. **Basics on Genetic Fuzzy Systems (GFS)**
   - **Introduction to Genetic Fuzzy System Research**
   - **An Example on a Real Application**

2. **Interpretability-Accuracy Tradeoff of Fuzzy Systems:** *Two contradictory objectives*
   - **Interpretability Issues in Fuzzy System Design**
   - **Applicability of MOGFSs to the I-A problem**

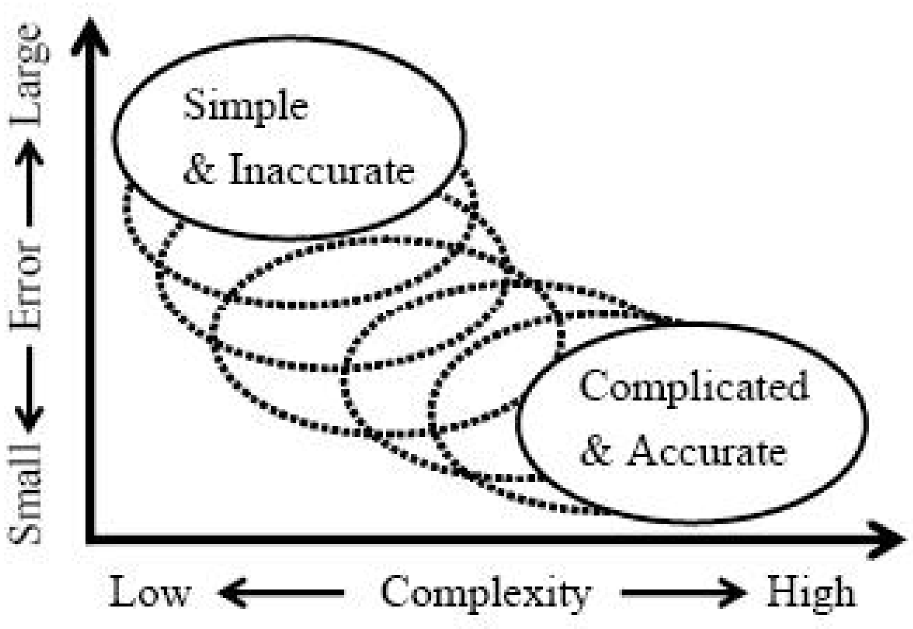
3. **Evolutionary Multiobjective Optimization (EMO)**
   - **Some Basic Concepts in Multiobjective Optimization**
   - **Framework of Evolutionary Multiobjective Optimization**

4. **Multiobjective Genetic Fuzzy Systems (MoGFS)**
   - **Overview of MoGFS Research (***some representative examples***)**
   - **New Research Directions in MoGFS**

Interpretability Issues in Fuzzy System Design

Complexity Criteria

- **Highly used criteria: Complexity criteria in the learning of FRBSs.**



Number of variables, labels, rules, conditions …

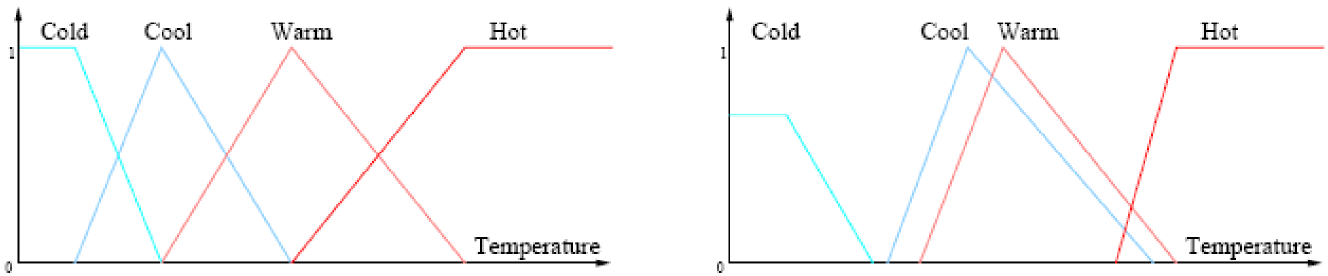
Interpretability Issues in Fuzzy System Design
Semantic Criteria

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

Interpretability considerations: semantic criteria

Semantics: the study of meanings

- Distinguishability: Each linguistic label has semantic meaning
- Number of elements: Compatible with human capabilities
- Coverage: Any element belongs to at least one fuzzy set
- Normalization: At least one element has unitary membership
- Complementarity: For each element, the sum of memberships is one



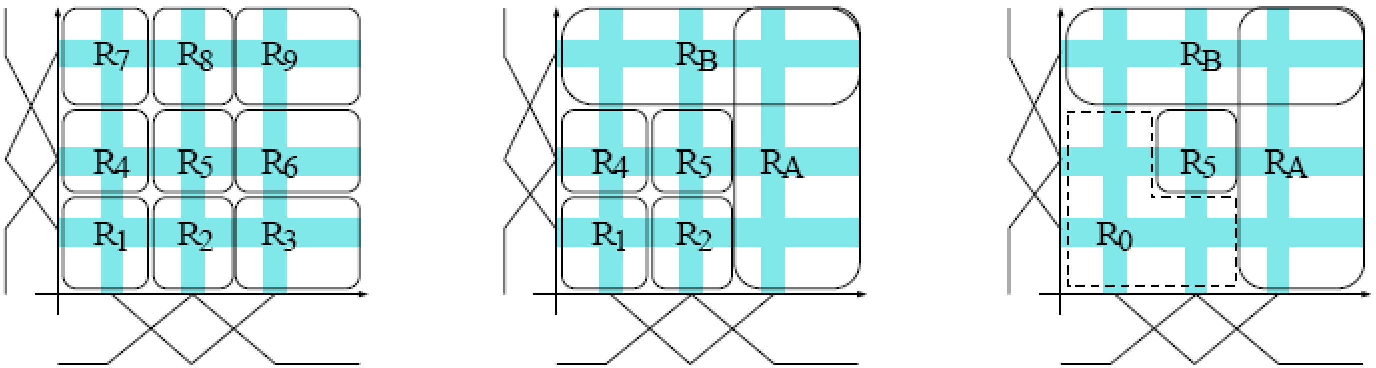
Interpretability Issues in Fuzzy System Design
Syntactic Criteria

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

Interpretability considerations: syntactic criteria

Syntax: the way in which linguistic elements are put together

- Completeness: for any input, at least one rule must fire
- Rule−base simplicity: Set of rules as small as possible
- Rule readability: small number of conditions in rule antecedents
- Consistency: rules firing simultaneously must have similar consequents

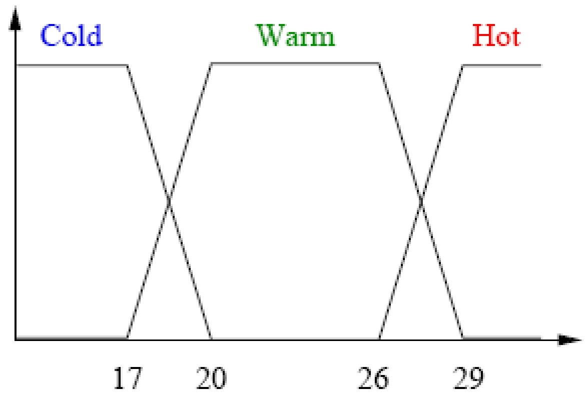


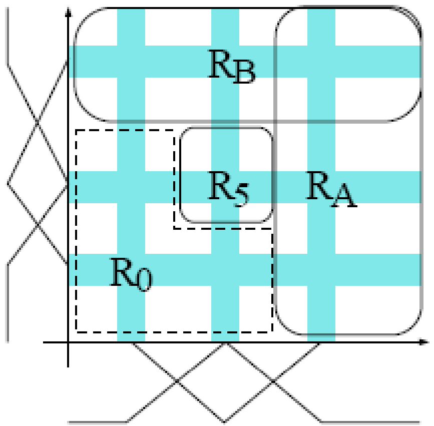
Interpretability Issues in Fuzzy System Design
Strategies to Satisfy Interpretability

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

Strategies to satisfy interpretability criteria

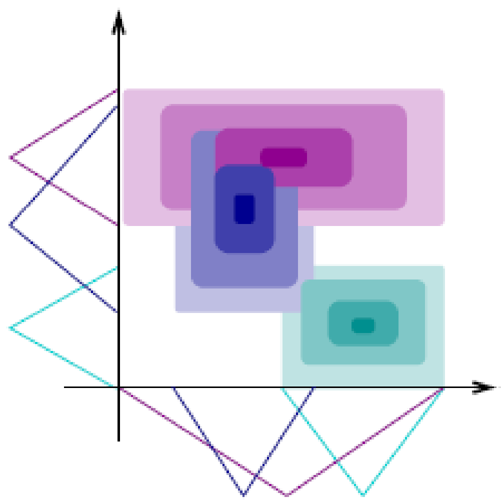
- Linguistic labels shared by all rules
- Normal, orthogonal membership functions
- Don't care conditions

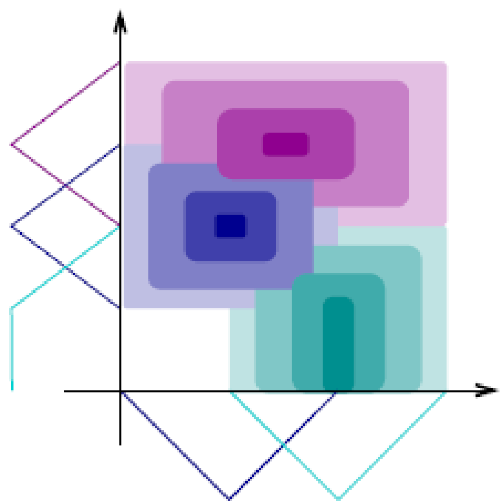




Interpretability Issues in Fuzzy System Design
Still not Clear Concepts

- **Interpretability quality:**





What is the most interpretable rule base?

A Taxonomy on the Existent Interpretability Measures for Linguistic FRBSs

Most works in C1 and C2 are applied to classification problems. They are the classic measures.

	Rule Base Level	Fuzzy Partition Level
Complexity-based Interpretability	**C1** Number of rules Number of conditions	**C2** Number of membership functions Number of features
Semantic-based Interpretability	**C3** Consistency of rules Rules fired at the same time Transparency of rule structure (rule weights, etc.) Cointension	**C4** Absolute Measures: Completeness or coverage, normalization, distinguishability, complementarity Relative Measures

There are few works in C3
Still an open problem

Most works in C4 impose absolute measures or restrictions. Relativity could be a new possibility.
Still an open problem.

A Taxonomy on the Existent Interpretability Measures for Linguistic FRBSs (2)

- Interpretability of FRBSs is still an open problem since there is no single (or global) comprehensive measure to quantify the interpretability of linguistic models

- To get a good global measure it would be necessary to consider appropriate measures from all of the quadrants, in order to take into account the different interpretability properties required for these kinds of systems together.

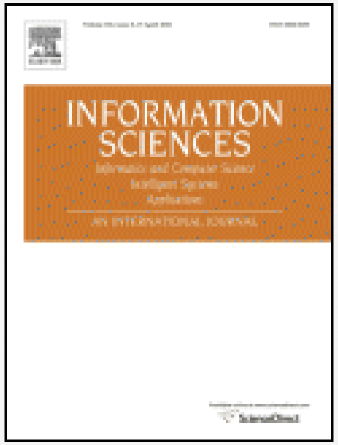
M.J. Gacto, R. Alcalá, F. Herrera

Interpretability of Linguistic Fuzzy Rule-Based Systems: An Overview on Interpretability Measures

Information Sciences, *doi: 10.1016/j.ins.2011.02.021*, ***in press (2011)***

A thematic website is being developed to maintain this study at:

http://sci2s.ugr.es/ ***(under construction)***



Applicability of MOGFSs to the I-A problem

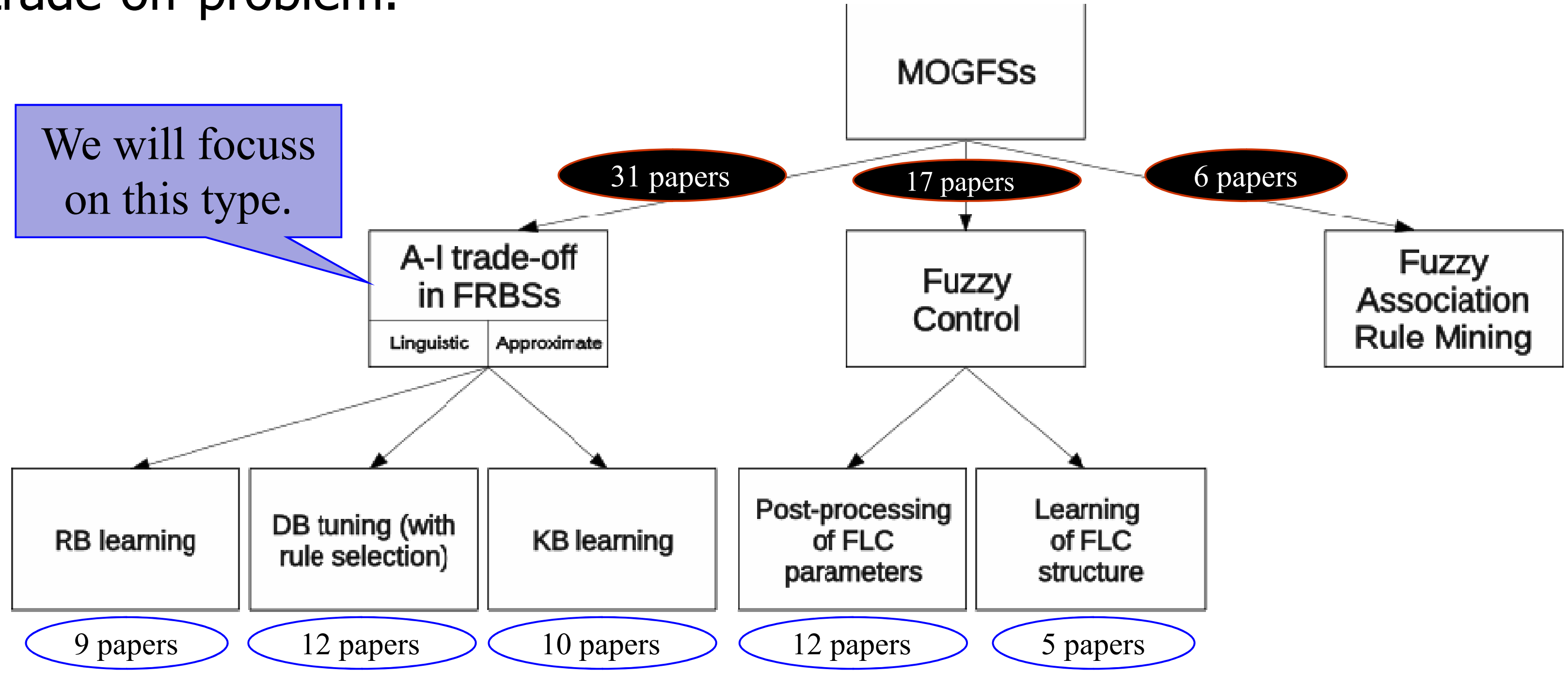
- The different measures from each quadrant could be optimized as different objectives within a multi-objective framework.

- They are contradictory to some degree. Not only accuracy is contradictory to interpretability. The different measures represent different properties and requirements.

- Together with accuracy, many interpretability objectives should be optimized at the same. Two different solutions:
  - Development of new EMO algorithms for many objective problems
    (incoming for future)
  - By grouping complexity measures and semantic measures into two respective indexes.
    (it would represent the present)

Applicability of MOGFSs to the I-A problem (2)

❑ In fact, a revision on the application of MOGFSs indicates that most of the approaches have been applied to the Interpretability-accuracy trade-off problem.



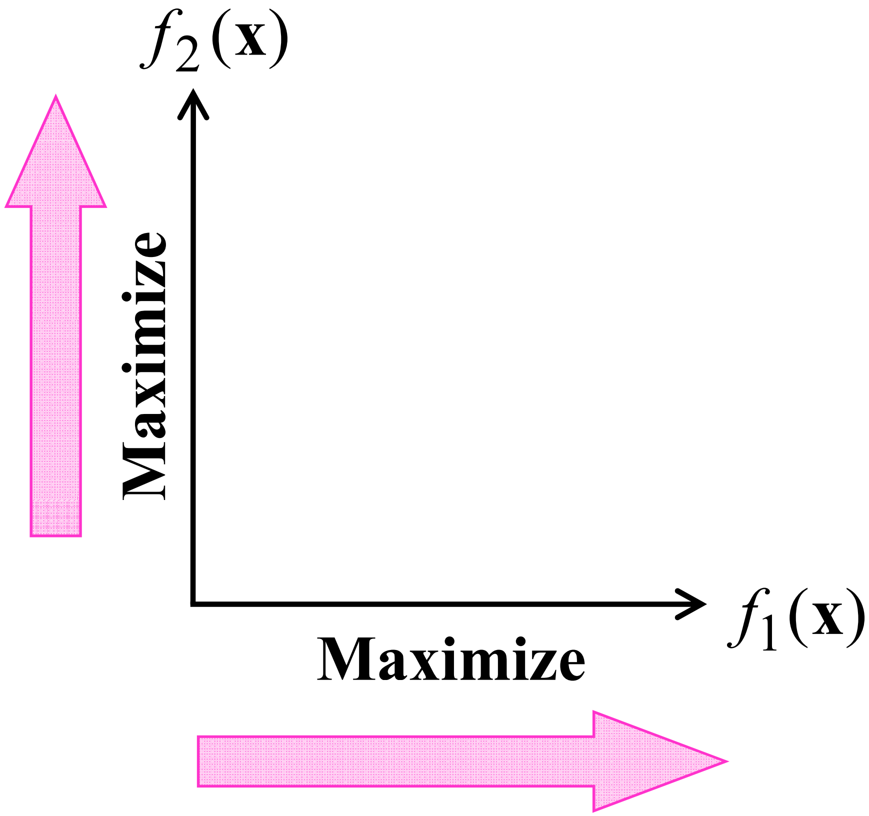
Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera. ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****, in submission, 2011 *(Available soon)*.**

Contents

Multiobjective Optimization

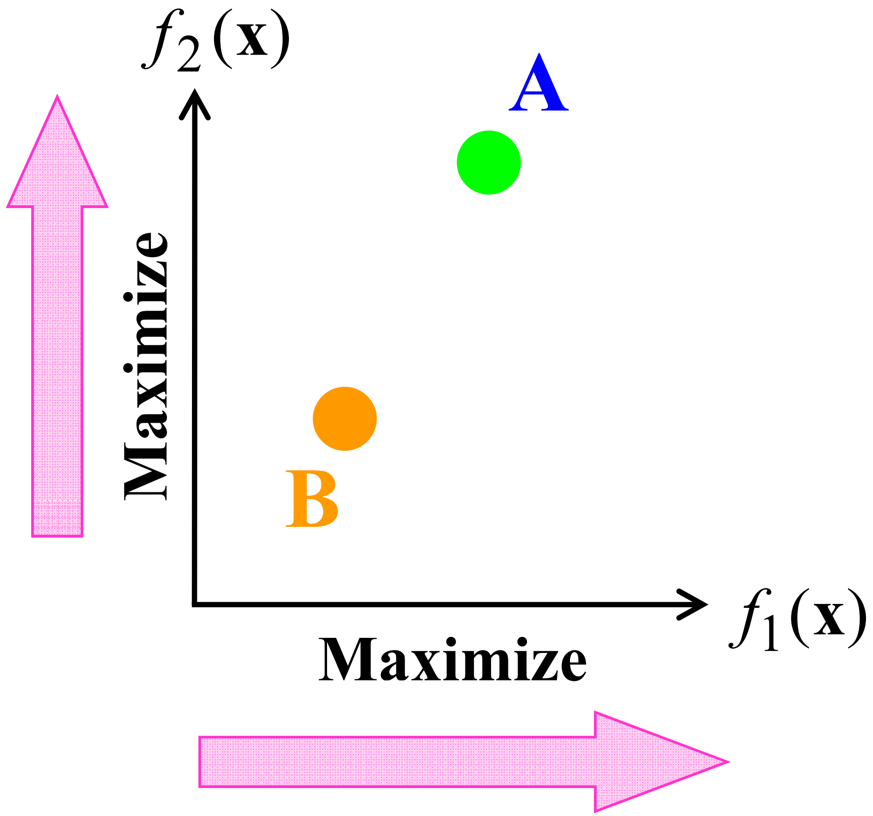
Two-Objective Maximization Problem:

$$\textbf{Maximize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$$



Comparison between Two Solutions

Maximize $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}))$



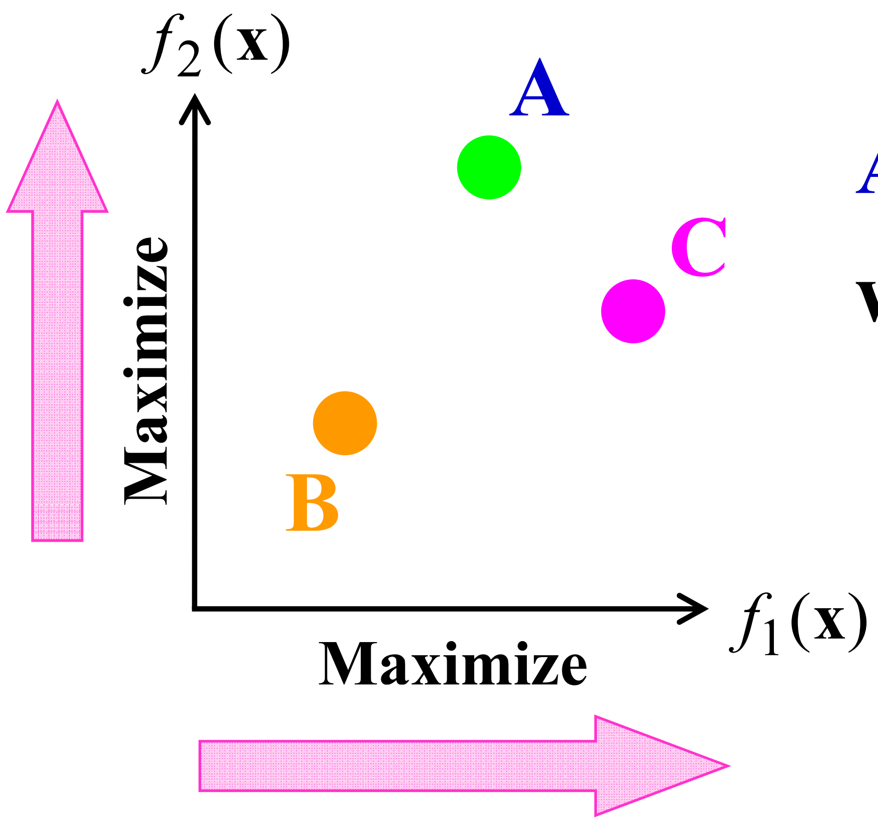
A dominates B

B is dominated by A

(A is better than B)

Comparison between Two Solutions

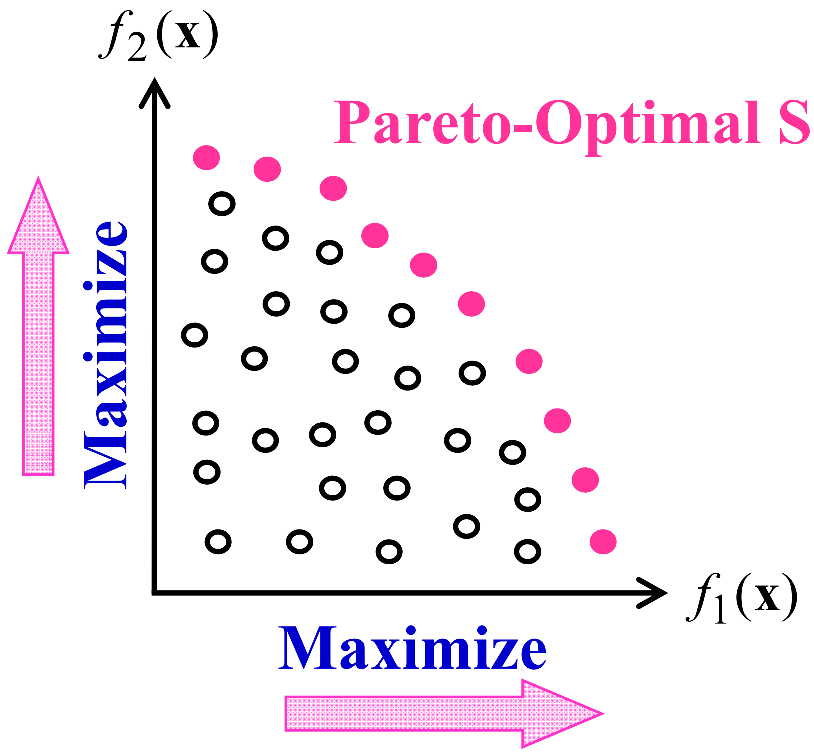
Maximize $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}))$



A and C are non-dominated with each other.

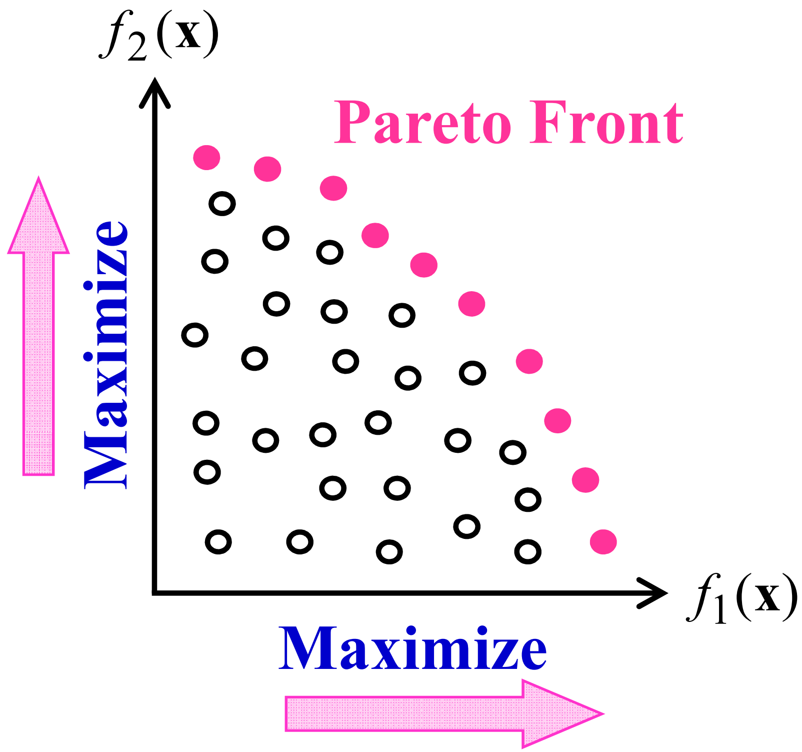
Pareto-Optimal Solutions

A Pareto-optimal solution is a solution that is not dominated by any other solutions.



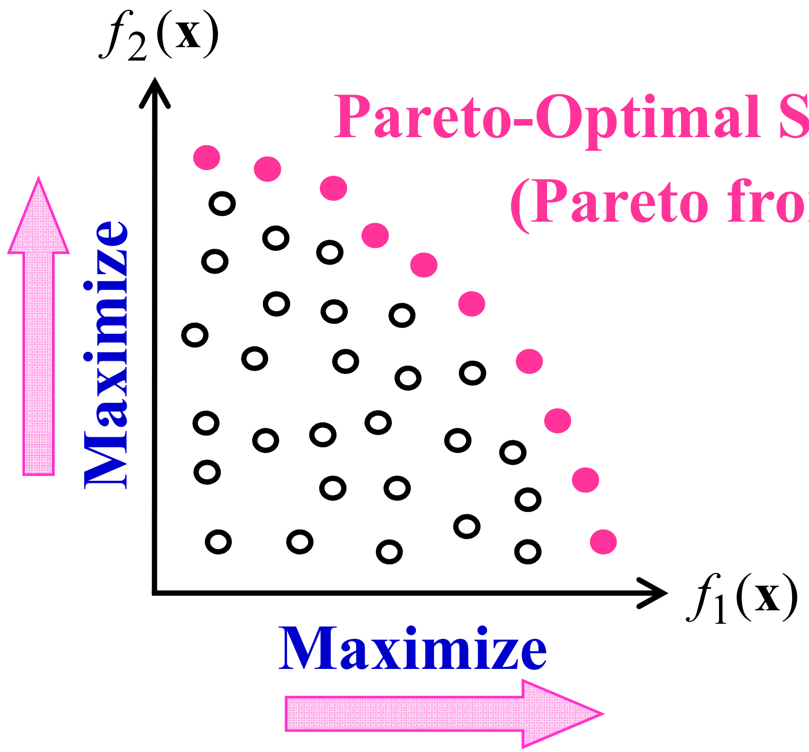
Pareto Front

The set of all Pareto-optimal solutions is called the Pareto front of the problem.



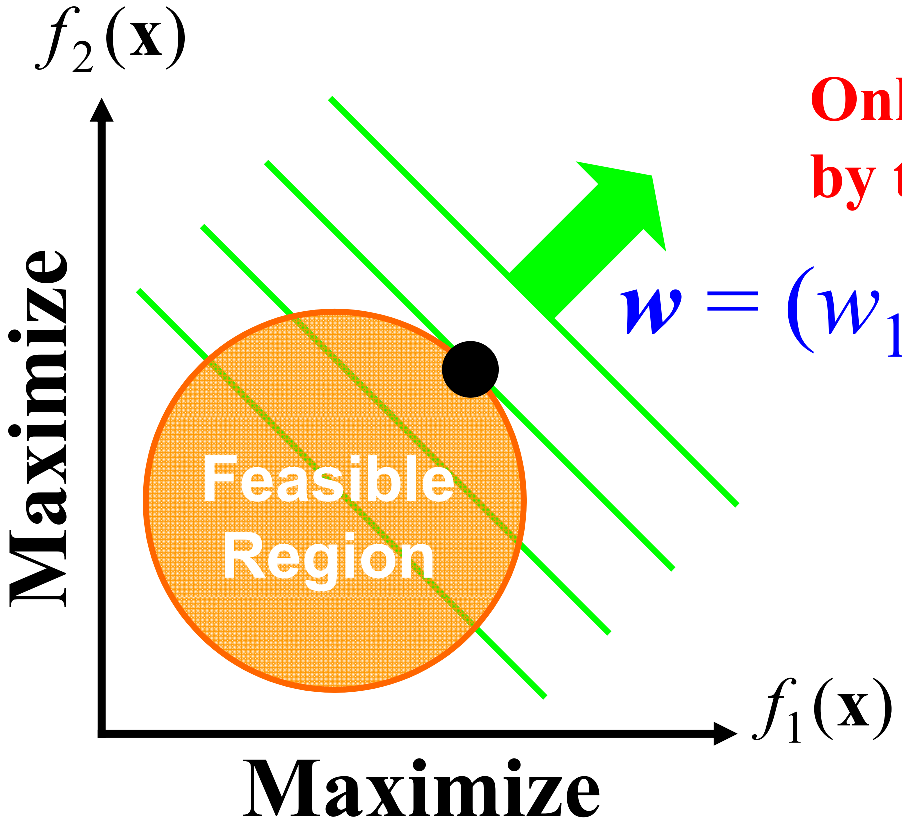
EMO Algorithms

Evolutionary multiobjective optimization (EMO) algorithms have been designed to search for Pareto-optimal solutions in their single run.



Comparison: Weighted Sum Approach

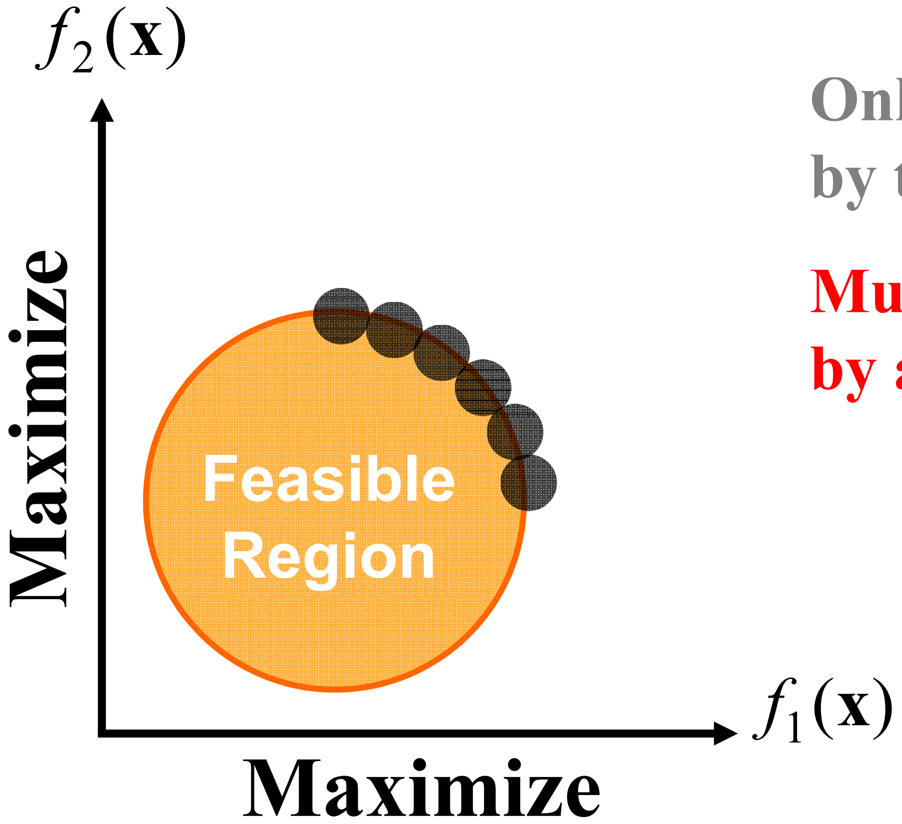
$$\text{Maximize} \quad g(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x})$$



Only a single solution is obtained by the weighted sum approach.

Comparison: EMO Approach

Maximize $f_1(\mathbf{x}),\ f_2(\mathbf{x})$

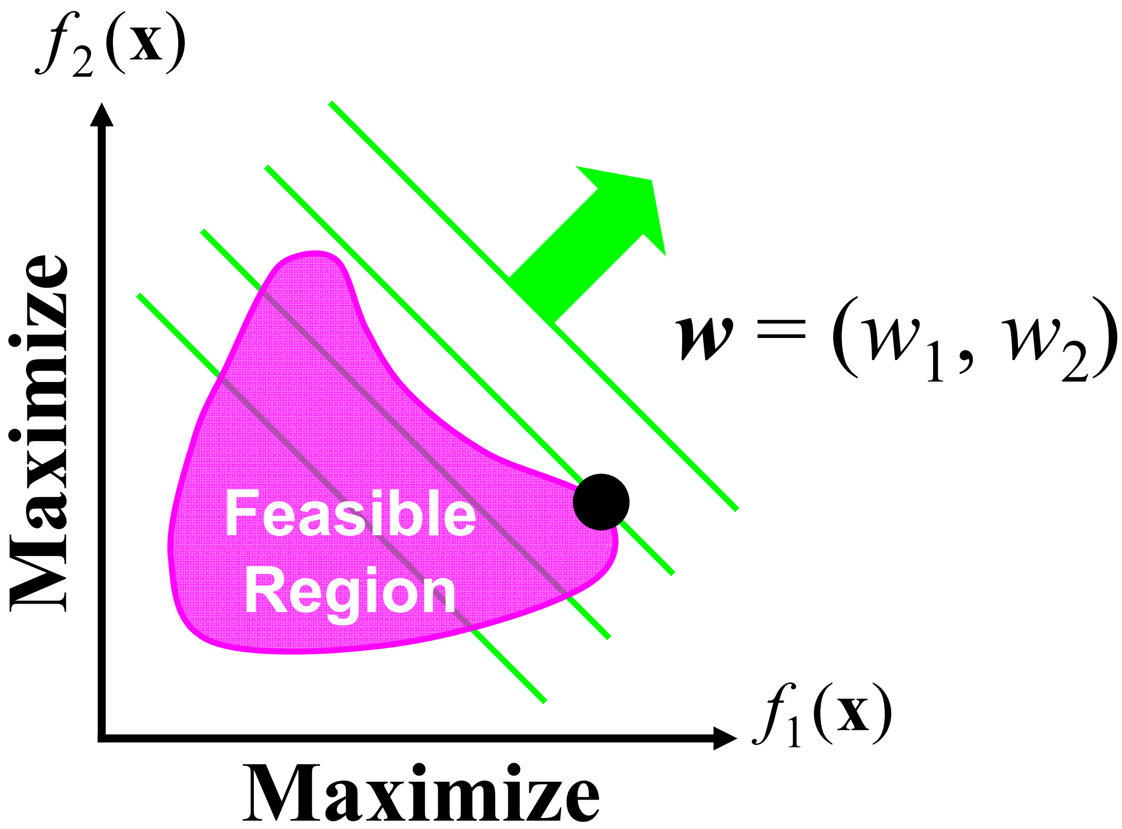


Only a single solution is obtained by the weighted sum approach.

Multiple solutions are obtained by an EMO algorithm.

Difficulties in Weighted Sum Approach

- This approach is sensitive to the weight vector specification.
- This approach can not find any Pareto-optimal solutions in a non-convex region of the Pareto front in the objective space.



Difficulties in Weighted Sum Approach

- This approach is sensitive to the weight vector specification.
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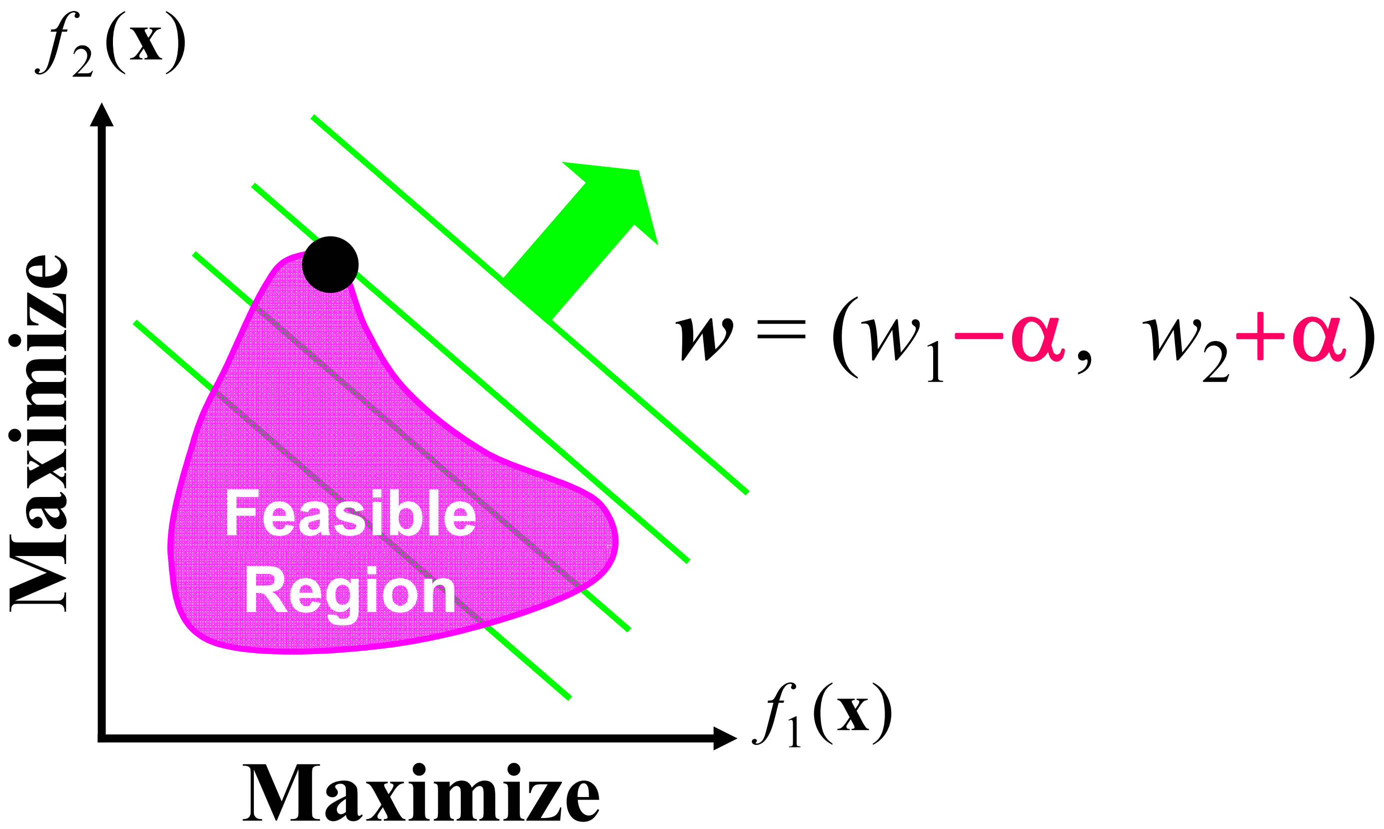
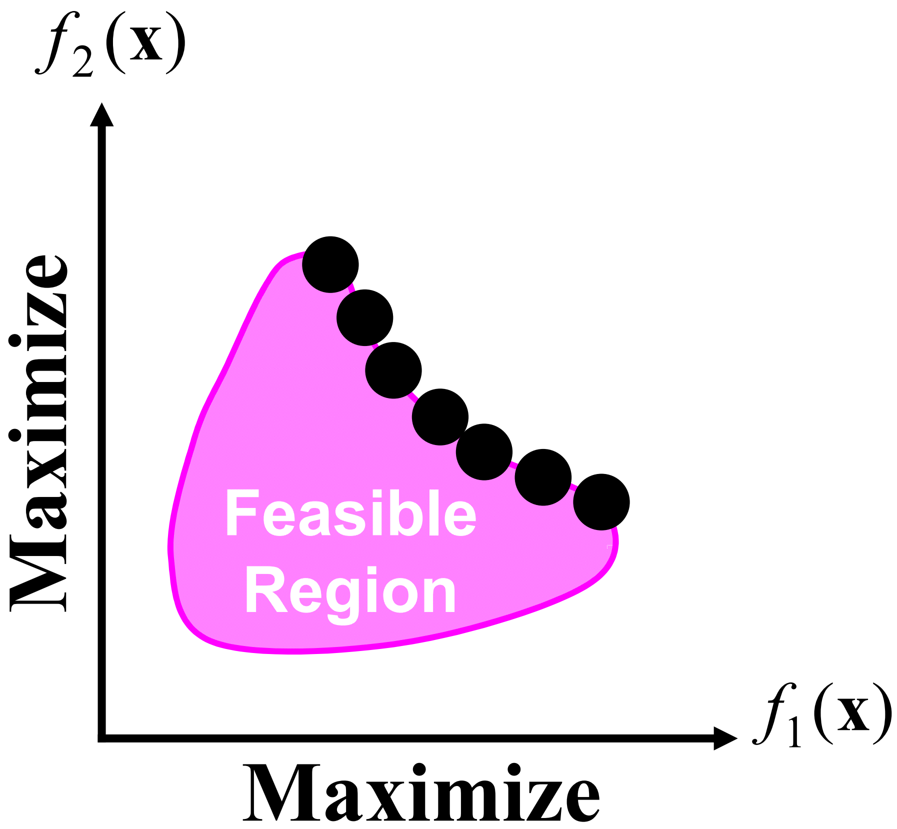


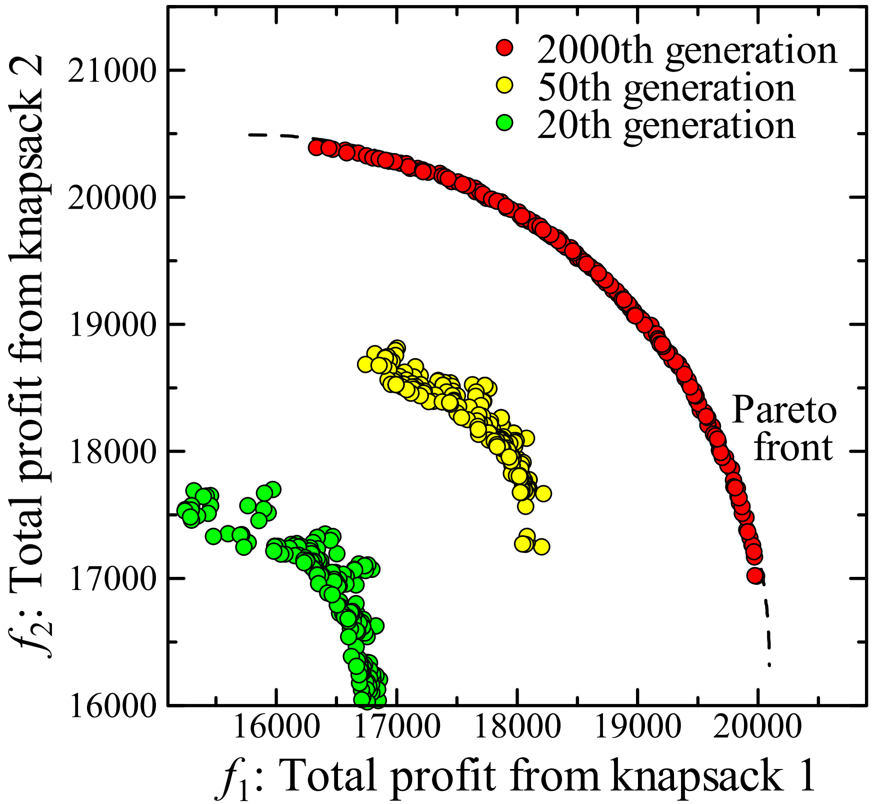
EMO Approach

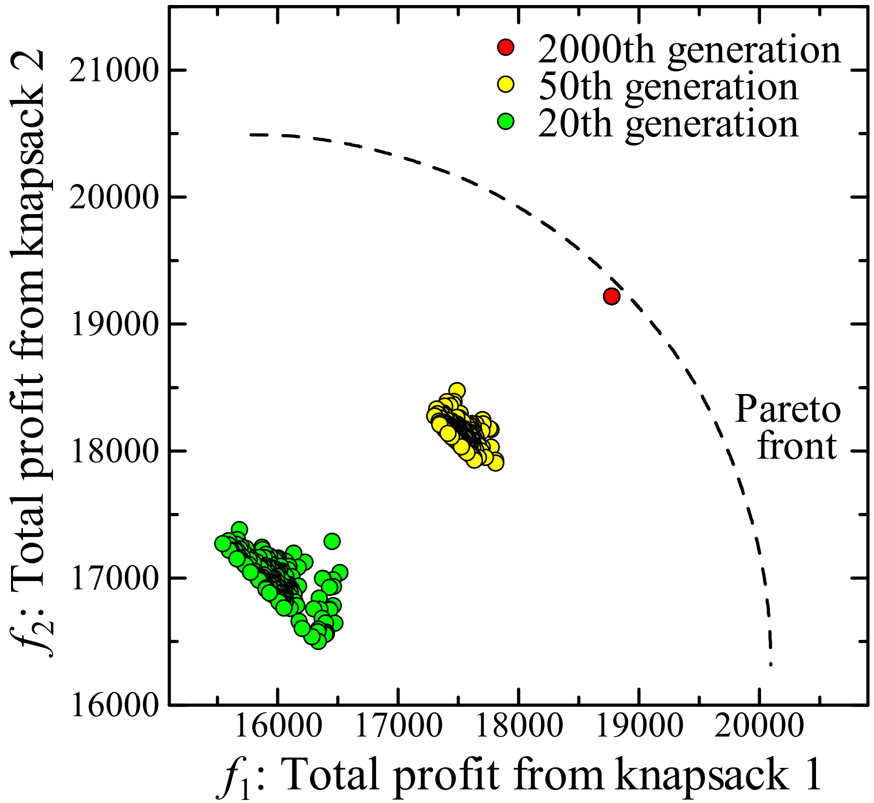
- EMO approach can find Pareto-optimal solutions even in a non-convex region of the Pareto front in the objective space.



Comparison of the Two Approaches

Two-objective maximization problem



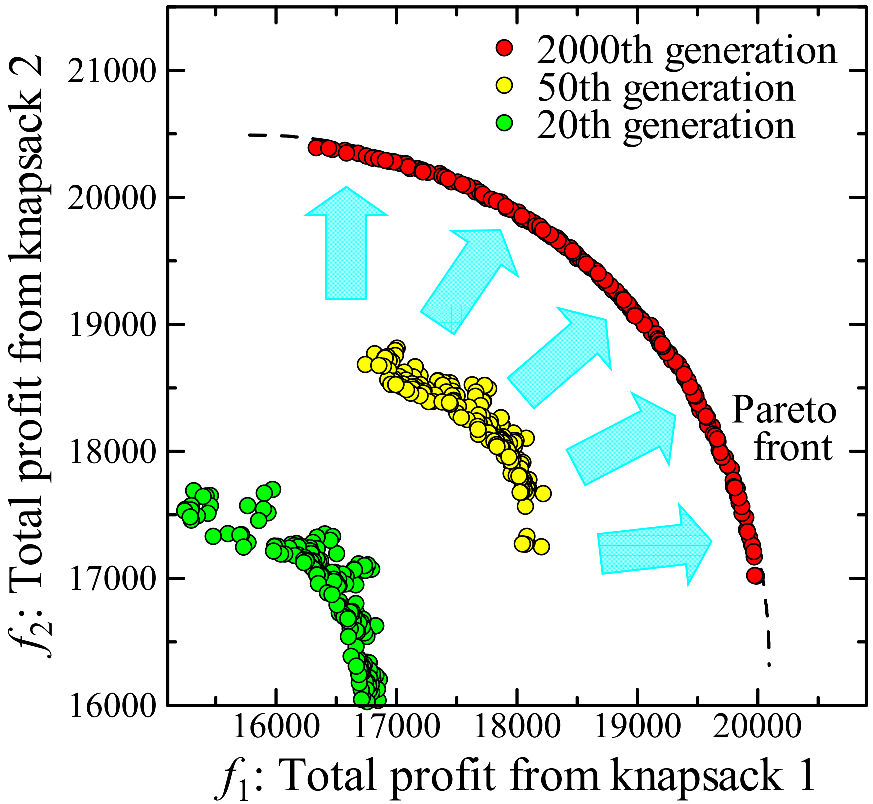


EMO Approach **Weighted Sum Approach**

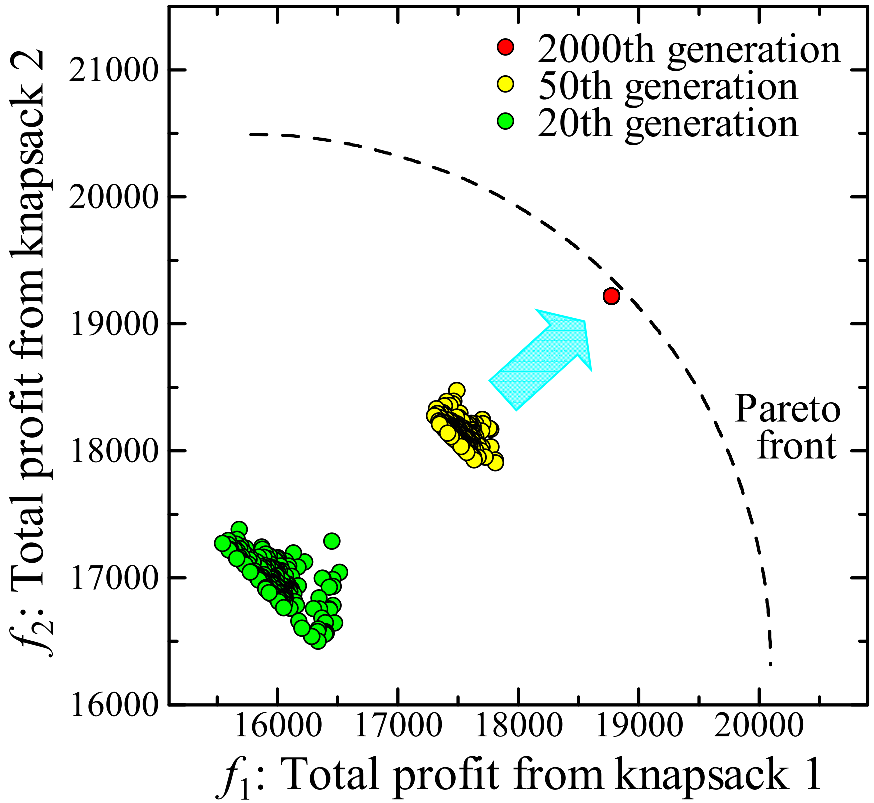
Experimental results of a single run of each approach

Search Direction in Each Approach

Two-objective maximization problem



EMO Approach



Weighted Sum Approach

Both the diversity and the convergence should be improved in EMO.

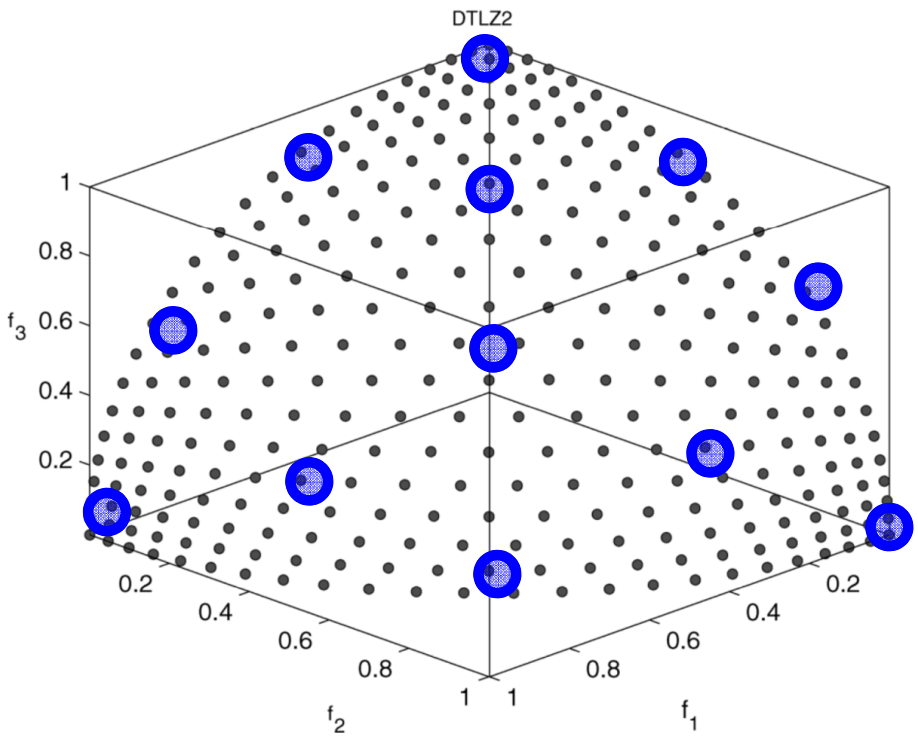
Highly Cited EMO Papers

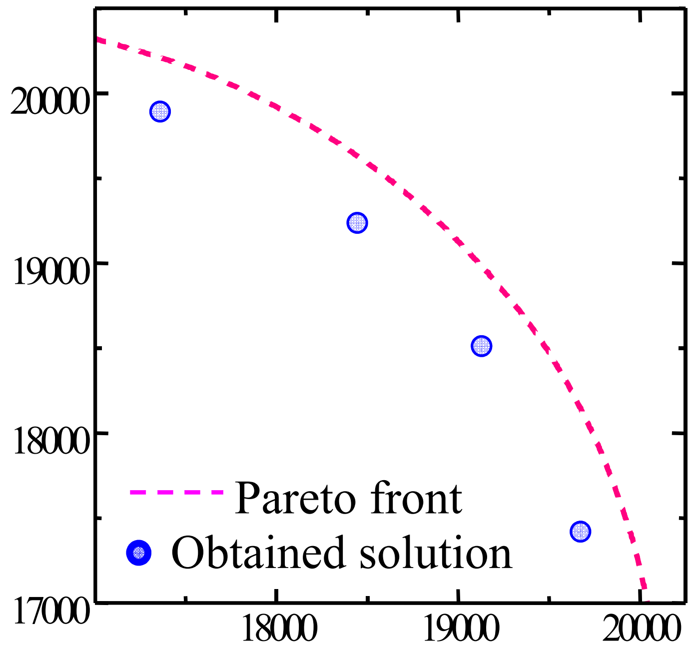
Two Dominant Algorithms: NSGA-II and SPEA

1. Deb K et al. (2002) A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE TEC*. NSGA-II
2. Zitzler E, Thiele L (1999) Multiobjective evolutionary algorithms: A comparative case study and the Strength Pareto approach. *IEEE TEC*. SPEA (=> SPEA2 in TIK-Report)
3. Fonseca CM, Fleming PJ (1998) Multiobjective optimization and multiple constraint handling with evolutionary algorithms (Part I): A unified formulation, *IEEE SMC Part A*.
4. Zitzler E, Thiele L, Laumanns M (2003) Performance assessment of multiobjective optimizers: An analysis and review. *IEEE TEC*.
5. Ishibuchi H, Murata T (1998) A multi-objective genetic local search algorithm and its application to flowshop scheduling, *IEEE SMC Part C*.

Goal of EMO Algorithms

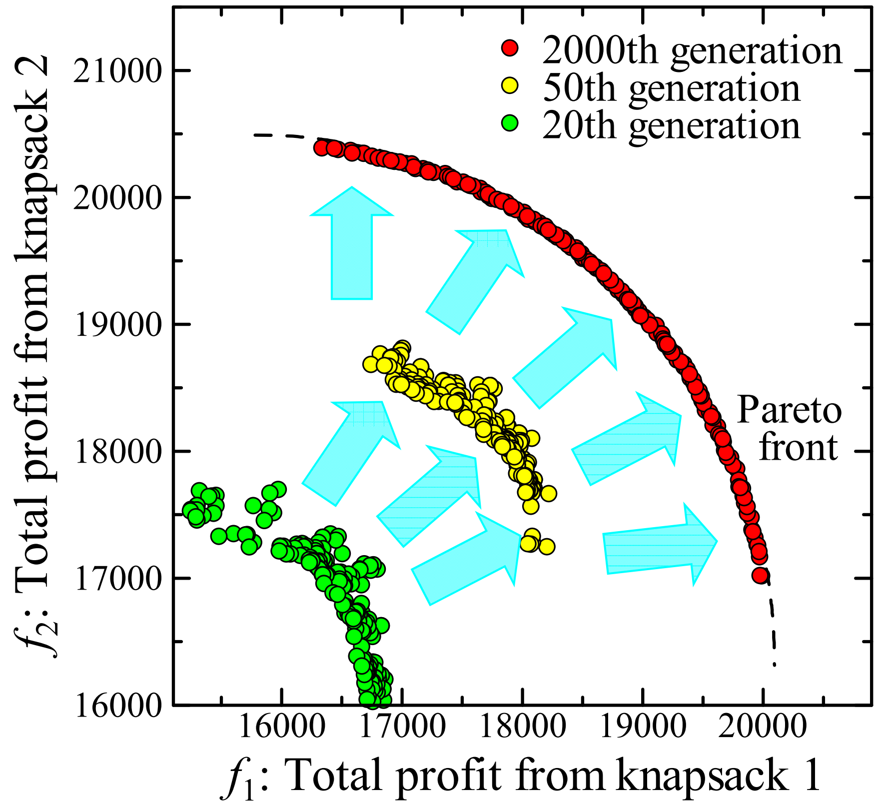
An EMO algorithm is designed to search for
  - all Pareto-optimal solutions
  - uniformly distributed Pareto optimal solutions
  - a solution set which approximates the Pareto front
in their single run.





Basic Ideas in EMO Algorithm Design

Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features.



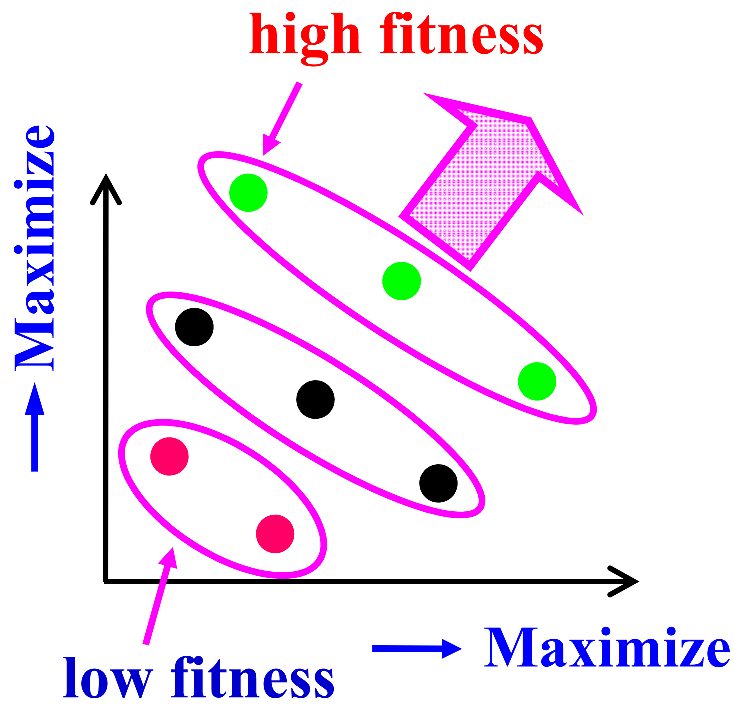
Desired search behavior of EMO algorithms

Basic Ideas in EMO Algorithm Design

Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features:

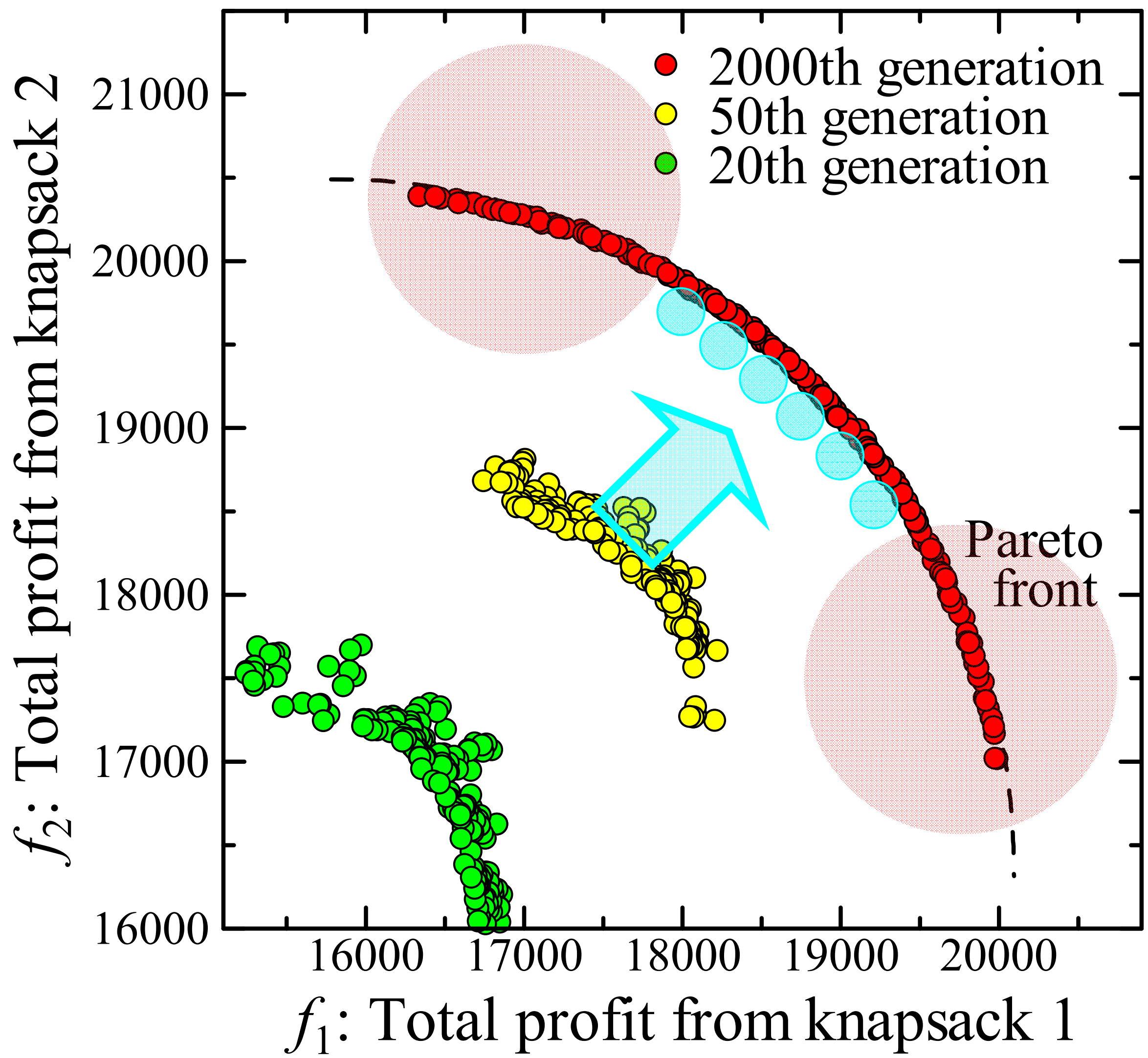
(1) Pareto Dominance

Converge to the Pareto front



Basic Ideas in Recent EMO Algorithms

1. Pareto Dominance



Basic Ideas in EMO Algorithm Design

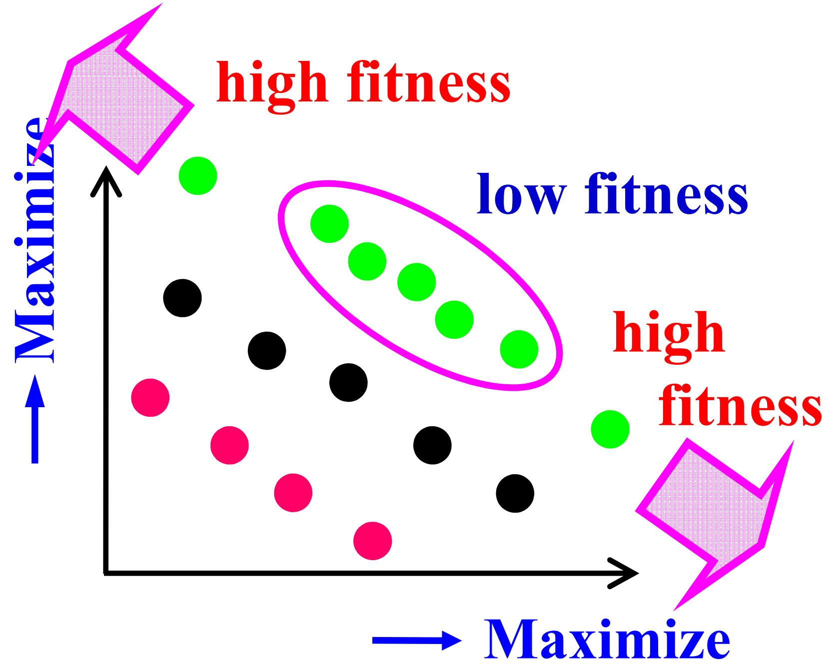
Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features:

(1) Pareto Dominance

Converge to the Pareto front

(2) Crowding

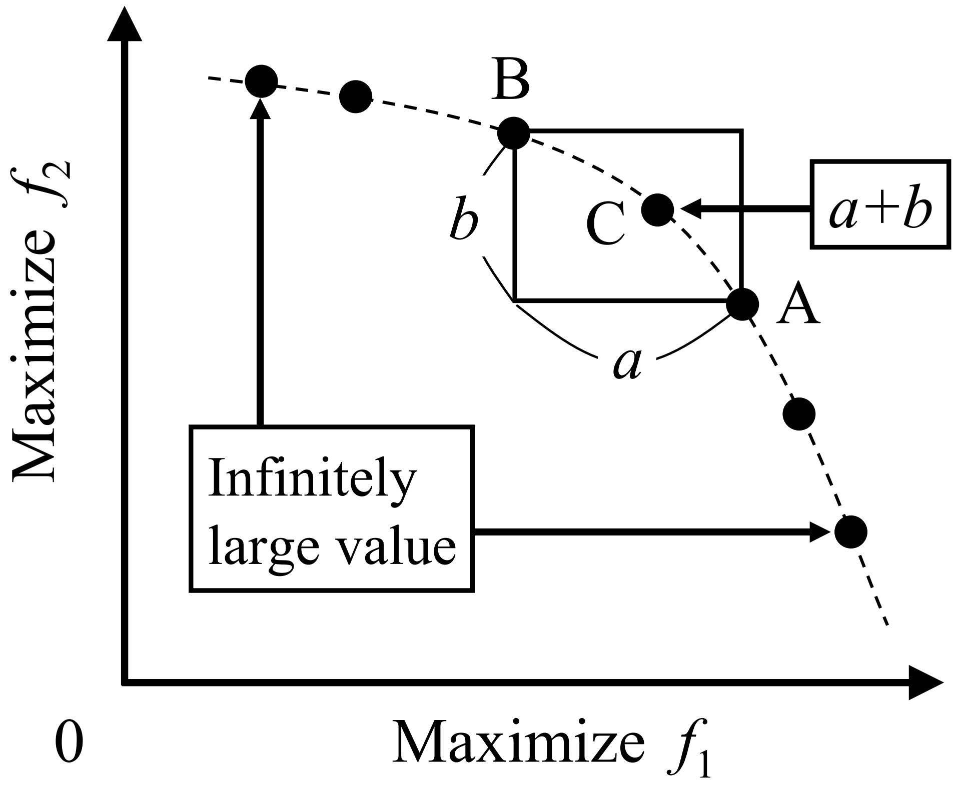
Diversity maintenance



Basic Ideas in EMO Algorithm Design

Example: Crowding Distance in NSGA-II

Distance between adjacent individuals



Crowding distance of C is (*a* + *b*)

Basic Ideas in EMO Algorithm Design

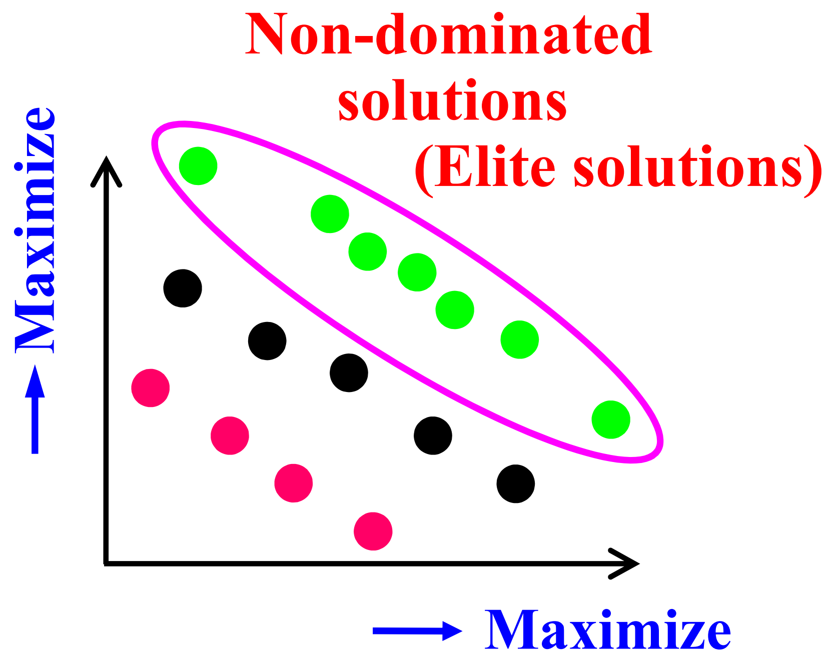
Recently developed well-known EMO algorithms such as NSGA-II and SPEA2 have some common features:

(1) Pareto Dominance

Converge to the Pareto front

(2) Crowding

Diversity maintenance

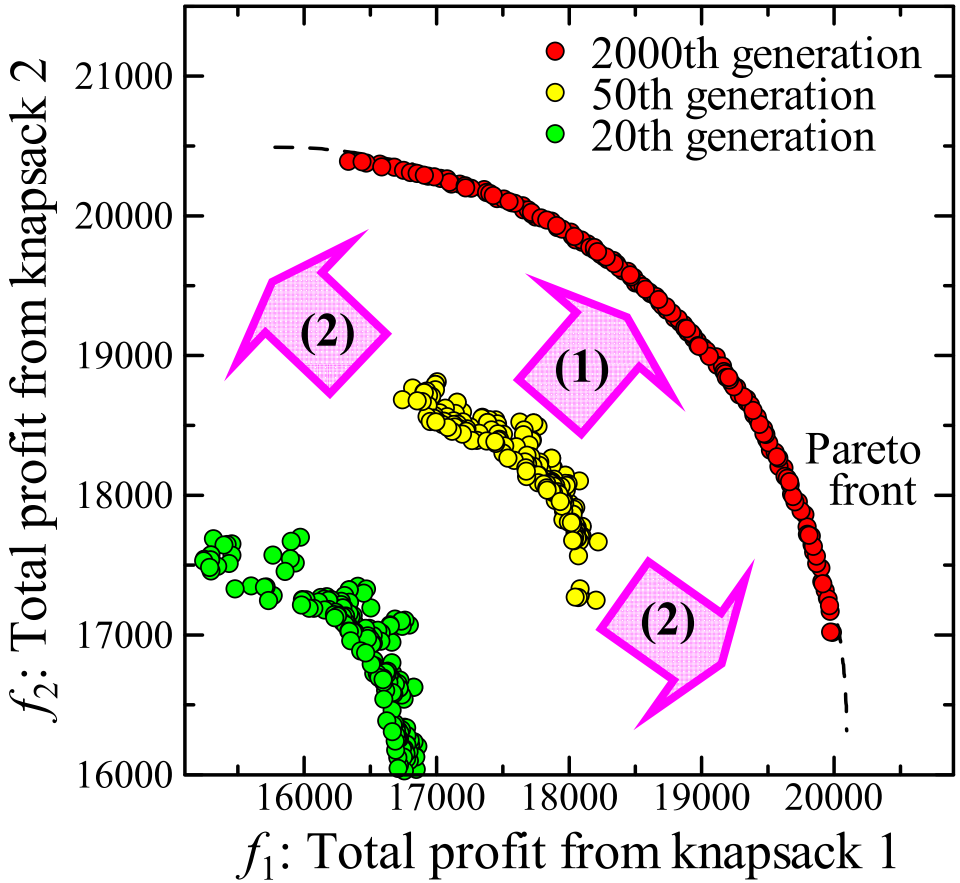


(3) Elitist Strategy

Non-dominated solutions are handled as elite solutions.

Basic Ideas in Recent EMO Algorithms

(1) Pareto Dominance (Convergence to the Pareto front)
(2) Crowding (Diversity Maintenance)
(3) Elite Strategy (Non-Dominated Solutions)



Hot Issues in EMO Research

Utilization of Decision Maker's Preference

- Preference is incorporated into EMO algorithms.
- Interactive EMO approaches seem to be promising.

Handling of Many Objectives by EMO Algorithms

- Pareto dominance-based algorithms do not work well.
- More selection pressure is needed.

Hybridization with Local Search

- Hybridization often improves the performance of EMO.
- Balance between local and genetic search is important.

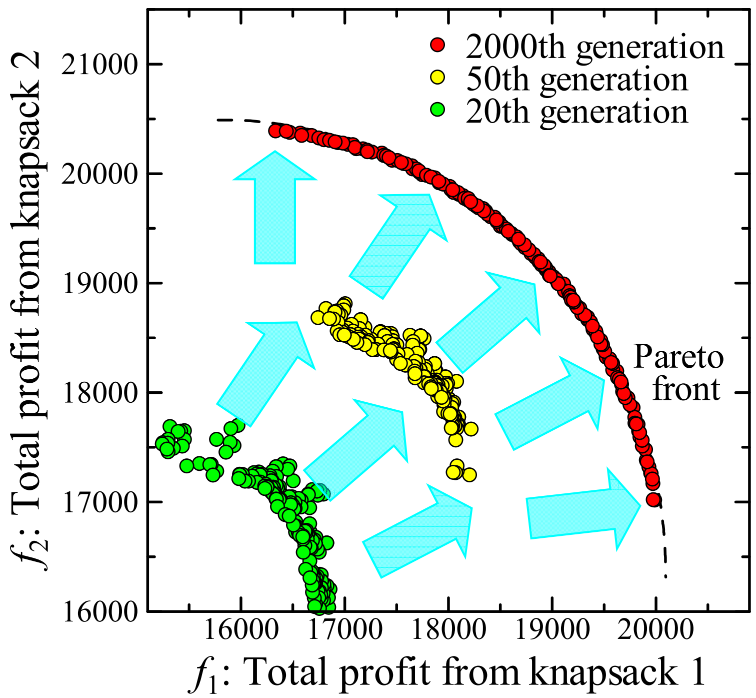
Design of New EMO Algorithms

- Indicator-based EMO algorithms
- Scalarizing function-based EMO algorithms
- Use of other search methods such as PSO, ACO and DE.

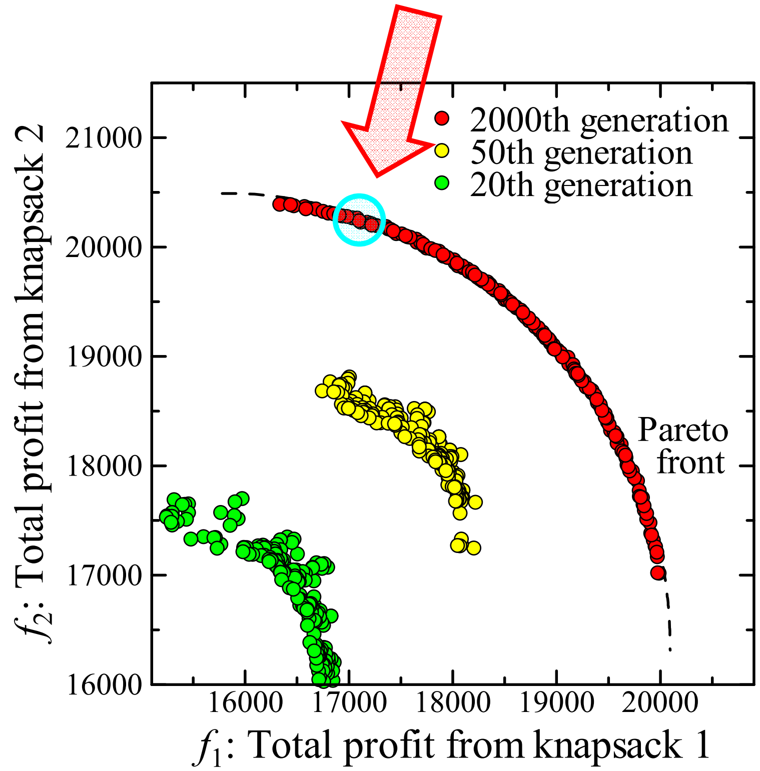
Hot Issue: Preference Incorporation
EMO Approach to Decision Making

Step 1: Evolutionary multiobjective optimization
==> Many non-dominated solutions (Candidates).

Step 2: Choice of a single solution by the decision maker.



Step 1

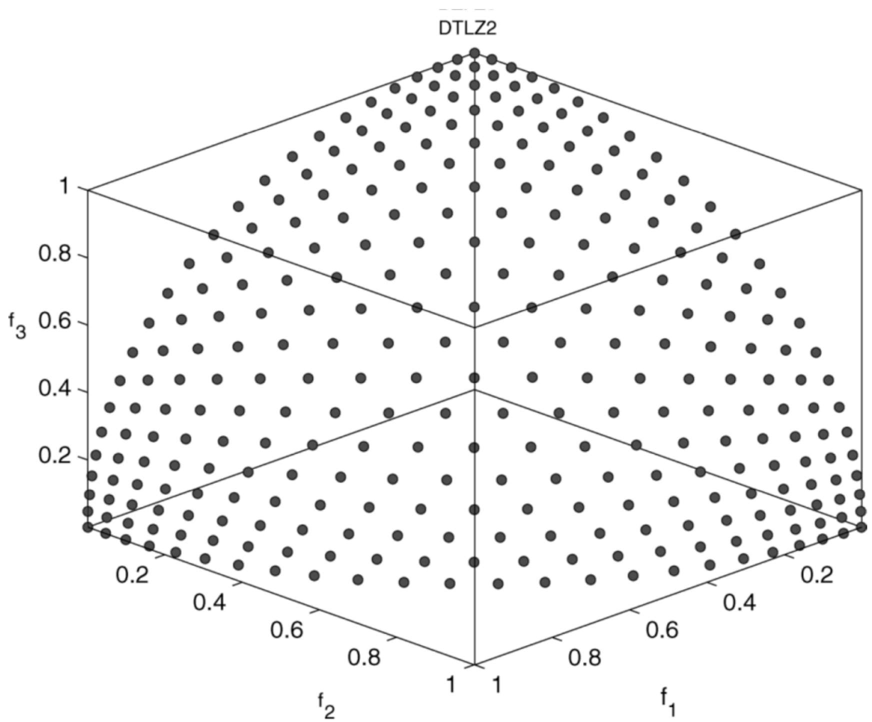


Step 2

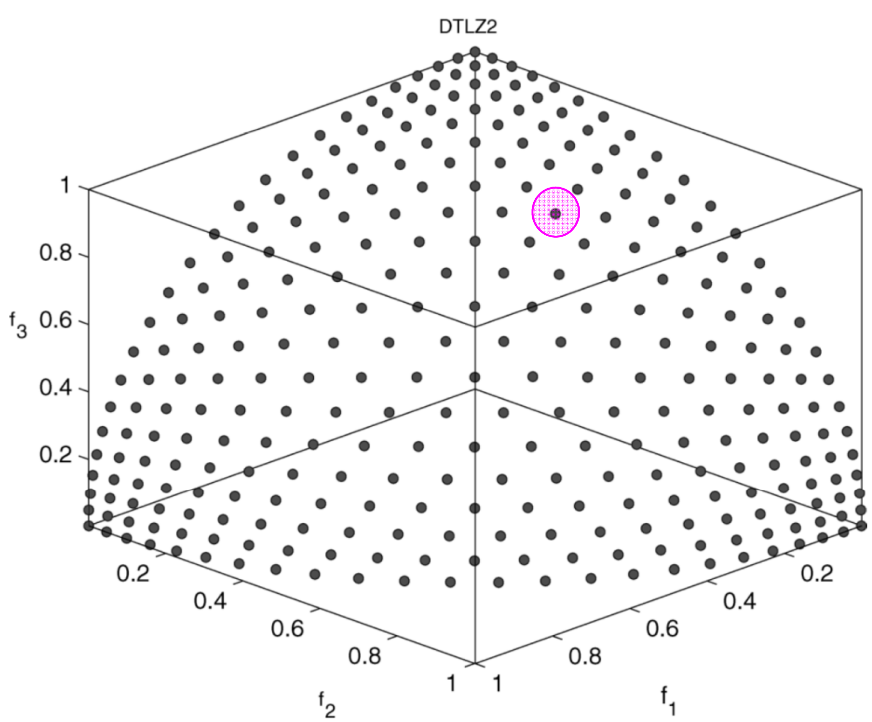
EMO Approach to Decision Making

Difficulty in Step 1: It is not always easy to find a set of non-dominate solutions that covers the entire Pareto front.

Difficulty in Step 2: It is not always easy for the DM to choose a single solution form a large number of alternatives.



Step 1



Step 2

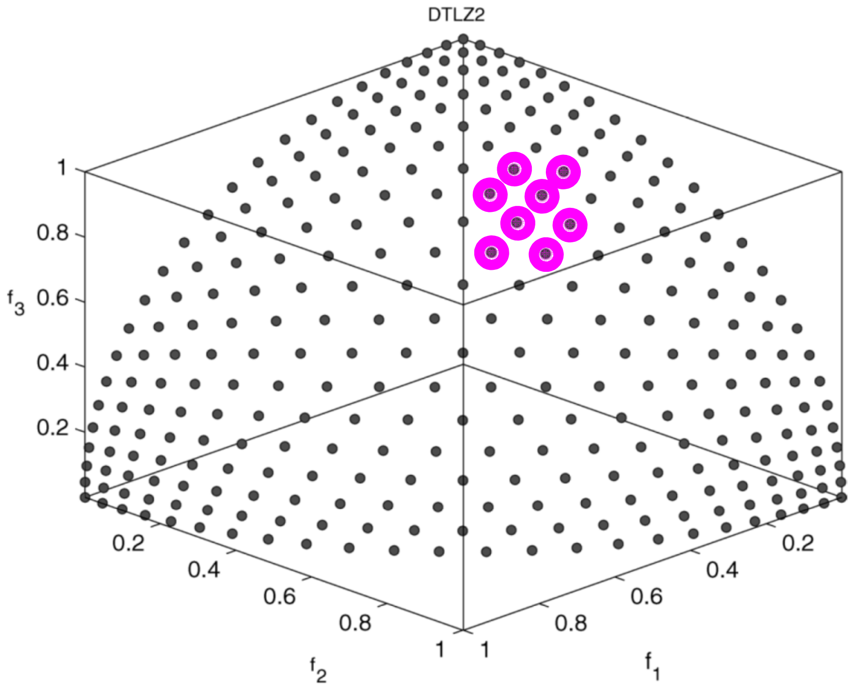
EMO Approach to Decision Making

Difficulty in Step 1: It is not always easy to find a set of non-dominate solutions that covers the entire Pareto front.

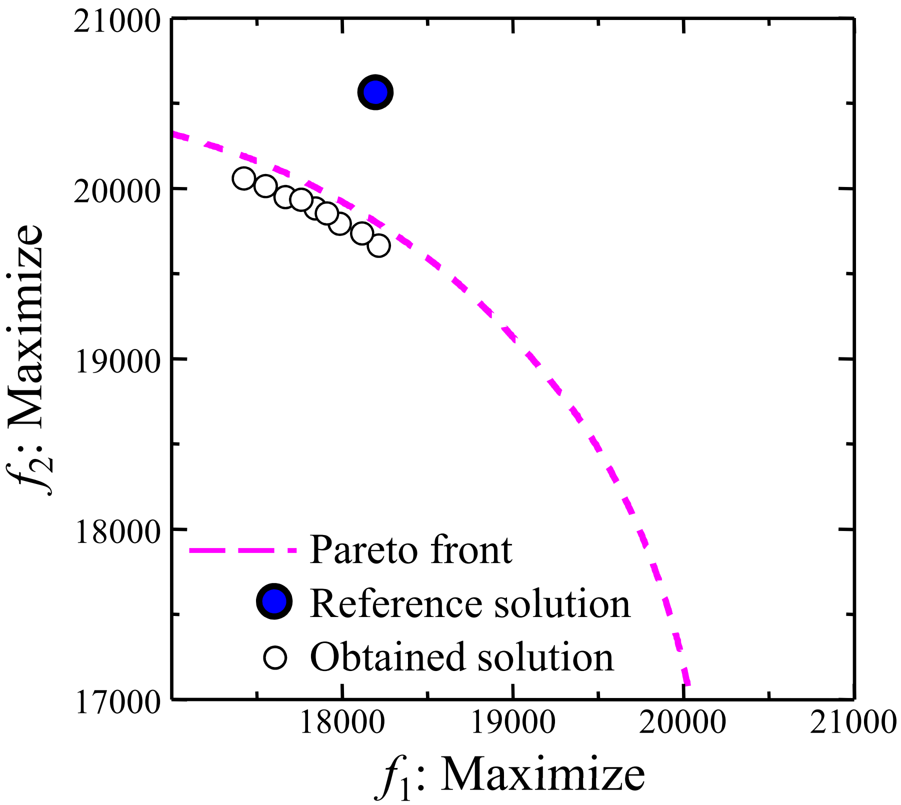
Difficulty in Step 2: It is not always easy for the DM to choose a single solution form a large number of alternatives.

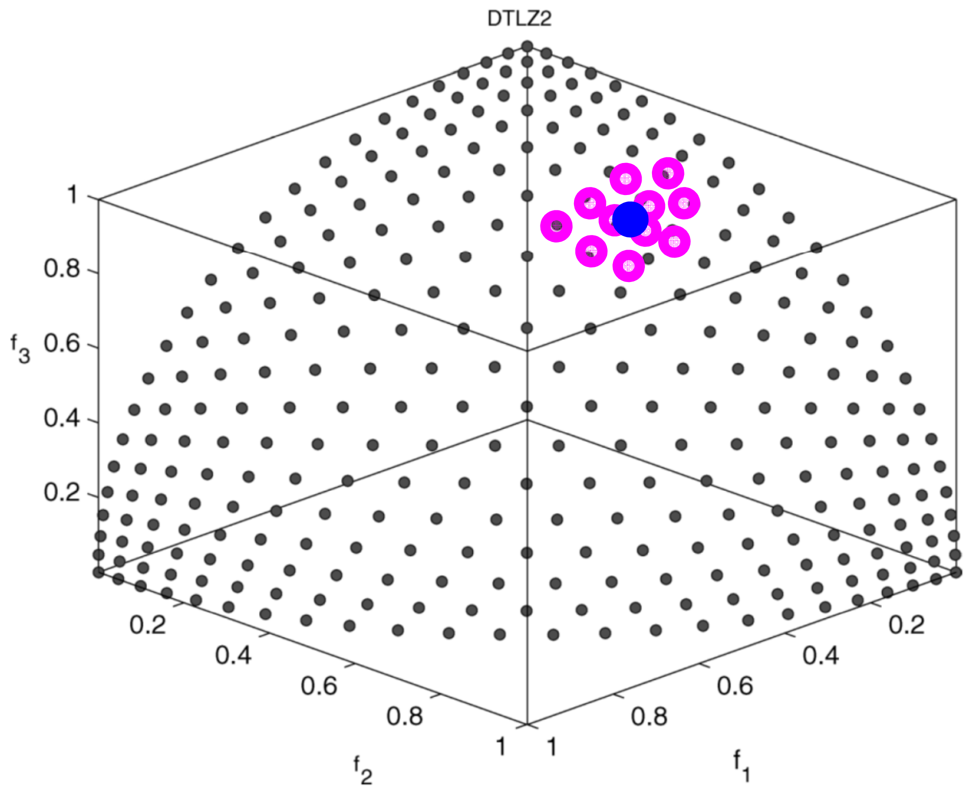
One idea to tackle these two difficulties:

To search for a small number of non-dominate solutions.



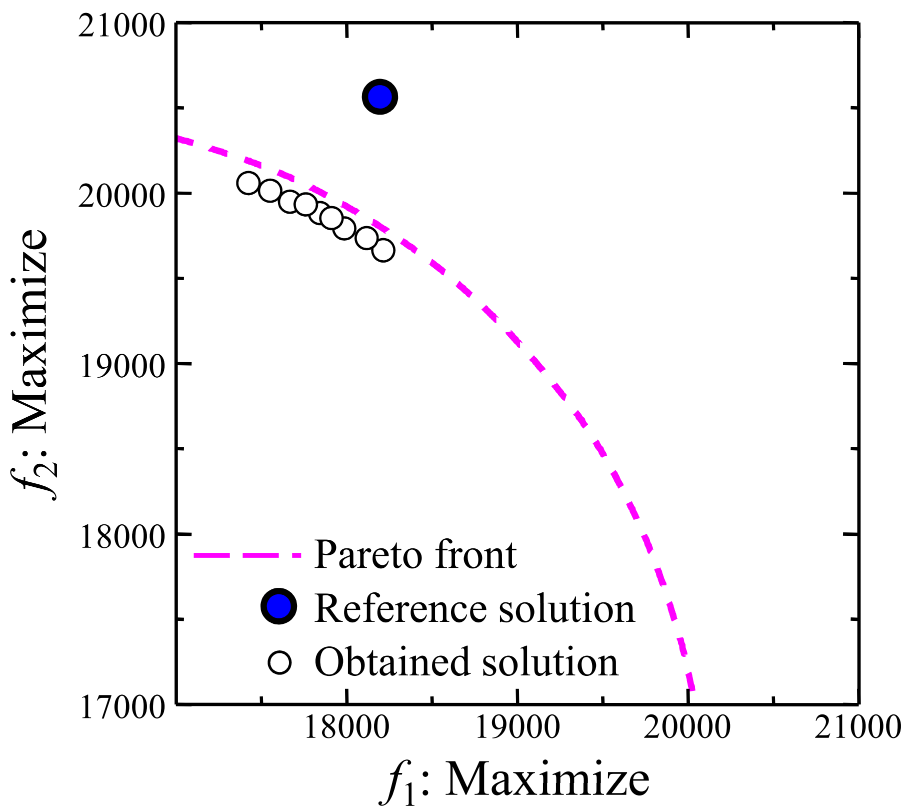
Utilization of Preference Information

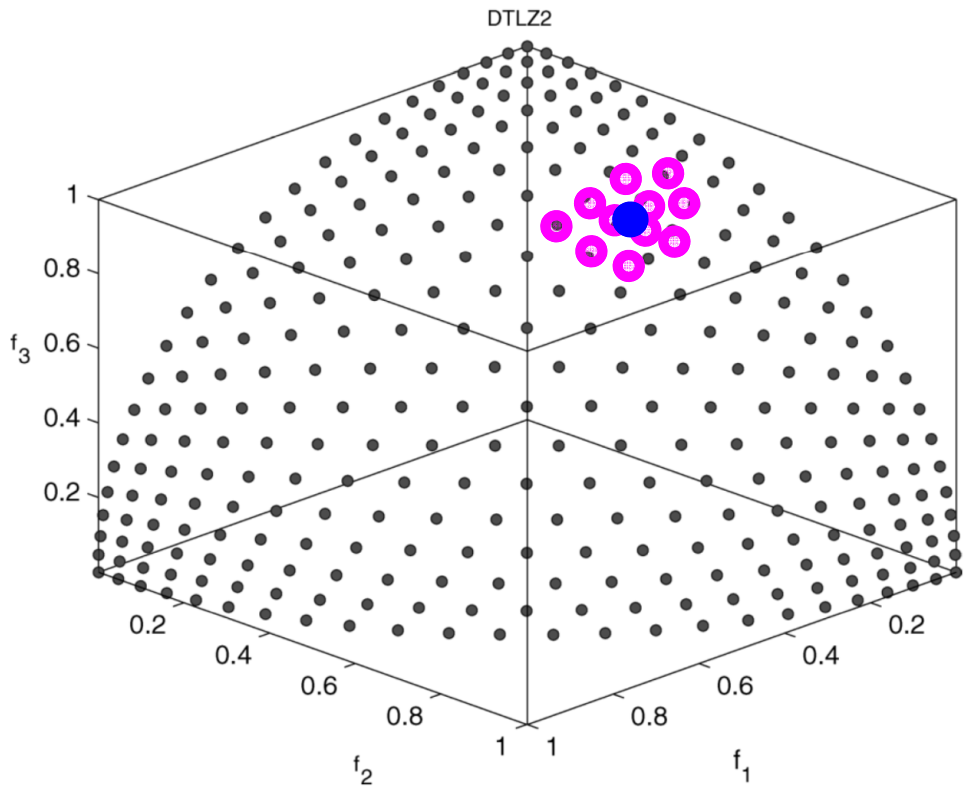




Basic Idea: Concentration on the preferred region of the Pareto front. The decision maker is not always interested in all the Pareto-front.

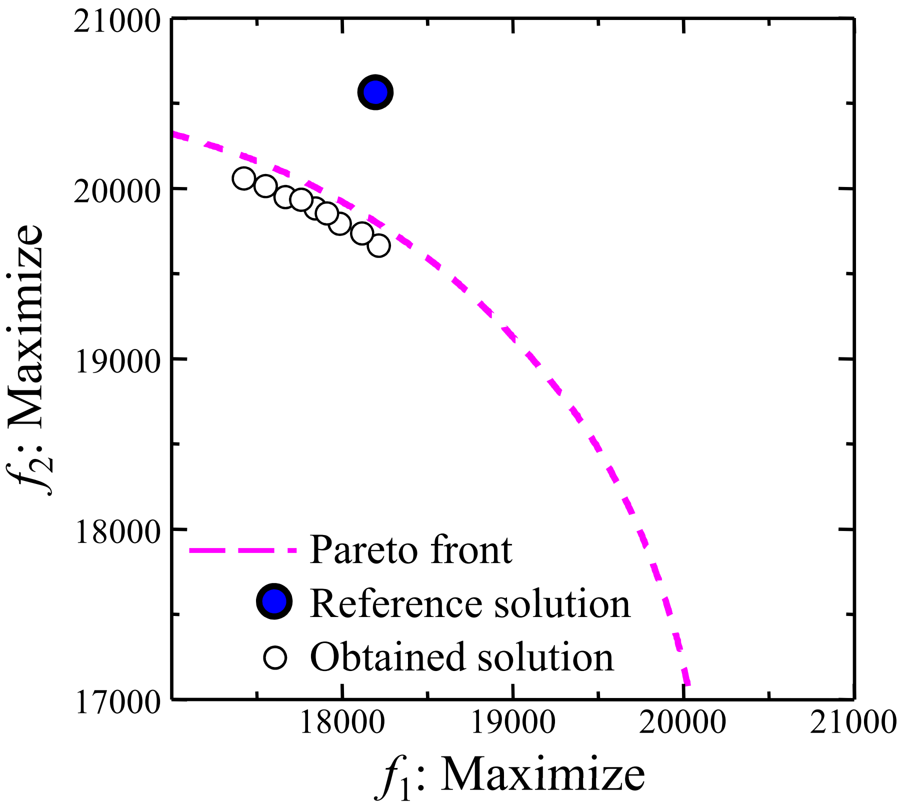
Utilization of Preference Information





Difficulty: **It is not easy to extract preference information from the decision maker (DM). It may be much simpler to compare different solutions. ==> Interactive Approaches.**

Extraction of Preference Information



Preference Extraction

(1) Relatively Easy Case

- Number of Objectives: Two
- Pareto Front: Known
- The DM knows the problem

(2) Very Difficult Case

- Number of Objectives: Many
- Pareto Front: Unknown
- The DM does not know the problem very well.

Example: Flight Tickets (Cost, # of Stops, Total Time)

Case 1: You are planning to buy a ticket to your home town.

Case 2: You are planning to buy a ticket to Easter Island.

Another Hot Issue:
Evolutionary Many-Objective Optimization

Why are many-objective problems difficult?

1. Many Objectives: Difficulty in Multiobjective Search

Selection pressure toward the Pareto front becomes very weak since almost all solutions are non-dominated.

2. Many Solutions: Difficulty in Approximation

A large number of non-dominated solutions are needed to approximate the entire Pareto front.

3. Many Solutions with Many Objectives: Presentation

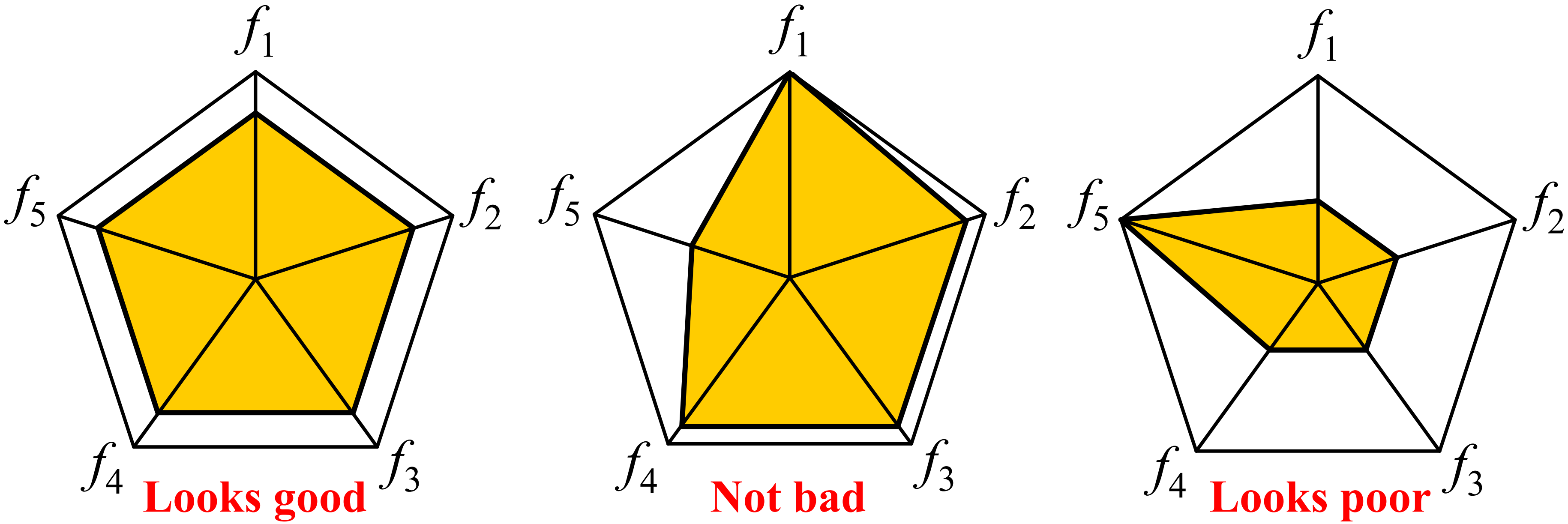
It is very difficult to present a large number of obtained solutions in the high-dimensional object space to the decision maker in a visually understandable manner.

Difficulties in Many-Objective Optimization

Q. **Why are many-objective problems hard for EMO ?**

A. **Solutions with many objectives are usually non-dominated with each other. This means very low selection pressure toward the Pareto front in Pareto dominance-based EMO.**

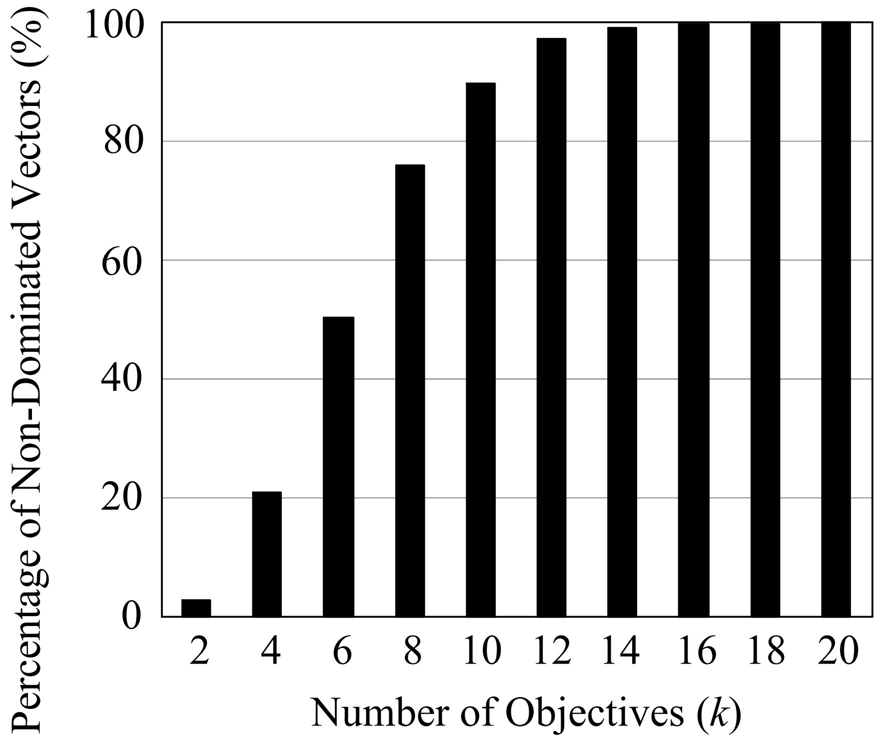
Five-Objective Maximization Example (Non-dominated Vectors)



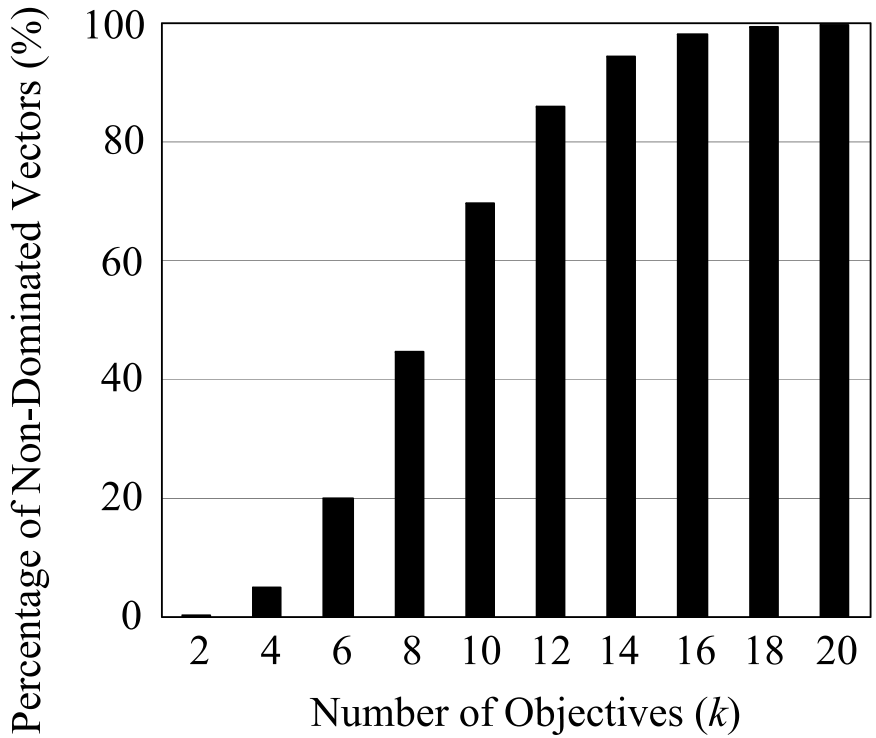
Difficulties in Many-Objective Optimization

Percentage of Non-Dominated Vectors

We randomly generate vectors in a *k*-dimensional space.



(1) Among 200 vectors.



(2) Among 2,000 vectors

Experimental Results of NSGA-II

Standard Implementation of NSGA-II

Generation Update: (100 + 100) ES

Current Population: 100 Individuals

Offspring Population: 100 Individuals

Next Population: The best 100 individuals from the current population and the offspring population.

Fitness Evaluation:

1st Criterion: Pareto Dominance

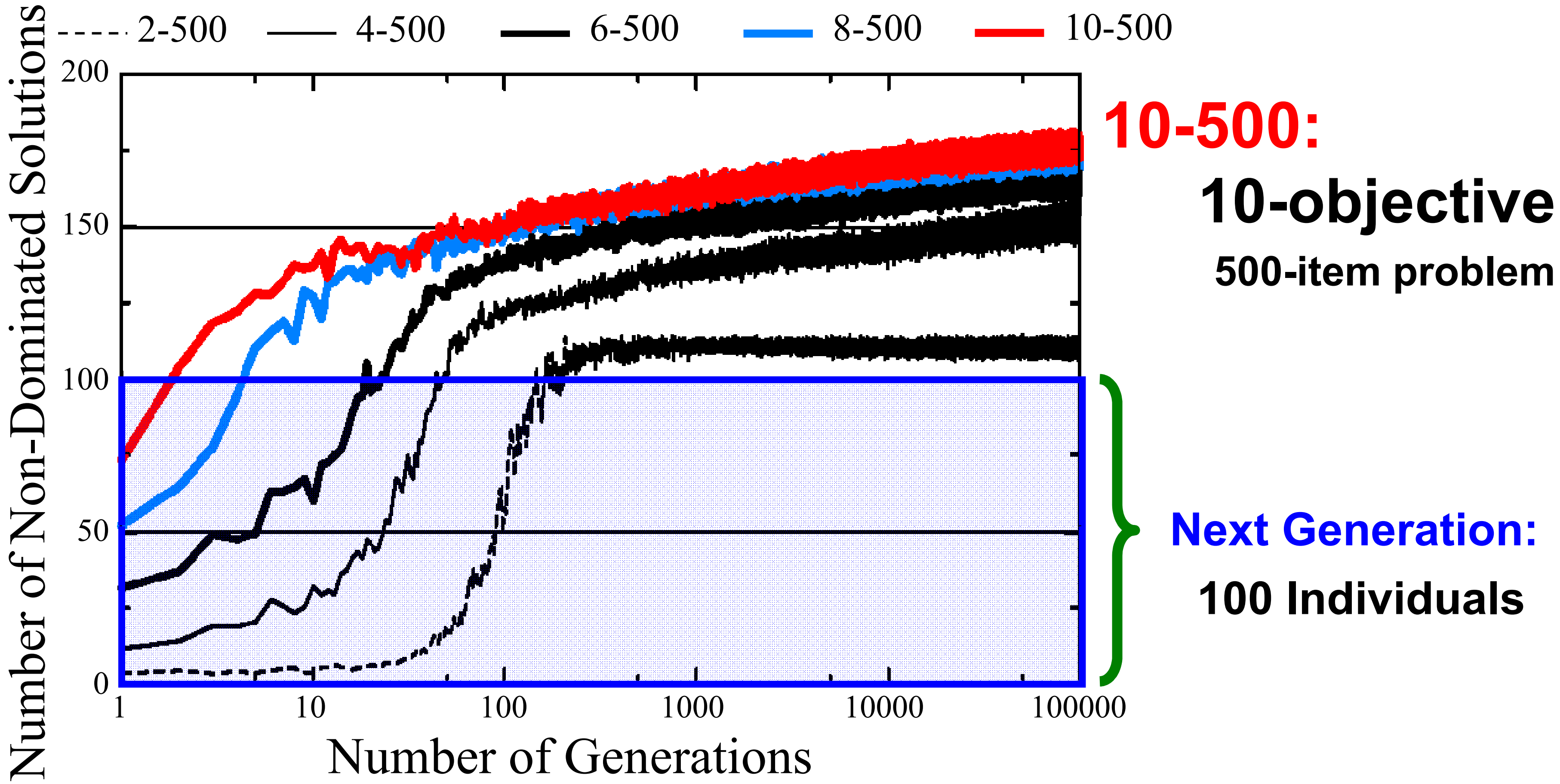
2nd Criterion: Crowding Distance

Test Problems

k-objective 500-item knapsack problems (k-500 problem)

k = 2, 4, 6, 8, 10

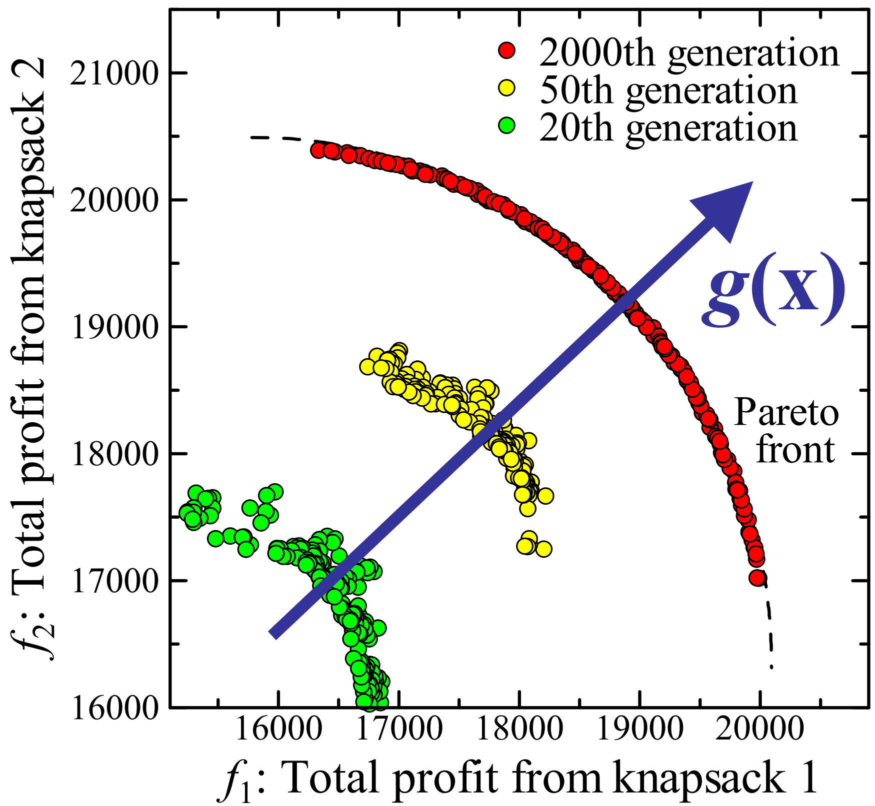
Number of Non-Dominated Solutions
(Among 200 solutions before the generation update in NSGA-II)



All individuals are non-dominated solutions after a few generations (10-500 problem) and after about 200 generations (2-500 problem).

Very Simple Measure of Convergence

The sum of the given objectives: $g(\mathbf{x}) = f_1(\mathbf{x}) + f_2(\mathbf{x})$

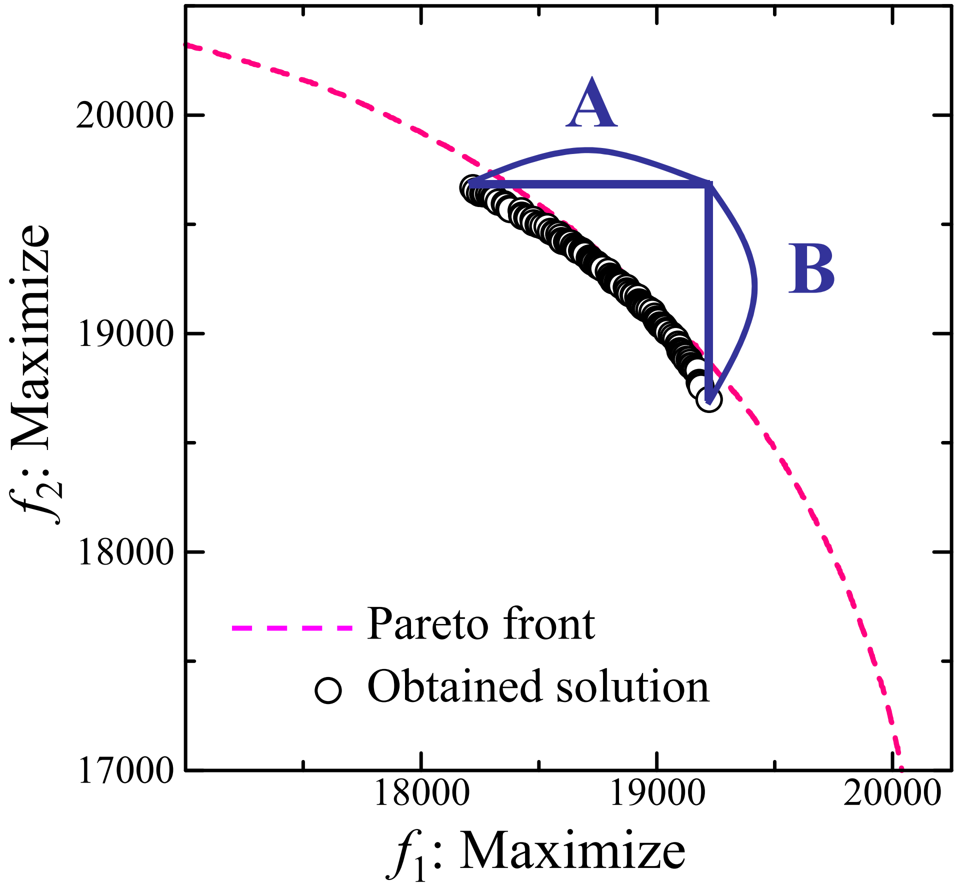


MaxSum

$= \text{Max } \{g(\mathbf{x})\}$

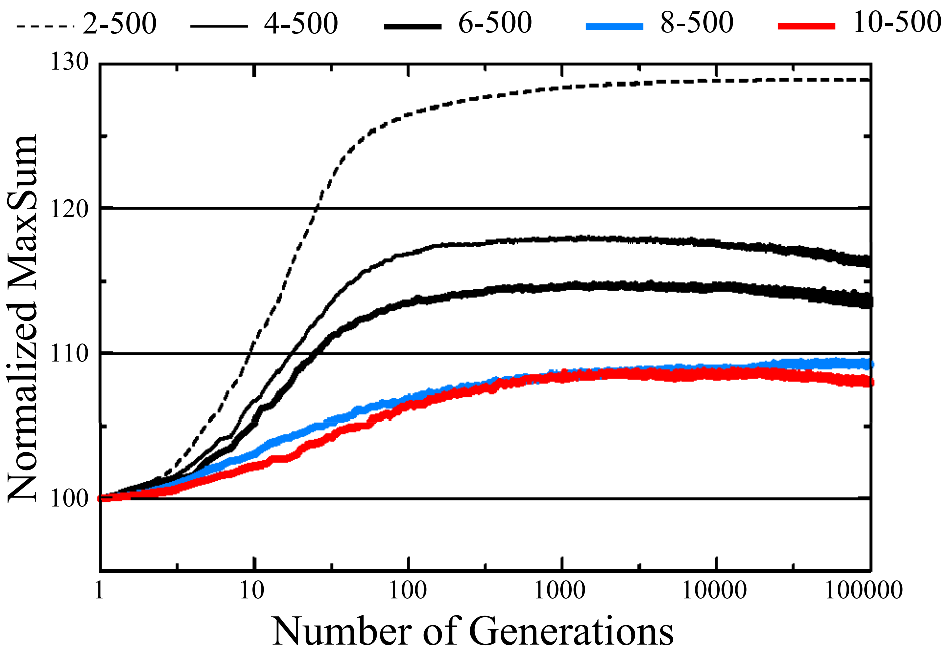
Very Simple Measure of Diversity

Range Measure

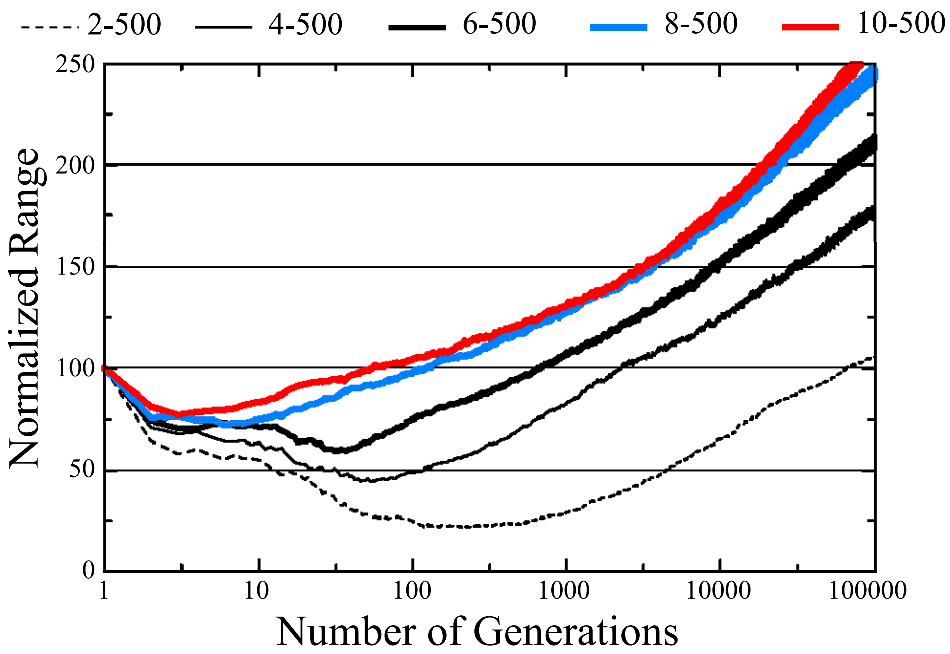


Range = A + B

Experimental Results of NSGA-II



MaxSum: Convergence



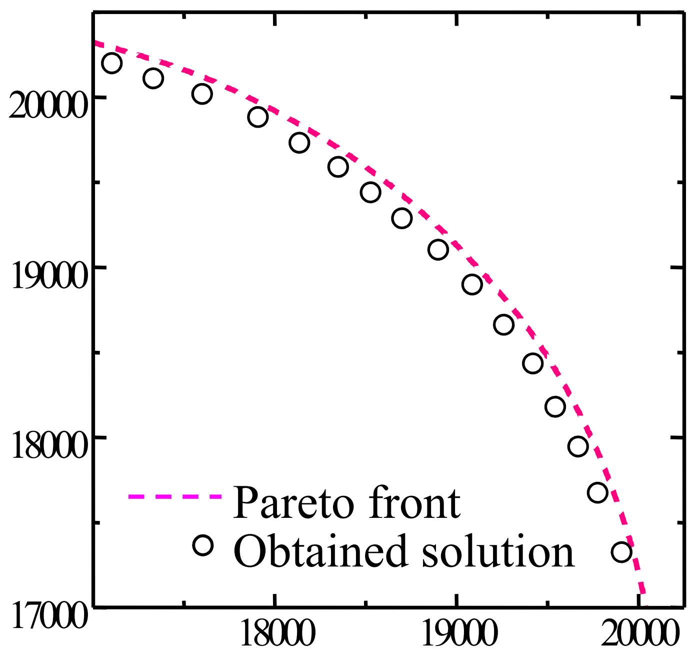
Range: Diversity of solutions

Observation: **Only the convergence was improved in the early generations. After that, only the diversity was improved.**

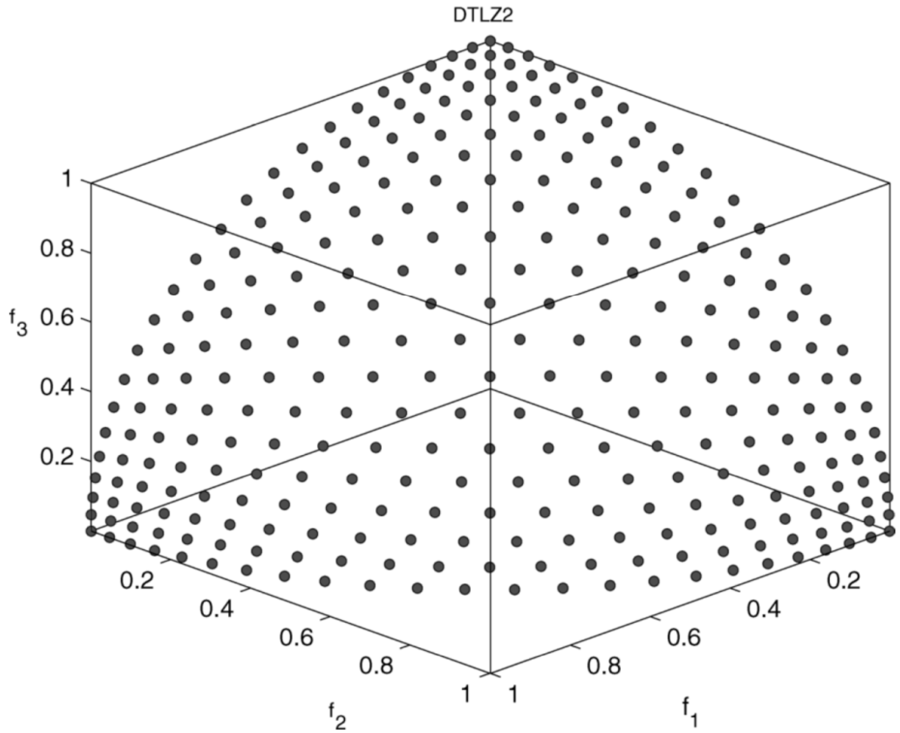
Approximation of the Pareto Front

Q: How many non-dominated solutions are needed to approximate the entire Pareto-front of the k-objective problem? (k = 2, 3, 4, ...)

A: Huge when k is large (It exponentially increases with k)



$k = 2$



$k = 3$

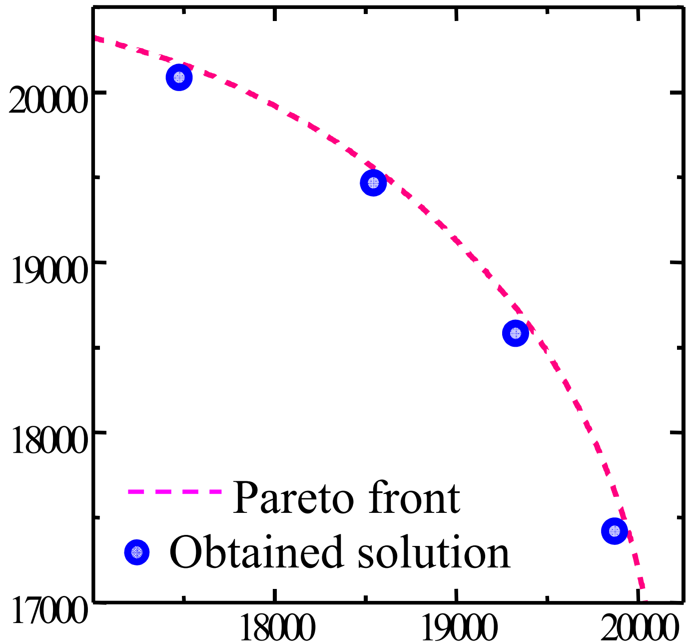
Approximation with Finite Solutions

Two Strategies for Many-Objective Problems

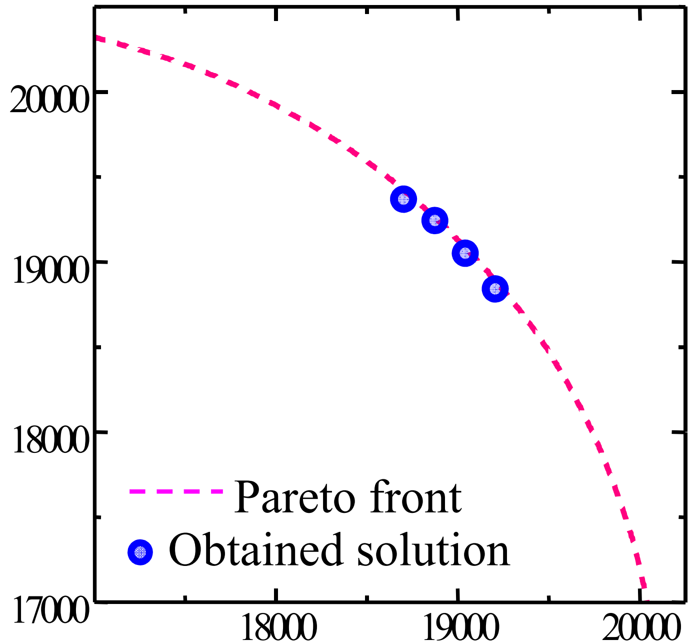
(1) Sparse approximation of the entire Pareto front.

(2) Dense approximation of only a part of the Pareto front.

Dense approximation of the entire Pareto front is impossible in the case of many objectives.



(1) Sparse Approximation



(2) Dense Approximation

Approximation with Finite Solutions

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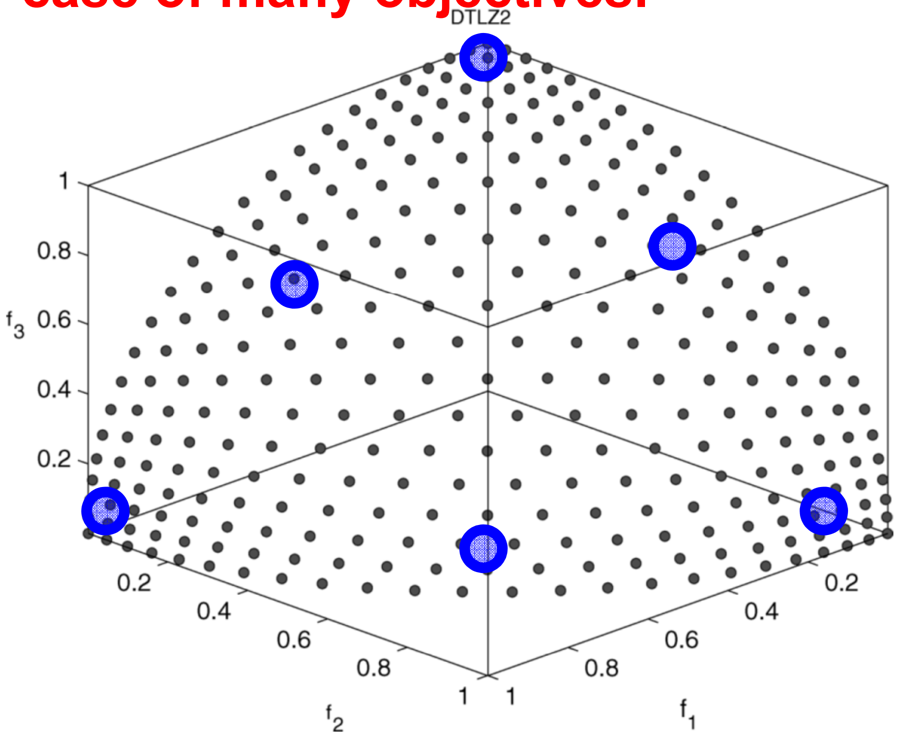
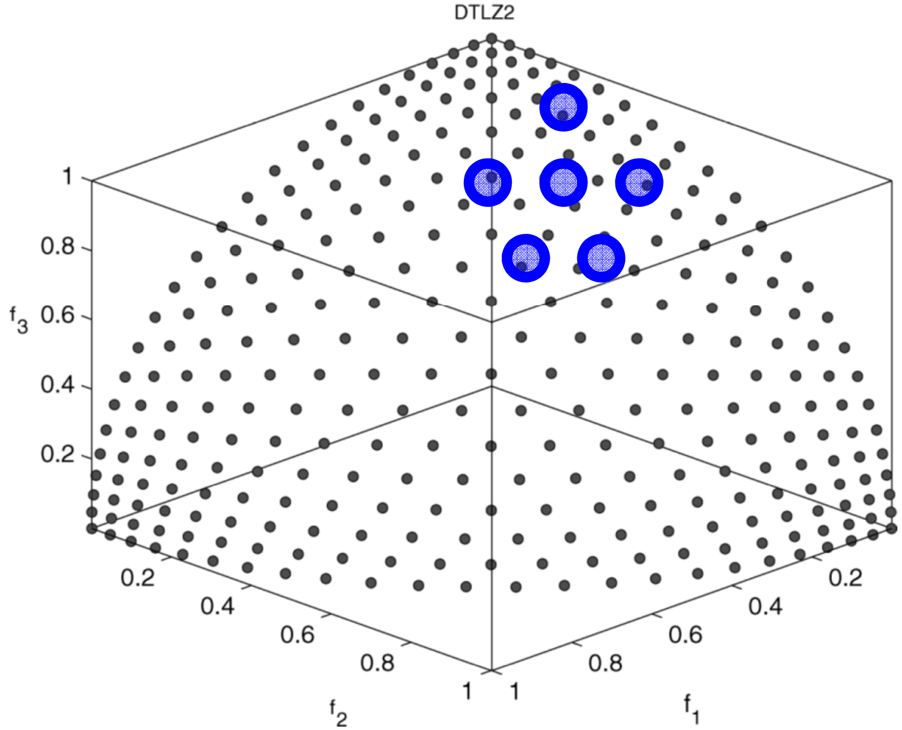


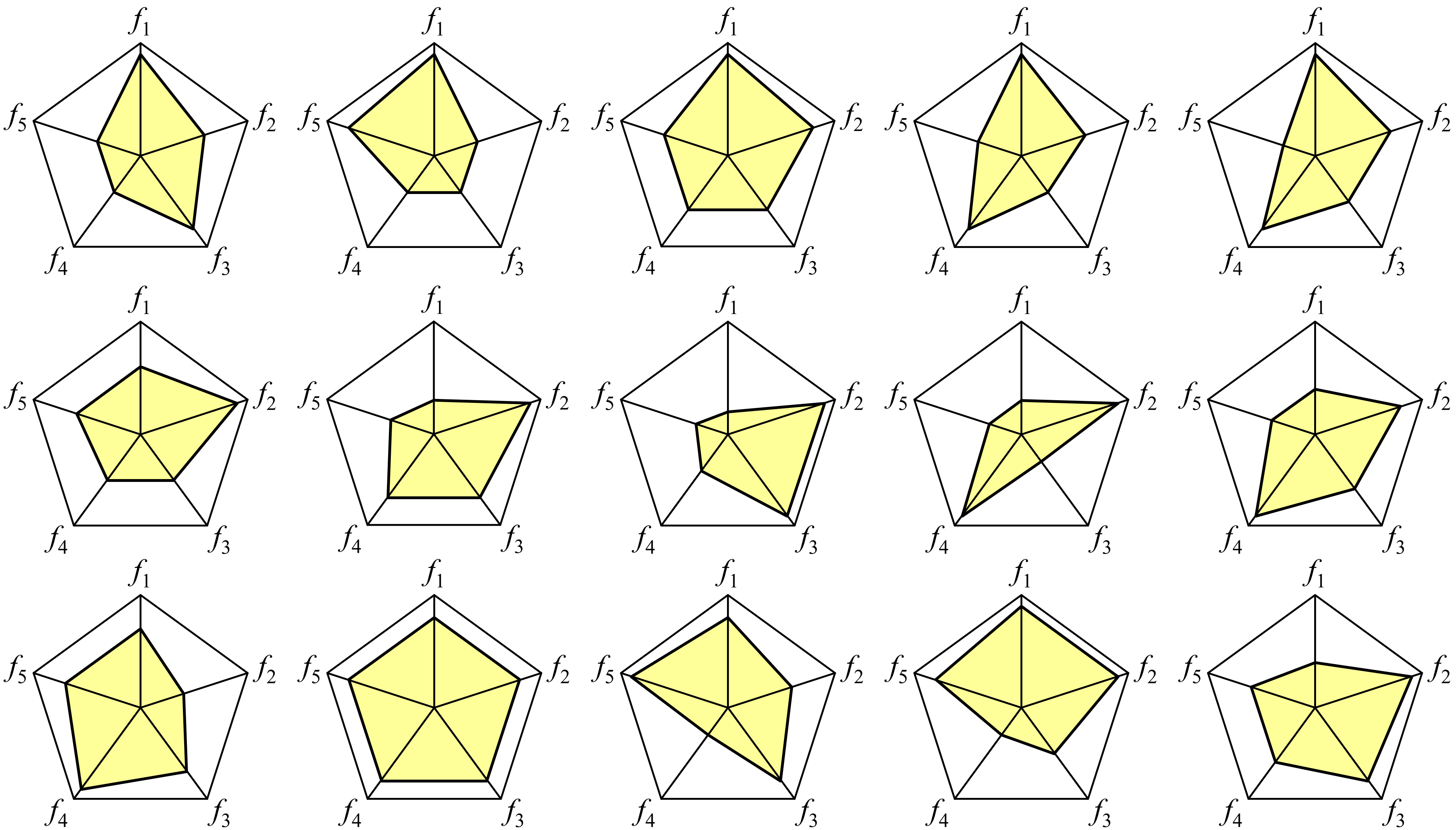
(1) Sparse Approximation



(2) Dense Approximation

Handling of Obtained Solutions

Difficulty: How to show a large number of non-dominated solutions.



Another Hot Issue: Hybridization
Multiobjective Memetic Algorithm (MOMA)

Powerful Approach to Single-Objective Optimization: MA

Evolutionary algorithm	+	Local search	=	Memetic algorithm

Multiobjective Memetic Algorithm: MOMA

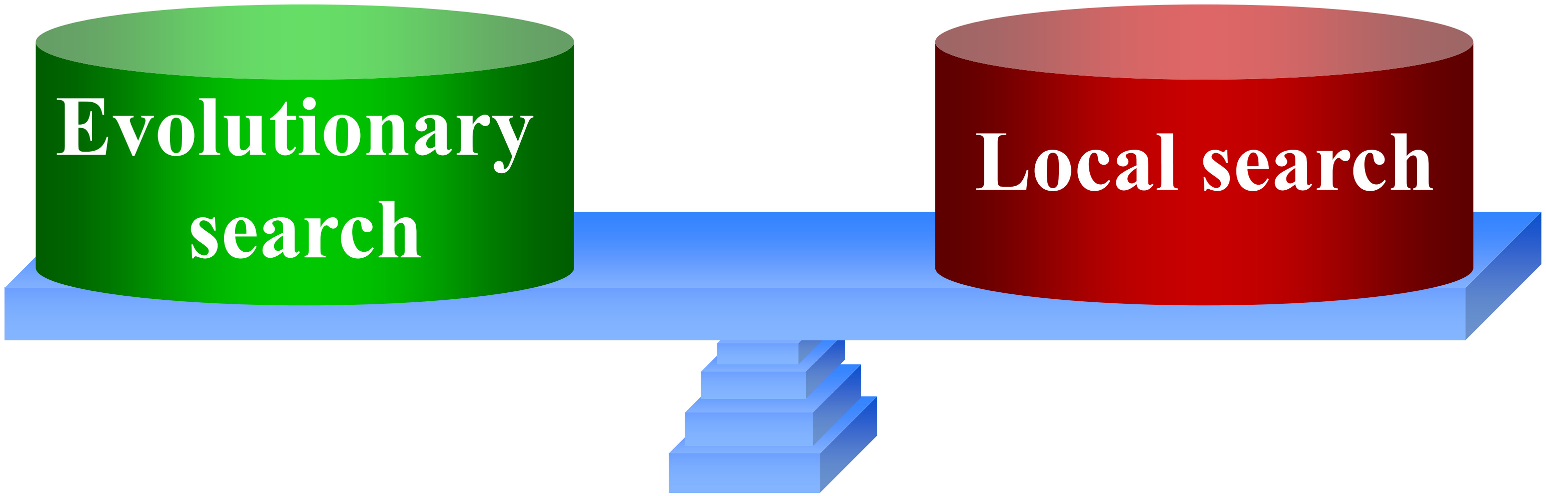
Multiobjective evolutionary algorithm	+	Multiobjective local search	=	Multiobjective memetic algorithm

Design of MA and MOMA

One important implementation issue:

Specification of the balance between evolutionary search and local search (or its dynamic adaptation).

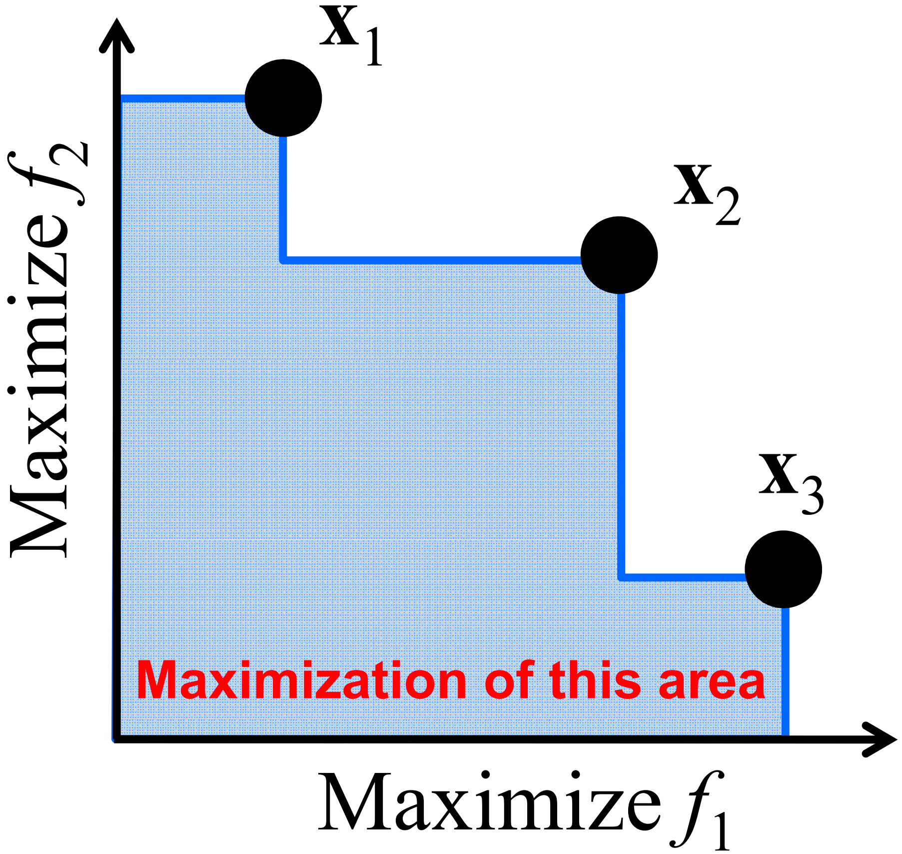
Ishibuchi H, Yoshida T, Murata T (2003) Balance between genetic search and local search in memetic algorithms for multiobjective permutation flowshop scheduling. *IEEE Trans. on Evolutionary Computation.*



New Trend in EMO Algorithm Design
IBEA: Indicator-Based Evolutionary Algorithm

Basic Idea

To maximize a performance indicator of a solution set (not a solution): Hypervolume is often used.



New Trend in EMO Algorithm Design
IBEA: Indicator-Based Evolutionary Algorithm

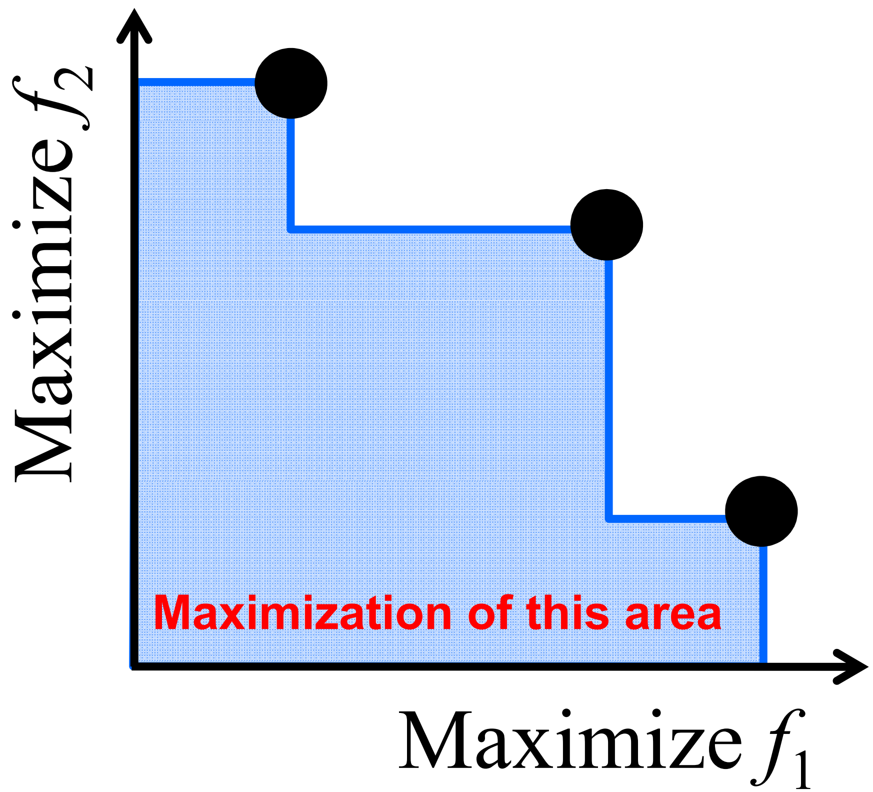
$$\text{Maximize } \boldsymbol{I}(\boldsymbol{S})$$ **(Maximization of an Indicator Function)**

$$\text{subject to } |\boldsymbol{S}| \leq \boldsymbol{N} \quad \text{where } \boldsymbol{S} \subset \{\mathbf{x} \mid \mathbf{x} \in \mathbf{X}\}$$

$\boldsymbol{S}$: **A set of solutions**

$\boldsymbol{N}$: **A pre-specified number of required solutions**

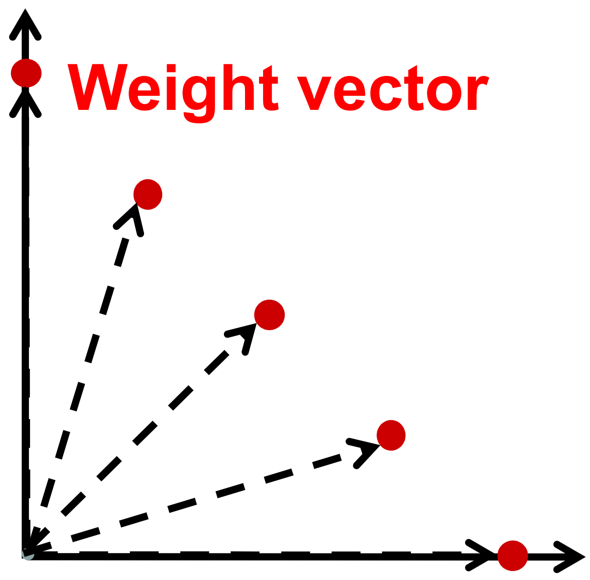
$\mathbf{X}$: **A feasible region**



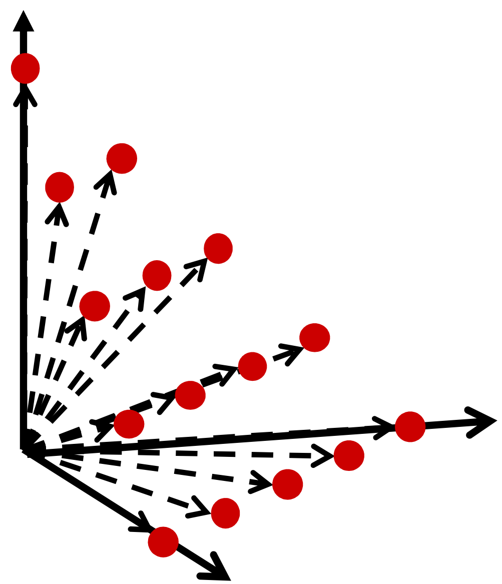
New Trend in EMO Algorithm Design
MOEA/D: Use of Scalarizing Functions

MOEA/D: Multi-objective evolutionary algorithm based on decomposition by Zhang and Li (IEEE TEC 2007)

Its Basic Idea (Decomposition): A multi-objective problem is handled as a set of scalarizing function optimization problems with different weight vectors.



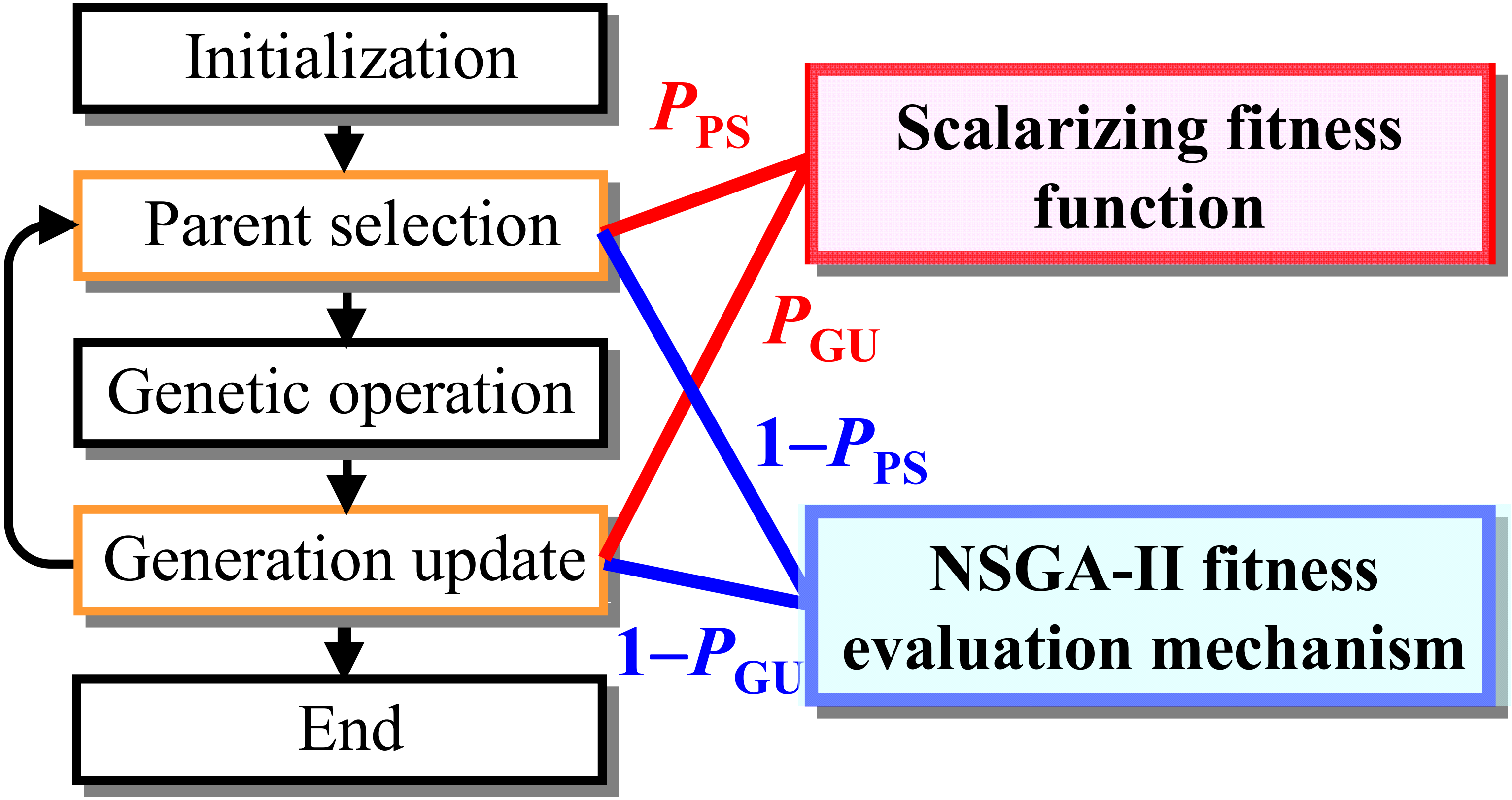
(a) Two-objective case



(b) Three-objective case

New Trend in EMO Algorithm Design

Hybrid Method: Use of Scalarizing Functions



Probability for scalarizing fitness functions:

Parent selection: P_{PS} **Generation update: P_{GU}**

Ishibuchi et al. (PPSN 2006)

New Trend in EMO Algorithm Design
Use of Other Meta-Heuristics (PSO, ACO, etc.)

Highly Cited Papers

[1] Coello CAC, Pulido GT, Lechuga MS (2004) Handling Multiple Objectives with Particle Swarm Optimization, IEEE TEC

[2] McMullen PR (2001) An Ant Colony Optimization Approach to Addressing a JIT Sequencing Problem with Multiple Objectives, Artificial Intelligence in Engineering

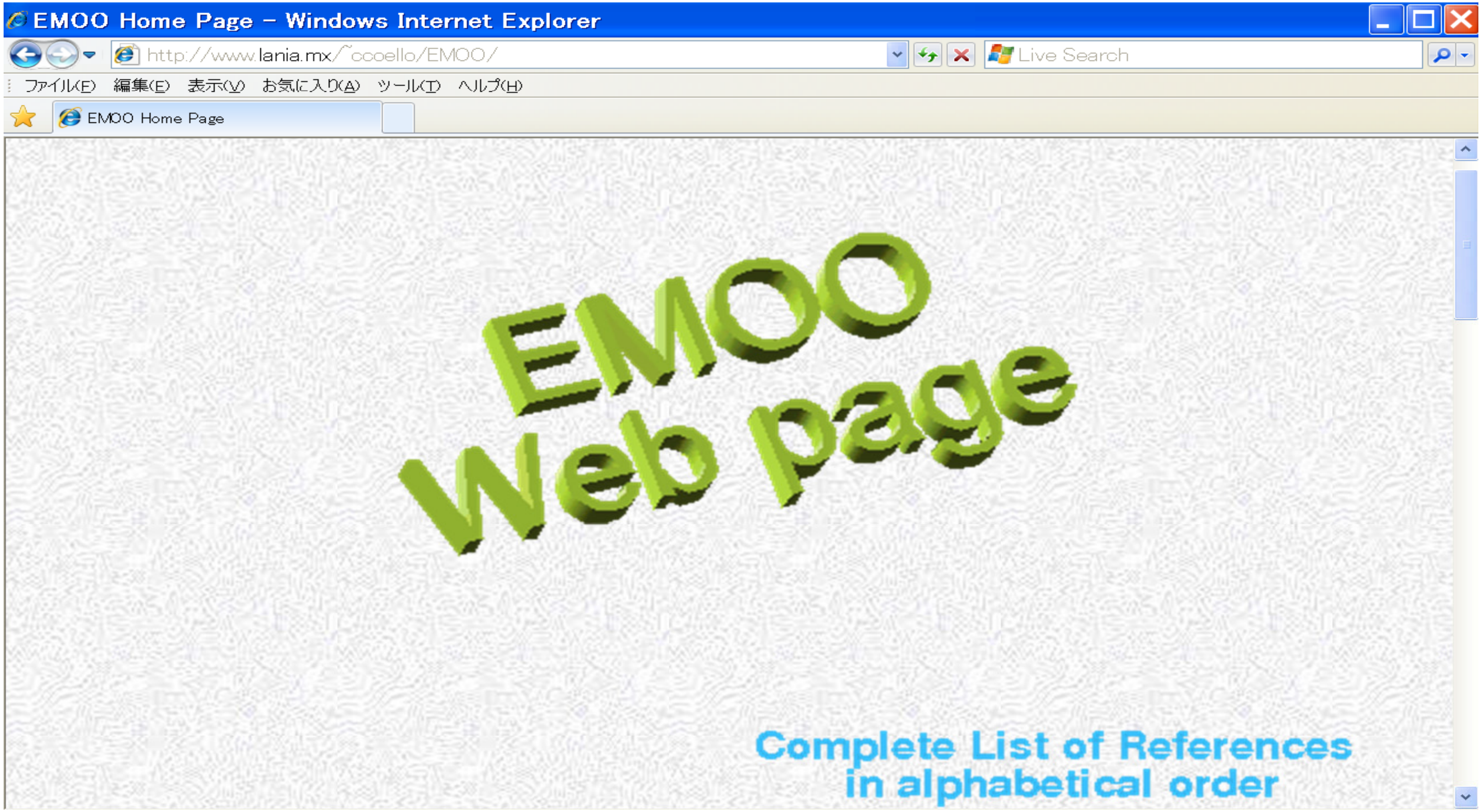
[3] Ray T, Liew KM (2002) A Swarm Metaphor for Multiobjective Design Optimization, Engineering Optimization

[4] Li XD (2003) A Non-Dominated Sorting Particle Swarm Optimizer for Multiobjective Optimization, GECCO 2003.

[5] Ho SL et al. (2005) A Particle Swarm Optimization-Based Method for Multiobjective Design Optimizations, IEEE Trans. on Magnetics

For More Information

Webpage for EMO Papers: EMOO



http://www.lania.mx/~ccoello/EMOO/

For More Information

Webpage for EMO Algorithms and Problems: PISA

http://www.tik.ee.ethz.ch/sop/pisa/

Contents

1. Basics on Genetic Fuzzy Systems (GFS)

- Introduction to Genetic Fuzzy System Research

- An Example on a Real Application

2. Interpretability-Accuracy Tradeoff of Fuzzy Systems: *Two contradictory objectives*

- Interpretability Issues in Fuzzy System Design

- Applicability of MOGFSs to the I-A problem

3. Evolutionary Multiobjective Optimization (EMO)

- Some Basic Concepts in Multiobjective Optimization

- Framework of Evolutionary Multiobjective Optimization

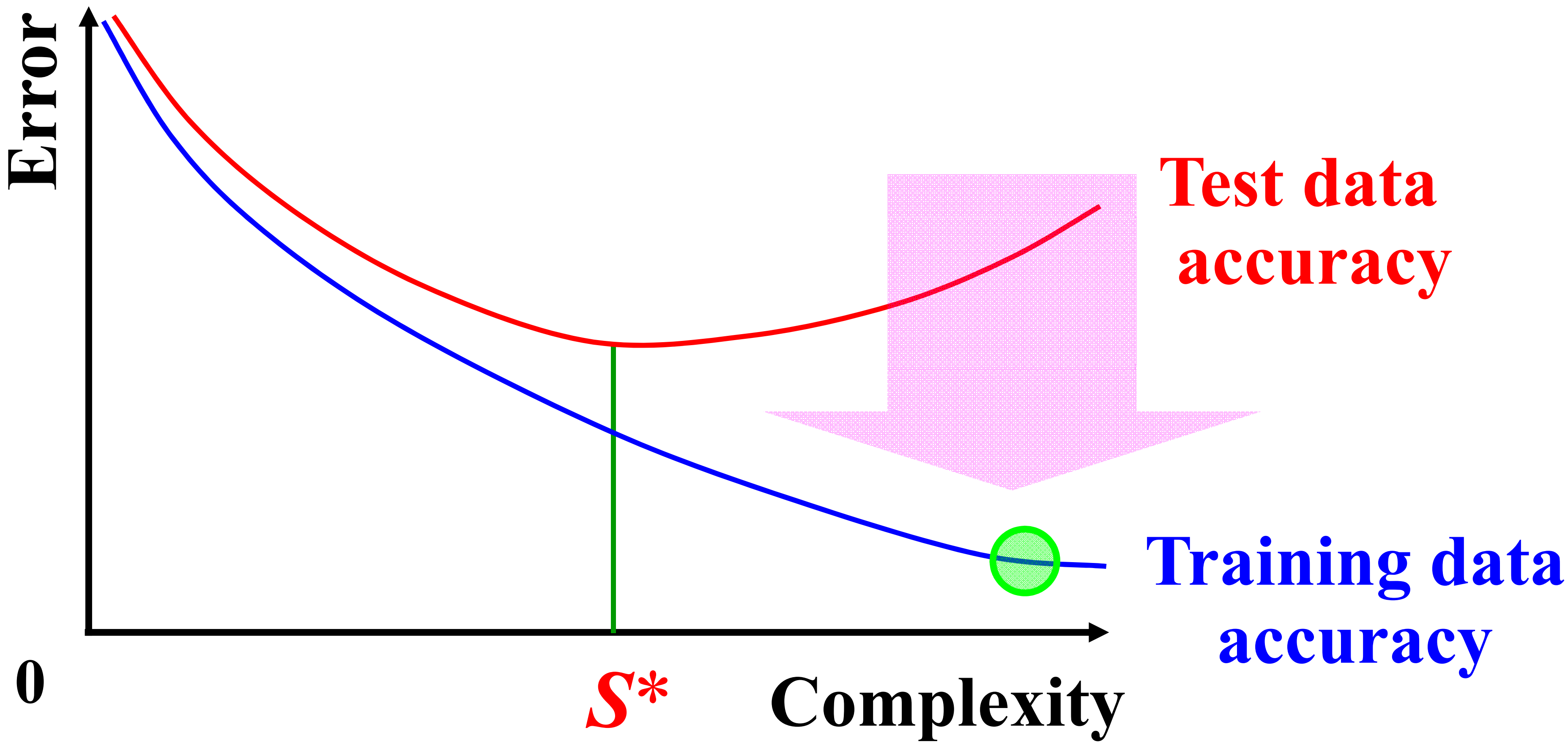
4. Multiobjective Genetic Fuzzy Systems (MoGFS)

- Overview of MoGFS Research (*some representative examples***)**

- New Research Directions in MoGFS

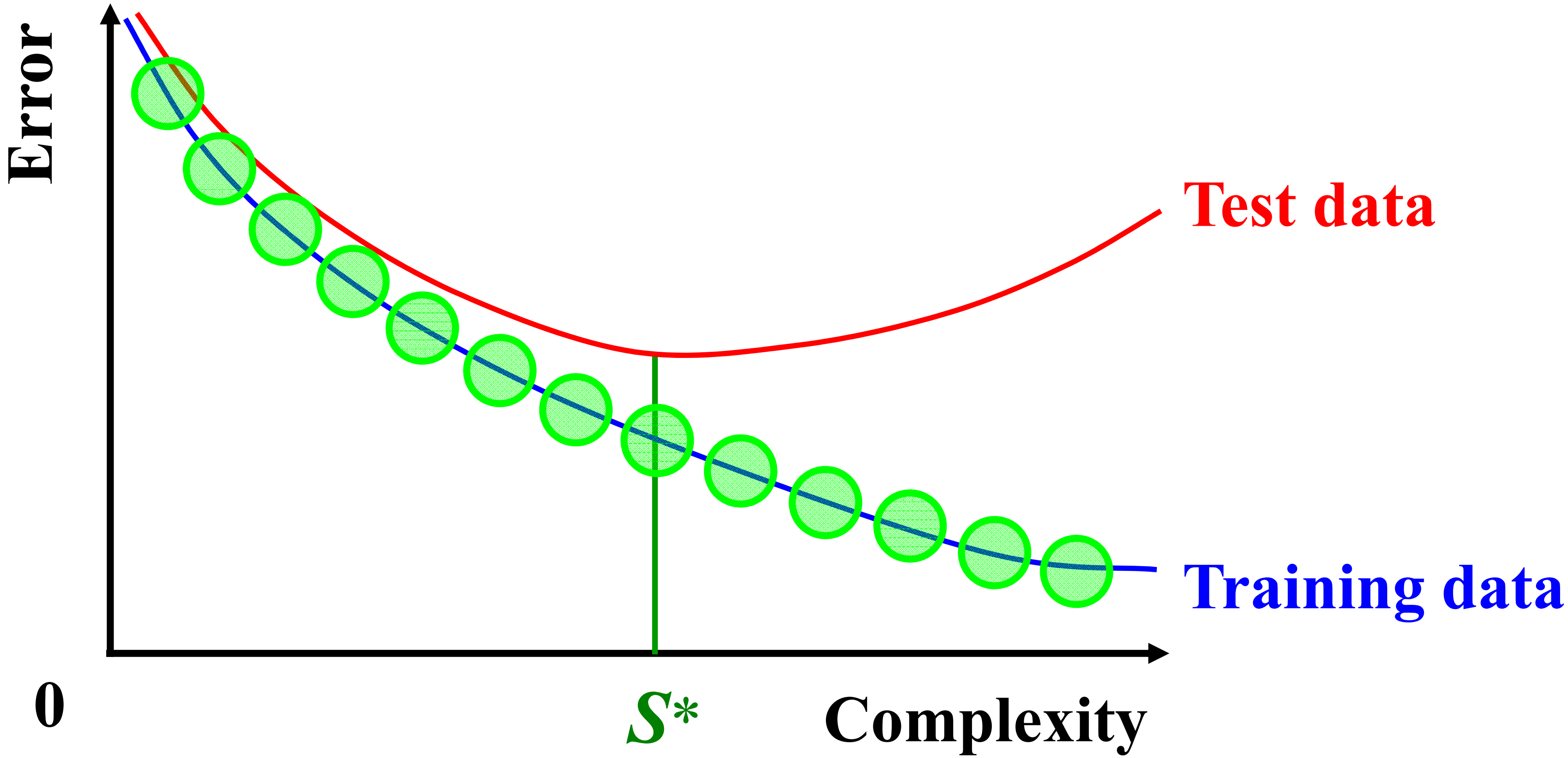
Main Motivations for MoGFSs
Overfitting and Poor Interpretability

Accuracy maximization → **Overfitting**



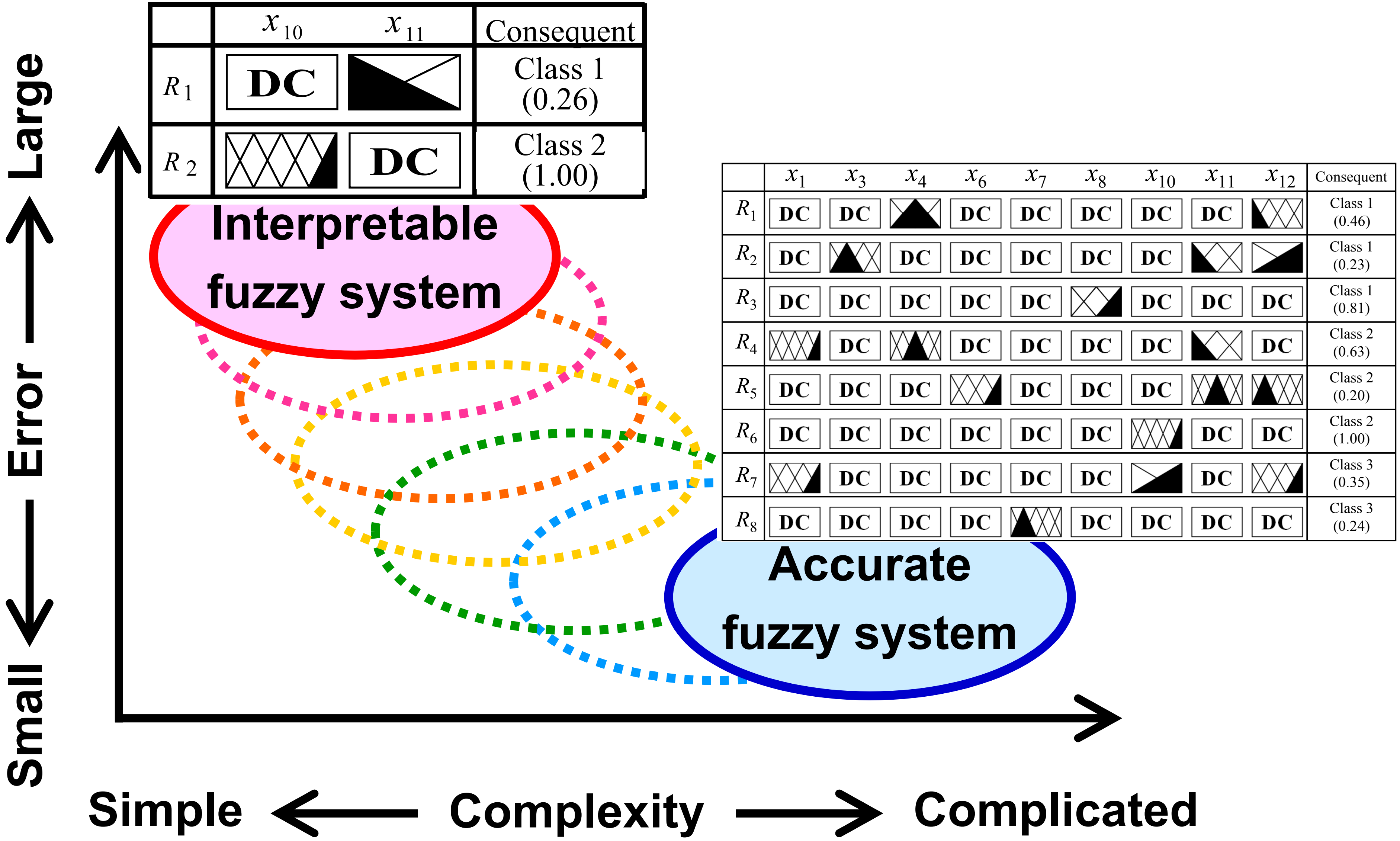
Multiobjective Design of Fuzzy Systems

Many non-dominated fuzzy systems can be obtained along the tradeoff surface by a single run of an EMO algorithm.



Main Motivations for MoGFSs

Deterioration in Interpretability



	x_{10}	x_{11}	Consequent
R_1	DC		Class 1 (0.26)
R_2		DC	Class 2 (1.00)

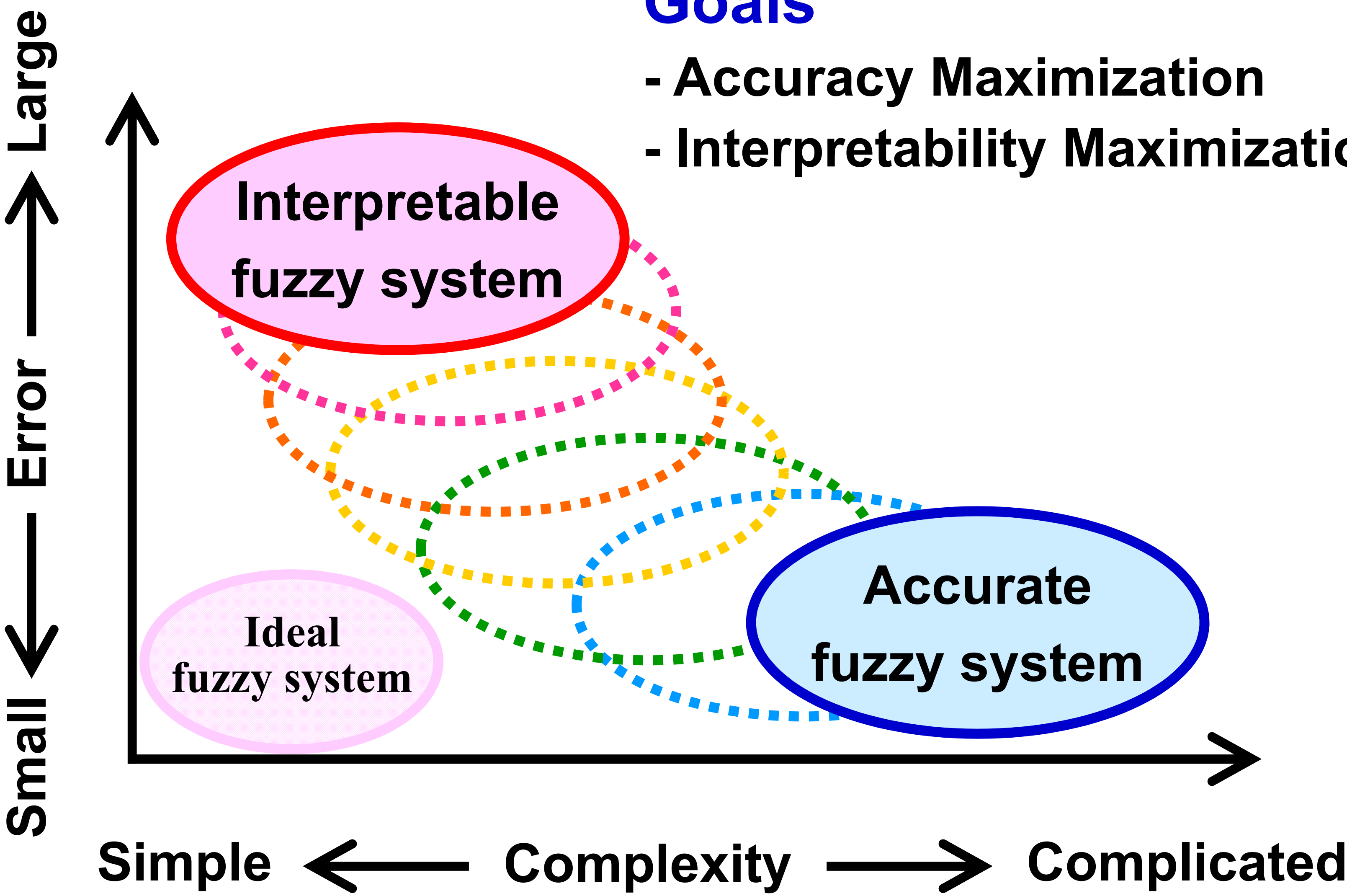
	x_1	x_3	x_4	x_6	x_7	x_8	x_{10}	x_{11}	x_{12}	Consequent
R_1	DC	DC		DC	DC	DC	DC	DC		Class 1 (0.46)
R_2	DC		DC	DC	DC	DC	DC			Class 1 (0.23)
R_3	DC	DC	DC	DC	DC		DC	DC	DC	Class 1 (0.81)
R_4		DC		DC	DC	DC	DC		DC	Class 2 (0.63)
R_5	DC	DC	DC		DC	DC	DC			Class 2 (0.20)
R_6	DC	DC	DC	DC	DC	DC		DC	DC	Class 2 (1.00)
R_7		DC	DC	DC	DC	DC		DC		Class 3 (0.35)
R_8	DC	DC	DC	DC		DC	DC	DC	DC	Class 3 (0.24)

Current Trend in Fuzzy System Design

Multiobjective Fuzzy System Design (Late 1990s -)

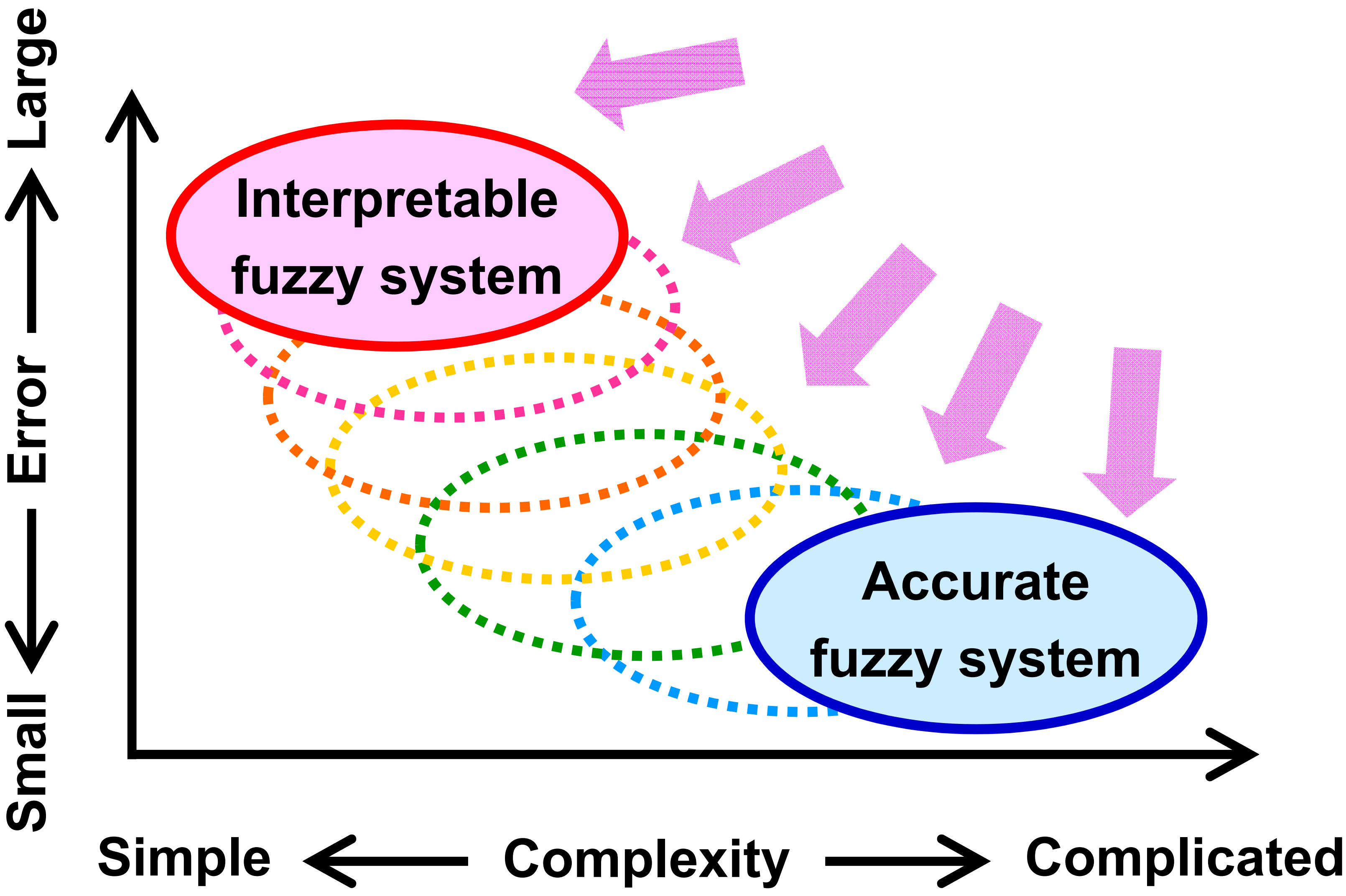
Goals

- Accuracy Maximization
- Interpretability Maximization



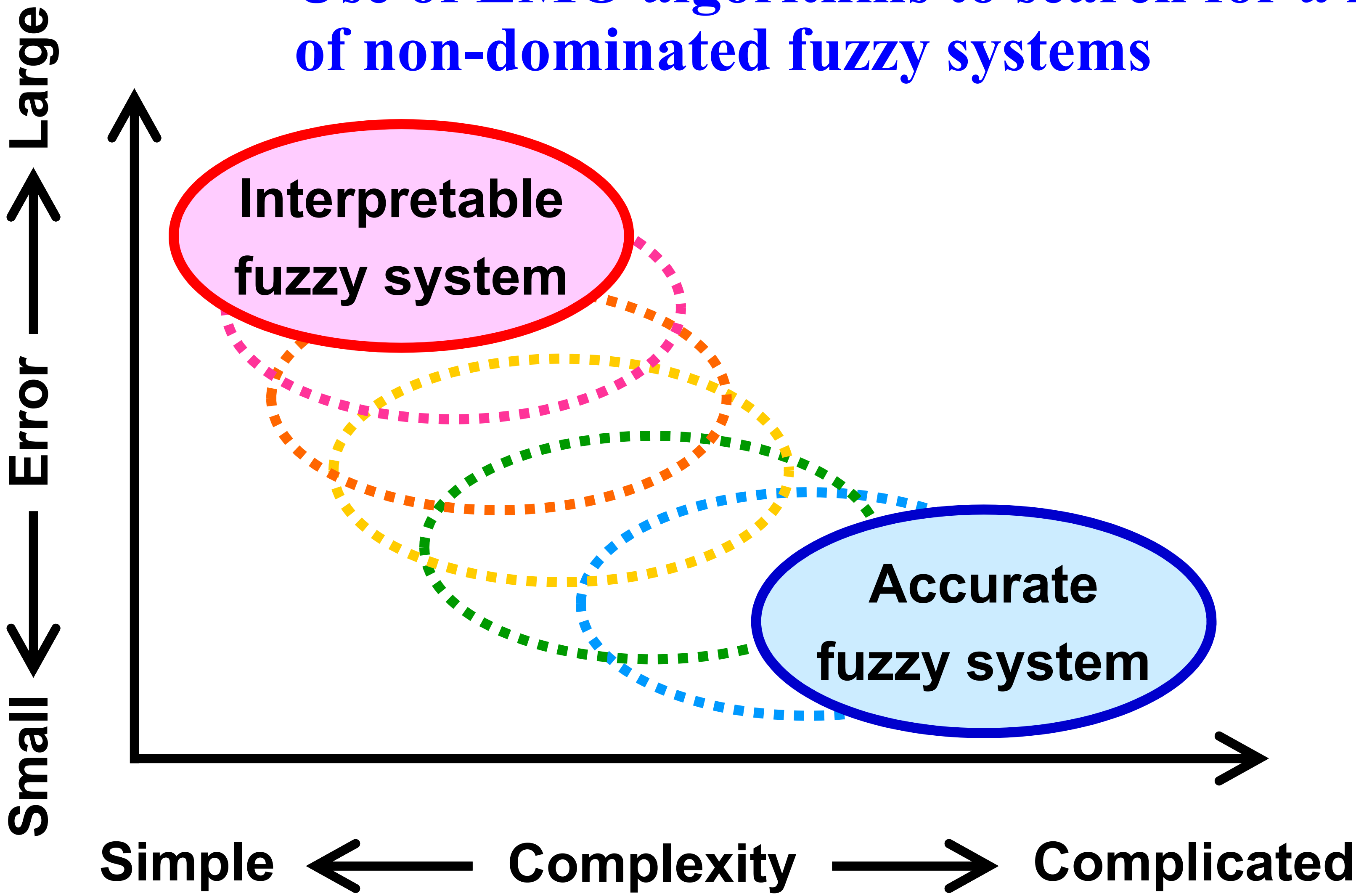
Direction of Fuzzy System Research

Multiobjective Fuzzy System Design (Late 1990s -)



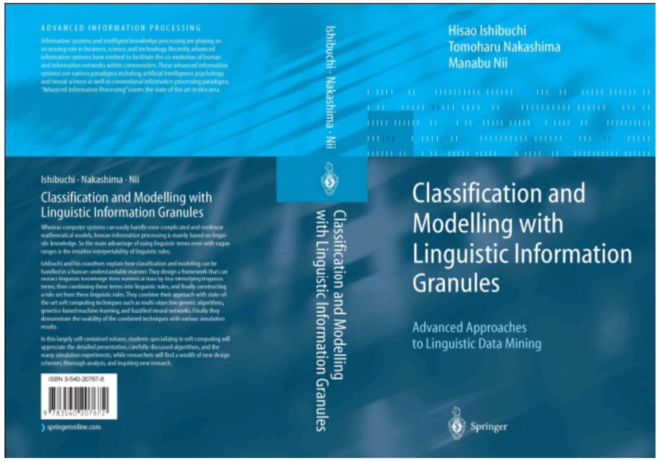
Multiobjective Design of Fuzzy Systems

Use of EMO algorithms to search for a number of non-dominated fuzzy systems

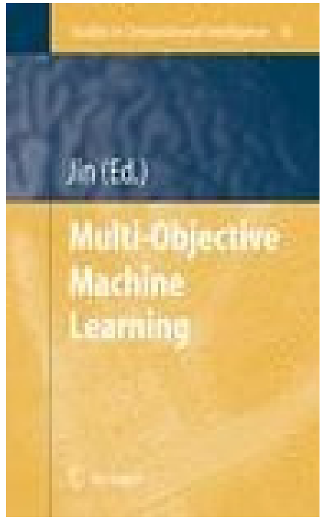


Multiobjective Genetic Fuzzy Systems

Bibliography



H. Ishibuchi, T. Nakashima, M. Hii. Classification and Modelling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer-Verlag, 2004.



Jin, Yaochu (Ed.)
Multi-Objective Machine Learning
Springer-Verlag, 2006

Literature (http://www.keel.es)

Multiobjective Genetic Algorithms and Rule Learning

http://sci2s.ugr.es/keel/specific.php?area=44

Highly Cited MoGFS Papers

[1] Ishibuchi et al. (1997) Single-objective and two-objective genetic algorithms for selecting linguistic rules for pattern classification problems. *Fuzzy Sets & Systems.*

[2] Ishibuchi et al. (2001) Three-objective genetics-based machine learning for linguistic rule extraction. *Information Sciences.*

[3] Ishibuchi & Yamamoto (2004) Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining. *Fuzzy Sets & Systems*.

[4] Wang et al. (2005) Multi-objective hierarchical genetic algorithm for interpretable fuzzy rule-based knowledge extraction. *Fuzzy Sets & Systems*.

[5] Johansen & Babuska (2003) Multiobjective identification of Takagi-Sugeno fuzzy models. *IEEE TFS.*

Different Models of Multiobjective GFSs
Bibliography on Interpretability/Accuracy

A-I trade-off			FRBS approach		Objectives		MOEA			
	Authors	Year	Rules	Type	#Obj.	Type	Name	Gen.	Type	Problem type
RB LEARNING	Ishibuchi et al.	1997,1998	MAM.	LING.	2	A+C	NON.	1st	N	CLAS.
	Ishibuchi et al.	2001	MAM.	LING.	3	A+C+C	GBML	1st	N	CLAS.
	Ishibuchi et al.	2004	MAM.	LING.	3	A+C+C	MOGLS	1st	N	CLAS.
	Setzkorn et al.	2005	MAM.	LING.	3	A+C+C	NON.	2nd	I •	CLAS.
	Ishibuchi et al.	2006	MAM.	LING.	2	A+C	NSGA-II	2nd	G	CLAS.
	Ishibuchi et al.	2007	MAM.	LING.	3	A+C+C	GBML	2nd	I †	CLAS.
	Cococcioni et al.	2007	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Xing et al.	2007	TSK	LING. ∗	2	A+C	PMOCCA	2nd	N	REG.,TS.
	Ducange et al.	2010	MAM.	LING.	3	A+A+C	NSGA-II	2nd	G	IMB. CLAS.
DB TUNING (+RULE SELECT.)	Wang et al.	2005	TSK	LING. ∗	5	A+C+C+S+S	MOHGA	1st	I ∘	REG.
	Alcalá et al.	2007	MAM.	LING.	2	A+C	*SPEA2ACC*	2nd	I •	REG.
	Gonzalez et al.	2007	TSK	APPROX.	2	A+C	NON.	2nd	I †	REG.
	Gomez et al.	2007	TSK	APPROX.	4	A+C+C+S	MONEA	2nd	N	REG.
	Pulkkinen et al.	2008	MAM.	LING.	3	A+C+C	NSGA-II	2nd	G	CLAS.
	Pulkkinen et al.	2008	MAM.	LING.	3	A+A+S	NSGA-II	2nd	G	CLAS.
	Guenounou et al.	2009	TSK	LING. ∗	2	A+C	NSGA-II	2nd	G	REG.
	Gacto et al.	2009	MAM.	LING.	2	A+C	VARIOUS	2nd	G	REG.
	Botta et al.	2009	MAM.	LING.	2	A+S	NSGA-II	2nd	G	REG.
	Marquez et al.	2009	MAM.	LING.	2	A+C	VARIOUS	2nd	I †•	REG.
	Marquez et al.	2010	MAM.	LING.	3	A+C+S	NON.	2nd	I †	REG.
	Gacto et al.	2010	MAM.	LING.	3	A+C+S	SPEA2-SI	2nd	I •	REG.
KB LEARNING	Cordón et al.	2003	MAM.	LING.	2	A+C	NON.	1st	N	CLAS.
	Alcalá et al.	2009	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Antonelli et al.	2009	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Antonelli et al.	2009	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Casillas et al.	2009	DNF-RULES	LING.	2	A+C	NON.	2nd	I †	REG.
	Pulkkinen et al.	2010	MAM.	LING.	2	A+C	NON.	2nd	I †	REG.
	Alcalá et al.	2010	MAM.	LING.	3	A+C+C	NSGA-II	2nd	g	CLAS.
	Antonelli et al.	2011	MAM.	LING.	3	A+C+S	(2+2)M-PAES	2nd	I ⋆	REG.
	Antonelli et al.	2011	MAM.	LING.	2	A+(C+S)	(2+2)M-PAES	2nd	I ⋆	REG.
	Alcalá et al.	2011	MAM.	LING.	2	A+C	NON.	2nd	I †	REG.

MAM. = Mamdani, TSK = Takagi-Sugeno-Kang, LING. = Linguistic, APPROX. = Approximate, ∗In the antecedent;
A = Accuracy, C = Complexity, S = Semantic aspects;
NON. = No name, N = New algorithm, I = Improved version, G = General use;
CLAS. = Classification, REG. = Regression, TS. = Time Series, IMB. = Imbalanced;
†NSGA-II based, ⋆PAES based, ∘MOGA based, •SPEA2 based.

- Most of them are based on 2nd gen. MOEAs
- Usually no more than 3 objectives
- Complexity at the beginning; Semantic aspects in the last years
- Most of them are Linguistic and Mamdani type based approaches
- KB learning in the last years *(granularity as important factor)*
- Most of them are improved versions of the most known MOEAs *(particularly in the case of KB learning)*

Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera. ***A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*****, in submission, 2011.**

Different Models of Multiobjective GFSs

Bibliography on Interpretability/Accuracy

	A-I trade-off		FRBS approach		Objectives		MOEA			
	Authors	Year	Rules	Type	#Obj.	Type	Name	Gen.	Type	Problem type
RB LEARNING	Ishibuchi et al.	1997,1998	MAM.	LING.	2	A+C	NON.	1st	N	CLAS.
	Ishibuchi et al.	2001	MAM.	LING.	3	A+C+C	GBML	1st	N	CLAS.
	Ishibuchi et al.	2004	MAM.	LING.	3	A+C+C	MOGLS	1st	N	CLAS.
	Setzkorn et al.	2005	MAM.	LING.	3	A+C+C	NON.	2nd	I •	CLAS.
	Ishibuchi et al.	2006	MAM.	LING.	2	A+C	NSGA-II	2nd	G	CLAS.
	Ishibuchi et al.	2007	MAM.	LING.	3	A+C+C	GBML	2nd	I †	CLAS.
	Cococcioni et al.	2007	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Xing et al.	2007	TSK	LING. ∗	2	A+C	PMOCCA	2nd	N	REG.,TS.
	Ducange et al.	2010	MAM.	LING.	3	A+A+C	NSGA-II	2nd	G	IMB. CLAS.
DB TUNING (+RULE SELECT.)	Wang et al.	2005	TSK	LING. ∗	5	A+C+C+S+S	MOHGA	1st	I ◦	REG.
	Alcalá et al.	2007	MAM.	LING.	2	A+C	$SPEA2_{ACC}$	2nd	I •	REG.
	Gonzalez et al.	2007	TSK	APPROX.	2	A+C	NON.	2nd	I †	REG.
	Gomez et al.	2007	TSK	APPROX.	4	A+C+C+S	MONEA	2nd	N	REG.
	Pulkkinen et al.	2008	MAM.	LING.	3	A+C+C	NSGA-II	2nd	G	CLAS.
	Pulkkinen et al.	2008	MAM.	LING.	3	A+A+S	NSGA-II	2nd	G	CLAS.
	Guenounou et al.	2009	TSK	LING. ∗	2	A+C	NSGA-II	2nd	G	REG.
	Gacto et al.	2009	MAM.	LING.	2	A+C	VARIOUS	2nd	G	REG.
	Botta et al.	2009	MAM.	LING.	2	A+S	NSGA-II	2nd	G	REG.
	Marquez et al.	2009	MAM.	LING.	2	A+C	VARIOUS	2nd	I †•	REG.
	Marquez et al.	2010	MAM.	LING.	3	A+C+S	NON.	2nd	I †	REG.
	Gacto et al.	2010	MAM.	LING.	3	A+C+S	SPEA2-SI	2nd	I •	REG.
KB LEARNING	Cordón et al.	2003	MAM.	LING.	2	A+C	NON.	1st	N	CLAS.
	Alcalá et al.	2009	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Antonelli et al.	2009	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Antonelli et al.	2009	MAM.	LING.	2	A+C	(2+2)M-PAES	2nd	I ⋆	REG.
	Casillas et al.	2009	DNF-RULES	LING.	2	A+C	NON.	2nd	I †	
	Pulkkinen et al.	2010	MAM.	LING.	2	A+C	NON.	2nd	I †	
	Alcalá et al.	2010	MAM.	LING.	3	A+C+C	NSGA-II	2nd	g	
	Antonelli et al.	2011	MAM.	LING.	3	A+C+S	(2+2)M-PAES	2nd	I ⋆	
	Antonelli et al.	2011	MAM.	LING.	2	A+(C+S)	(2+2)M-PAES	2nd	I ⋆	
	Alcalá et al.	2011	MAM.	LING.	2	A+C	NON.	2nd	I †	

MAM. = Mamdani, TSK = Takagi-Sugeno-Kang, LING. = Linguistic, APPROX. = Approximate, ∗I
A = Accuracy, C = Complexity, S = Semantic aspects;
NON. = No name, N = New algorithm, I = Improved version, G = General use;
CLAS. = Classification, REG. = Regression, TS. = Time Series, IMB. = Imbalanced;
†NSGA-II based, ⋆PAES based, ◦MOGA based, •SPEA2 based.

In the following we will see in deep a representative example for each type:

- FIRST TYPE: RB Learning
- SECOND TYPE: DB Tuning + Rule Select.
- THIRD TYPE: KB Learning

Michela Fazzolari, Rafael Alcalá, Yusuke Nojima, Hisao Ishibuchi, Francisco Herrera. *A review on the application of Multi-Objective Genetic Fuzzy Systems: current status and further directions*, in submission, 2011.

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

FIRST TYPE: RULE BASE LEARNING - CLASSIFICATION

H. Ishibuchi, T. Yamamoto, Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining, *Fuzzy Sets and Systems, Vol. 141, pp. 59-88 (2004)*

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Two-Stage Approach

1. Heuristic Rule Extraction

   A pre-specified number of candidate fuzzy rules are extracted from numerical data using a heuristic rule evaluation criterion (data mining).

2. Multiobjective Genetic Fuzzy Rule Selection

   A small number of fuzzy rules are selected from the extracted candidate rules using a multi-objective genetic algorithm (evolutionary optimization).

H. Ishibuchi and T. Yamamoto, "Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining," ***Fuzzy Sets and Systems***, Vol. 141, pp. 59-88 (2004).

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

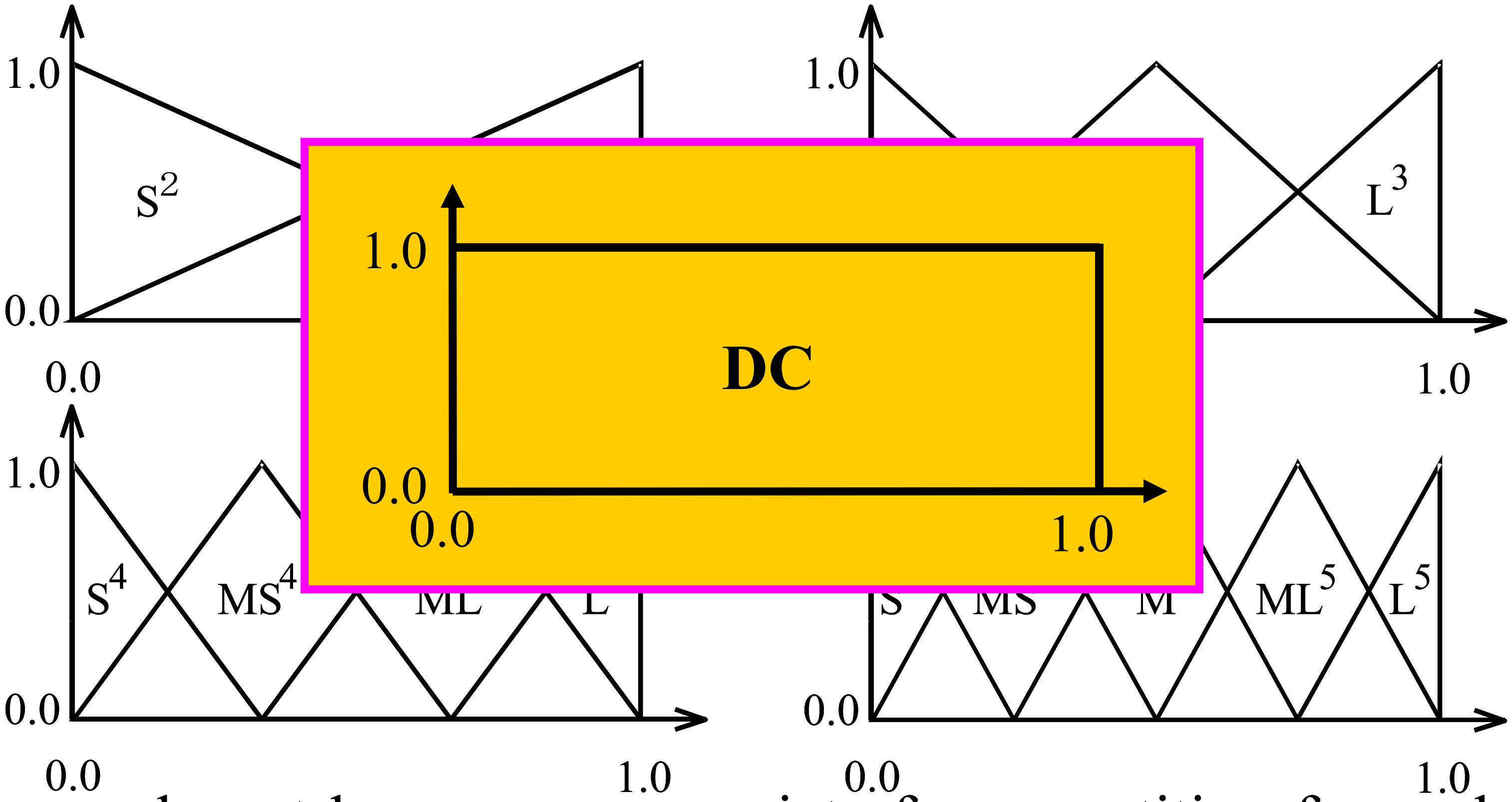
Fuzzy Rules for n-dimensional Problems

If x_1 is A_1 and … and x_n is A_n
then Class C with CF

A_i :	Antecedent fuzzy set
Class C :	Consequent class
CF :	Rule weight (Certainty factor)

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Antecedent Fuzzy Sets (Multiple Partitions)



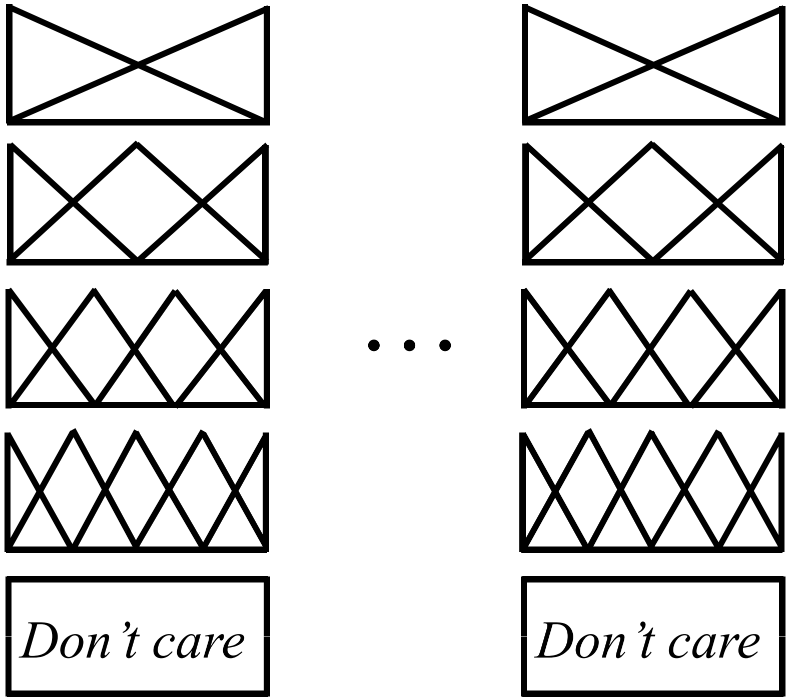
Usually we do not know an appropriate fuzzy partition for each input variable.

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Possible Fuzzy Rules

Total number of possible fuzzy rules



x_1 x_n

$$(14+1)\times \cdots \times(14+1)=(15)^n$$

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Examined Fuzzy Rules

They only examine short fuzzy rules with only a few antecedent conditions.

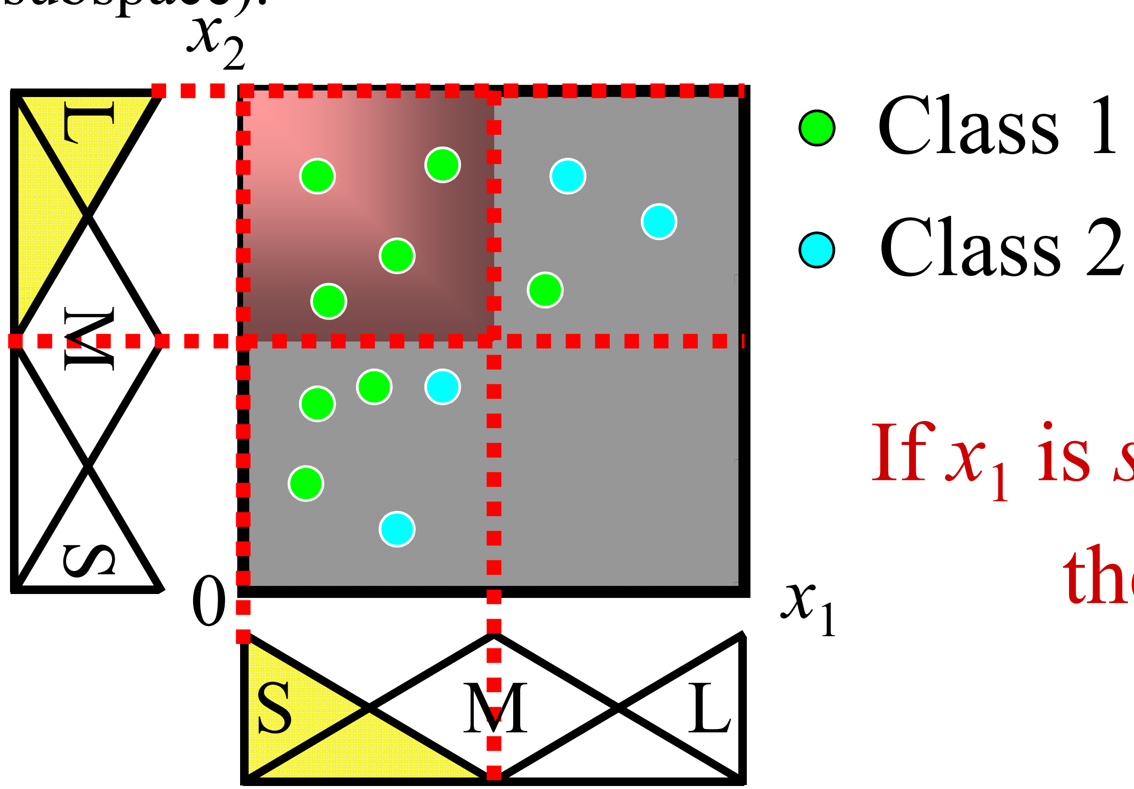
If x_1 is *small* and x_{48} is *large*

then Class 1 with 0.58

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Consequent Class

The consequent class of each fuzzy rule is determined by compatible training patterns (i.e., the dominant class in the corresponding fuzzy subspace).



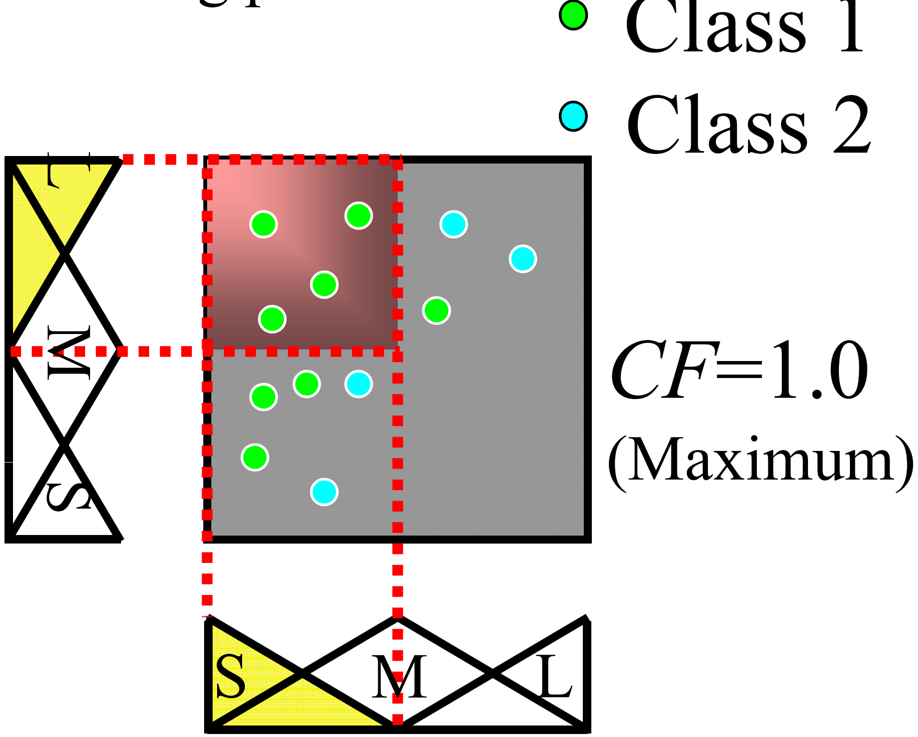
If x_1 is *small* and x_2 is *large* then Class 1 with 1.0

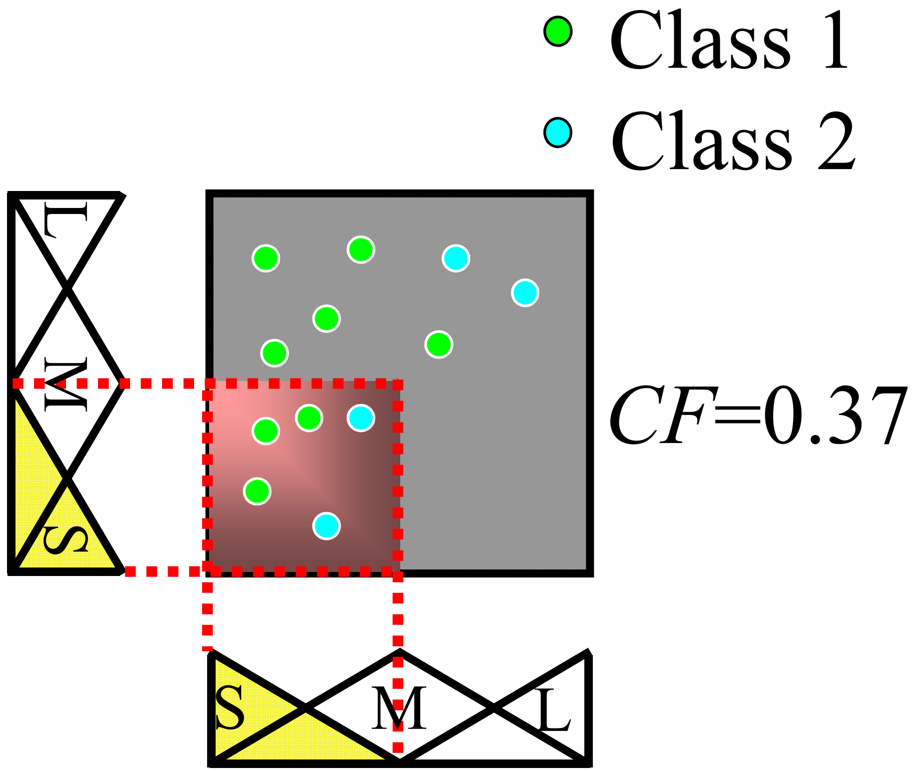
Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Rule Weight **(Certainty Factor)**

The rule weight *CF* of each fuzzy rule is calculated from compatible training patterns.



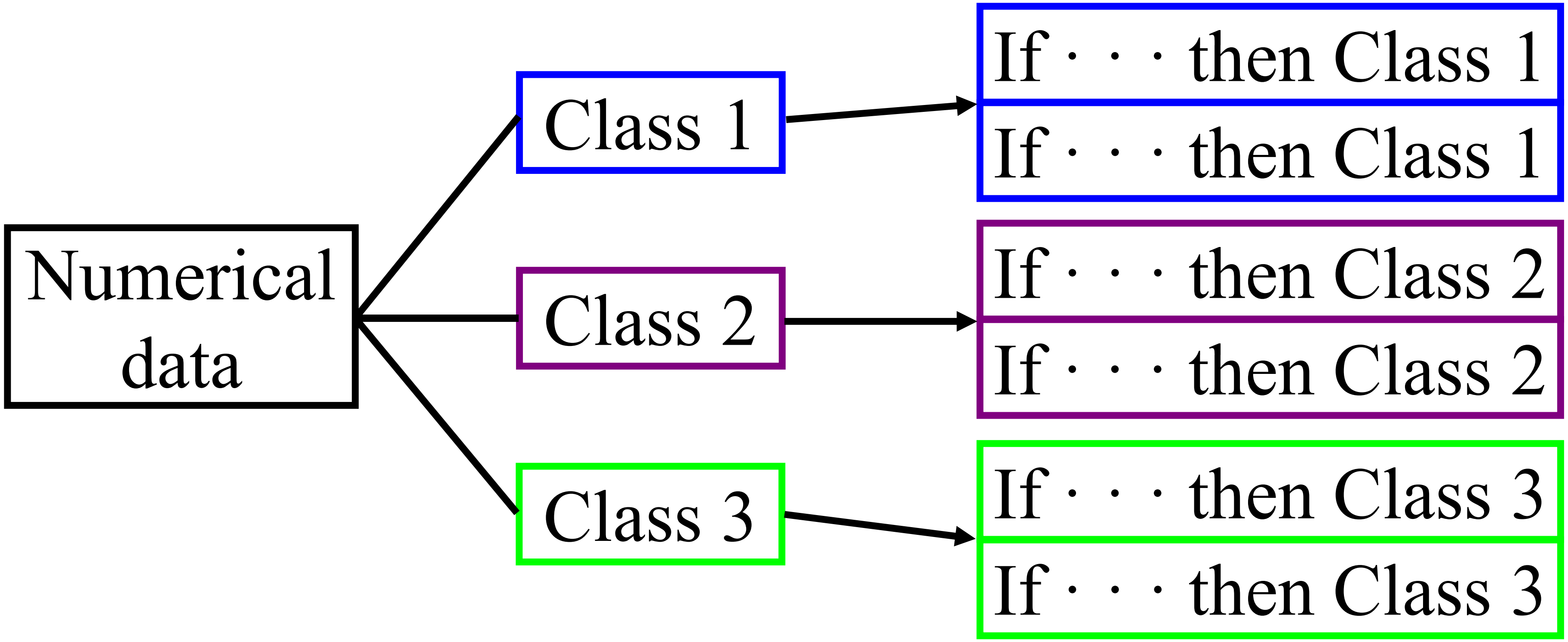


Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Heuristic Rule Extraction

They extract a pre-specified number of the best fuzzy rules with respect to a pre-specified heuristic rule evaluation criterion.



Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Heuristic Rule Extraction

Possible fuzzy rules: $(15)^n$ rules

Restriction on the rule length :

Only short fuzzy rules

Rule evaluation criterion:

The best rules for each class

300 fuzzy rules for each class

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Two-Stage Approach

1. Heuristic Rule Extraction

   A pre-specified number of candidate fuzzy rules are extracted from numerical data using a heuristic rule evaluation criterion (data mining).

2. Multiobjective Genetic Fuzzy Rule Selection

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Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Implementation of Multiobjective approach

Coding: $S = s_1 s_2 \cdots s_N$

N: Total number of candidate rules

s_j={0, 1}: Inclusion or exclusion of the j-th rule

Objectives: $f_1(S)$, $f_2(S)$, $f_3(S)$

$f_1(S)$: Number of correctly classified patterns by S

$f_2(S)$: Number of selected rules in S

$f_3(S)$: Total number of antecedent conditions in S

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Comparison of Four Approaches

(1) Two-objective approach

Maximize $f_1(S)$ and minimize $f_2(S)$

(2) Weighted sum of the two objectives

Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S)$

(3) Three-objective approach

Maximize $f_1(S)$ and minimize $f_2(S)$, $f_3(S)$

(4) Weighted sum of the three objectives

Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S) - w_3 \cdot f_3(S)$

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

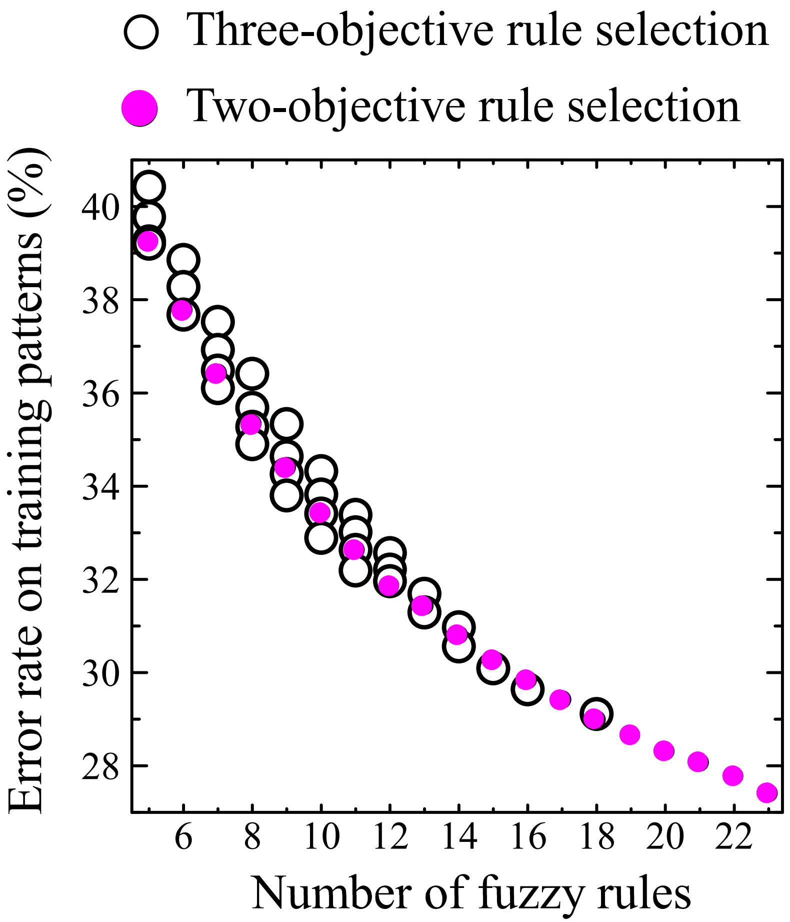
Data Sets

Data set	Attributes	Patterns	Classes	Length
Breast W	9	683*	2	3
Diabetes	8	768	2	3
Glass	9	214	6	3
Heart C	13	297*	5	3
Iris	4	150	3	3
Sonar	60	208	2	2
Wine	13	178	3	3

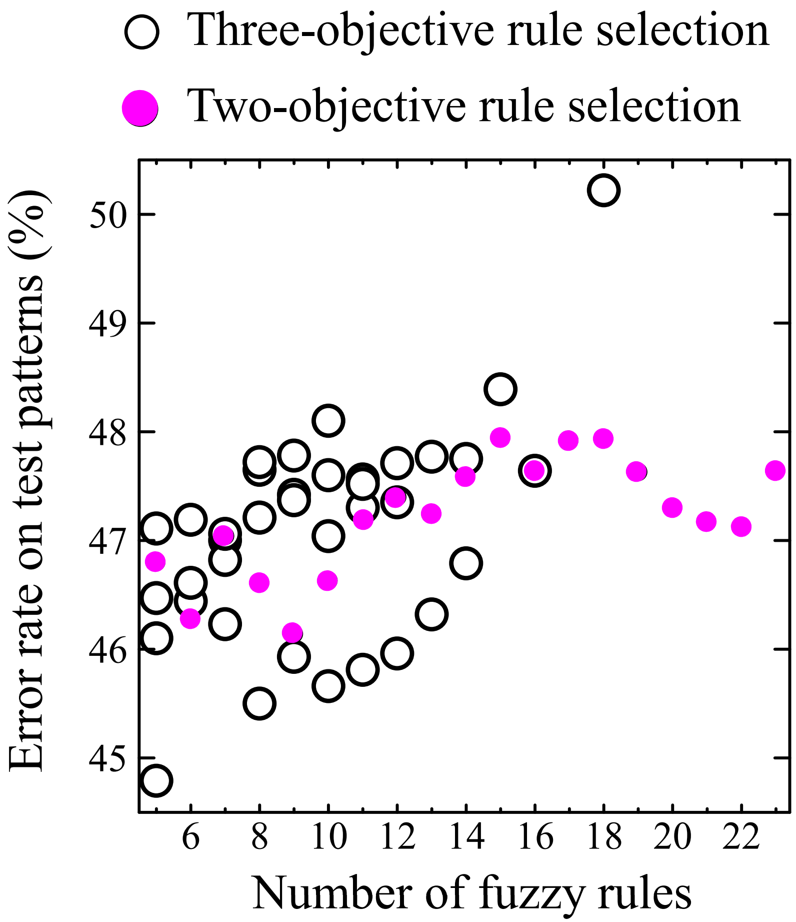
Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Experimental Results (Cleveland Heart)



(a) Error rates on **training data**



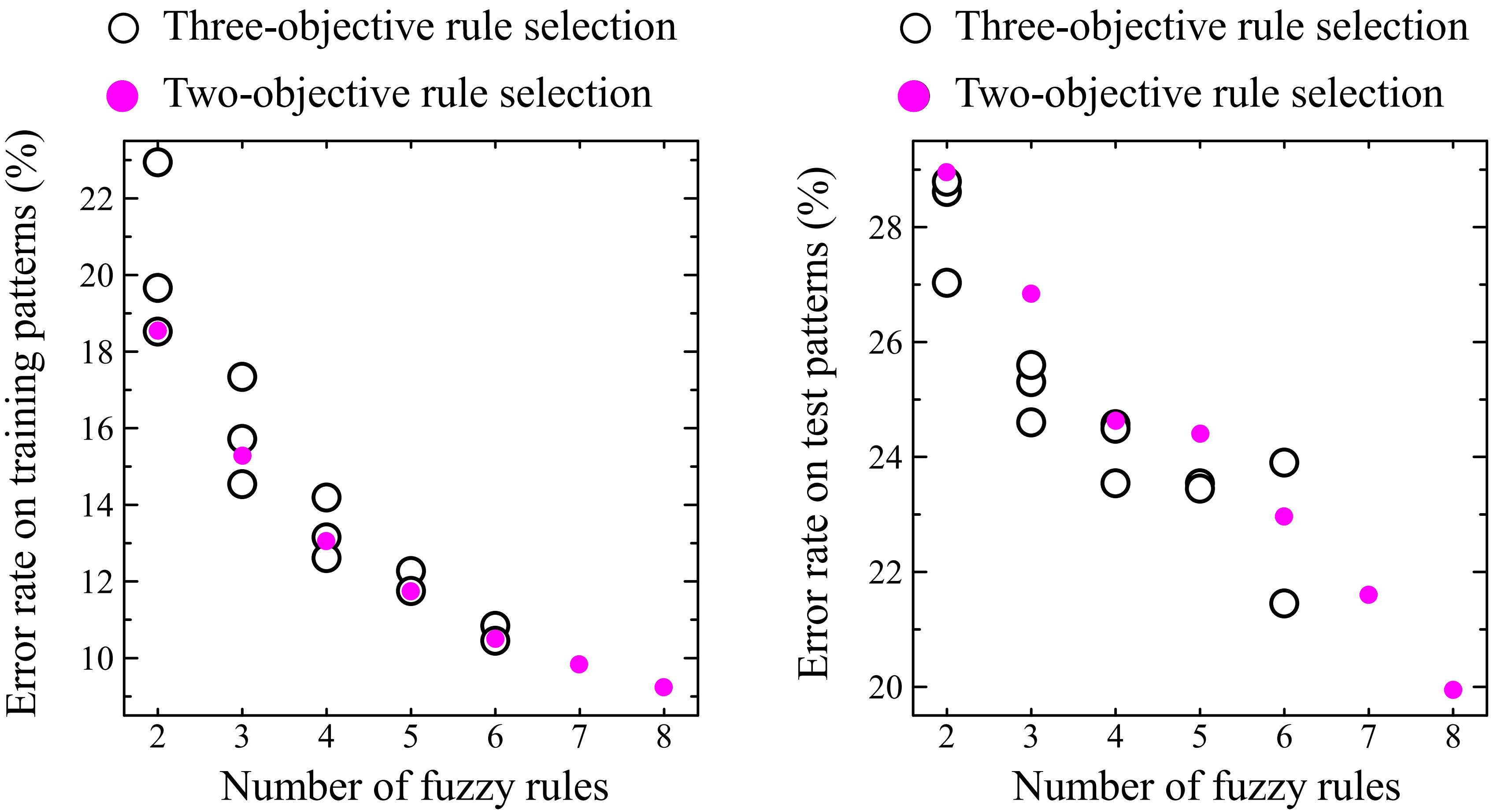
(b) Error rates on **test data**

We can observe the overfitting due to the increase in the number of fuzzy rules.

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Experimental Results (Sonar)



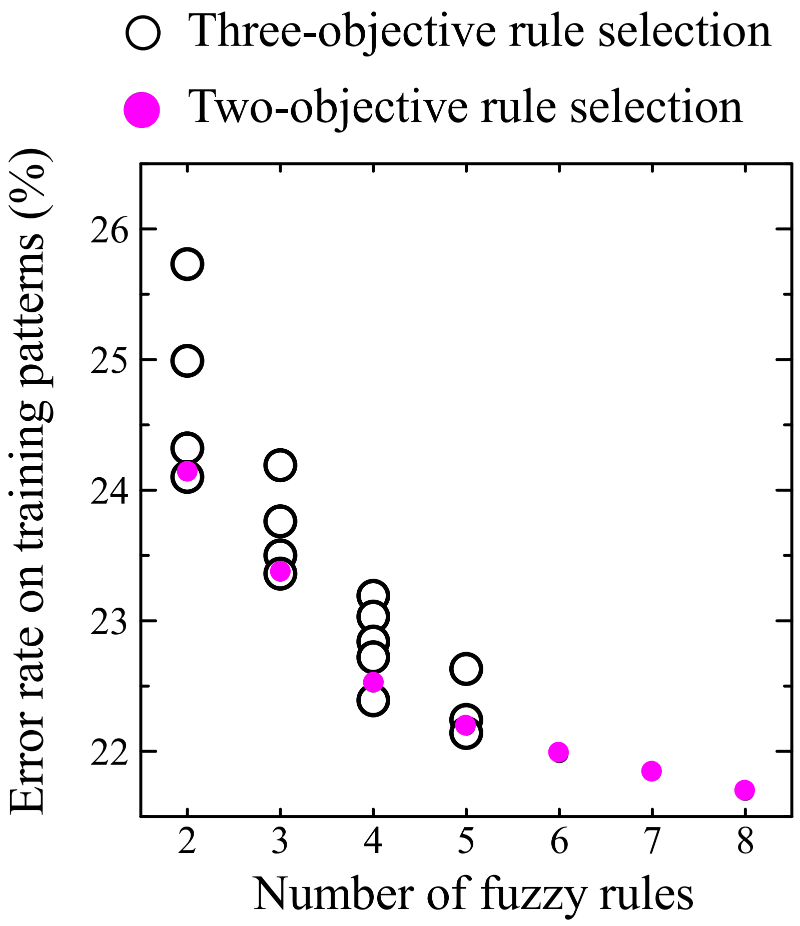
(a) Error rates on **training data** (b) Error rates on **test data**

The generalization ability is increased by increasing the number of fuzzy rules (i.e., the overfitting is not observed).

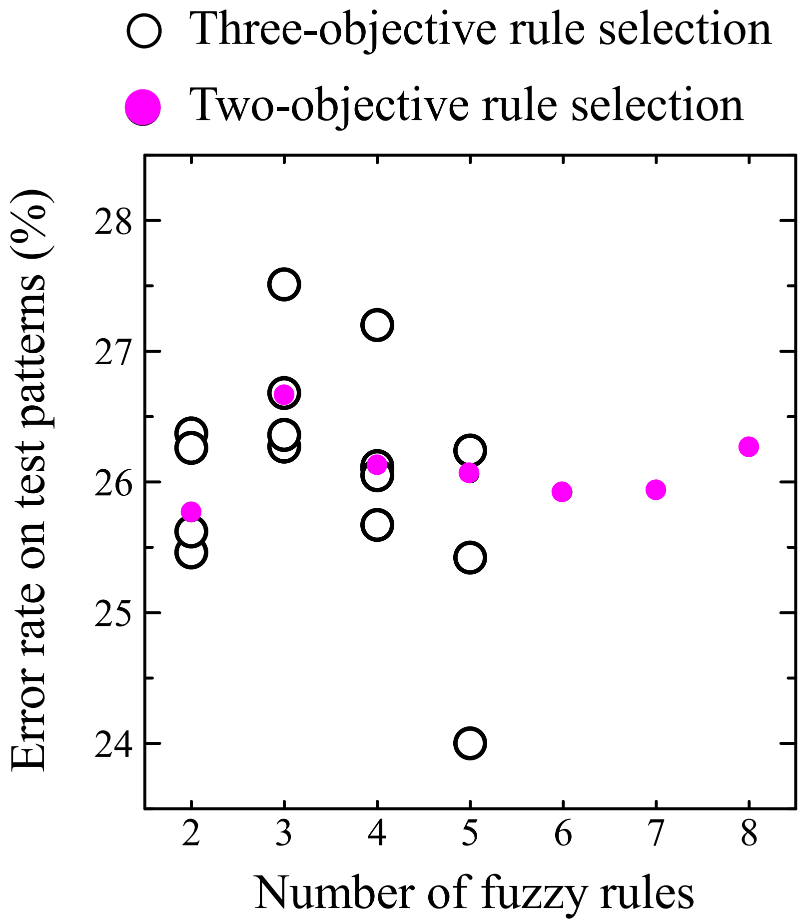
Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Experimental Results (Diabetes)



(a) Error rates on **training data**

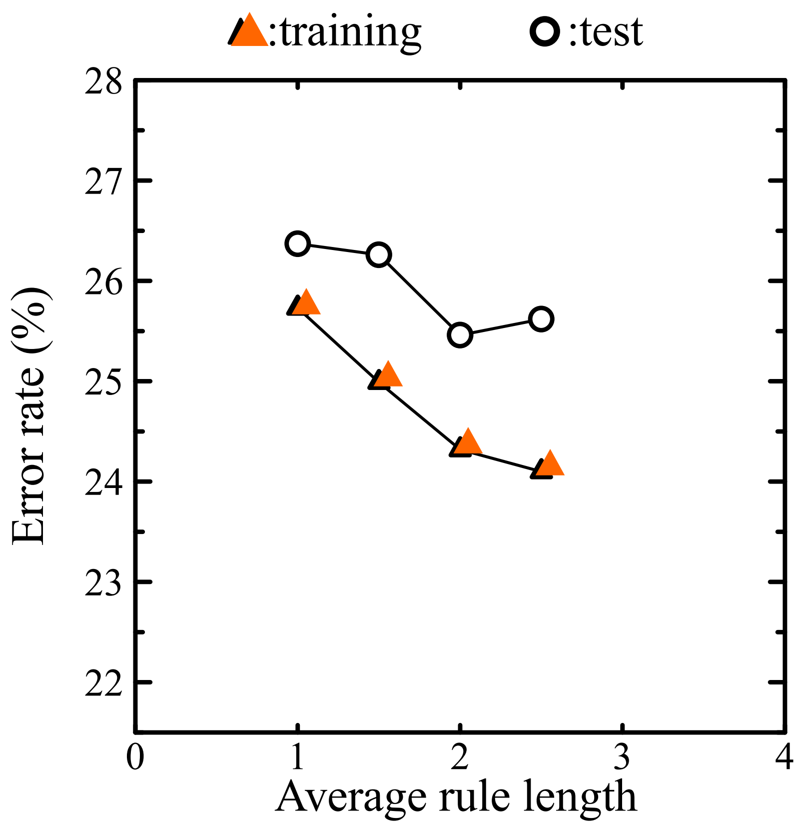


(b) Error rates on **test data**

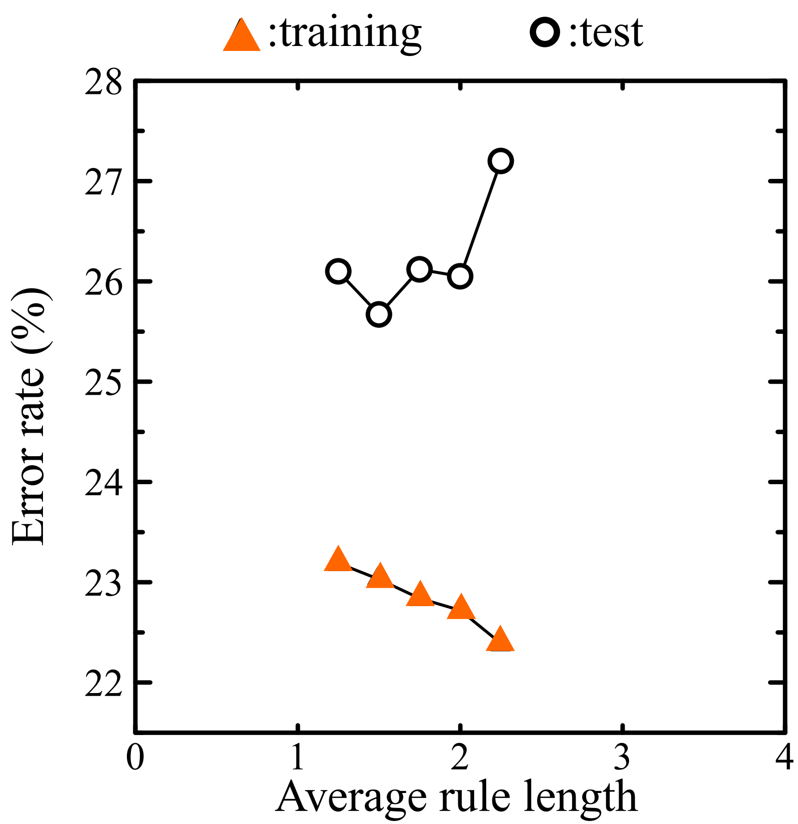
The effect of the increase in the number of fuzzy rules is not clear.

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Experimental Results (Diabetes)



(a) Rule sets with two rules



(b) Rule sets with four rules

We can observe the overfitting due to the increase in the rule length in the right figure for rule sets with four fuzzy rules.

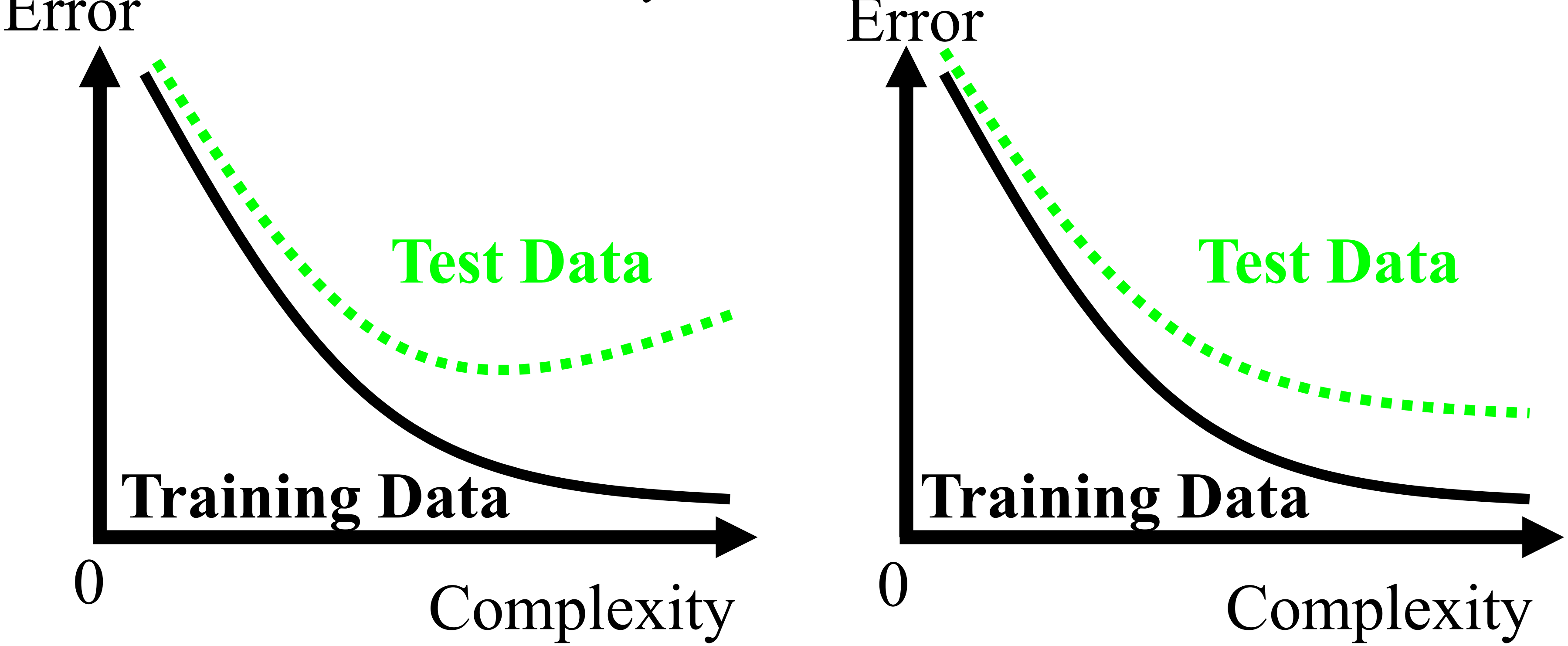
Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Observation

(1) Experimental results showed that each test problem has a different tradeoff structure.

(2) Knowledge on the tradeoff structure is useful in the design of fuzzy rule-based classification systems.



Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

SECOND TYPE: DATA BASE TUNING (+ RULE SELECT.) - REGRESSION

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15:5 (2007) 539–557*

M.J. Gacto, R. Alcalá, F. Herrera, Adaptation and Application of Multi-Objective Evolutionary Algorithms for Rule Reduction and Parameter Tuning of Fuzzy Rule-Based Systems. *Soft Computing 13:5 (2009) 419-436*

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15:5 (2007) 539–557,*

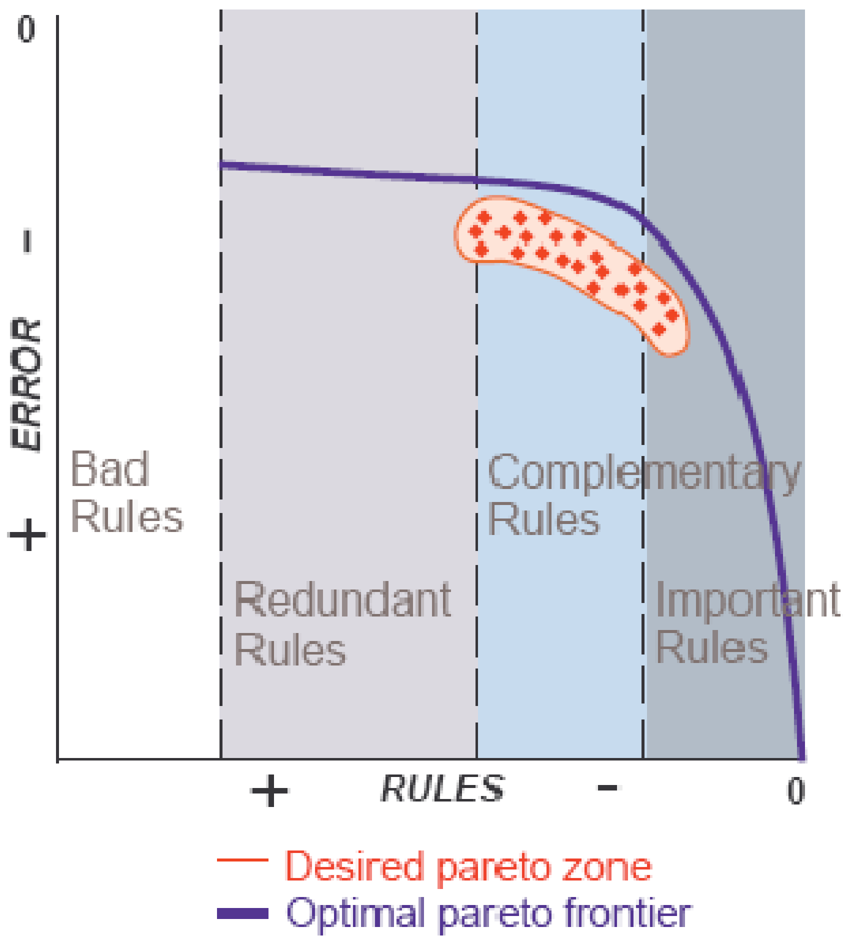
Multi-objective EAs are powerful tools to generate GFSs but they are based on getting a large, well distributed and spread off, Pareto set of solutions

- **The two criteria to optimize in GFSs are accuracy and interpretability. The former is more important than the latter, so many solutions in the Pareto set are not useful**
- **Solution: Inject knowledge through the MOEA run to bias the algorithm to generate the desired Pareto front part**

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Pareto front classification in an interpretability-accuracy GFSs:



- Bad rules zone: solutions with bad performance rules. Removing them improves the accuracy, so no Pareto solutions are located here
- Redundant rules zone: solutions with irrelevant rules. Removing them does not affect the accuracy and improves the interpretability
- Complementary rules zone: solutions with neither bad nor irrelevant rules. Removing them slightly decreases the accuracy
- Important rules zone: solutions with essential rules. Removing them significantly decreases the accuracy

Different Models of Multiobjective GFSs
MODEL 2: Multiobjective Tuning and Rule Selection

Accuracy-oriented modifications performed:

- **Restart the genetic population at the middle of the run time, keeping the individual with the highest accuracy as the only one in the external population and generating all the new individuals with the same number of rules it has**

- **In each MOGA step, the number of chromosomes in the external population considered for the binary tournament is decreased, focusing the selection on the higher accuracy individuals**

Different Models of Multiobjective GFSs
MODEL 2: Multiobjective Tuning and Rule Selection

Obtained results for the medium voltage line problem:

Multi-objective genetic tuning + rule selection method:

Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
WM	65	57605	2841	+	57934	4733	+
WM+T	65	18602	1211	+	22666	3386	+
WM+S	40.8	41086	1322	+	59942	4931	+
WM+TS	41.9	14987	391	+	18973	3772	+
NSGAII	41.0	14488	965	+	18419	3054	+
$NSGAII_{ACC}$	48.1	16321	1636	+	20423	3138	+
SPEA2	**33**	13272	1265	+	17533	3226	+
$SPEA2_{ACC}$	34.5	**11081**	1186	*	**14161**	2191	*

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

STUDY ON SEVERAL ALTERNATIVE APPROACHES AND IMPROVEMENTS:
ADAPTATION AND APPLICATION OF MOEAs

M.J. Gacto, R. Alcalá, F. Herrera,
Adaptation and Application of Multi-Objective Evolutionary Algorithms for Rule Reduction and Parameter Tuning of Fuzzy Rule-Based Systems, *Soft Computing 13:5 (2009) 419-436*,

- **To perform the study we have applied six different approaches based on the two most known and successful MOEAs:**
  - **Application of *SPEA2* and *NSGA-II***
  - **Two versions of NSGA-II for finding knees, $NSGA\text{-}II_A$ and $NSGA\text{-}II_U$**
  - **Two extensions for specific application, $SPEA2_{Acc}$ and $SPEA2_{Acc2}$**

Method	Description
WM	Wang & Mendel algorithm
T	Tuning of Parameters
S	Rule Selection
TS	Tuning & Selection
Application of standard MOEAs for general use	
TS-SPEA2	Tuning & Selection by SPEA2
TS-NSGA-II	Tuning & Selection by NSGA-II
TS-NSGA-II$_A$	Tuning & Selection by NSGA-II$_{angle}$
TS-NSGA-II$_U$	Tuning & Selection by NSGA-II$_{utility}$
Extended MOEAs for specific application	
TS-SPEA2$_{Acc}$	Accuracy-Oriented SPEA2
TS-SPEA2$_{Acc^2}$	Extension of SPEA2$_{Acc}$

- **Two objectives are considered: MSE and Number of Rules**

- **Proper operators have to be selected.**

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

NSGA-II FOR FINDING KNEES

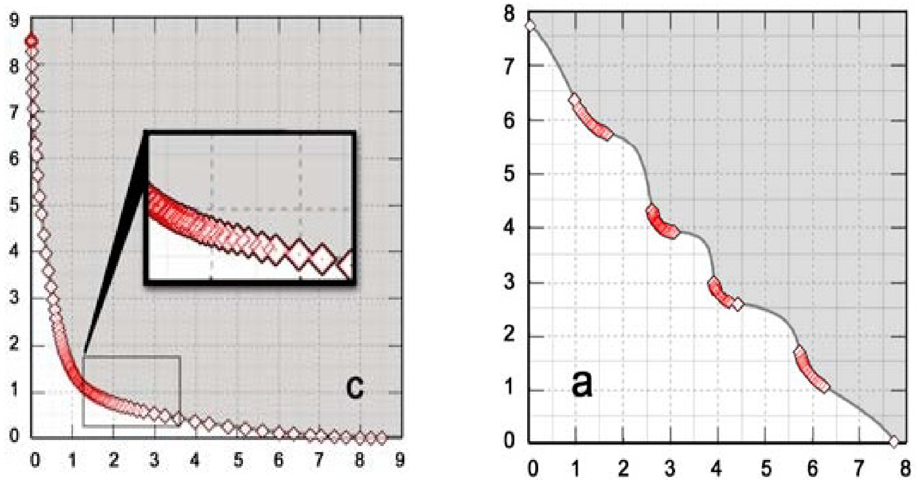
J. Branke, K. Deb, H. Dierolf, and M. Osswald, "Finding Knees in Multi-objective Optimization," Proc. Parallel Problem Solving from Nature Conf. - PPSN VIII, LNCS 3242, (Birmingham, UK, 2004) 722–731.

- **A variation of NSGAII in order to find knees in the Pareto front by replacing the crowding measure by either an angle-based measure or an utility-based measure**

Two different approaches

Angle Based Approach

Utility Based Approach



- **In our case, a knee could represent the best compromise between accuracy and number of rules.**

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Extension of $SPEA2_{Acc}$ ($SPEA2_{Acc2}$)

A New Crossover Operator for the Rule Part

- **Objective: to improve the search with a more intelligent operator replacing the HUX crossover in $SPEA2_{ACC}$**
- **Once BLX is applied a normalized euclidean distance is calculated between the centric point of the MFs used by each rule of the offpring and each parent**
- **The closer parent determines if this rule is selected or not for this offpring**
- **Whit this crossover operator, mutation can be particularly used to remove rules**

Different Models of Multiobjective GFSs
MODEL 2: Multiobjective Tuning and Rule Selection

Obtained results for the medium voltage line problem:

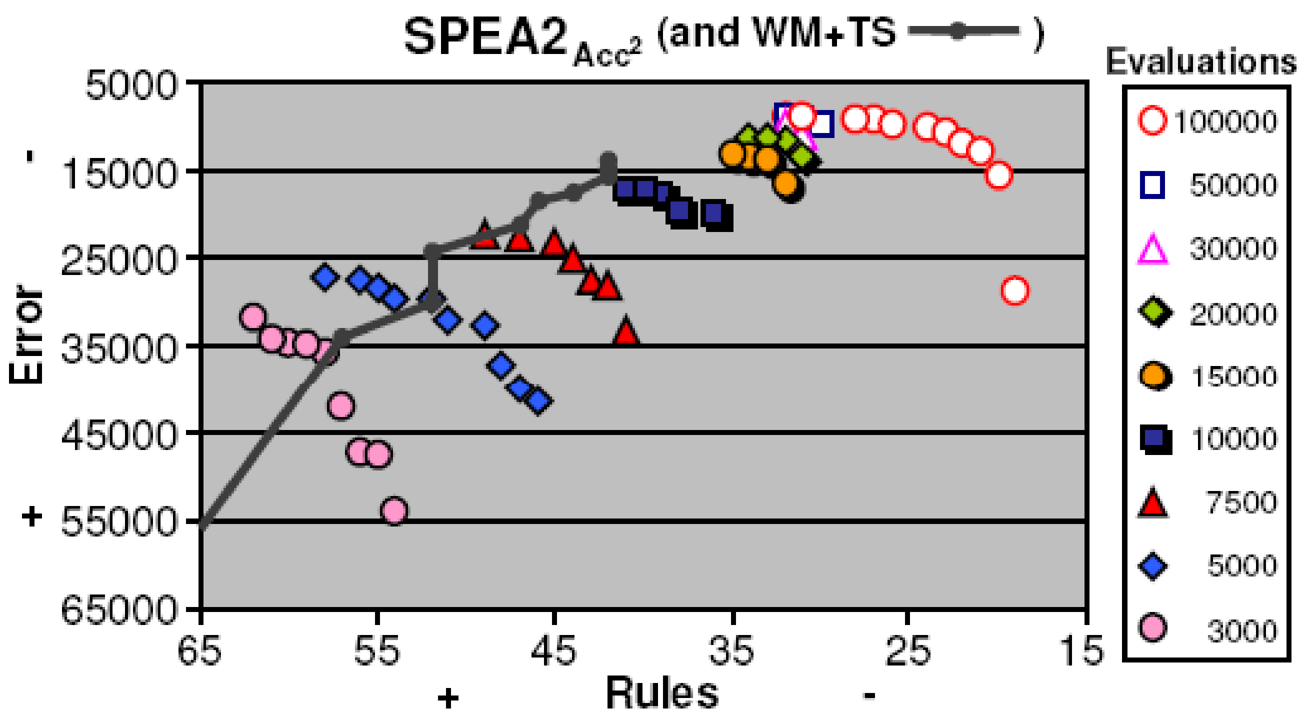
Method	#R	MSE_{tra}	σ_{tra}	t	MSE_{tst}	σ_{tst}	t
			100,000 evaluations				
WM	**65.0**	**57605**	**2841**	+	**57934**	**4733**	**+**
T	**65.0**	**17020**	**1893**	**+**	**21027**	**4225**	**+**
S	**40.9**	**41158**	**1167**	**+**	**42988**	**4441**	**+**
TS	**41.3**	**13387**	**1153**	**+**	**17784**	**3344**	**+**
TS-SPEA2	**28.9**	**11630**	**1283**	+	**15387**	**3108**	**+**
TS-NSGA-II	**31.4**	**11826**	**1354**	**+**	**16047**	**4070**	**+**
TS-NSGA-II$_A$	**29.7**	**11798**	**1615**	**+**	**16156**	**4091**	**+**
TS-NSGA-II$_U$	**30.7**	**11954**	**1768**	**+**	**15879**	**4866**	**+**
TS-SPEA2$_{Acc}$	**32.3**	**10714**	**1392**	=	**14252**	**3181**	=
TS-SPEA2$_{Acc2}$	**29.8**	**10325**	**1121**	*	**13935**	**2759**	*

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

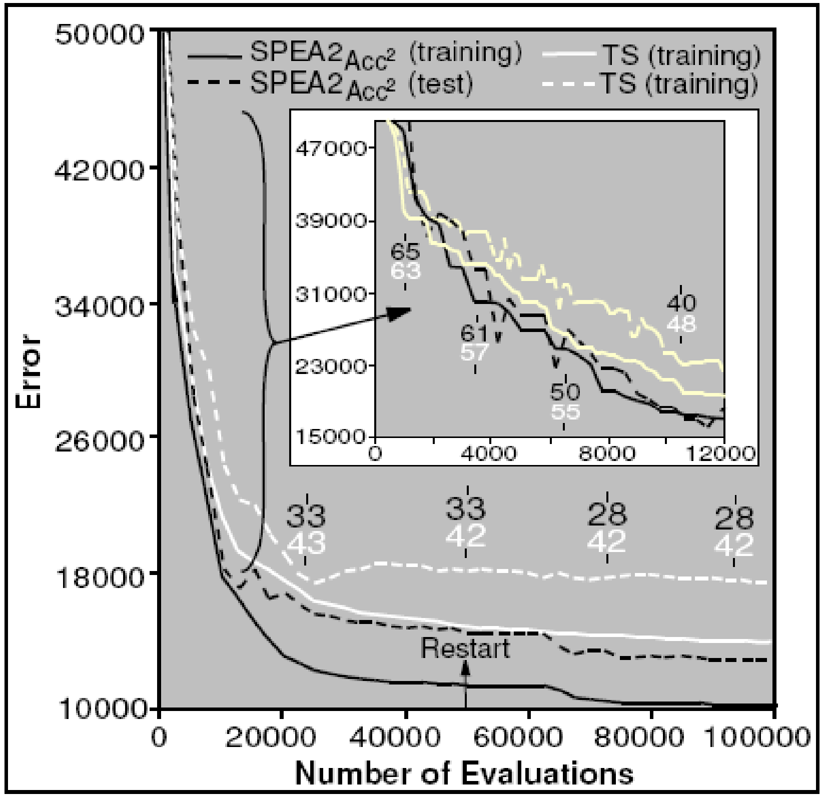
Comparison of the SPEA2acc² and classical GA for for the medium voltage line problem:

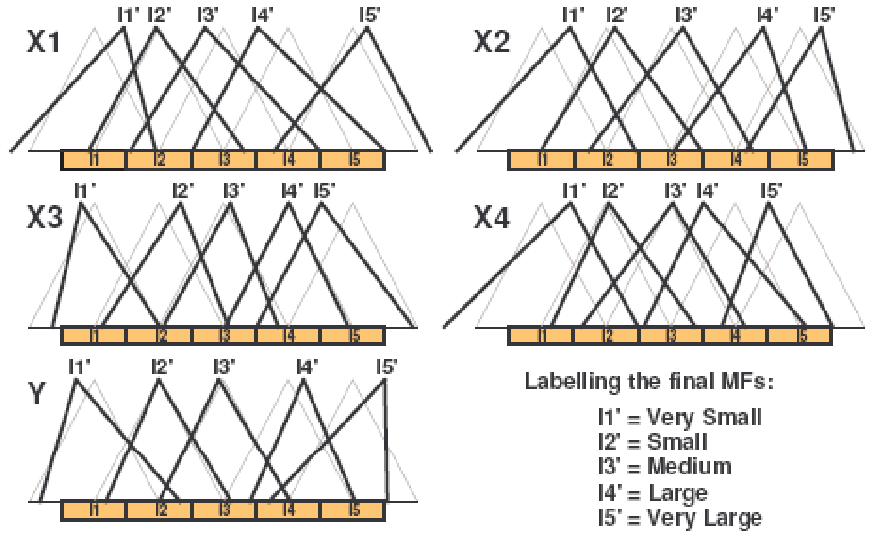


Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Convergence and an example model





Labelling the final MFs:

l1' = Very Small
l2' = Small
l3' = Medium
l4' = Large
l5' = Very Large

#R: 28 *MSE-tra: 8232* *MSE-tst: 14670*

X1	X2	X3	X4	Y
l1'	l1'	l1'	l1'	l1'
l1'	l1'	l1'	l2'	l2'
l2'	l1'	l1'	l1'	l1'
l2'	l1'	l1'	l2'	l2'
l2'	l1'	l2'	l2'	l2'
l2'	l2'	l2'	l1'	l2'
l2'	l3'	l3'	l1'	l3'
l3'	l2'	l1'	l1'	l1'
l3'	l2'	l1'	l2'	l2'

X1	X2	X3	X4	Y
l3'	l2'	l1'	l3'	l2'
l3'	l2'	l2'	l3'	l3'
l3'	l3'	l2'	l2'	l2'
l3'	l3'	l3'	l2'	l3'
l3'	l4'	l3'	l3'	l3'
l4'	l2'	l2'	l2'	l2'
l4'	l3'	l2'	l1'	l2'
l4'	l3'	l2'	l3'	l3'
l4'	l3'	l2'	l4'	l3'

X1	X2	X3	X4	Y
l4'	l3'	l3'	l2'	l3'
l4'	l3'	l3'	l3'	l4'
l4'	l4'	l3'	l2'	l3'
l4'	l4'	l4'	l2'	l4'
l4'	l4'	l4'	l4'	l5'
l4'	l5'	l4'	l2'	l3'
l4'	l5'	l5'	l3'	l5'
l5'	l2'	l2'	l5'	l4'
l5'	l2'	l3'	l2'	l3'
l5'	l4'	l3'	l5'	l5'

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

THIRD TYPE: KNOLEDGE BASE LEARNING - REGRESSION

R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, F. Marcelloni, A Multi-Objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems, *IEEE Transactions on Fuzzy Systems* 17:5 (2009) 1106-1122, doi:10.1109/TFUZZ.2009.2023113

Different Models of Multiobjective GFSs

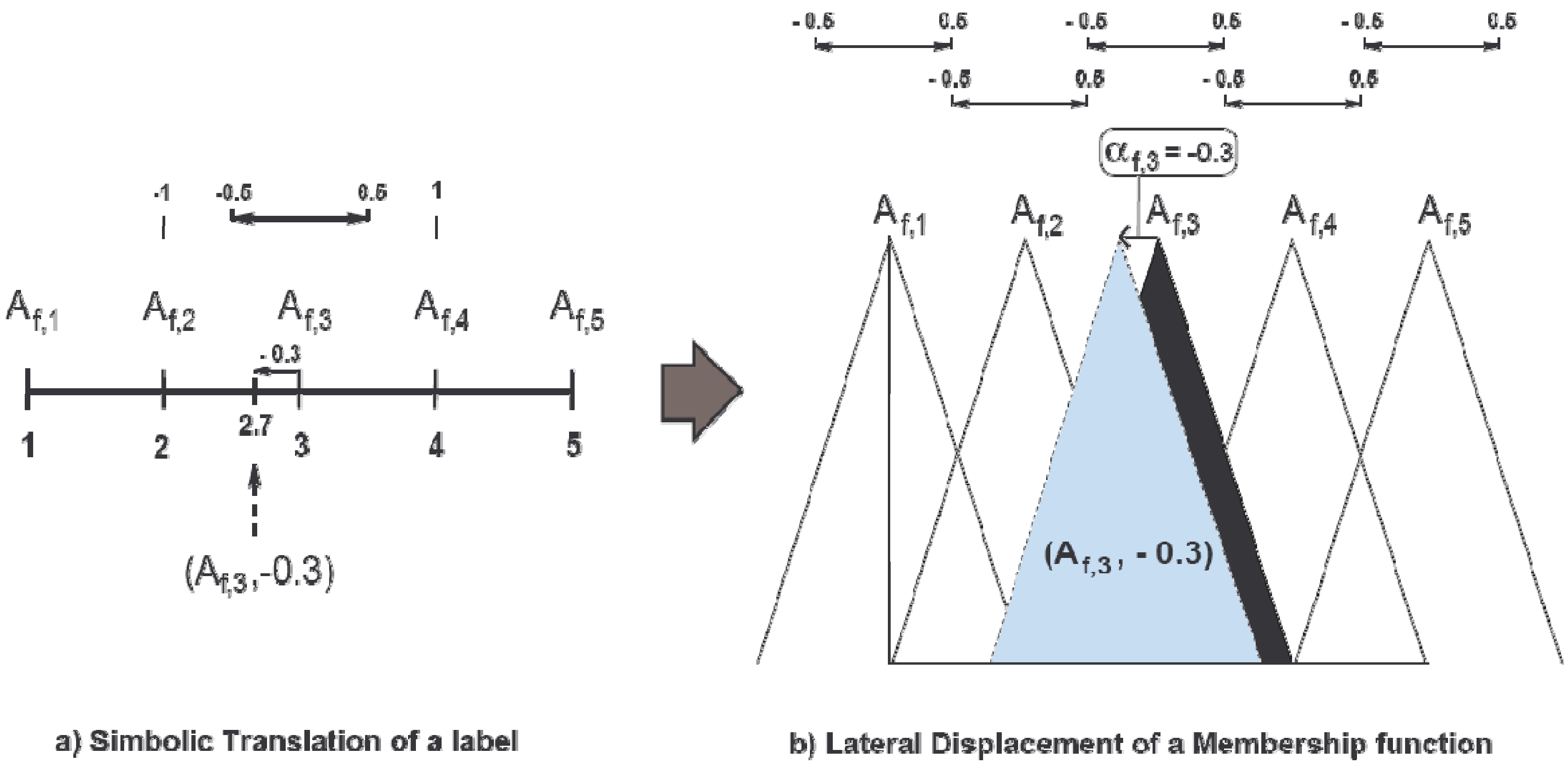
MODEL 3: Multiobjective Learning of DB and RB

R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, F. Marcelloni, A Multi-Objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems 17:5 (2009) 1106-1122, *IEEE Transactions on Fuzzy Systems, doi:10.1109/TFUZZ.2009.2023113*,

- **Rule bases and parameters of the membership functions of the associated linguistic labels are learnt concurrently.**
- **Accuracy and interpretability are measured in terms of approximation error (MSE) and rule base complexity (#Conditions), respectively.**
- **To manage the size of the search space, the linguistic 2-tuple representation model, which allows the symbolic translation of a label by only considering one parameter, has been exploited**

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB



This proposal decreases the tuning complexity, since the 3 parameters per label of the classical tuning are reduced to only 1 translation parameter (the tuning is applied to the level of linguistic partitions)

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Coding Scheme and Operators

- **A double coding scheme (C = C_{RB} + C_{DB})**

$$\underbrace{\overbrace{j_1^1 \;\cdots\; j_F^1 \;\; j_{F+1}^1}^{R^1} \;\cdots\; \overbrace{j_1^M \;\cdots\; j_F^M \;\; j_{F+1}^M}^{R^M}}_{C_{RB}\text{ (integer coding)}} \;\; \underbrace{\overbrace{\alpha_{1,1} \;\cdots\; \alpha_{1,T_1}}^{P_1} \;\cdots\; \overbrace{\alpha_{F+1,1} \;\cdots\; \alpha_{F+1,T_{F+1}}}^{P_{F+1}}}_{C_{DB}\text{ (real coding)}}$$

- **Crossover operator: one point + BLX-α crossovers (2 offsprings)**

- **Mutation operators:**
  - **Rule Adding: It adds γ random rules to the RB, where γ is randomly chosen in [1, γ_{max}]**

Different Models of Multiobjective GFSs
MODEL 3: Multiobjective Learning of DB and RB

Operators and Selection Schemes

- **Modify RB:** **It randomly changes δ elements of the RB part. The number δ is randomly generated in $[1, \delta_{max}]$**

- **Modify DB:** **It changes a gene value at random in the DB part**

PAES, NSGA-II and SOGA were applied using this representation and crossover

```
[p1, p2] = selection(archive/population);

if (rand() < Pcross)
        [s1, s2] = crossover(p1,p2);
        PmRB = 0.01;
else
        s1 = p1;
        s2 = p2;
        PmRB = 1;
endif
Loop i=1,2
        if (rand() < PmRB)
                if (rand <PmAdd)
                        si = add_rule();
                else
                        si = modify_rule_base();
                endif
        endif
        if (rand() < PmDB)
                si = mutate_DB();
        endif
endLoop
```

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Analysed Methods

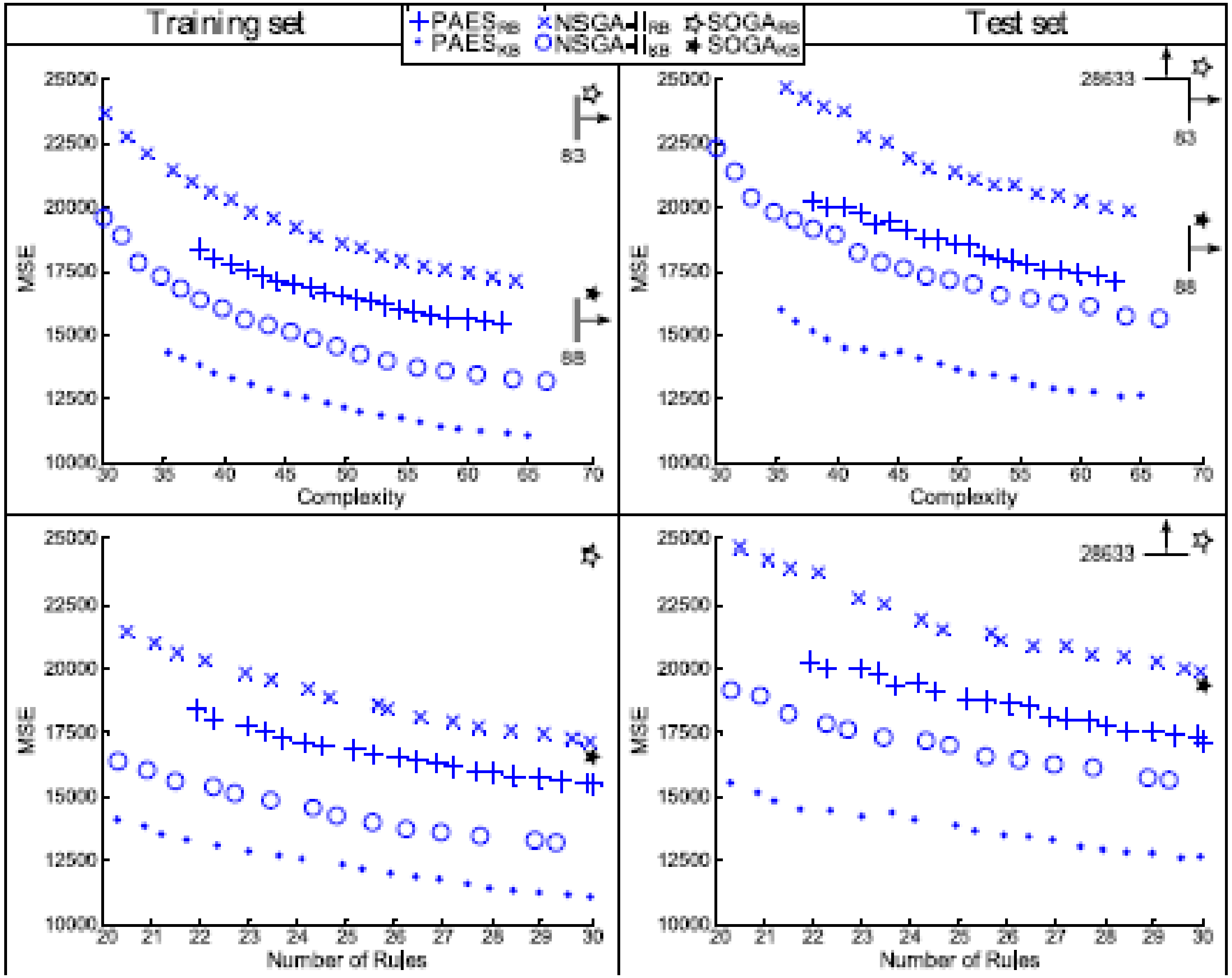
Method	Description	Pop. size
$SOGA_{RB}$	**Rule Base learning with SOGA**	**64**
$NSGA\text{-}II_{RB}$	**Rule Base learning with NSGA-II**	**64**
$PAES_{RB}$	**Rule Base learning with PAES**	**64**
$SOGA_{KB}$	**(Rule Base + Data Base) learning with SOGA**	**64**
$NSGA\text{-}II_{KB}$	**(Rule Base + Data Base) learning with NSGA-II**	**64**
$PAES_{KB}$	**(Rule Base + Data Base) learning with PAES**	**64**

- **Different population sizes were probed for these MOEAs showing better results when the population used for parent selection has similar sizes than those considered by single objective oriented algorithms.**
- **300,000 evaluations to allow complete convergence in all the algorithms**

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Average Pareto Fronts and average solution by SOGA
(medium voltage lines problem)



5 Data partitions 80% - 20%
6 Runs per partition
A total of 30 Runs
Test t-student α = 0.05

1. **Most accurate solution is selected from each Pareto**
2. **Average values are computed and represented**
3. **These solutions are no more used**
4. **Repeat to extract the desired avarage Pareto**

Only the first 20 solutions are considered

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Statistical Analysis

Statistical comparison among MOEAs

Method	Using the Pareto most accurate solution (FIRST) # R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t	Using the Pareto median solution (MEDIAN) # R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t	Using the Pareto simplest solution (LAST) # R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t
NSGA-II$_{RB}$	30/64	17116	4283	+	19834	4996	+	25/48	18853	4672	+	21533	5149	+	18/30	23649	5852	+	26660	6342	+
PAES$_{RB}$	30/63	15454	3882	+	17135	4234	+	27/51	16378	4112	+	18472	4740	+	22/38	18352	4631	+	20238	5419	+
NSGA-II$_{KB}$	29/67	13137	3378	+	15587	4806	+	23/46	15073	4126	+	17581	5853	+	17/29	21629	12156	+	25716	14722	+
PAES$_{KB}$	30/65	**11044**	**2771**	*	**12607**	**3106**	*	25/50	**12133**	**3380**	*	**13622**	**3353**	*	20/35	**14297**	**4449**	*	**15951**	**4405**	*

Statistical comparison of the best MOEA with SOGA

Method	# R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t
SOGA$_{RB}$	30/83	24340	8450	+	28633	11861	+
SOGA$_{KB}$	30/88	16502	5136	⋄	19112	6273	⋄
PAES$_{KB}$ (FIRST)	30/65	**11044**	**2771**	-	**12607**	**3106**	-
PAES$_{KB}$ (MEDIAN)	25/50	12133	3380	-	13622	3353	-
PAES$_{KB}$ (LAST)	20/35	14297	4449	=‡	15951	4405	-

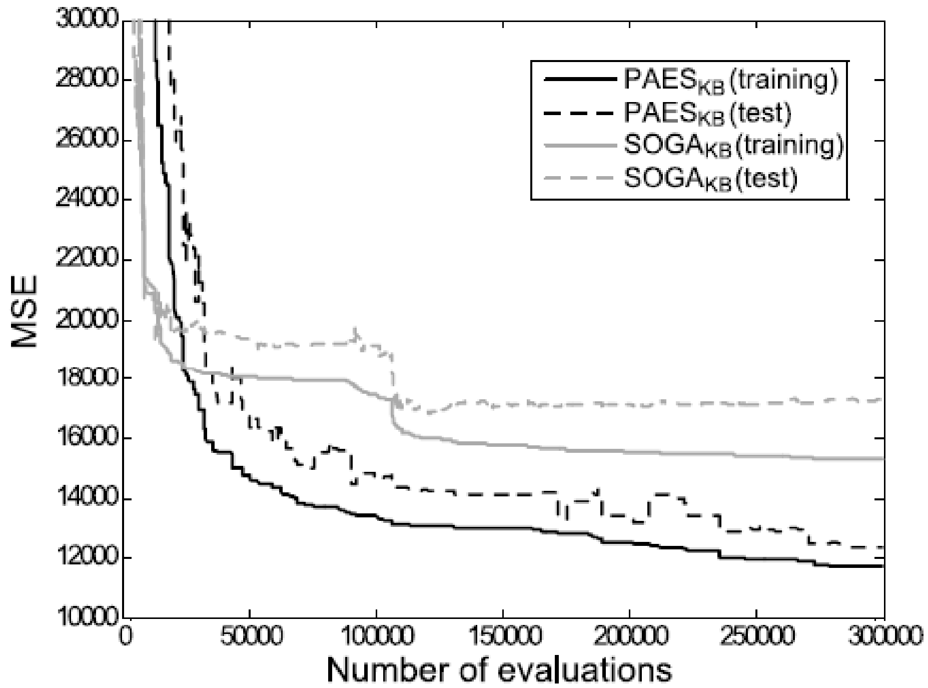
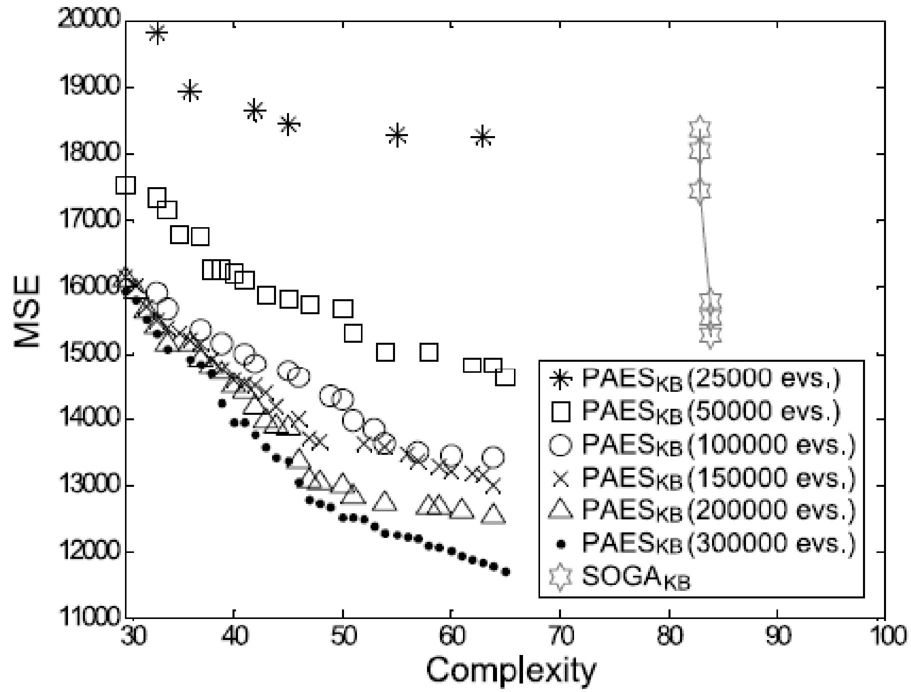
‡ It is (-) with 91% confidence

REMINDER
5 Data partitions 80% - 20%
6 Runs per partition
A total of 30 Runs
Test t-student α = 0.05

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Convergence

Evolution of SOGA and $PAES_{KB}$ (FIRST)	Pareto evolution of $PAES_{KB}$ and SOGA evolution
	

Different Models of Multiobjective GFSs

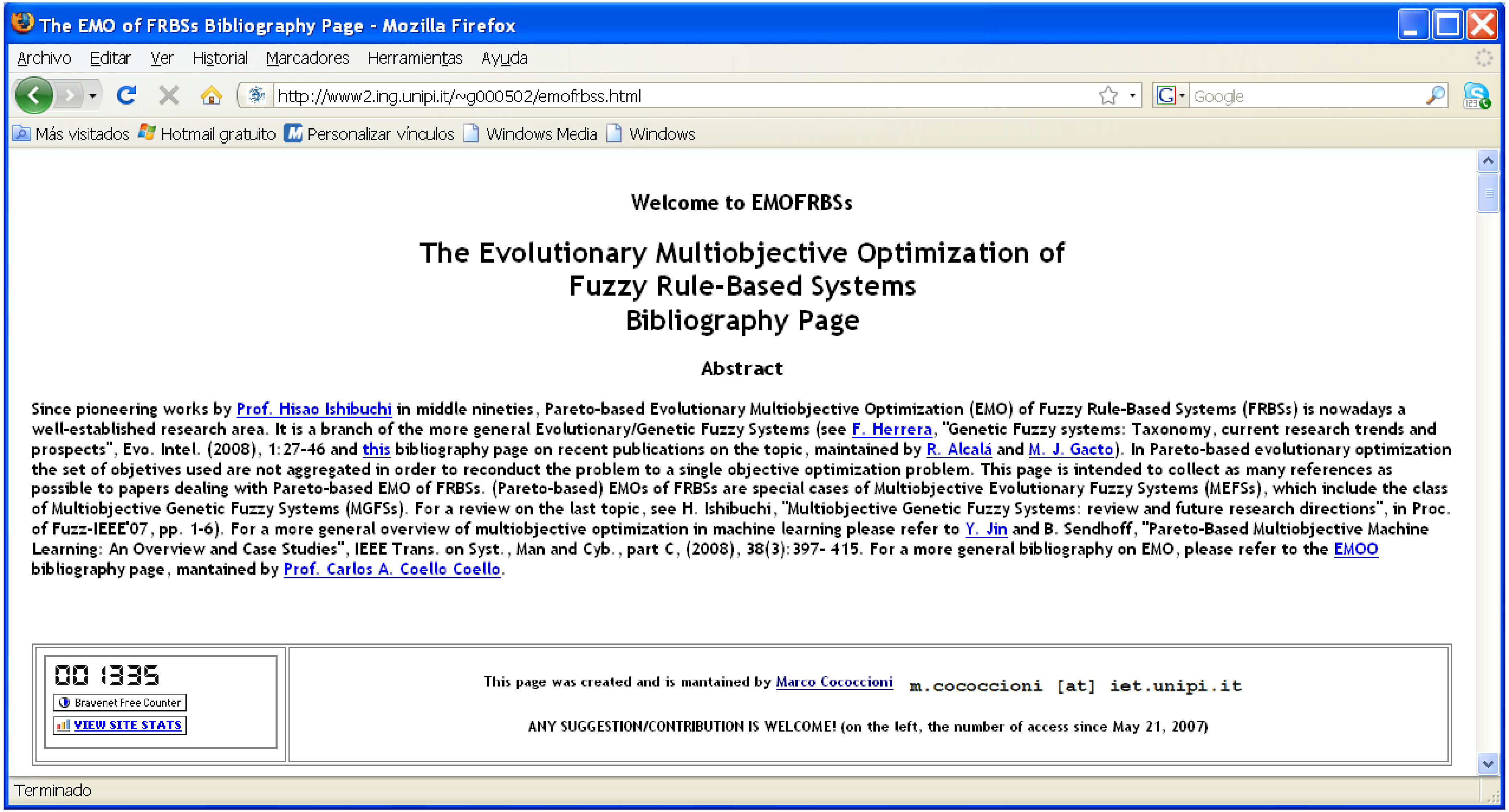
MODEL 3: Multiobjective Learning of DB and RB

- **The models obtained by these new approaches presented a better trade-off than those obtained by only considering performance measures.**

- **Between both multi-objective experimented, namely a modified (2+2)PAES and the classical NSGA-II, the modified (2+2)PAES has shown a better behavior than NSGA-II.**

- **Finally, the linguistic 2-tuples representation presented has shown a good positive synergy.**

Webpage of EMOFRBSs



The EMO of FRBSs Bibliography Page - Mozilla Firefox

Archivo Editar Ver Historial Marcadores Herramientas Ayuda

http://www2.ing.unipi.it/~g000502/emofrbss.html

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Welcome to EMOFRBSs

The Evolutionary Multiobjective Optimization of Fuzzy Rule-Based Systems Bibliography Page

Abstract

Since pioneering works by Prof. Hisao Ishibuchi in middle nineties, Pareto-based Evolutionary Multiobjective Optimization (EMO) of Fuzzy Rule-Based Systems (FRBSs) is nowadays a well-established research area. It is a branch of the more general Evolutionary/Genetic Fuzzy Systems (see F. Herrera, "Genetic Fuzzy systems: Taxonomy, current research trends and prospects", Evo. Intel. (2008), 1:27-46 and this bibliography page on recent publications on the topic, maintained by R. Alcalá and M. J. Gacto). In Pareto-based evolutionary optimization the set of objetives used are not aggregated in order to reconduct the problem to a single objective optimization problem. This page is intended to collect as many references as possible to papers dealing with Pareto-based EMO of FRBSs. (Pareto-based) EMOs of FRBSs are special cases of Multiobjective Evolutionary Fuzzy Systems (MEFSs), which include the class of Multiobjective Genetic Fuzzy Systems (MGFSs). For a review on the last topic, see H. Ishibuchi, "Multiobjective Genetic Fuzzy Systems: review and future research directions", in Proc. of Fuzz-IEEE'07, pp. 1-6). For a more general overview of multiobjective optimization in machine learning please refer to Y. Jin and B. Sendhoff, "Pareto-Based Multiobjective Machine Learning: An Overview and Case Studies", IEEE Trans. on Syst., Man and Cyb., part C, (2008), 38(3):397- 415. For a more general bibliography on EMO, please refer to the EMOO bibliography page, mantained by Prof. Carlos A. Coello Coello.

00 1335 Bravenet Free Counter VIEW SITE STATS	This page was created and is mantained by Marco Cococcioni m.cococcioni [at] iet.unipi.it ANY SUGGESTION/CONTRIBUTION IS WELCOME! (on the left, the number of access since May 21, 2007)

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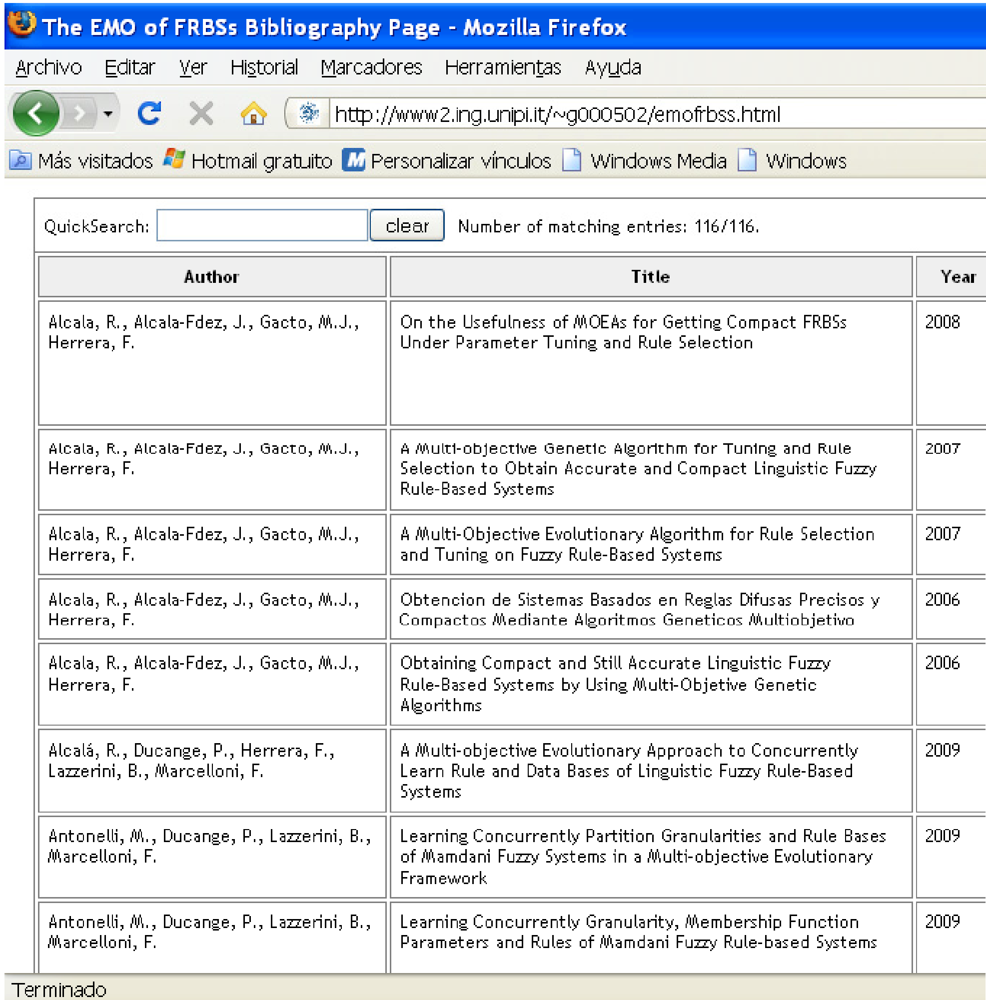
http://www.iet.unipi.it/m.cococcioni/emofrbss.html

Webpage of EMOFRBSs: List of 116 MGFSs contributions



The EMO of FRBSs Bibliography Page - Mozilla Firefox

Archivo Editar Ver Historial Marcadores Herramientas Ayuda

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QuickSearch: clear Number of matching entries: 116/116.

Author	Title	Year	Journal/Proceedings	Reftype	DOI/URL
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	On the Usefulness of MOEAs for Getting Compact FRBSs Under Parameter Tuning and Rule Selection	2008	in: Multi-objective Evolutionary Algorithms for Knowledge Discovery from Data Bases, Ghosh, A., Dehuri, S., Ghosh, S. (eds), Studies in Computational Intelligence, 2008Multi-objective Evolutionary Algorithms for Knowledge Discovery from Data Bases	inbook	
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	A Multi-objective Genetic Algorithm for Tuning and Rule Selection to Obtain Accurate and Compact Linguistic Fuzzy Rule-Based Systems	2007	International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems	article	
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	A Multi-Objective Evolutionary Algorithm for Rule Selection and Tuning on Fuzzy Rule-Based Systems	2007			
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	Obtencion de Sistemas Basados en Reglas Difusas Precisos y Compactos Mediante Algoritmos Geneticos Multiobjetivo	2006			
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	Obtaining Compact and Still Accurate Linguistic Fuzzy Rule-Based Systems by Using Multi-Objective Genetic Algorithms	2006			
Alcalá, R., Ducange, P., Herrera, F., Lazzerini, B., Marcelloni, F.	A Multi-objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems	2009			
Antonelli, M., Ducange, P., Lazzerini, B., Marcelloni, F.	Learning Concurrently Partition Granularities and Rule Bases of Mamdani Fuzzy Systems in a Multi-objective Evolutionary Framework	2009			
Antonelli, M., Ducange, P., Lazzerini, B., Marcelloni, F.	Learning Concurrently Granularity, Membership Function Parameters and Rules of Mamdani Fuzzy Rule-based Systems	2009			

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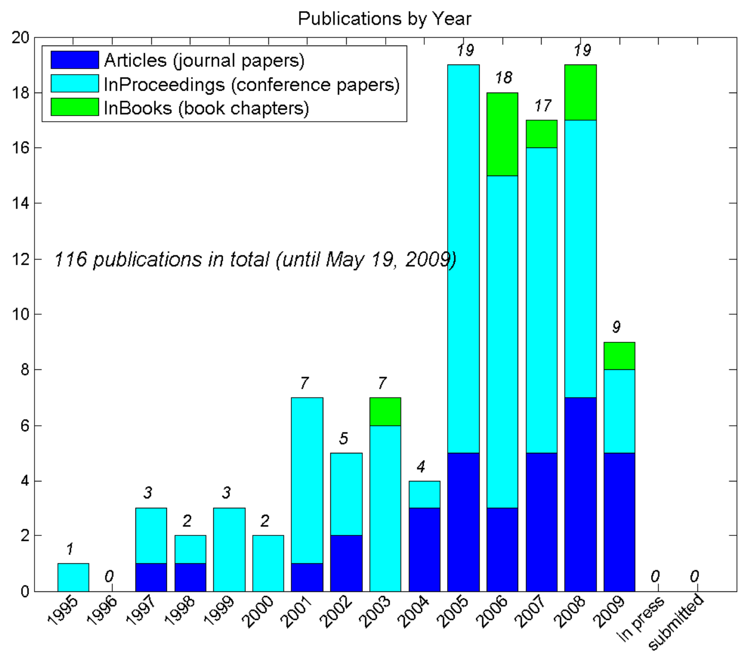


http://www.iet.unipi.it/m.cococcioni/emofrbss.html

Recent Keynote materials

A vision on the current state-of-the-art

Available at http://sci2s.ugr.es/gfs/#six

Current and Future Research Directions in MGFSs

1) Development of New MGFS Methods with Improved Algorithms

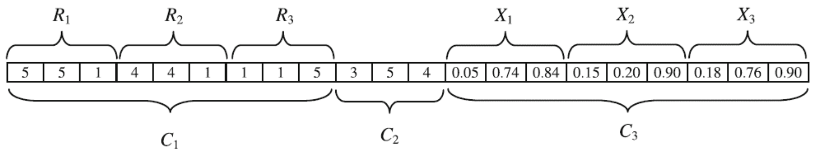
- Particular algorithms for multiobjective input selection
- Particular algorithms for multiobjective fuzzy partition learning
- . . .

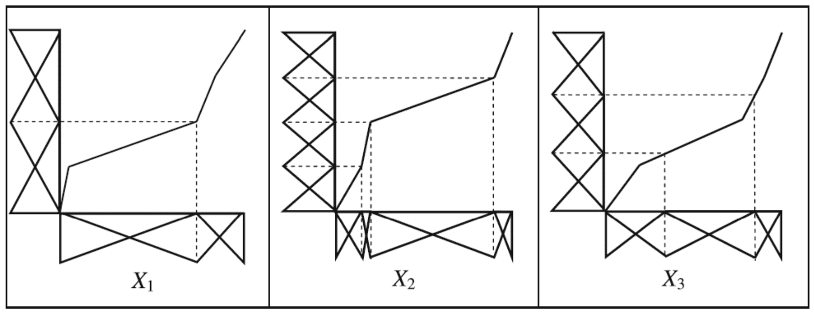
An example for learning granularities and selecting conditions can be found in:

M. Antonelli, P. Ducange, B. Lazzerini, and F. Marcelloni, "Learning concurrently partition granularities and rule bases of Mamdani fuzzy systems in a multi-objective evolutionary framework," Int. J. Approx. Reason., vol. 50, n. 7, pp. 1066–1080, 2009.

M. Antonelli, P. Ducange, B. Lazzerini, and F. Marcelloni, "Multi-objective evolutionary learning of granularity, membership function parameters and rules of Mamdani fuzzy systems," Evolutionary Intelligence, vol. 2, n. 1-2, pp. 21–37, 2009.

Exploiting the concept of virtual partitions with modified PAES





Current and Future Research Directions in MGFSs (2)

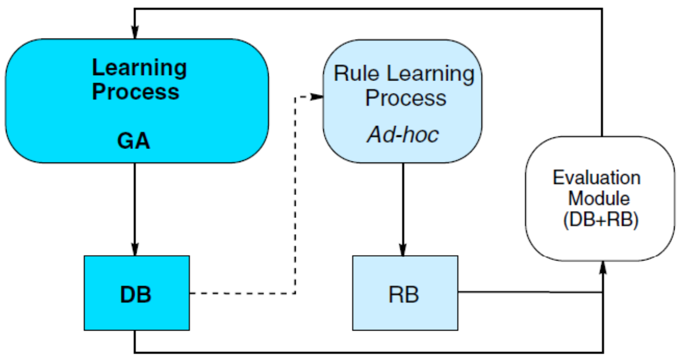
1) Development of New MGFS Methods with Improved Algorithms (2)

An example for learning granularities and for selecting variables can be found in:

R. Alcala, M. J. Gacto, and F. Herrera, "A Fast and Scalable Multi-Objective Genetic Fuzzy System for Linguistic Fuzzy Modeling in High-Dimensional Regression Problems," IEEE Transactions on Fuzzy System, doi: 10.1109/TFUZZ.2011.2131657, in press (2011).

Exploiting the embedded learning of the DB with improved SPEA2

$$C_1 = (L^1, \ldots, L^N) \quad C_2 = (\alpha^1, \ldots, \alpha^N)$$



2) Performance evaluation of MOGFSs

- **Visualization of Pareto-Optimal Fuzzy Systems**
- **How to compare MGFSs**
  - A statistical Analysis is needed
  - Use of non-parametric statistical tests

Evaluation indexes in the EMO framework evaluate the exploration and exploitation capabilities of the MOEA. But we are also interested in generalization capabilities of the FRBSs

Current and Future Research Directions in MGFSs (3)

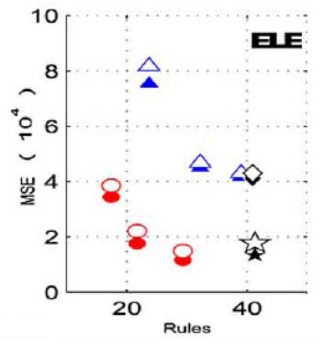
2) Performance evaluation of MOGFSs

- **How to compare MGFSs**

A recent possibility to apply non-parametric statistical tests:

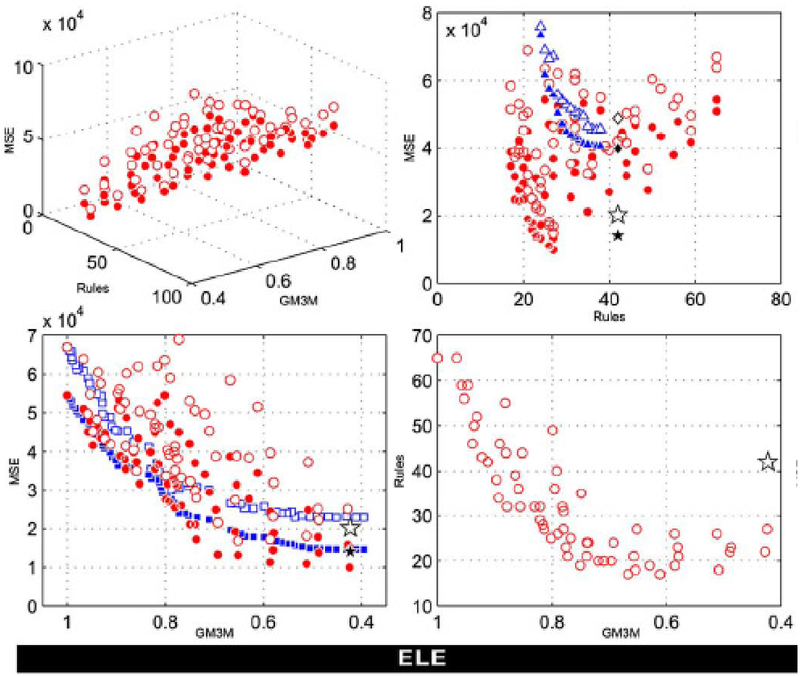
R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, and F. Marcelloni, "A Multi-objective evolutionary approach to concurrently learn rule and data bases of linguistic fuzzy rule-based systems," IEEE Trans. Fuzzy. Syst., vol. 17, n. 5, pp. 1106–1122, 2009.

Analyzing the averages on three representative points by non-parametric statistical tests for bi-objective problems (FIRST, MEDIAN, LAST)



An extension for the case of more than two objectives:

M. J. Gacto, R. Alcala, and F. Herrera, "Integration of an index to preserve the semantic interpretability in the multi-objective evolutionary rule selection and tuning of linguistic fuzzy systems," IEEE Trans. Fuzzy. Syst. vol. 18, n.3, pp. 515-531, 2010.



Projections on bi-objective planes.Then, representative points can be obtained in the new non-dominated solutions

Current and Future Research Directions in MGFSs (4)

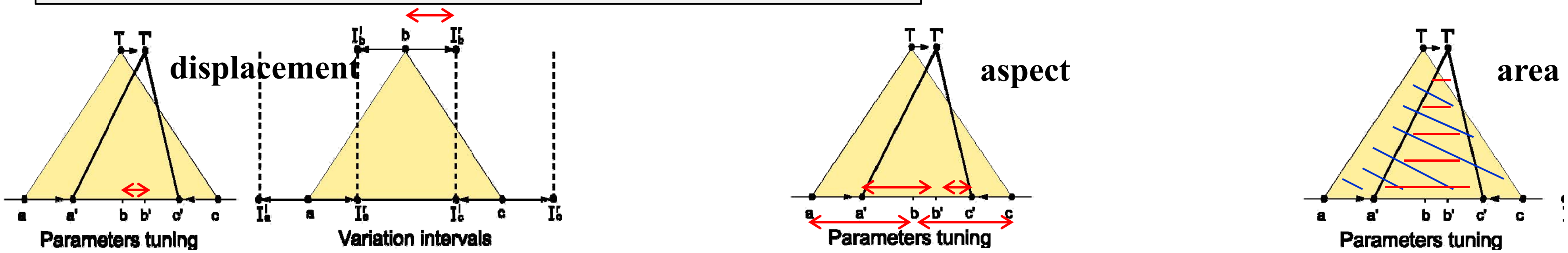
3) Reliable Interpretability Measures *(Formulations of the Interpretability)*

- We need well established and accepted measures
- Use of new ones for C3 (semantic-RB) as cointension or number of fired rules

The use of relative measures for C4 (semantic-DB) could be promising. First proposal in:

M. J. Gacto, R. Alcala, and F. Herrera, "Integration of an index to preserve the semantic interpretability in the multi-objective evolutionary rule selection and tuning of linguistic fuzzy systems," IEEE Trans. Fuzzy. Syst. vol. 18, n.3, pp. 515-531, 2010.

Measuring the differences to a given linguistic partition (obtained from experts or automatically by using absolute measures): GM3M index based on three metrics



Some recent approaches are also using this kind of measures:

M. Antonelli, P. Ducange, B. Lazzerini, and F. Marcelloni, "Learning knowledge bases of multi-objective evolutionary fuzzy systems by simultaneously optimizing accuracy, complexity and partition integrity, Soft Computing, DOI: 10.1007/s00500-010-0665-0, in press.

Current and Future Research Directions in MGFSs (5)

4) Objective dimensionality

- New EMO algorithms
- Aggregation or selection of a reasonable set of significant measures

5) Scalability issues

- High Dimensinality (handling the length of the rules)
- Large scale problems (using a reduced subset of examples)

■ **Some approaches dealing with large scale problems:**

- *M.A. de Vega, J.M Bardallo, F.A. Marquez, A. Peregrin, "Parallel distributed two-level evolutionary multiobjective methodology for granularity learning and membership functions tuning in linguistic fuzzy systems," in Proc. of ISDA 2009, pp. 134–139.* → ***Parallelization***
- *M. Cococcioni, B. Lazzerini, F. Marcelloni, "On reducing computational overhead in multi-objective genetic Takagi–Sugeno fuzzy systems," Appl. Soft Computing 11:1 (2011), 675-688.* → ***Fitness estimation***
- *M. Antonelli, P. Ducange, F. Marcelloni, "Exploiting a coevolutionary approach to concurrently select training instances and learn rule bases of Mamdani fuzzy dystems," in Proc. of WCCI 2010, 1366–1372.* → ***Instance Selection***

Current and Future Research Directions in MGFSs (6)

5) Scalability issues (2)

- **Some approaches dealing with high dimensional problems:**

• *H. Ishibuchi, and T. Yamamoto, "Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining," Fuzzy Sets and Systems, vol. 141, pp. 59–88, 2004.*

Imposing a maximum rule lenght

• *M. Antonelli, P. Ducange, B. Lazzerini, F. Marcelloni, "Multi-objective Evolutionary Generation of Mamdani Fuzzy Rule-Based Systems based on Rule and Condition Selection," in Proc. of GEFS 2011.*

Condition selection by specific approach

- **An approach dealing with both high dimensional and large scale problems:**

• *R. Alcala, M. J. Gacto, F. Herrera, "A Fast and Scalable Multi-Objective Genetic Fuzzy System for Linguistic Fuzzy Modeling in High-Dimensional Regression Problems," IEEE Trans. on Fuzzy Systems, doi: 10.1109/TFUZZ.2011.2131657, in press (2011).*

Using a specific approach for variable selection and fitness stimation by using a short subset of the examples

Current and Future Research Directions in MGFSs (7)

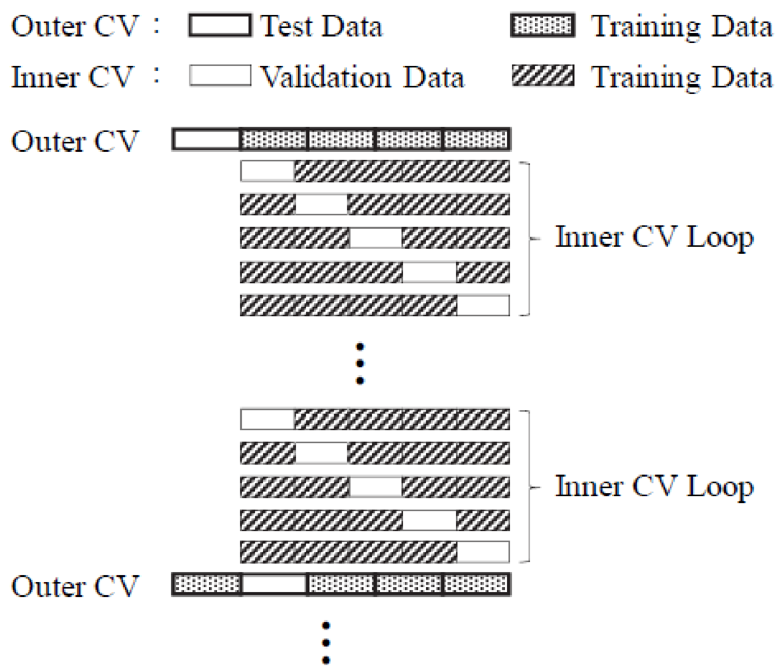
6) Automatic selection of the most suitable solution

- Determining those solutions with the best generalization ability
- Only training data can be took into account

■ **A recent approach on this topic:**

• *Ishibuchi H, Nakashima Y, Nojima Y, Double cross-validation for performance evaluation of multi-objective genetic fuzzy systems. In GEFS 2011, pp 31-38.*

Using a double cross-validation with two cross-validation loops. The inner loop uses the training data to determine the complexity of the systems with the best validation measure, which is used to select the solutions used for the outer loop.



Evolutionary Multi-Objective Design of Fuzzy Rule-Based Systems

Thank you very much for your attention !!!
Questions?

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