FUZZ-IEEE 2009 Tutorial, Jeju Island, Korea
Morning Session: 9:00-12:30, August 20, 2009

Evolutionary Multi-Objective Design of Fuzzy Rule-Based Systems

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5. Multiobjective Genetic Fuzzy Systems (MoGFS)
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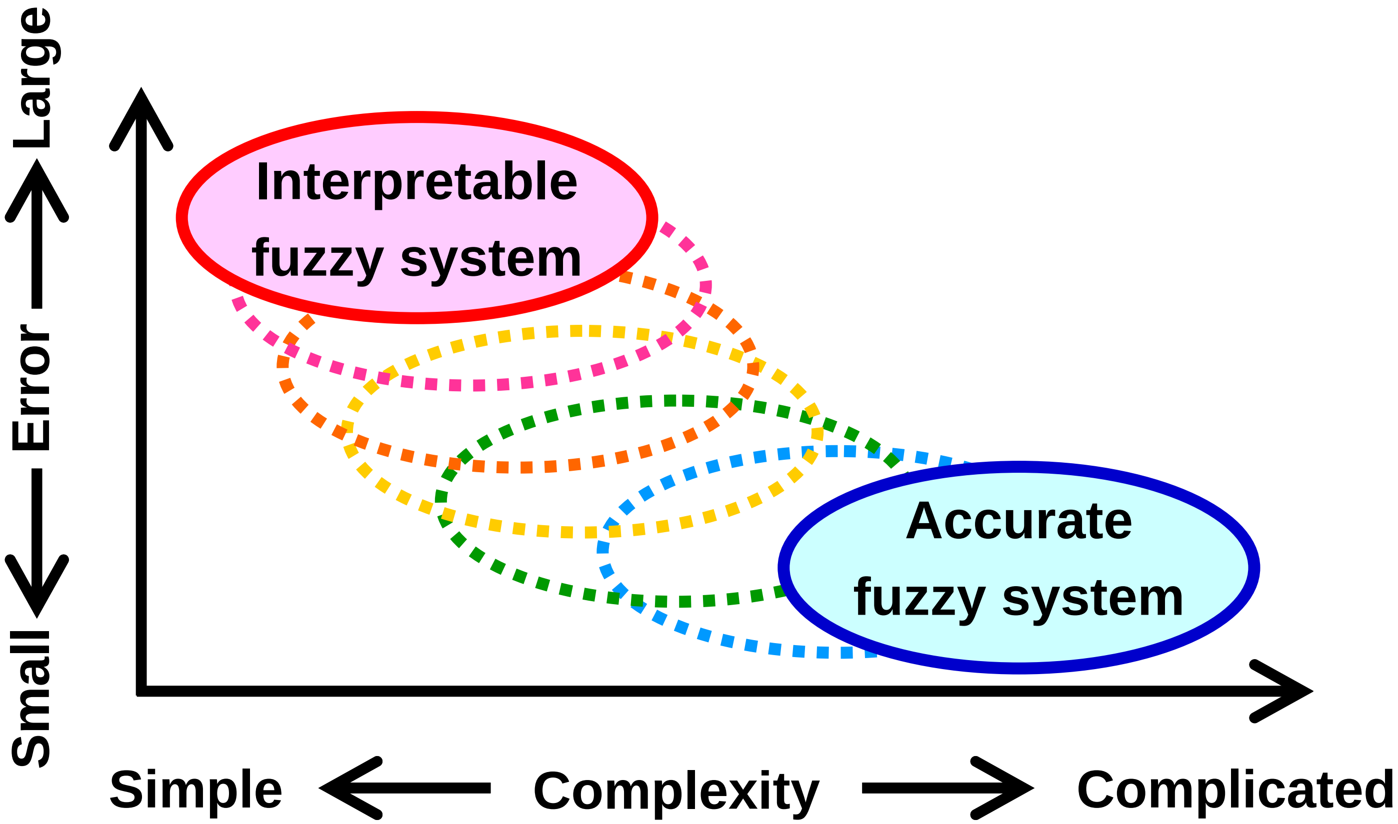
Fuzzy System Design

Two Goals in Fuzzy Rule-Based System Design:

(1) Accuracy Maximization (Error Minimization)
(2) Interpretability Maximization (Complexity Minimization)



Fuzzy System Design

Main Streams in Fuzzy System Design:

1970s - 1980s (Linguistic Knowledge Extraction)

- Fuzzy systems were designed by human experts.
- Fuzzy systems were linguistic rule-based systems.
- Fuzzy systems were highly interpretable.

Early 1990s - (Learning from Numerical Data)

- Fuzzy systems were designed from numerical data.
- Various neural and genetic approaches were proposed.

Mid 1990s - (Interpretability-Accuracy Tradeoff)

- Interpretability maintenance was taken into account.
- Interpretability-accuracy tradeoff was discussed.

Late 1990s - (Multiobjective Design)

- Evolutionary multiobjective algorithms were used.
- Multiple non-dominated fuzzy systems were generated.

Fuzzy Systems in 1970s - 1980s

Linguistic Systems with High Interpretability

- Fuzzy systems were design by human experts.
- Fuzzy systems were linguistic rule-based systems.
- Fuzzy systems were highly interpretable.

Less expensive controllers than others

Fuzzy Boom in Japan in the Late 1980s

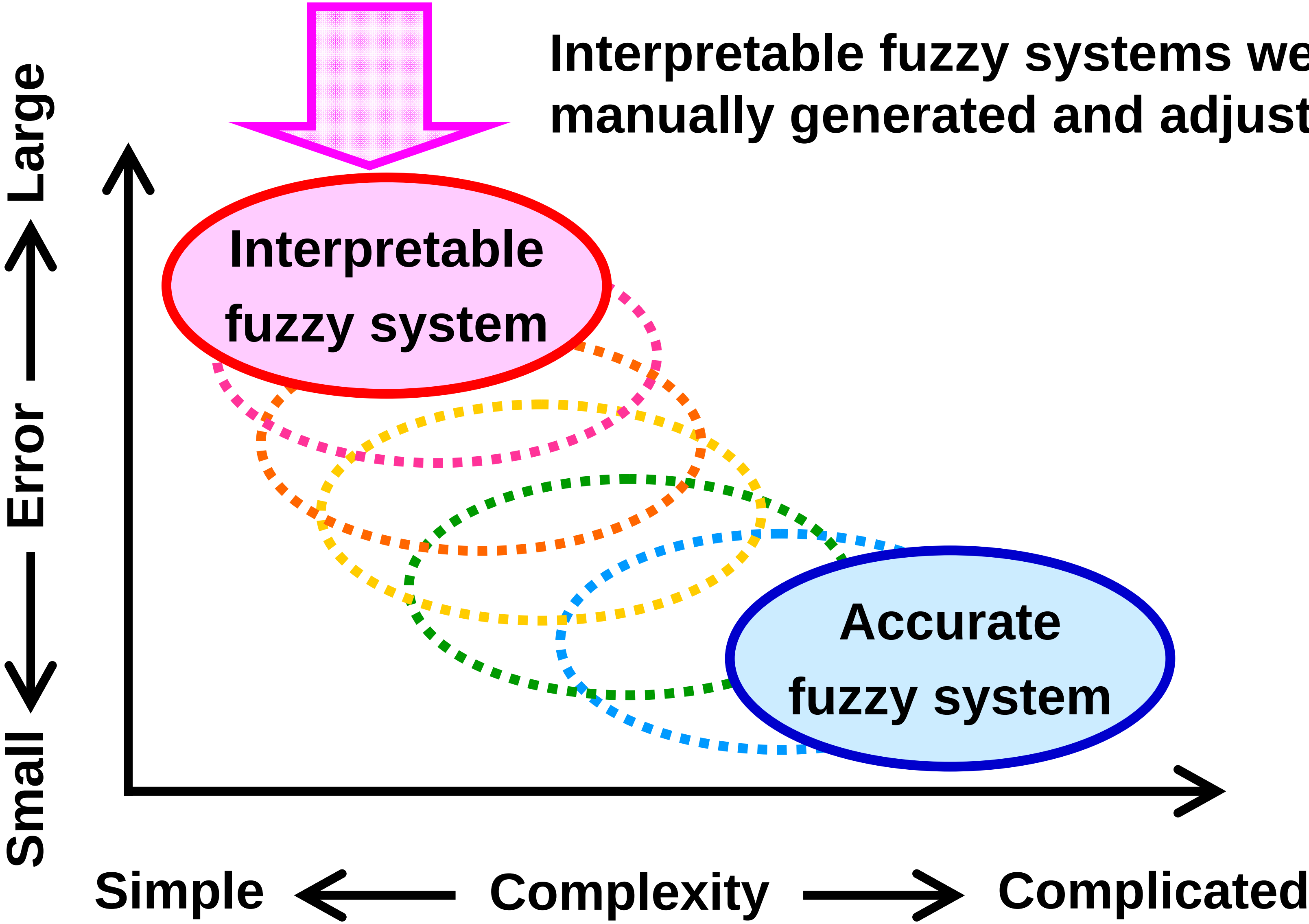
- Fuzzy air conditioner
- Fuzzy air cleaner
- Fuzzy rice cooker
- Fuzzy camera
- Fuzzy refrigerator
- Fuzzy ATM
- Fuzzy vacuum cleaner
- Fuzzy oven
- Fuzzy washing machine
- Fuzzy copy machine
- Fuzzy dryer
- Fuzzy automated cruise

More than 200 real-world applications in the SOFT website.

Direction of Fuzzy System Research

Fuzzy Systems in 1970s - 1980s

Interpretable fuzzy systems were manually generated and adjusted.



Difficulties in Fuzzy System Design

Difficulties in Fuzzy System Design by Human Experts

- Human experts are not always available.
- Knowledge extraction from human experts are time-consuming.
- Designed fuzzy systems do not always work well.

↓

Fuzzy System Design from Numerical Data

Highly Cited Papers

[1] Takagi T, Sugeno M: Fuzzy Identification of Systems and Its Applications to Modeling and Control, IEEE TSMC (1985)

[2] Wang LX, Mendel JM: Generating Fuzzy Rules by Learning from Examples, IEEE TSMC (1992)

Fuzzy Systems in the Early 1990s

Nonlinear Systems with High Accuracy

- Universal approximators of nonlinear functions
- Neural approaches to parameter learning
- Genetic approaches to parameter and structure learning

Increasing Popularity of Neural Networks and Genetic Algorithms

[1] D. E. Rumelhart, J. L. McClelland and the PDP Research Group: *Parallel Distributed Processing*, MIT Press (1986).

[2] D. E. Goldberg: *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley (1989).

Highly Cited Neuro-Fuzzy Papers

[1] Jang JSR: ANFIS - Adaptive-Network-Based Fuzzy Inference System, IEEE Trans. on SMC (1993)

[2] Lin CT, Lee CSG: Neural-Network-Based Fuzzy-Logic Control and Decision System, IEEE Trans. on Computers (1991)

[3] Jang JSR, Sun CT: Neuro-Fuzzy Modeling and Control, Proceedings of The IEEE (1995)

[4] Horikawa S, Furuhashi T, Uchikawa Y: On Fuzzy Modeling using Fuzzy Neural Networks with the Back-Propagation Algorithm, IEEE TNN (1992)

[5] Berenji HR, Khedkar P: Learning and Tuning Fuzzy-Logic Controllers through Reinforcements, IEEE TNN (1992)

Highly Cited Genetic Fuzzy Papers

[1] Homaifar A, Mccormick E: Simultaneous Design of Membership Functions and Rule Sets for Fuzzy Controllers using Genetic Algorithms, IEEE TFS (1995)

[2] Karr CL, Gentry EJ: Fuzzy Control of pH using Genetic Algorithms, IEEE TFS (1993)

[3] Ishibuchi H, et al.: Selecting Fuzzy If-Then Rules for Classification Problems using Genetic Algorithms, IEEE TFS (1995)

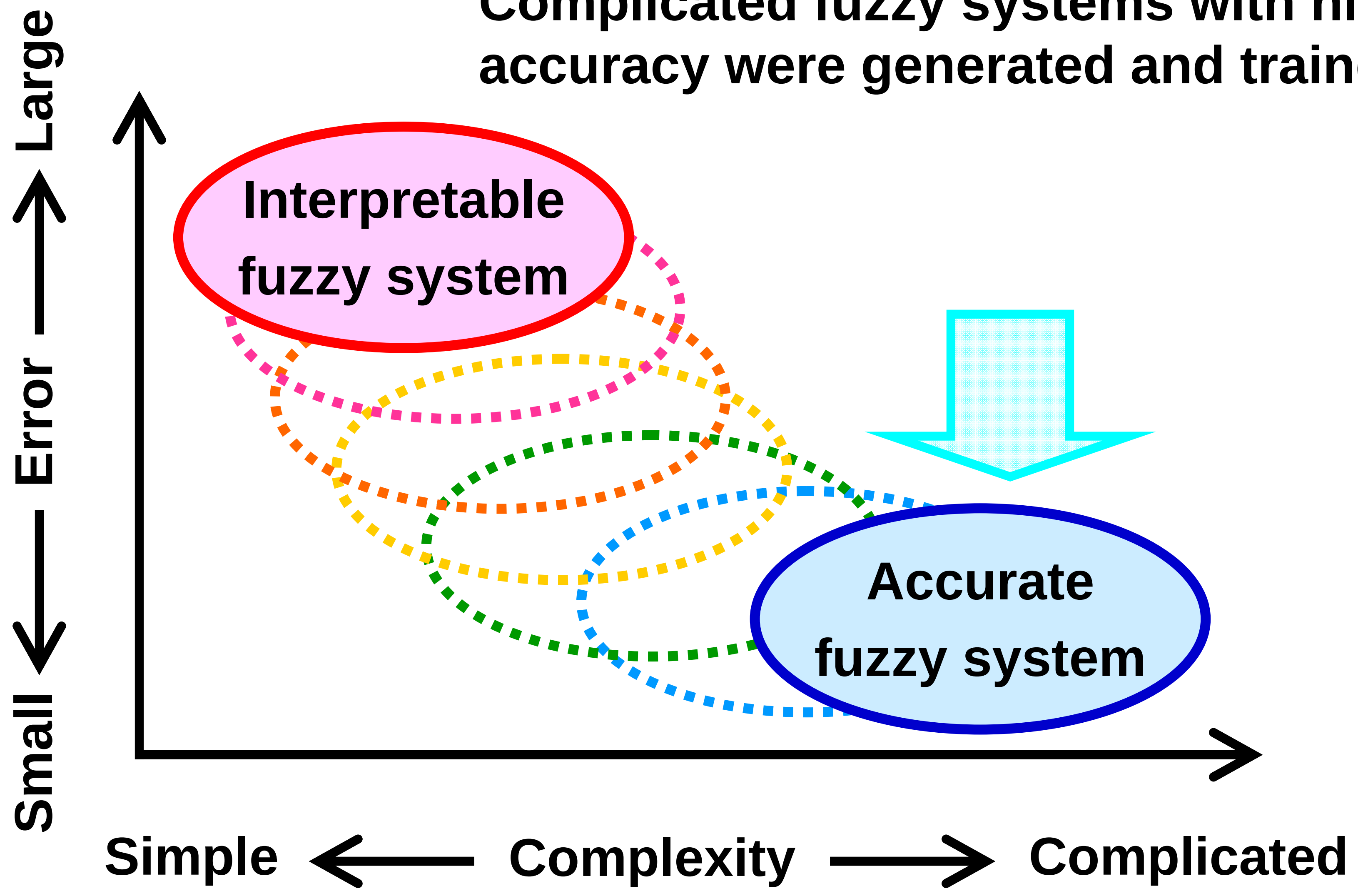
[4] Ishibuchi H, et al.: Performance Evaluation of Fuzzy Classifier Systems for Multidimensional Pattern Classification Problems, IEEE Trans. on SMC Part B (1999)

[5] Park D, Kandel A, Langholz G: Genetic-based New Fuzzy-Reasoning Models with Application to Fuzzy Control, IEEE Trans. on SMC (1994)

Direction of Fuzzy System Research

Fuzzy Systems in the Early 1990s

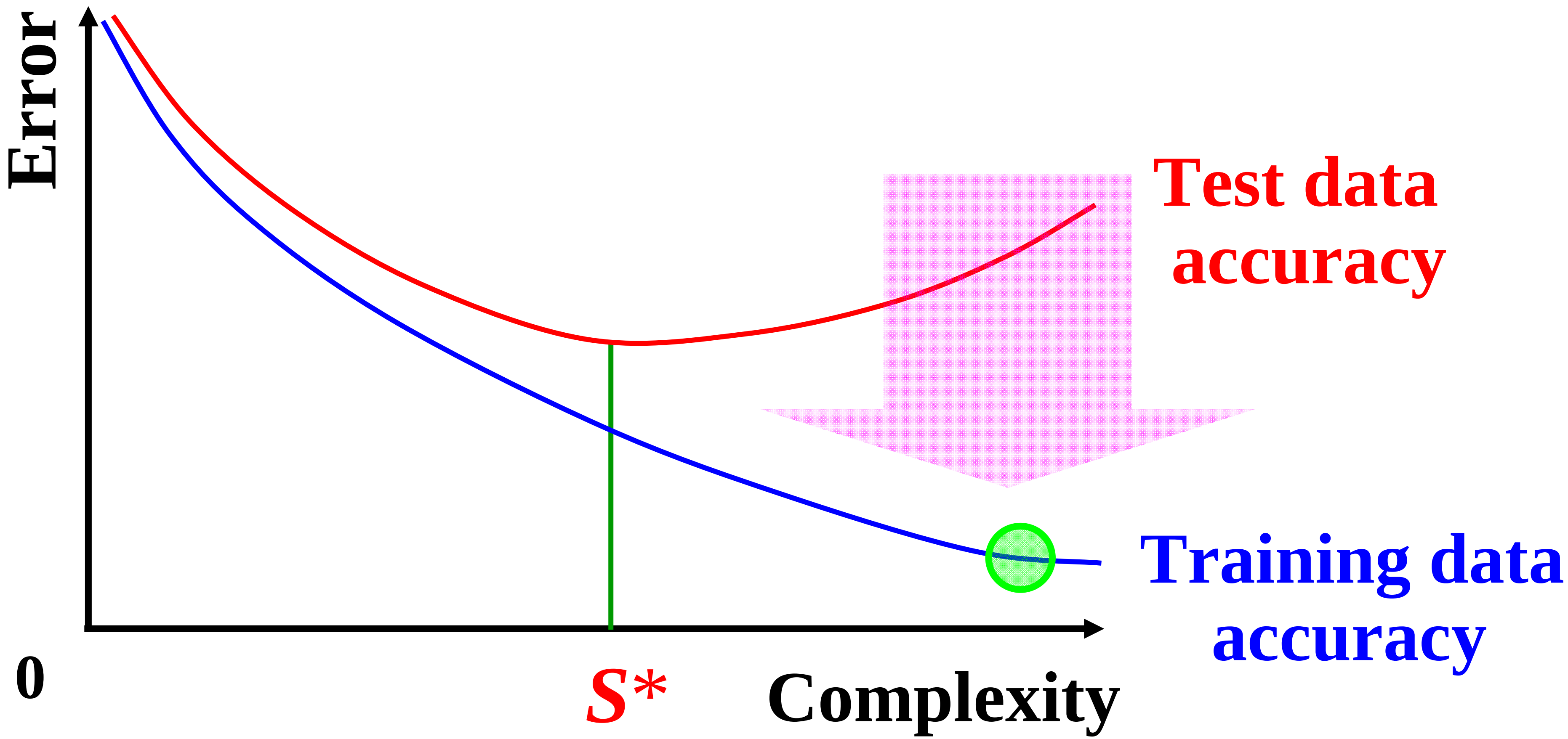
Complicated fuzzy systems with high accuracy were generated and trained.



Difficulties in Accuracy Maximization

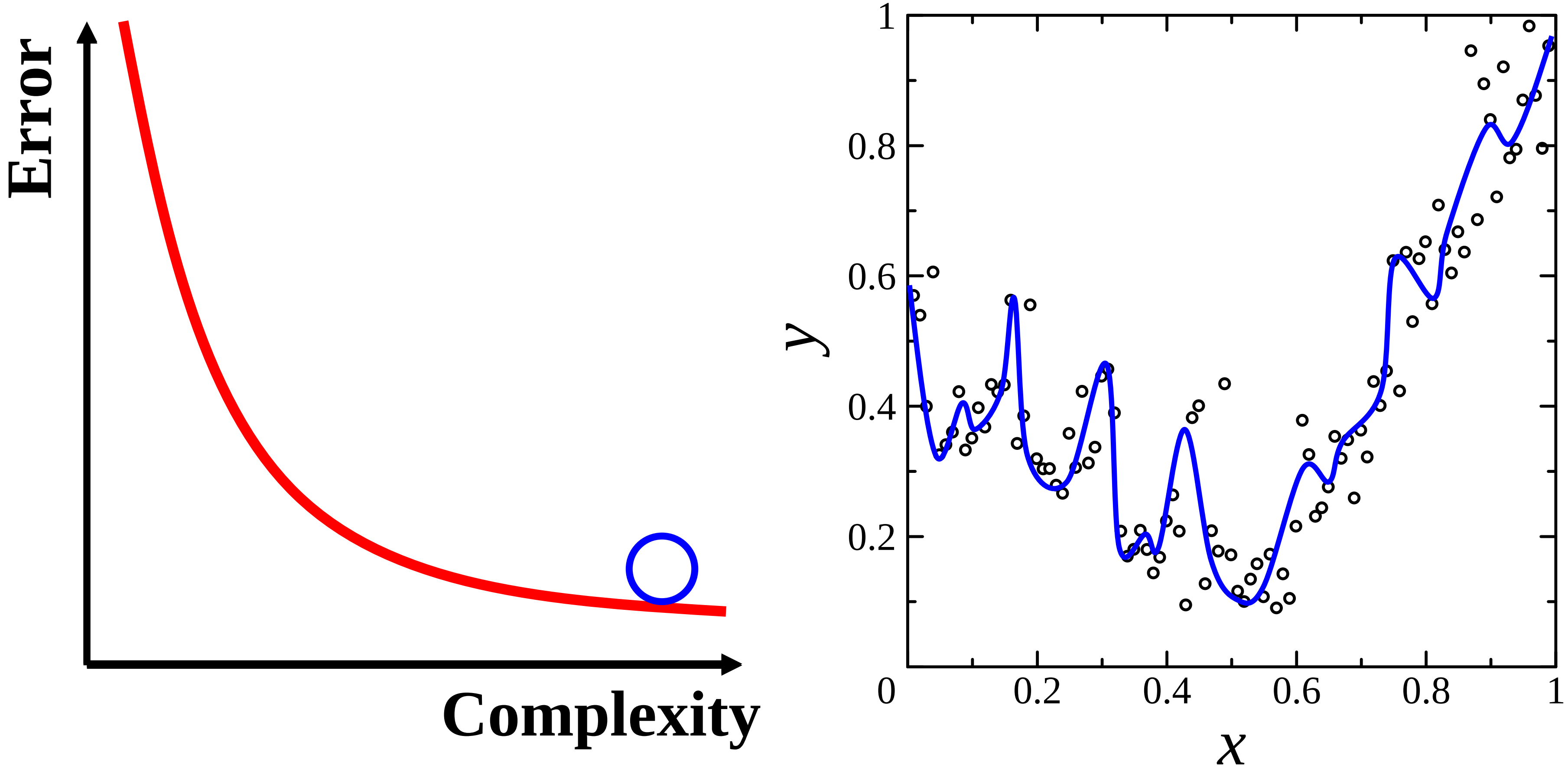
Overfitting and Poor Interpretability

Accuracy maximization → **Overfitting**

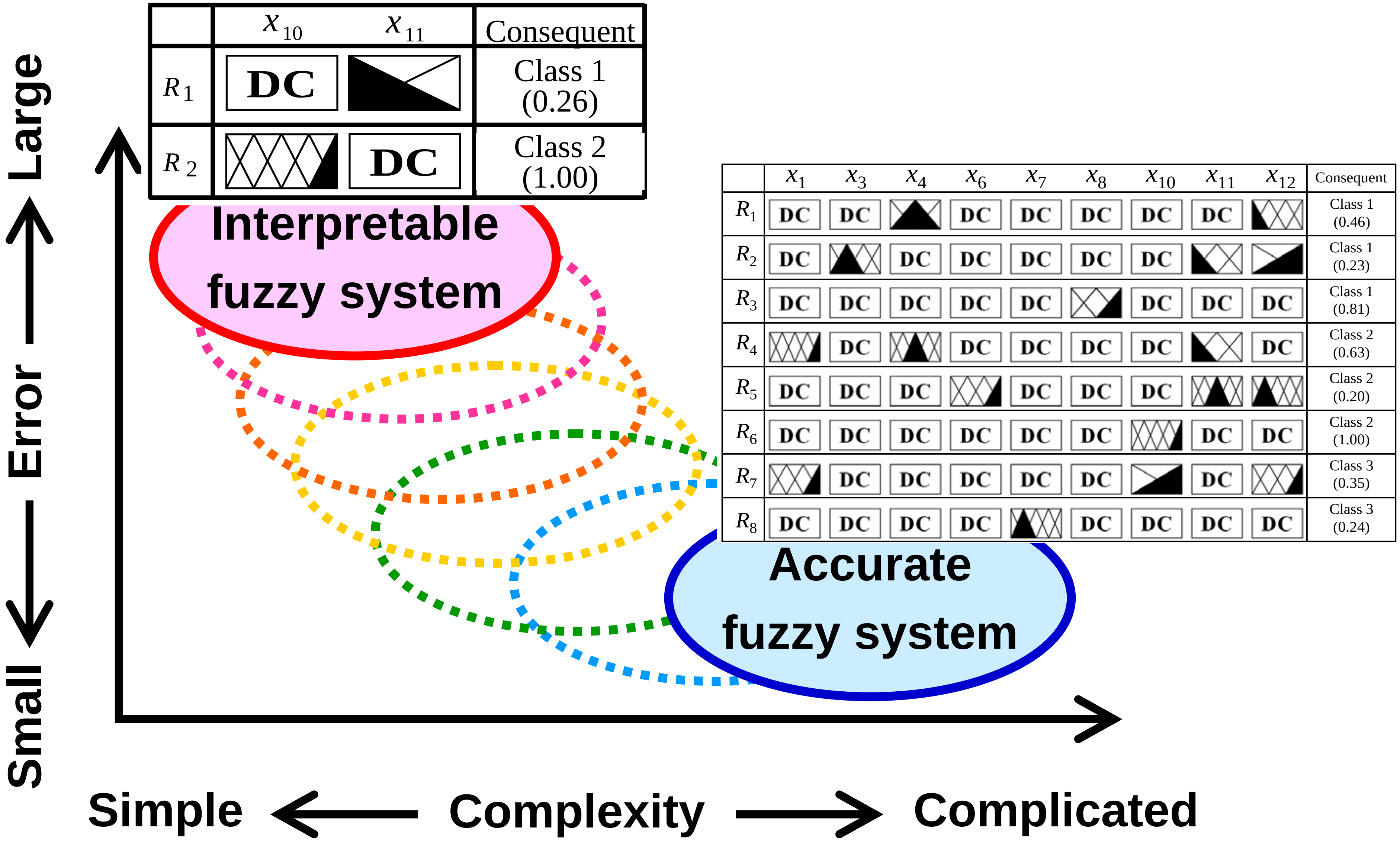


Overfitting to Training Data

Explanation of Overfitting to Training Data



Deterioration in Interpretability



Fuzzy Systems in the Mid 1990s

Compromise between Interpretability and Accuracy

(Search for a good interpretability-accuracy tradeoff)

Basic Idea

To combine the error minimization and the complexity minimization into a single scalar objective function

Example: Combination of the average error rate and the number of fuzzy rules

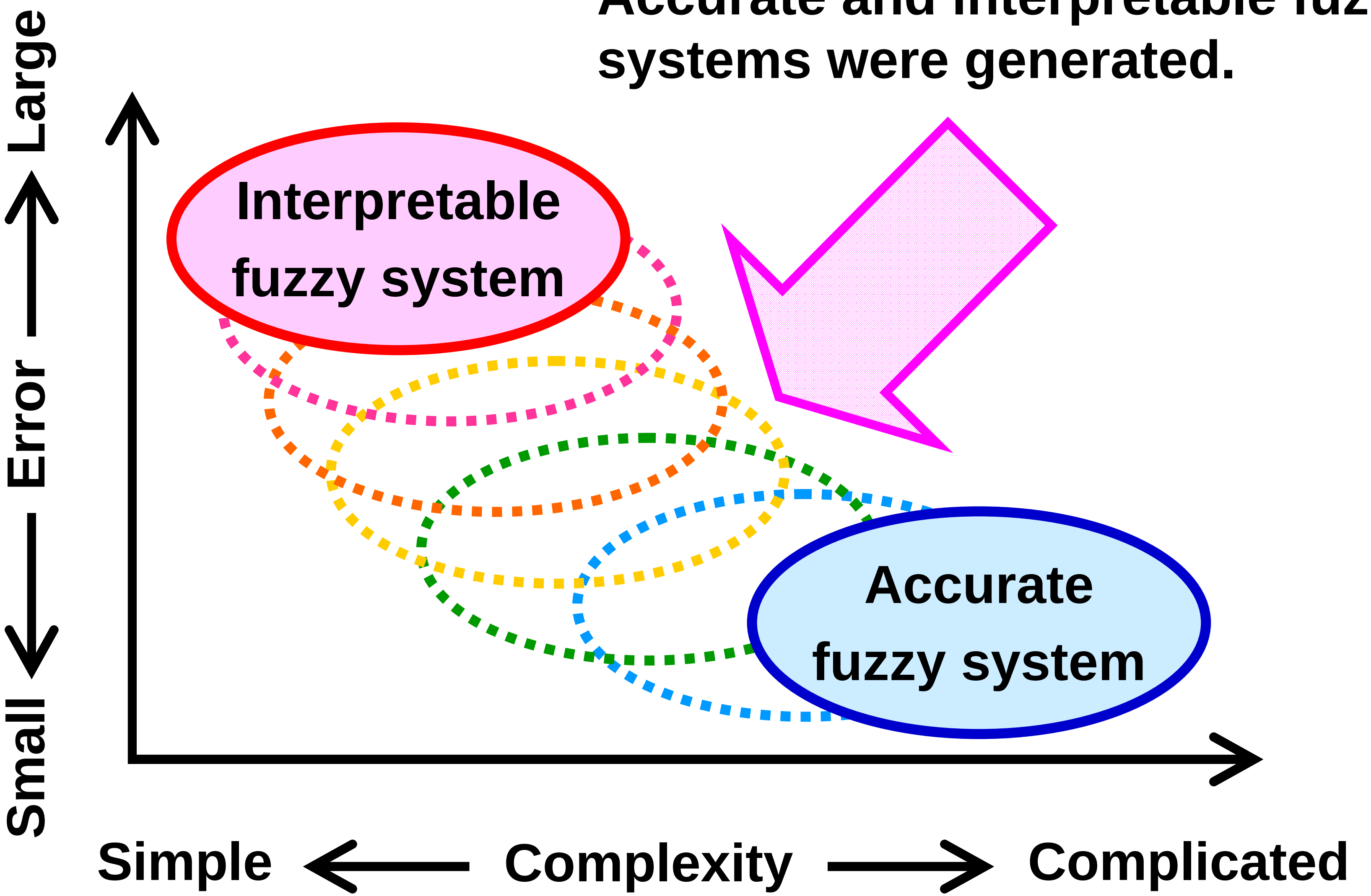
Example of a scalar objective function: Weighted sum

$$f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S)$$

Direction of Fuzzy System Research

Fuzzy Systems in the Mid 1990s

Accurate and interpretable fuzzy systems were generated.



Highly Cited I-A Tradeoff Papers

[1] Ishibuchi H, et al.: **Selecting fuzzy if-then rules for classification problems using genetic algorithms**, IEEE TFS (1995) (Weighted sum of the accuracy and the number of fuzzy rules)

[2] Setnes M et al.: **Similarity Measures in Fuzzy Rule Base Simplification**, IEEE TSMC-Part B (1998)

[3] Setnes M, Roubos H: **GA-fuzzy modeling and classification: Complexity and performance**, IEEE TFS (2000)

[4] Setnes M, et al.: **Rule-based modeling: recision and transparency**, IEEE TSMC-Part C (1998)

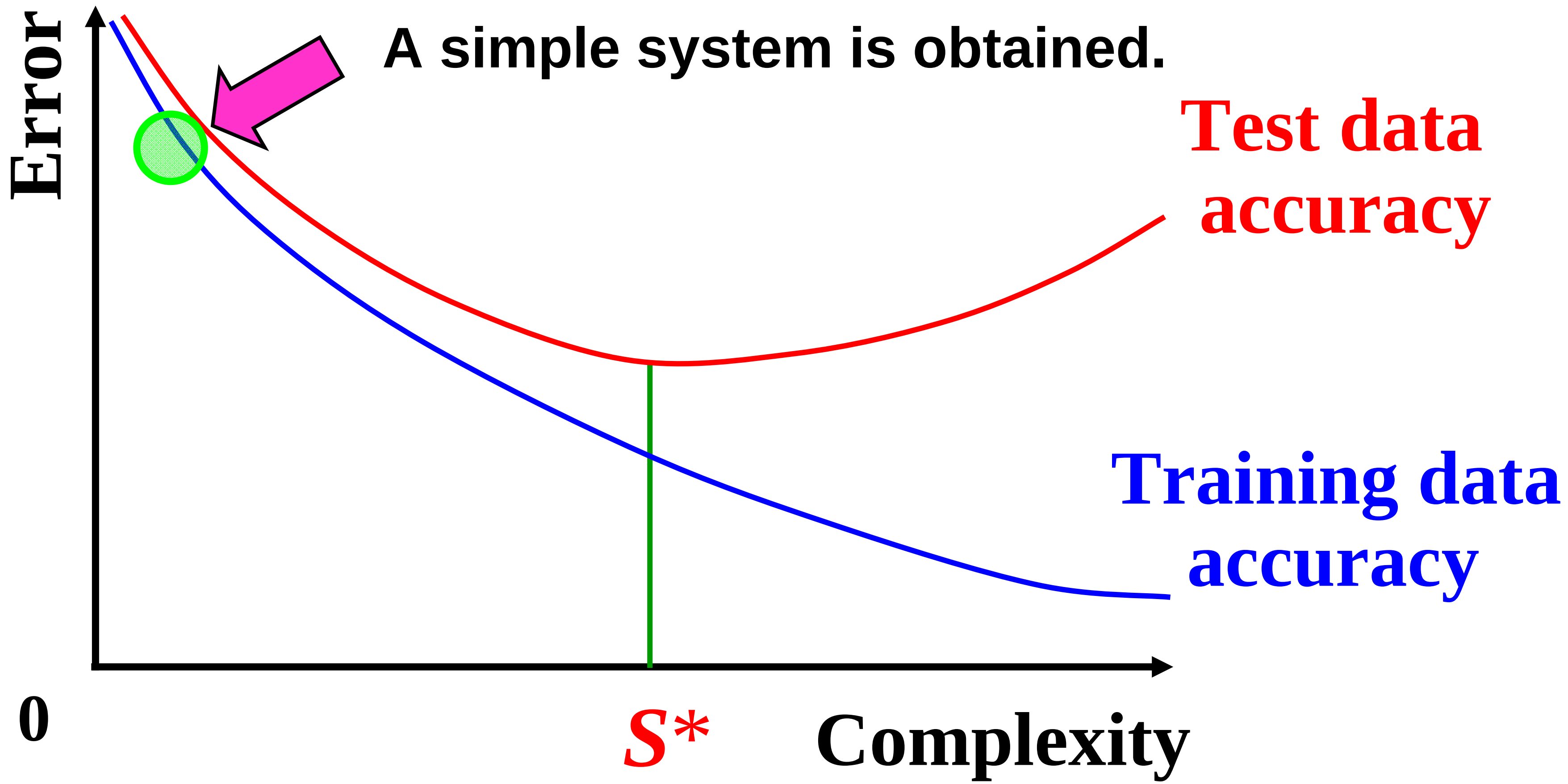
[5] Jin YC: **Fuzzy modeling of high-dimensional systems: Complexity reduction and interpretability improvement**, IEEE TFS (2000)

Difficulty in Weighted Sum Approach

Sensitivity of the Result to the Weight Vector Specification

Minimize $w_1 \cdot Error + w_2 \cdot Complexity$

When the weight for the complexity minimization is large:

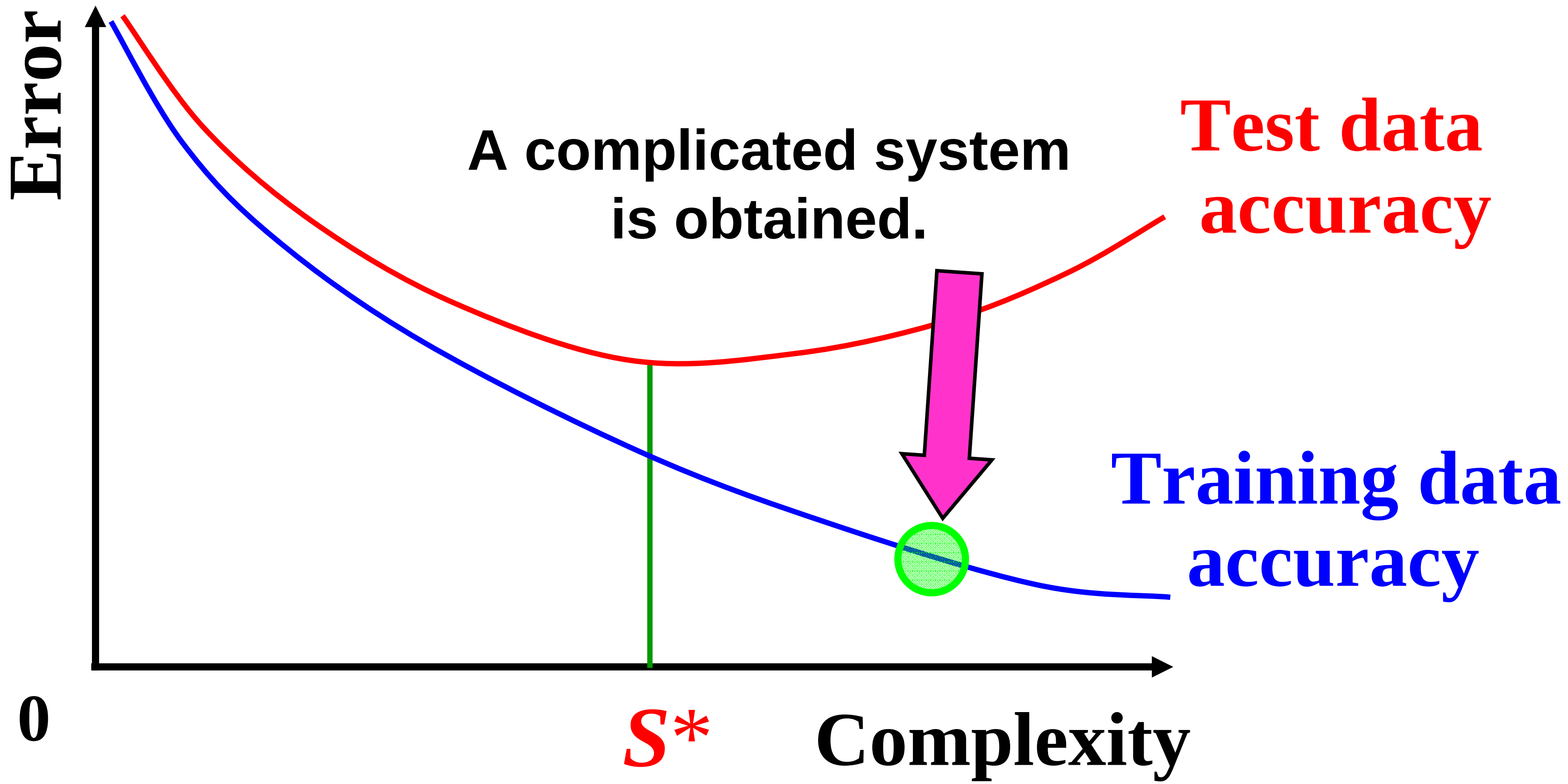


Difficulty in Weighted Sum Approach

Sensitivity of the Result to the Weight Vector Specification

Minimize $w_1 \cdot Error + w_2 \cdot Complexity$

When the weight for the error minimization is large:

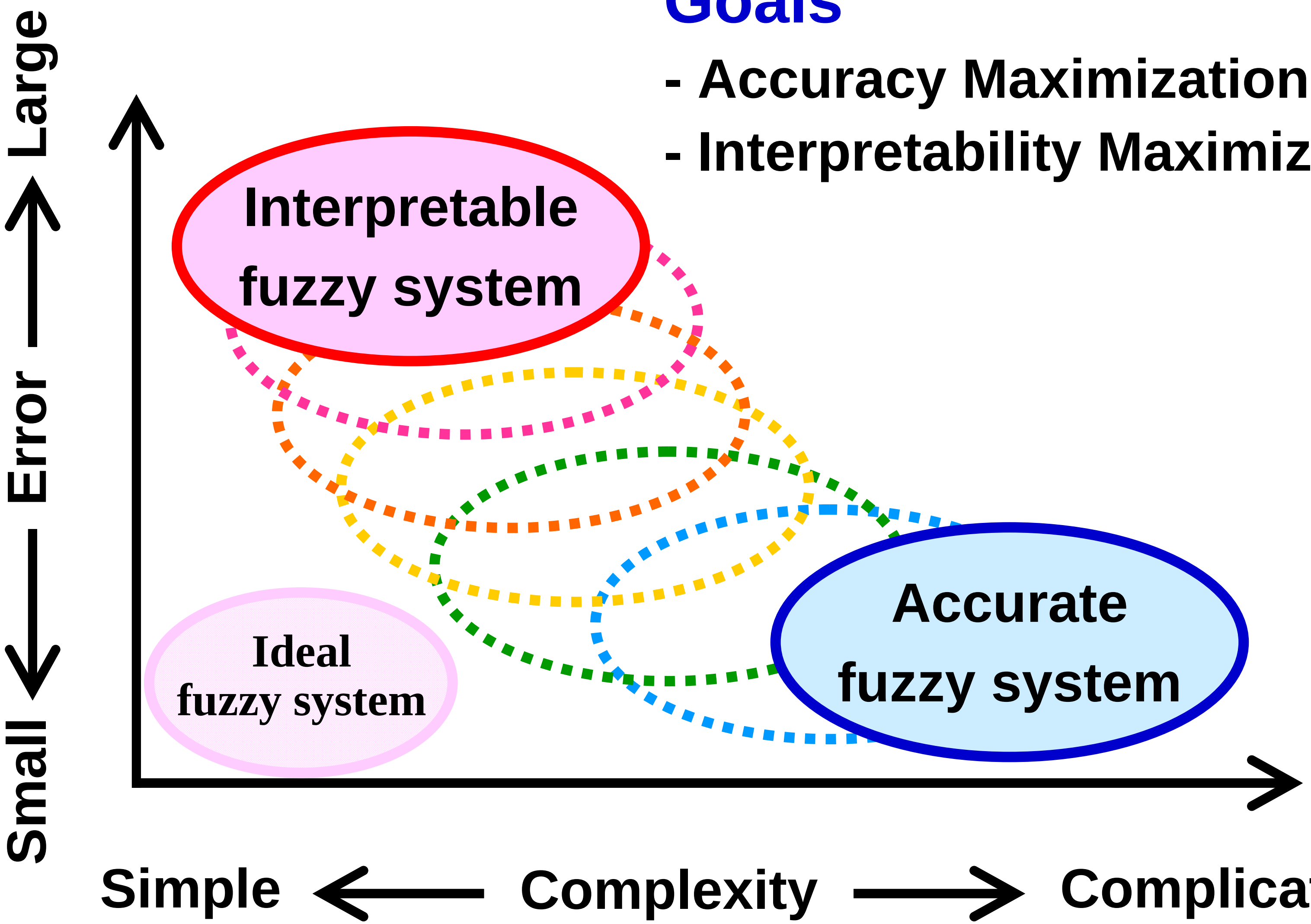


Current Trend in Fuzzy System Design

Multiobjective Fuzzy System Design (Late 1990s -)

Goals

- Accuracy Maximization
- Interpretability Maximization



Multiobjective Fuzzy System Design

Basic Idea

To search for a number of non-dominated fuzzy systems with respect to the accuracy maximization and the interpretability maximization (instead of searching for a single fuzzy system).

Aggregation Approach

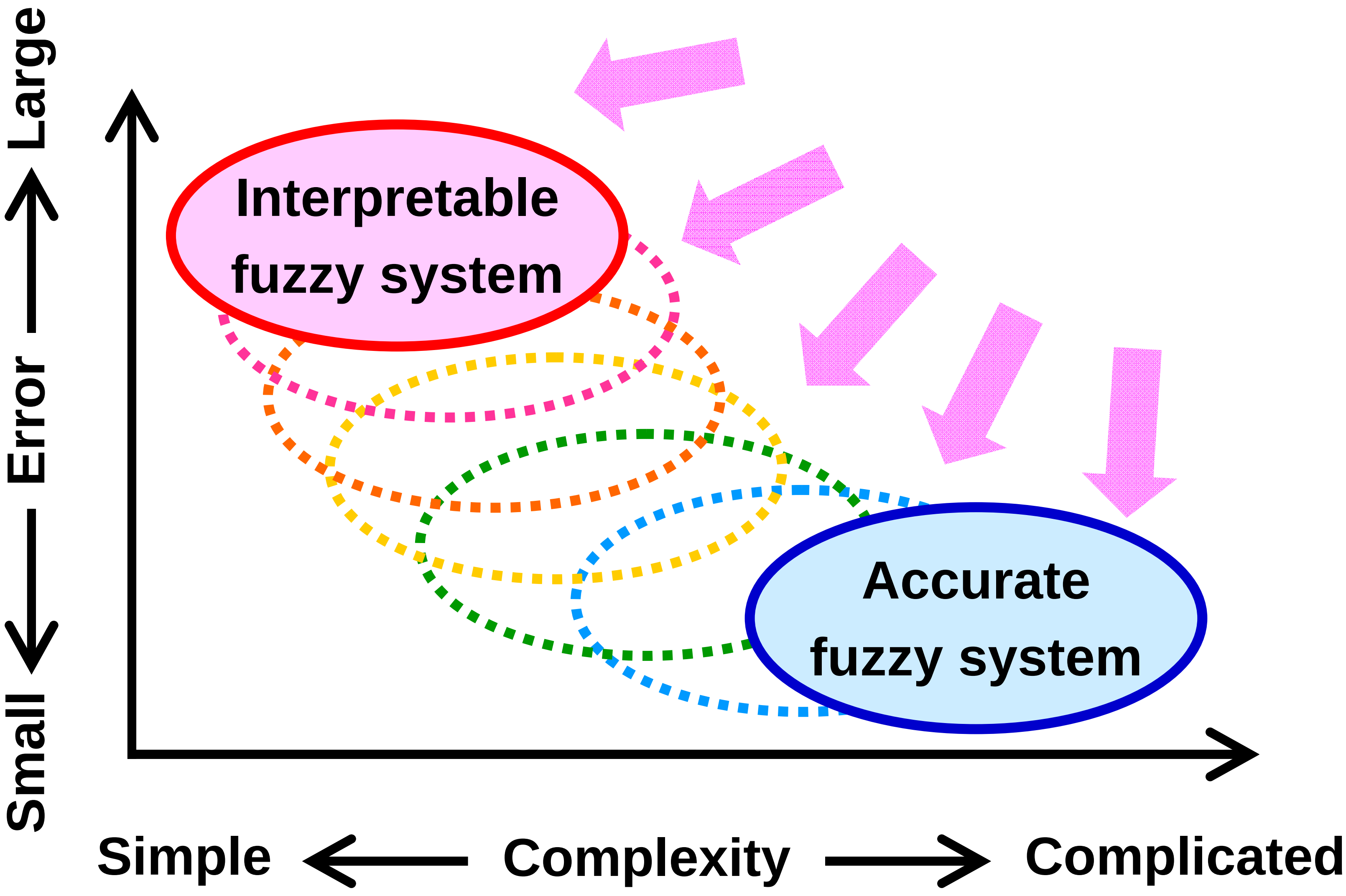
$$f(S) = w_1 \cdot f_{\text{Error}}(S) + w_2 \cdot f_{\text{Complexity}}(S)$$

Multiobjective Approach

$$\text{Minimize } \{ f_{\text{Error}}(S),\ f_{\text{Complexity}}(S) \}$$

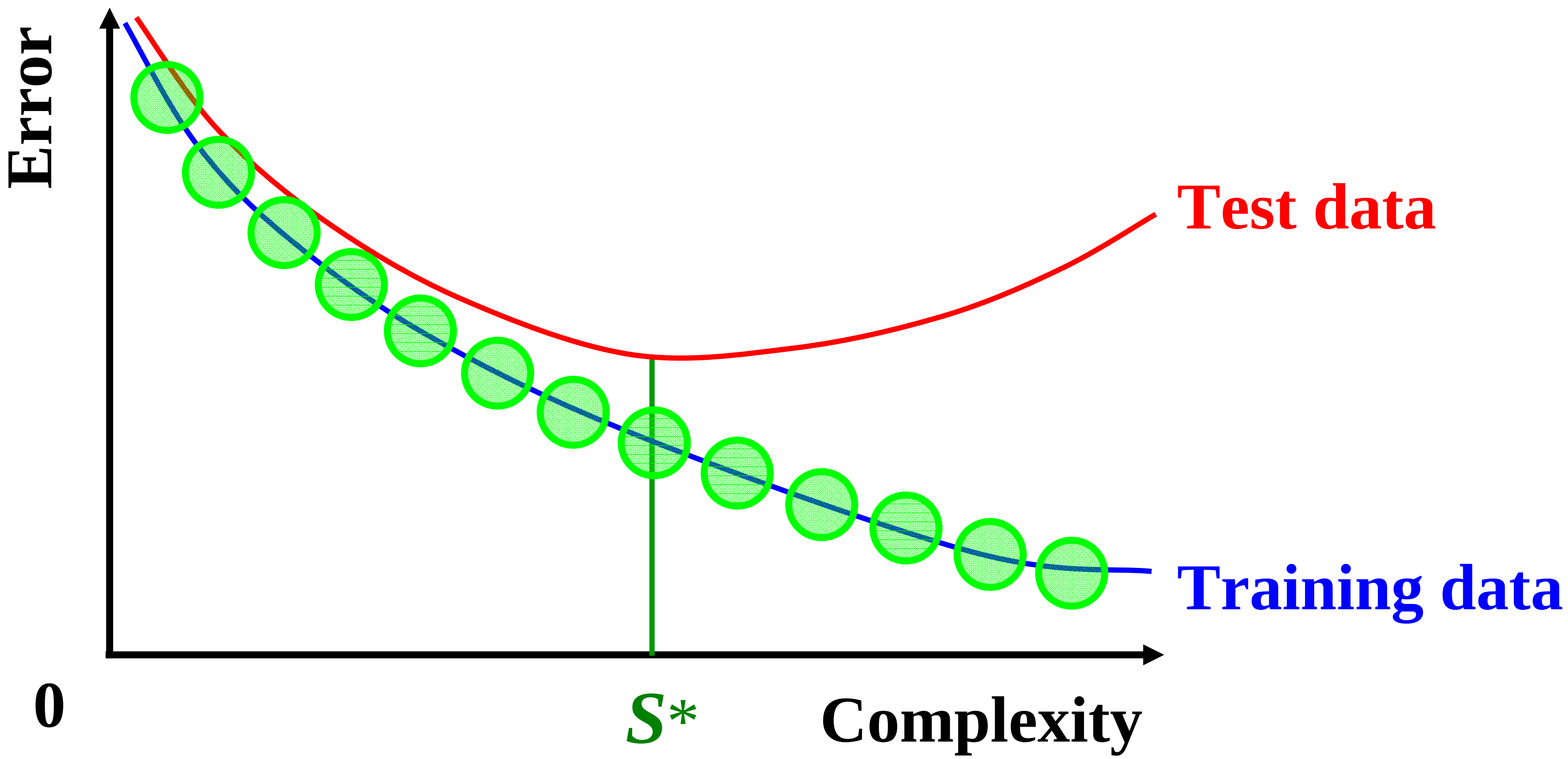
Direction of Fuzzy System Research

Multiobjective Fuzzy System Design (Late 1990s -)



Multiobjective Design of Fuzzy Systems

Many non-dominated fuzzy systems can be obtained along the tradeoff surface by a single run of an EMO algorithm.



Highly Cited MoGFS Papers

[1] Ishibuchi et al. (1997) Single-objective and two-objective genetic algorithms for selecting linguistic rules for pattern classification problems. *Fuzzy Sets & Systems.*

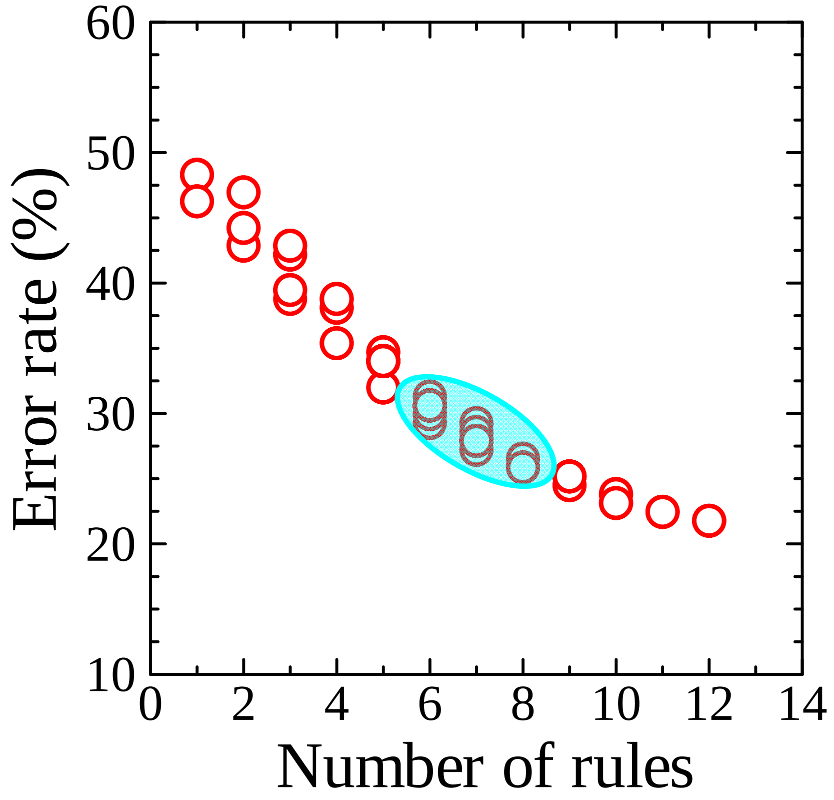
[2] Ishibuchi et al. (2001) Three-objective genetics-based machine learning for linguistic rule extraction. *Information Sciences.*

[3] Ishibuchi & Yamamoto (2004) Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining. *Fuzzy Sets & Systems*.

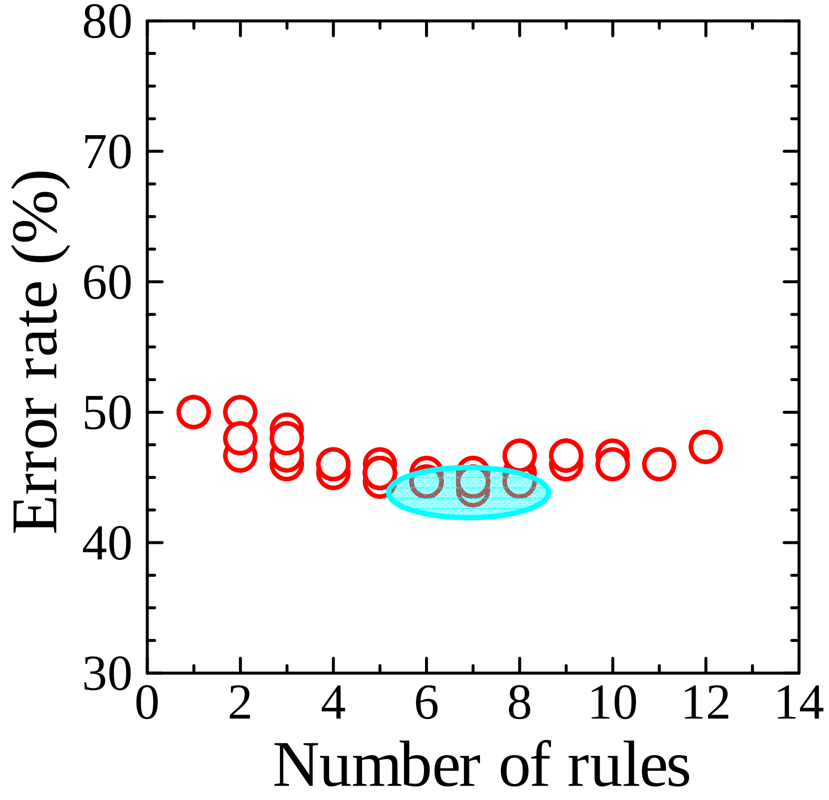
[4] Wang et al. (2005) Multi-objective hierarchical genetic algorithm for interpretable fuzzy rule-based knowledge extraction. *Fuzzy Sets & Systems*.

[5] Johansen & Babuska (2003) Multiobjective identification of Takagi-Sugeno fuzzy models. *IEEE TFS.*

Example: Obtained Rule Sets (Heart C)



Training data accuracy

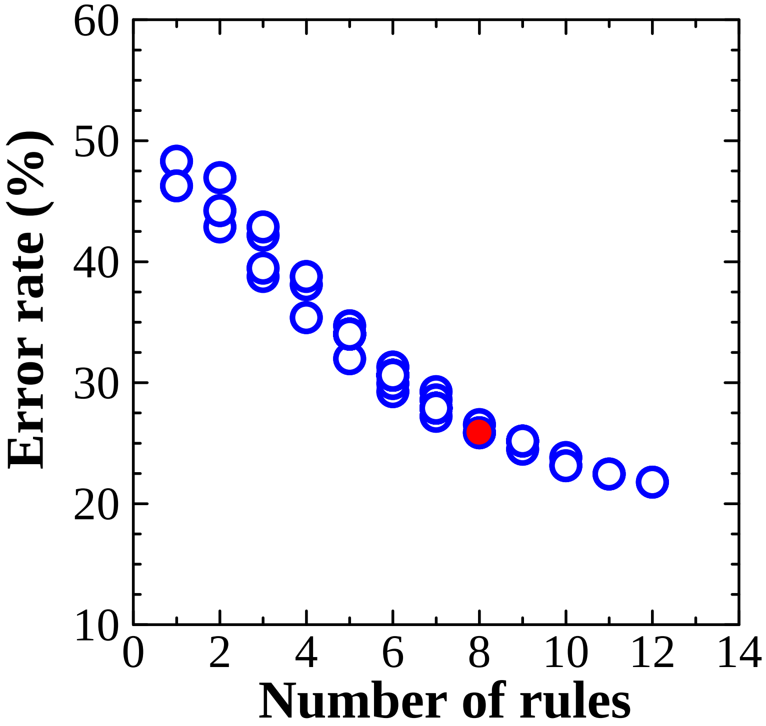


Testing data accuracy

Obtained rule sets help us to find the optimal complexity of fuzzy systems. (Rule sets with six, seven and eight rules may be good)

A Rule Set with High-Generalization Ability

A rule set with eight fuzzy rules

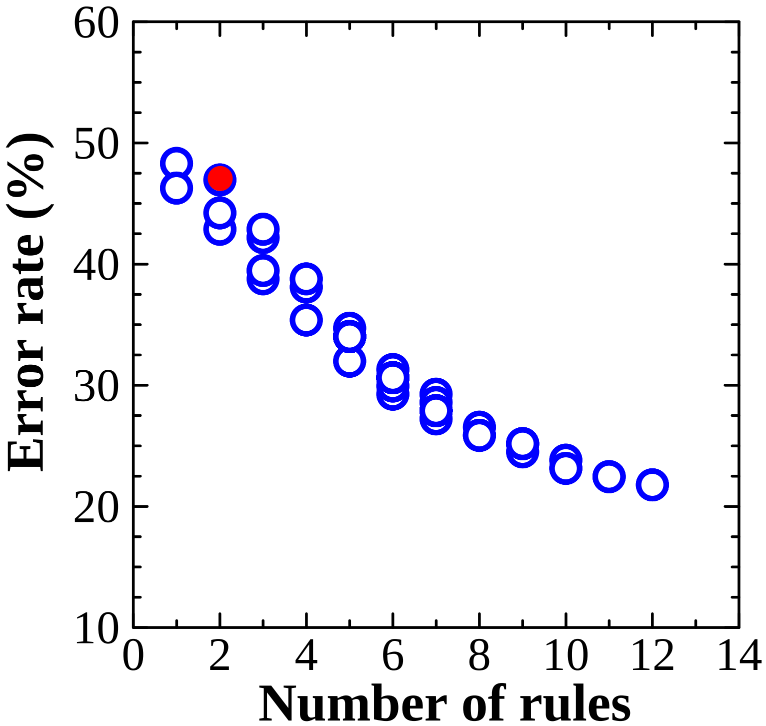


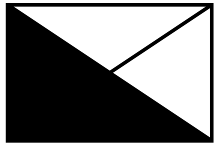
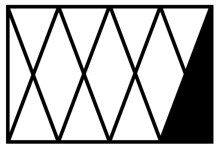
	x_1	x_3	x_4	x_6	x_7	x_8	x_{10}	x_{11}	x_{12}	Consequent
R_1	DC	DC		DC	DC	DC	DC	DC		Class 1 (0.46)
R_2	DC		DC	DC	DC	DC	DC			Class 1 (0.23)
R_3	DC	DC	DC	DC	DC		DC	DC	DC	Class 1 (0.81)
R_4		DC		DC	DC	DC	DC		DC	Class 2 (0.63)
R_5	DC	DC	DC		DC	DC	DC			Class 2 (0.20)
R_6	DC	DC	DC	DC	DC	DC		DC	DC	Class 2 (1.00)
R_7		DC	DC	DC	DC	DC		DC		Class 3 (0.35)
R_8	DC	DC	DC	DC		DC	DC	DC	DC	Class 3 (0.24)

Some human users may prefer simpler rule sets.

A Rule Set with High Interpretability

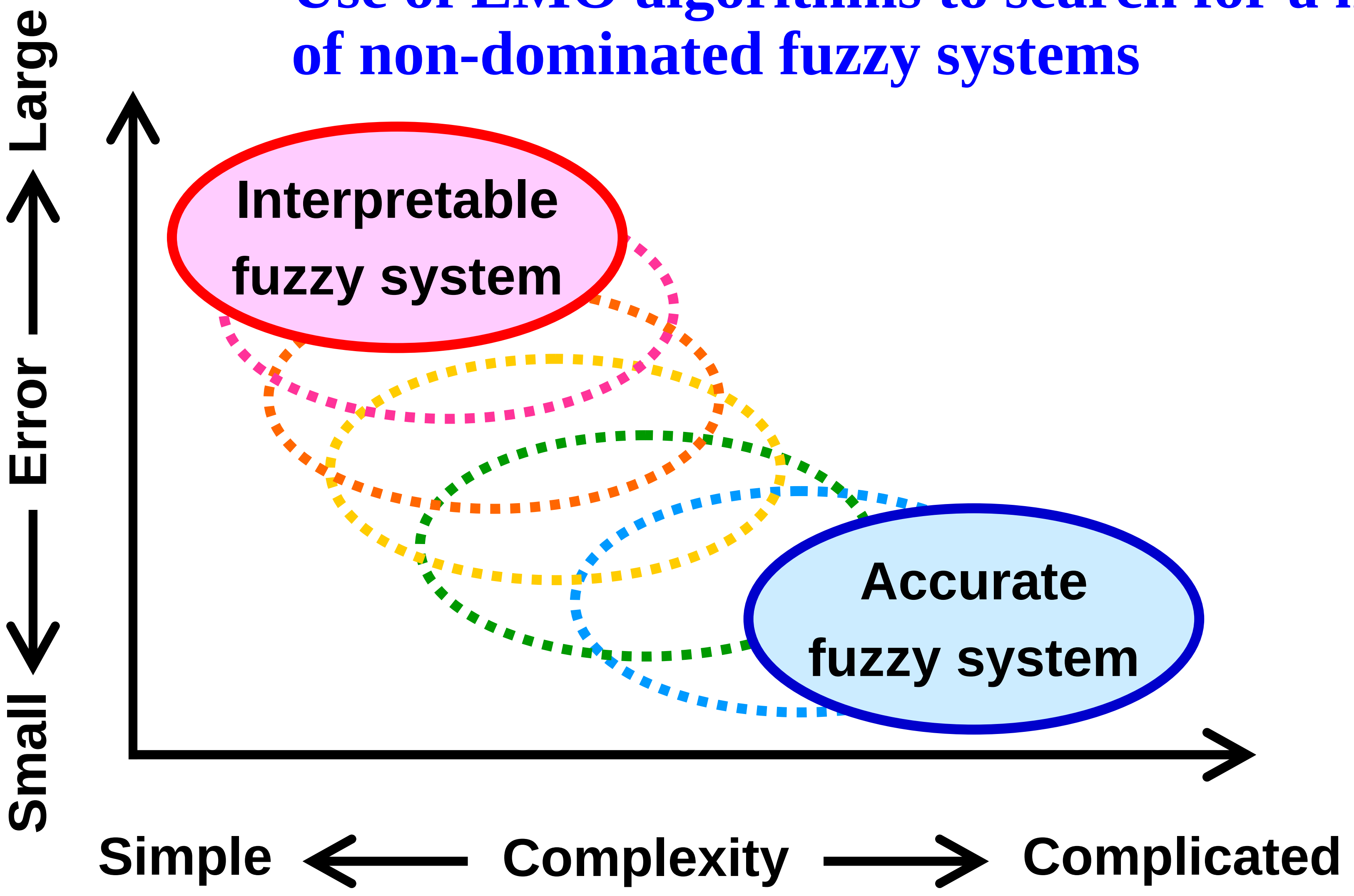
A very simple rule set with only two fuzzy rules



	x_{10}	x_{11}	Consequent
R_1	**DC**		Class 1 (0.26)
R_2		**DC**	Class 2 (1.00)

Multiobjective Fuzzy System Design

Use of EMO algorithms to search for a number of non-dominated fuzzy systems



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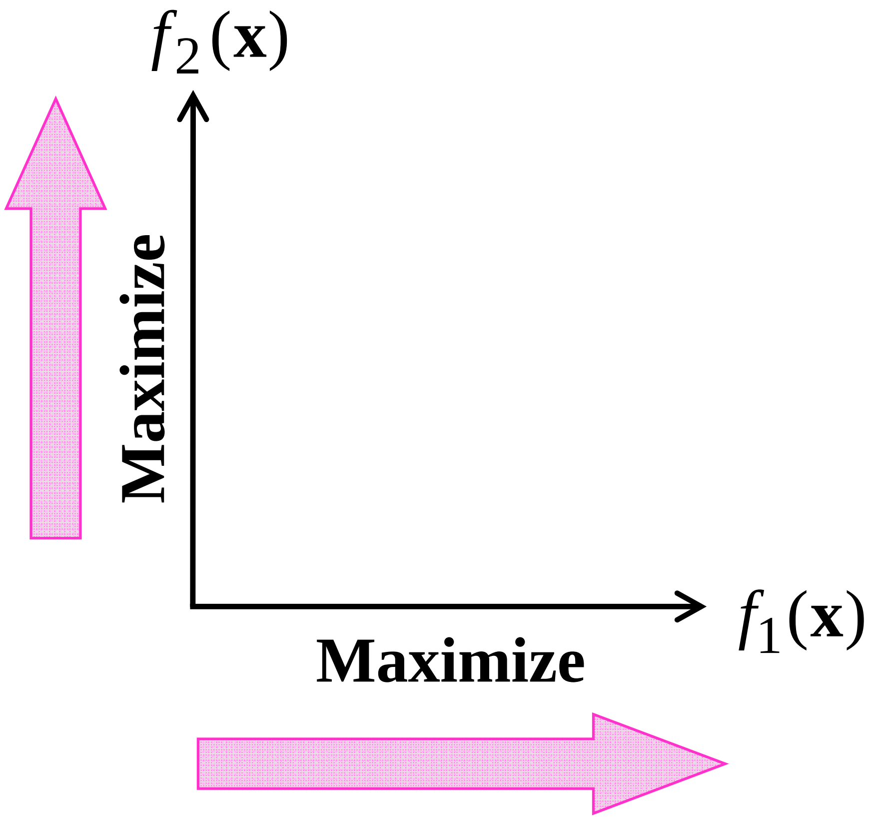
Multiobjective Optimization

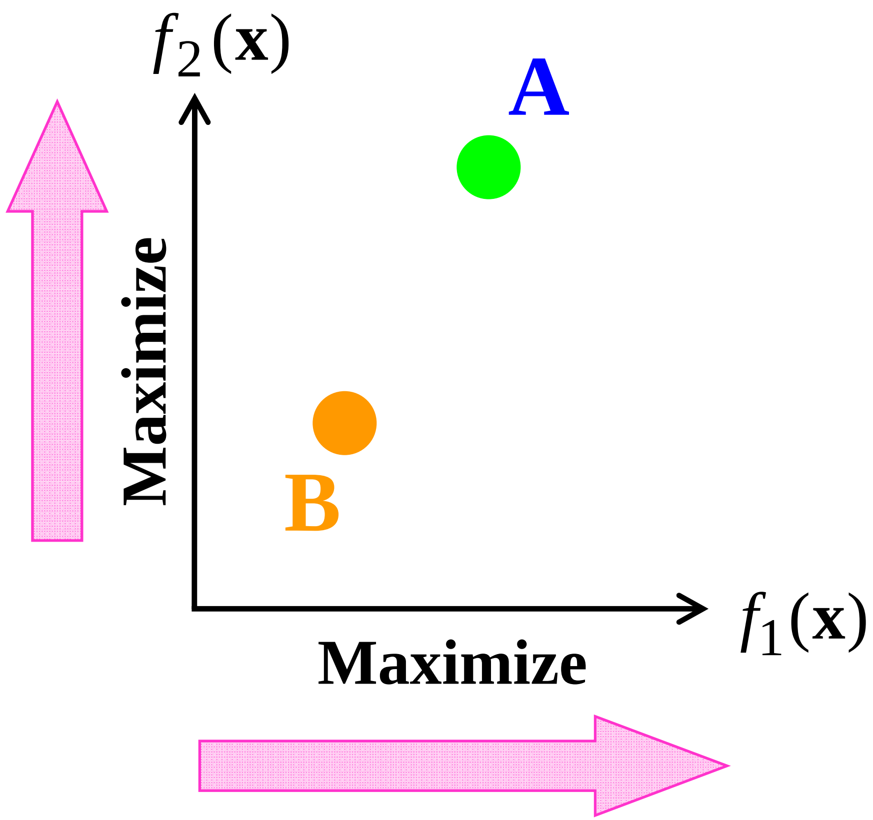
Two-Objective Maximization Problem:

$$\textbf{Maximize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$$



Comparison between Two Solutions

$$\textbf{Maximize} \quad \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$$



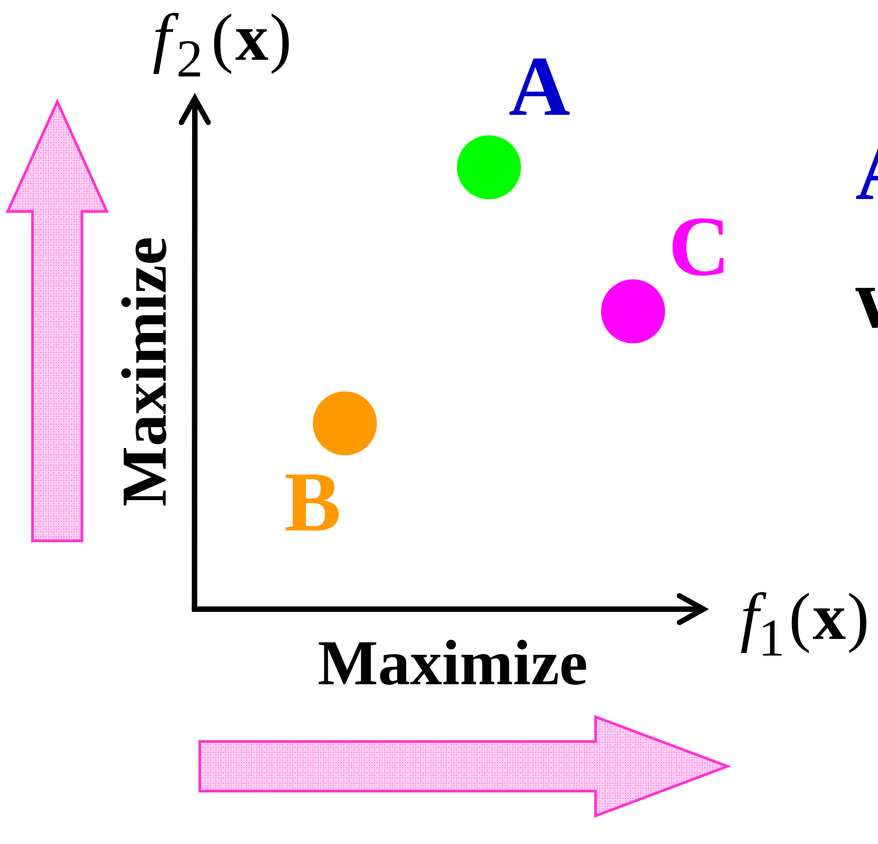
A dominates B

B is dominated by A

(A is better than B)

Comparison between Two Solutions

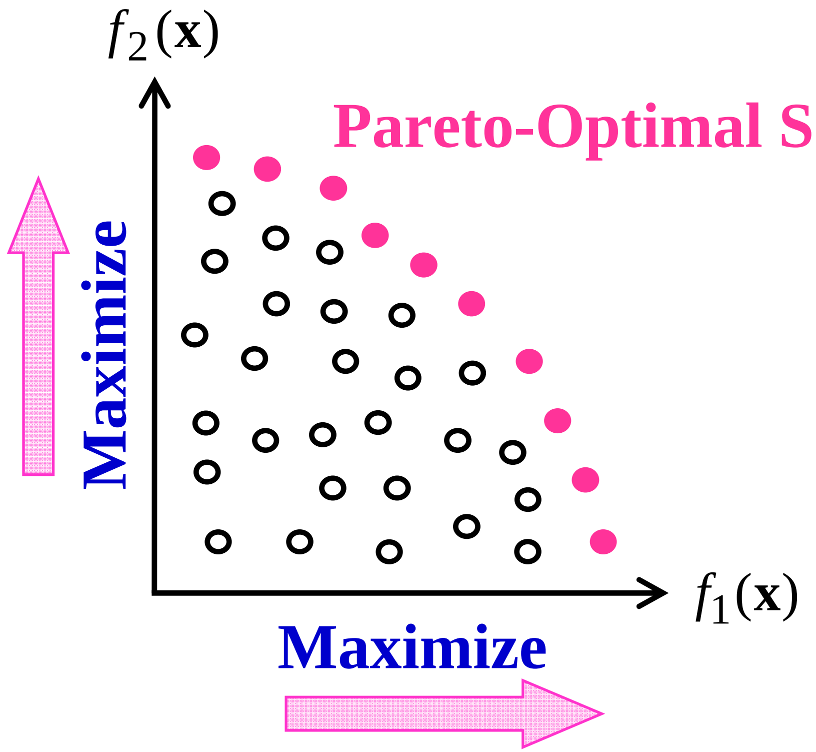
Maximize $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}),\ f_2(\mathbf{x}))$



A and C are non-dominated with each other.

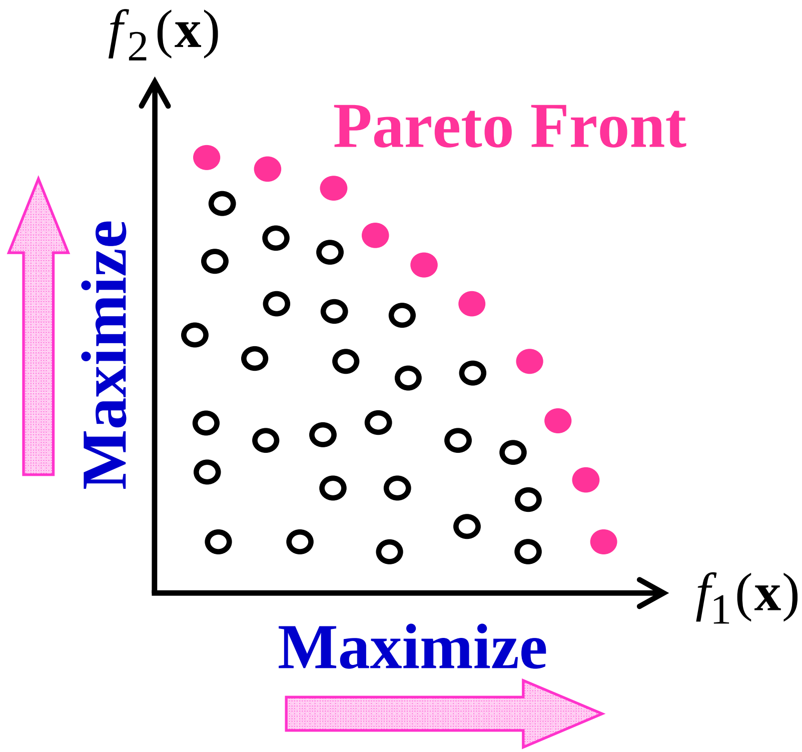
Pareto-Optimal Solutions

A Pareto-optimal solution is a solution that is not dominated by any other solutions.



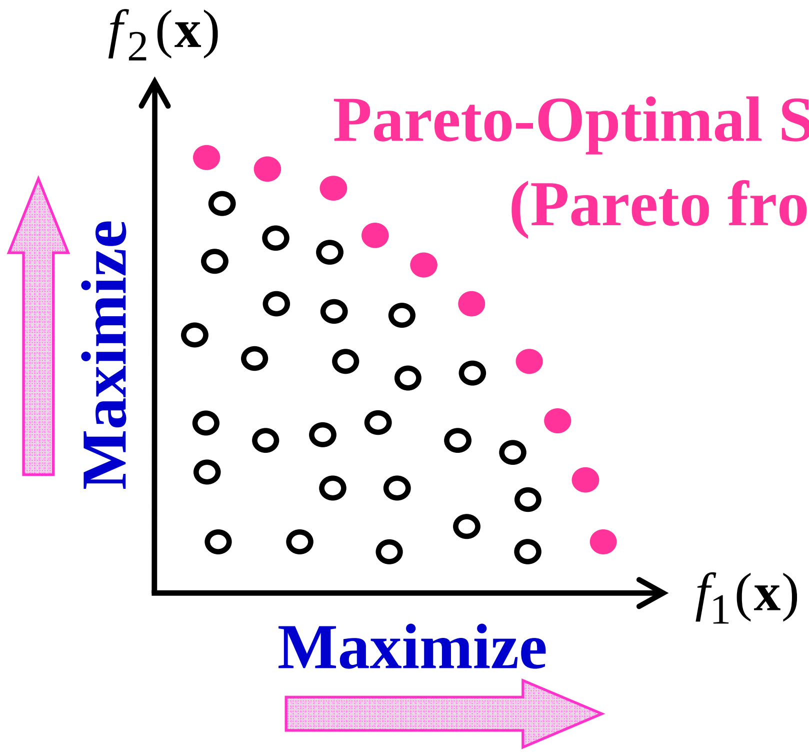
Pareto Front

The set of all Pareto-optimal solutions is called the Pareto front of the problem.



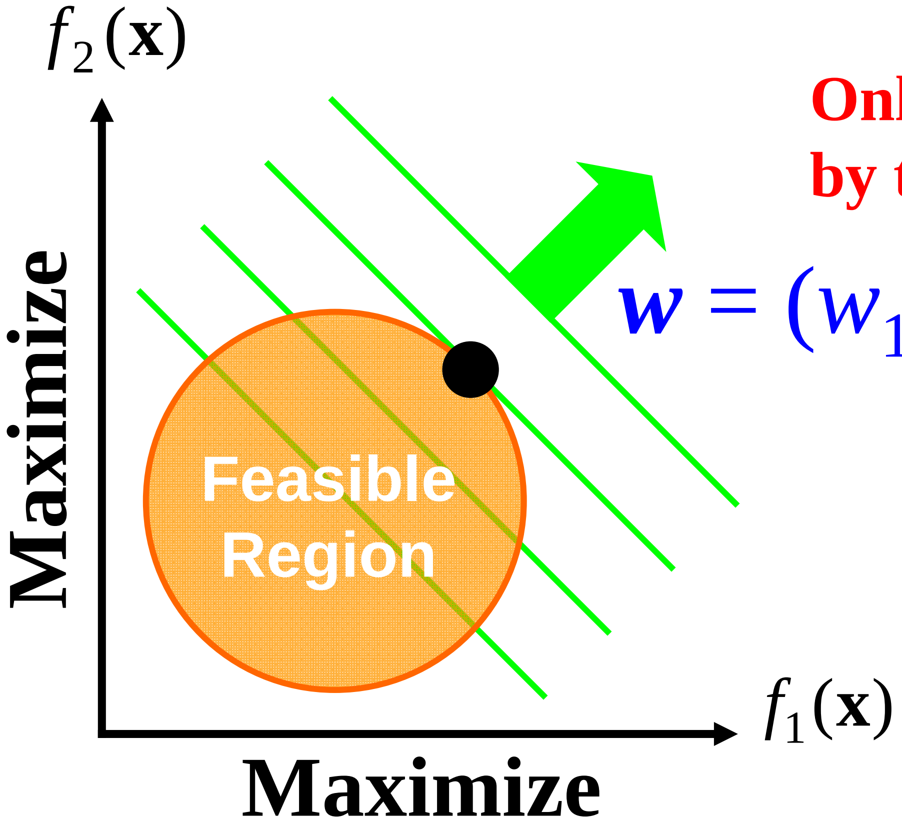
EMO Algorithms

Evolutionary multiobjective optimization (EMO) algorithms have been designed to search for Pareto-optimal solutions in their single run.



Comparison: Weighted Sum Approach

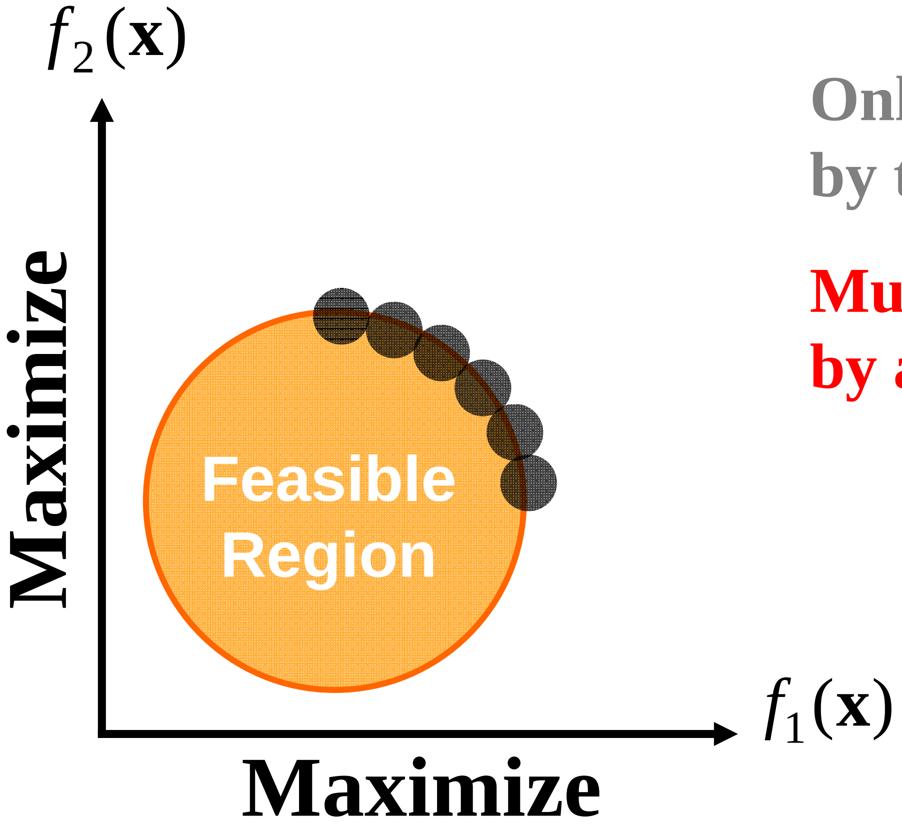
Maximize $g(\mathbf{x}) = w_1 f_1(\mathbf{x}) + w_2 f_2(\mathbf{x})$



Only a single solution is obtained by the weighted sum approach.

Comparison: EMO Approach

Maximize $f_1(\mathbf{x}),\ \ f_2(\mathbf{x})$

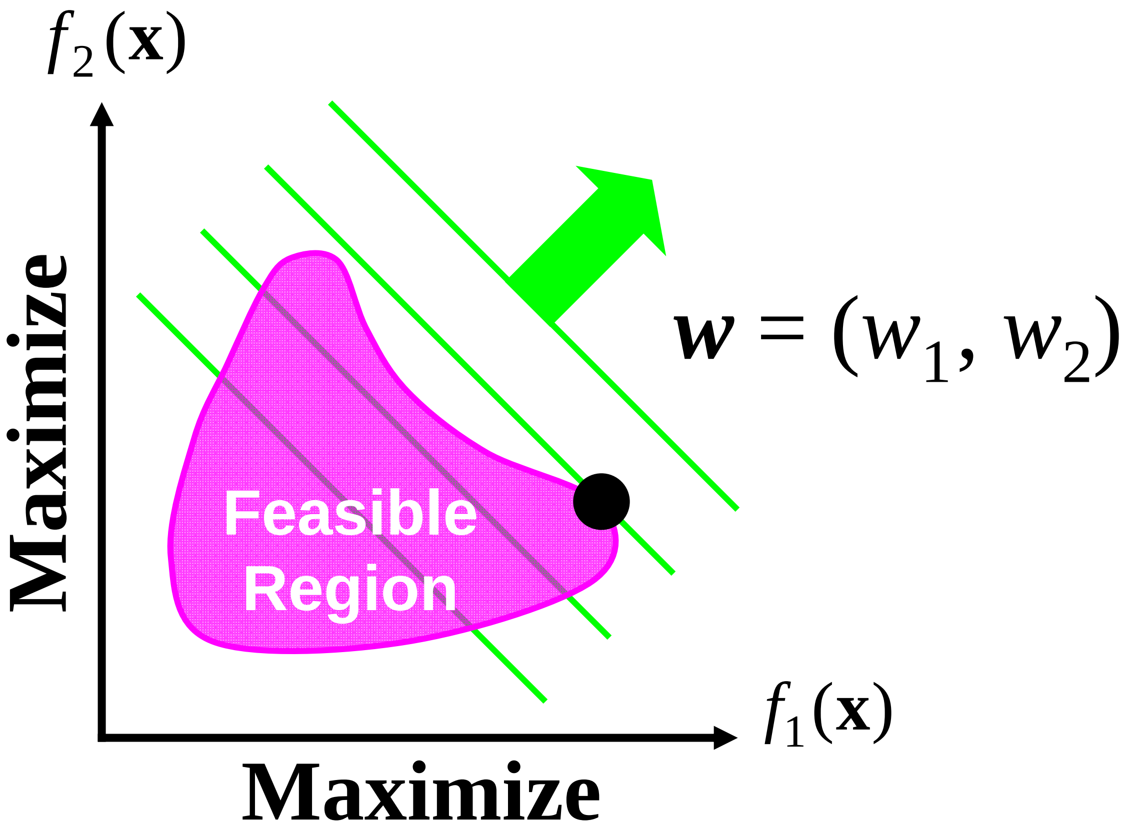


Only a single solution is obtained by the weighted sum approach.

Multiple solutions are obtained by an EMO algorithm.

Difficulties in Weighted Sum Approach

- This approach is sensitive to the weight vector specification.
- This approach can not find any Pareto-optimal solutions in a non-convex region of the Pareto front in the objective space.



Difficulties in Weighted Sum Approach

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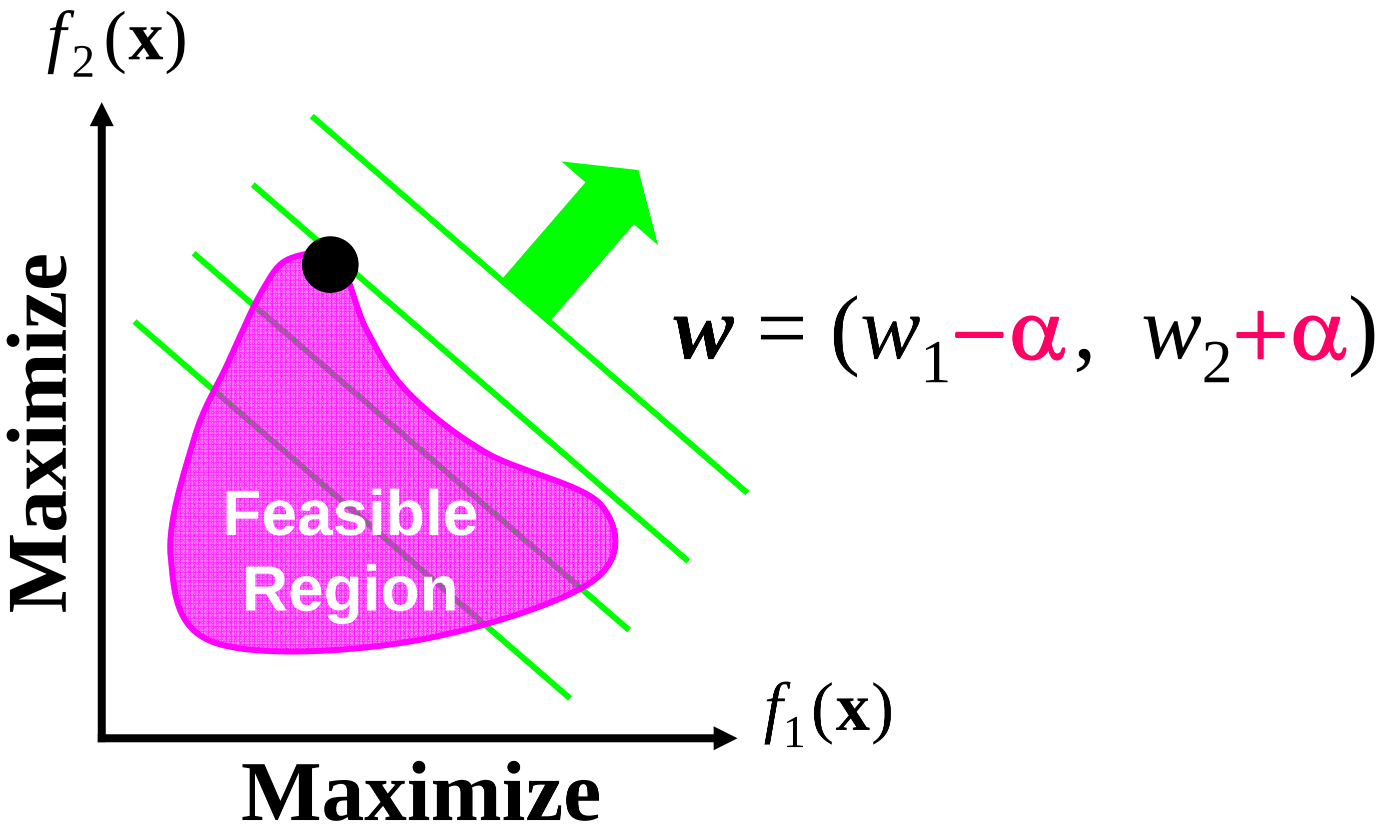
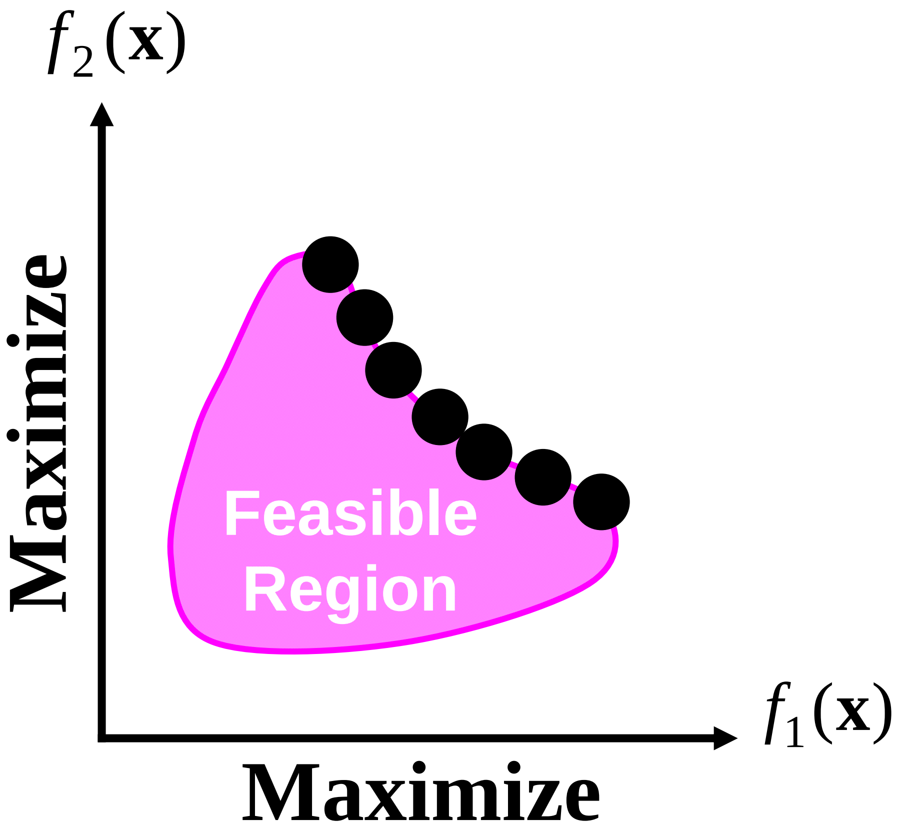


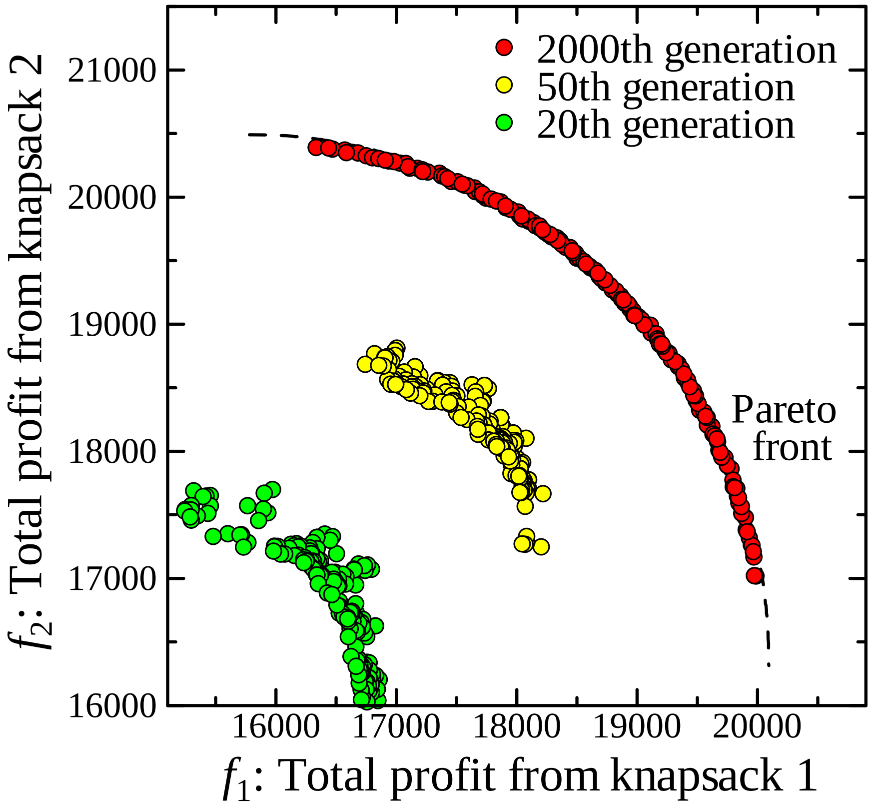
EMO Approach

- **EMO approach can find Pareto-optimal solutions even in a non-convex region of the Pareto front in the objective space.**

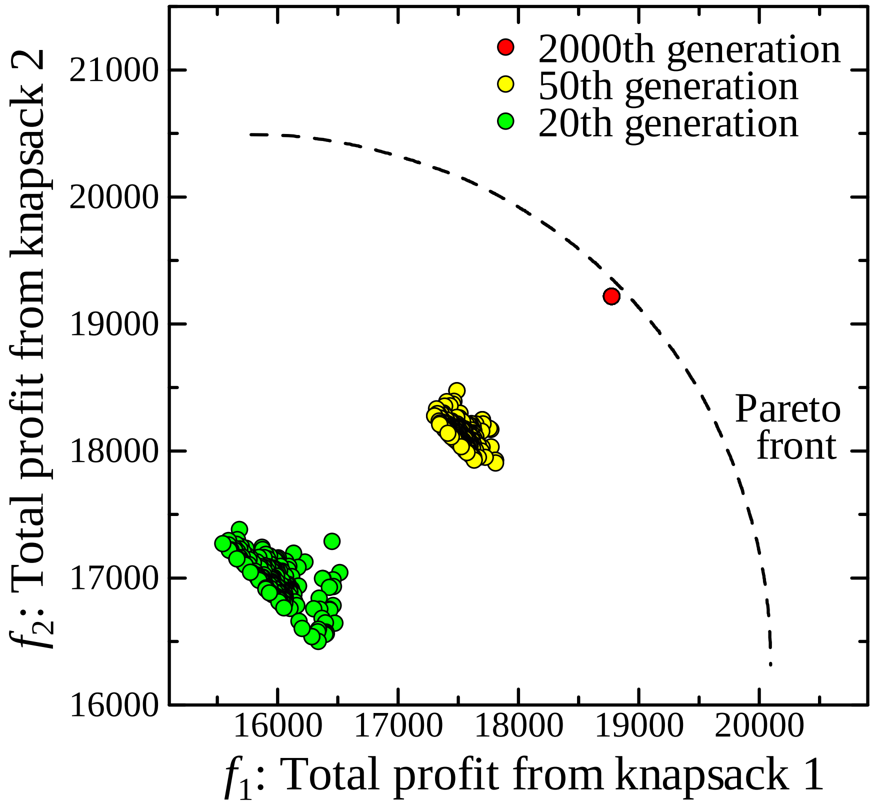


Comparison of the Two Approaches

Two-objective maximization problem



EMO Approach

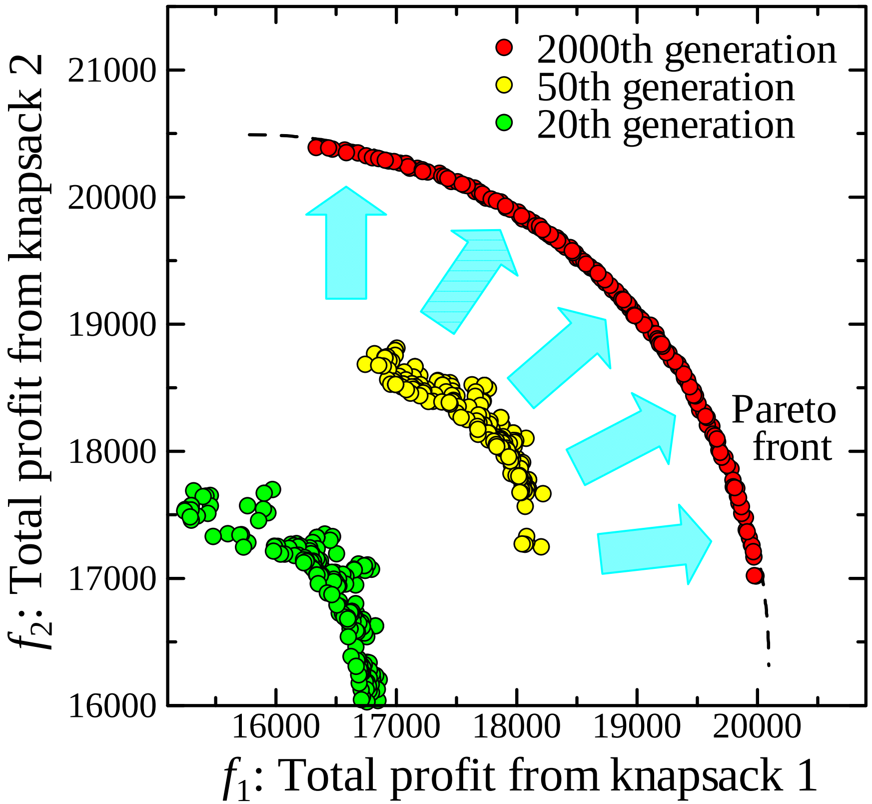


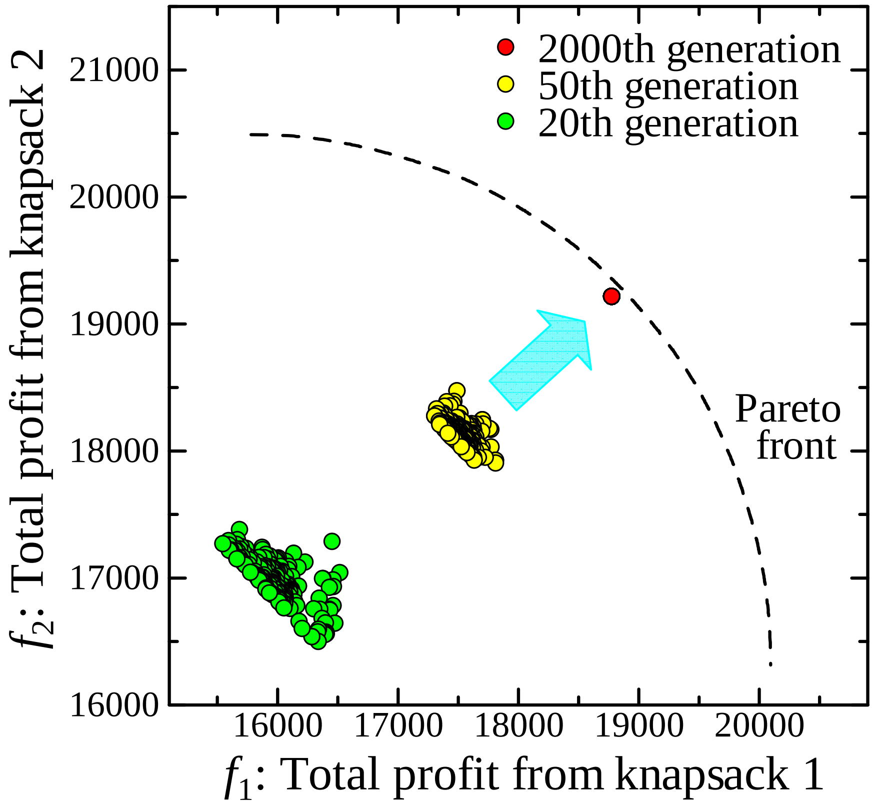
Weighted Sum Approach

Experimental results of a single run of each approach

Search Direction in Each Approach

Two-objective maximization problem





EMO Approach **Weighted Sum Approach**

Both the diversity and the convergence should be improved in EMO.

Highly Cited EMO Papers

Two Dominant Algorithms: NSGA-II and SPEA

1. Deb K et al. (2002) **A fast and elitist multiobjective genetic algorithm: NSGA-II.** *IEEE TEC*. **NSGA-II**

2. Zitzler E, Thiele L (1999) **Multiobjective evolutionary algorithms: A comparative case study and the Strength Pareto approach.** *IEEE TEC*. **SPEA** (=> SPEA2 in TIK-Report)

3. Fonseca CM, Fleming PJ (1998) Multiobjective optimization and multiple constraint handling with evolutionary algorithms (Part I): A unified formulation, *IEEE SMC Part A*.

4. Zitzler E, Thiele L, Laumanns M (2003) Performance assessment of multiobjective optimizers: An analysis and review. *IEEE TEC*.

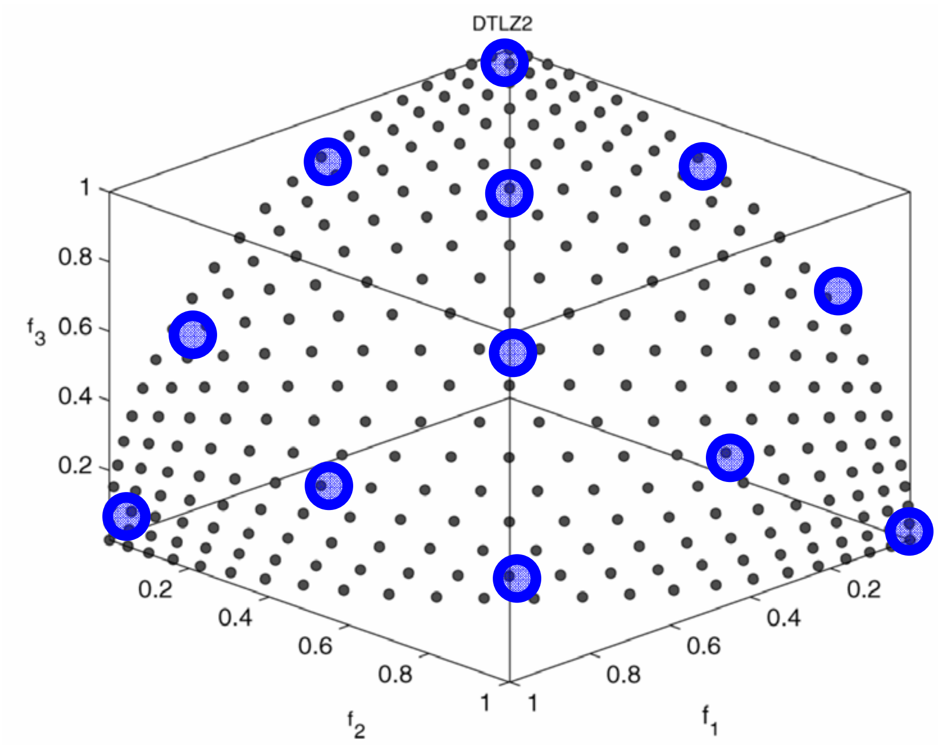
5. Ishibuchi H, Murata T (1998) A multi-objective genetic local search algorithm and its application to flowshop scheduling, *IEEE SMC Part C*.

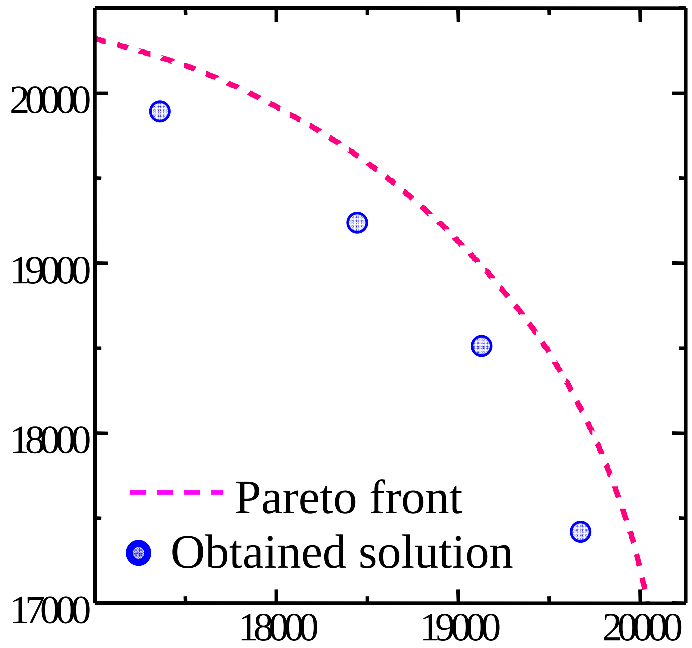
Goal of EMO Algorithms

An EMO algorithm is designed to search for

- **all Pareto-optimal solutions**
- **uniformly distributed Pareto optimal solutions**
- **a solution set which approximates the Pareto front**

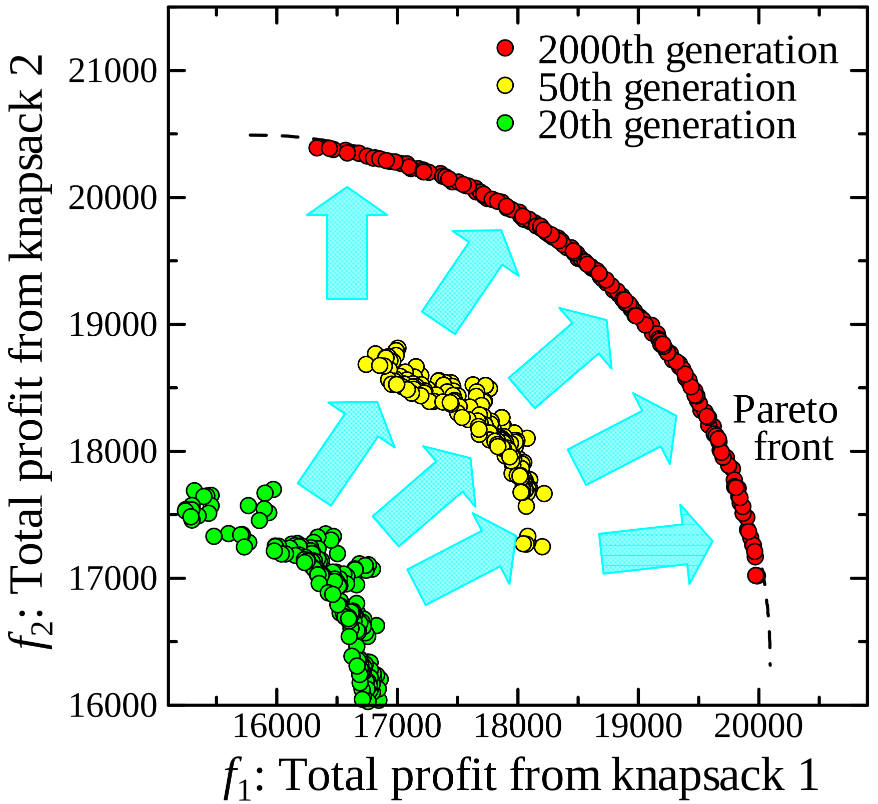
in their single run.





Basic Ideas in EMO Algorithm Design

Recently developed well-known EMO algorithms such as NSGA-II and SPEA have some common features.



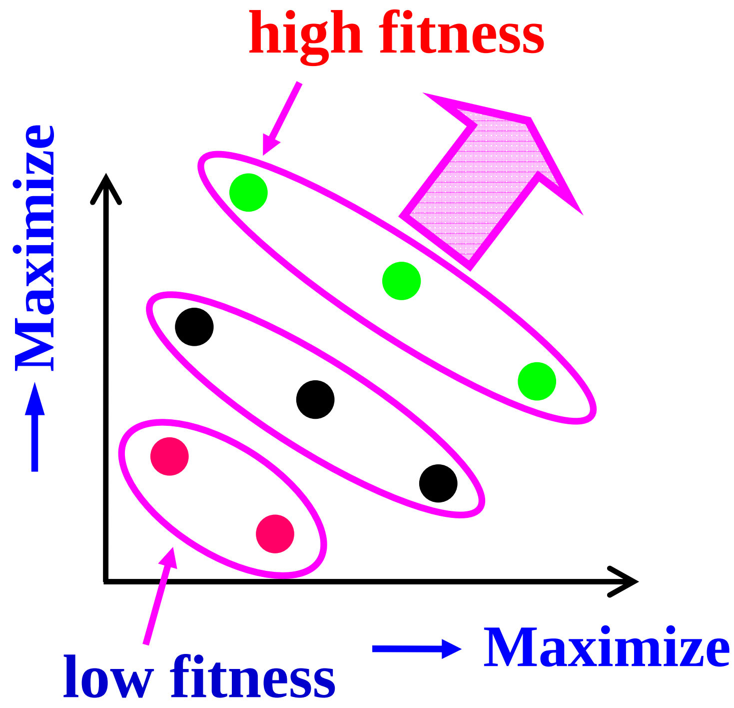
Desired search behavior of EMO algorithms

Basic Ideas in EMO Algorithm Design

Recently developed well-known EMO algorithms such as NSGA-II and SPEA have some common features:

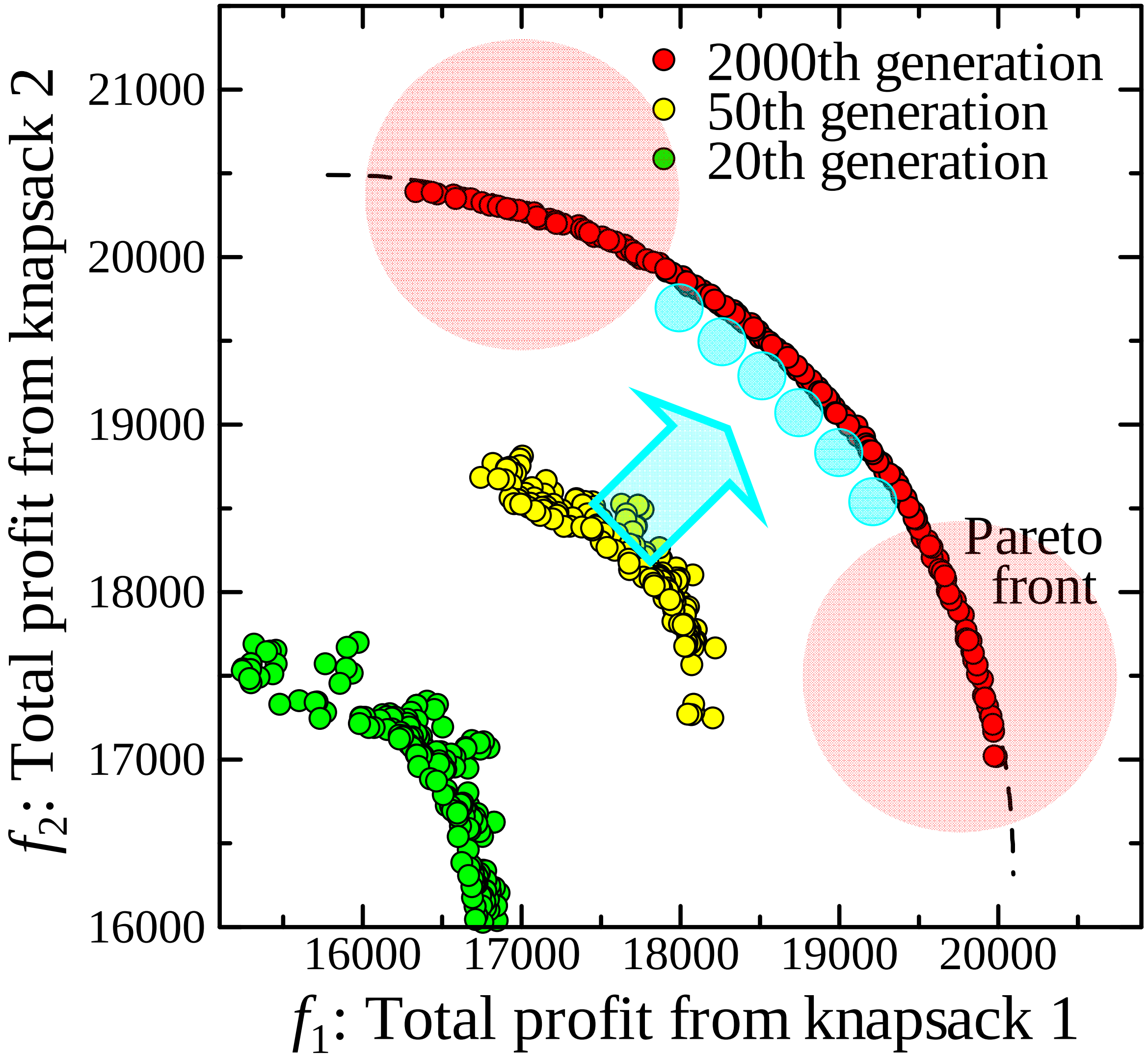
(1) Pareto Dominance

Converge to the Pareto front



Basic Ideas in Recent EMO Algorithms

1. Pareto Dominance



Basic Ideas in EMO Algorithm Design

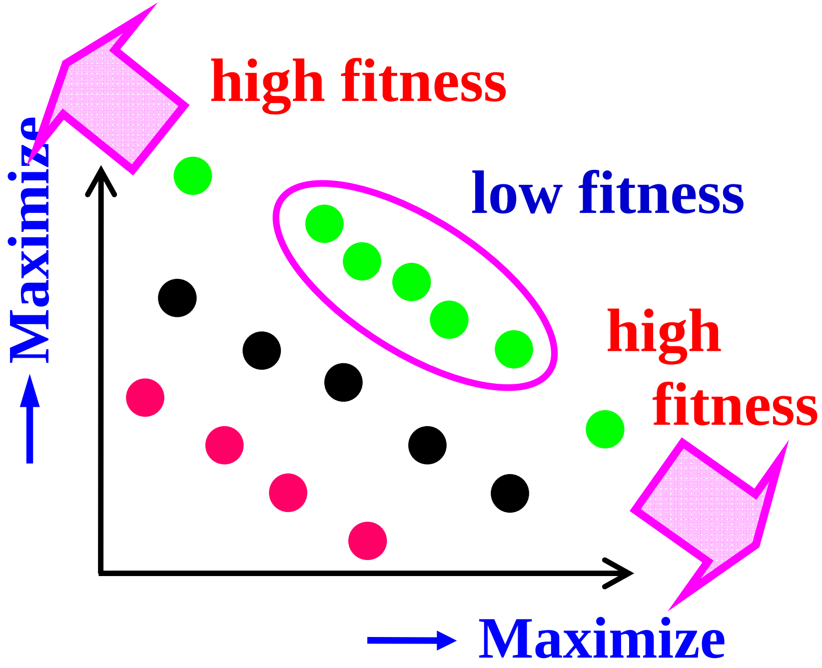
Recently developed well-known EMO algorithms such as NSGA-II and SPEA have some common features:

(1) Pareto Dominance

Converge to the Pareto front

(2) Crowding

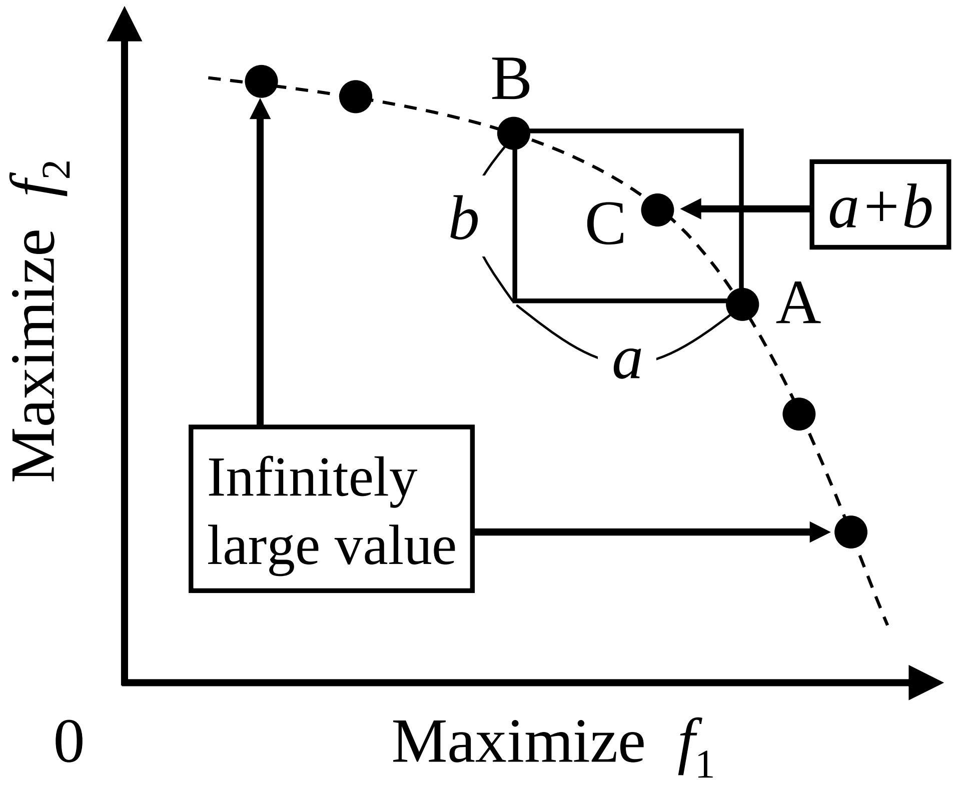
Diversity maintenance



Basic Ideas in EMO Algorithm Design

Example: Crowding Distance in NSGA-II

Distance between adjacent individuals



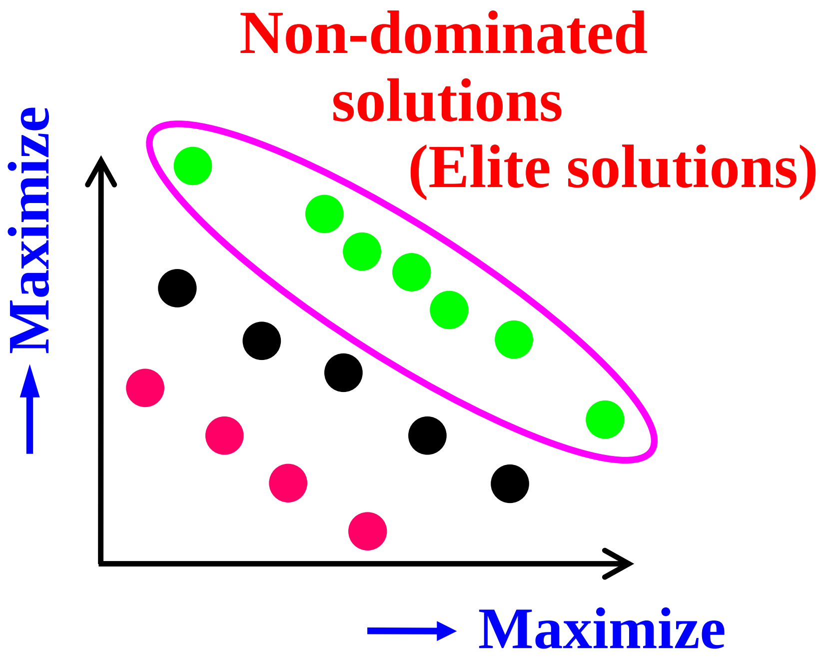
Crowding distance of C is (*a* + *b*)

Basic Ideas in EMO Algorithm Design

Recently developed well-known EMO algorithms such as NSGA-II and SPEA have some common features:

(1) Pareto Dominance

Converge to the Pareto front



(2) Crowding

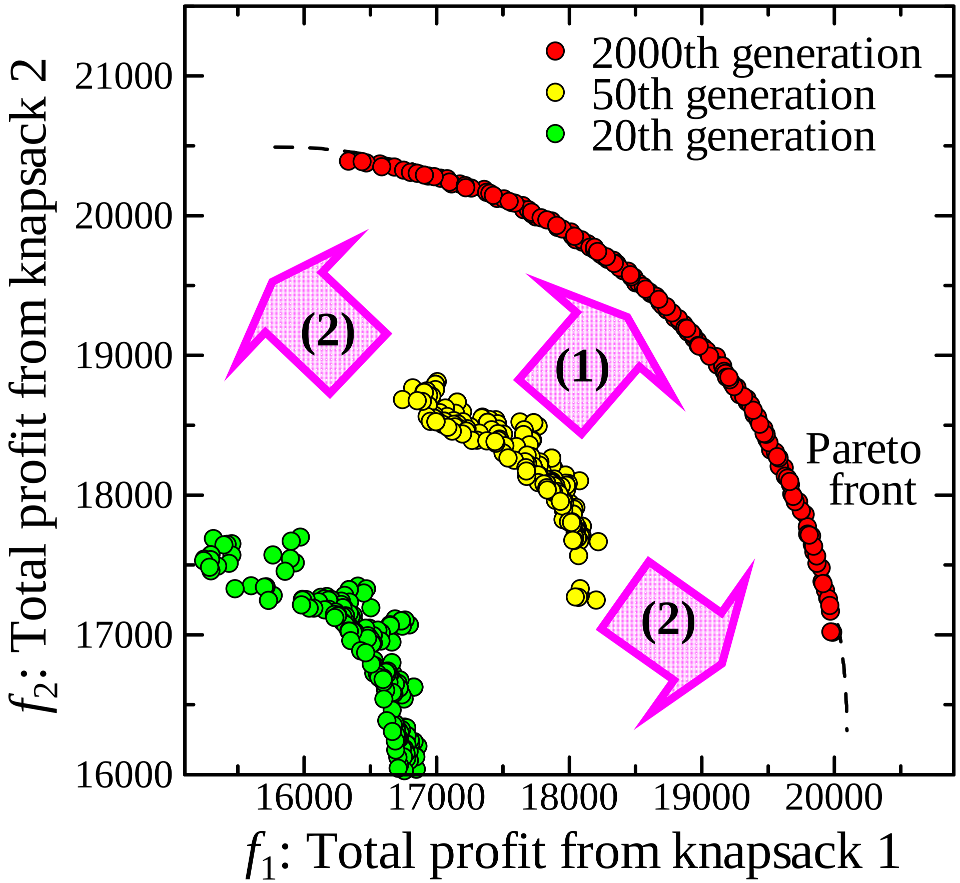
Diversity maintenance

(3) Elitist Strategy

Non-dominated solutions are handled as elite solutions.

Basic Ideas in Recent EMO Algorithms

(1) Pareto Dominance (Convergence to the Pareto front)
(2) Crowding (Diversity Maintenance)
(3) Elite Strategy (Non-Dominated Solutions)



Hot Issues in EMO Research

Utilization of Decision Maker's Preference

- Preference is incorporated into EMO algorithms.
- Interactive EMO approaches seem to be promising.

Handling of Many Objectives by EMO Algorithms

- Pareto dominance-based algorithms do not work well.
- More selection pressure is needed.

Hybridization with Local Search

- Hybridization often improves the performance of EMO.
- Balance between local and genetic search is important.

Design of New EMO Algorithms

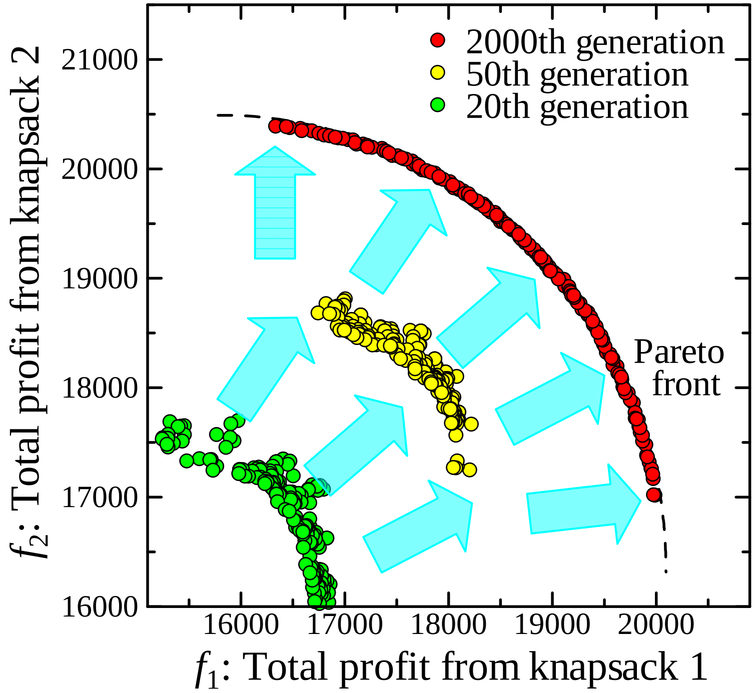
- Indicator-based EMO algorithms
- Scalarizing function-based EMO algorithms
- Use of other search methods such as PSO, ACO and DE.

Hot Issue: Preference Incorporation

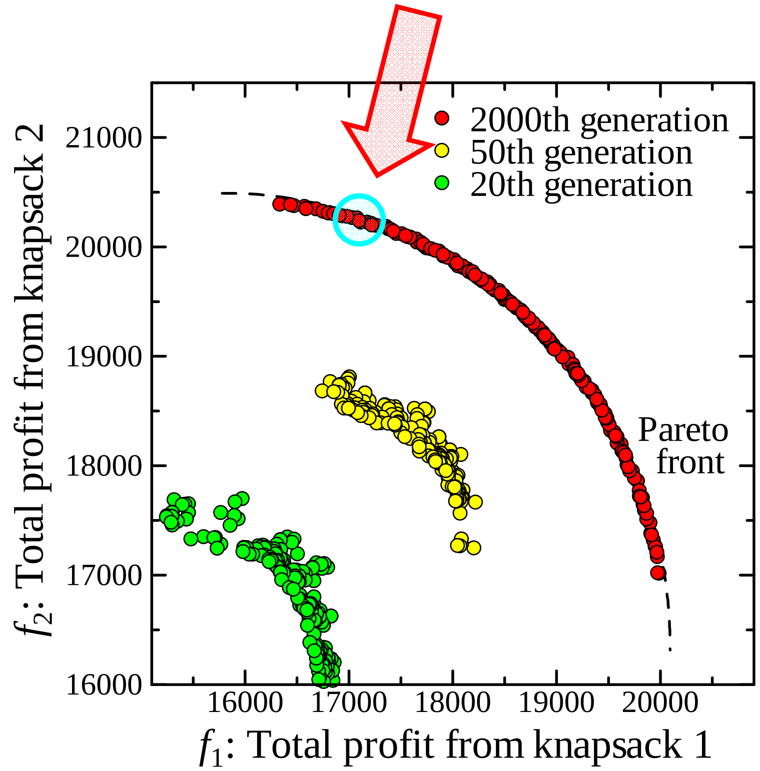
EMO Approach to Decision Making

Step 1: Evolutionary multiobjective optimization
==> Many non-dominated solutions (Candidates).

Step 2: Choice of a single solution by the decision maker.



Step 1

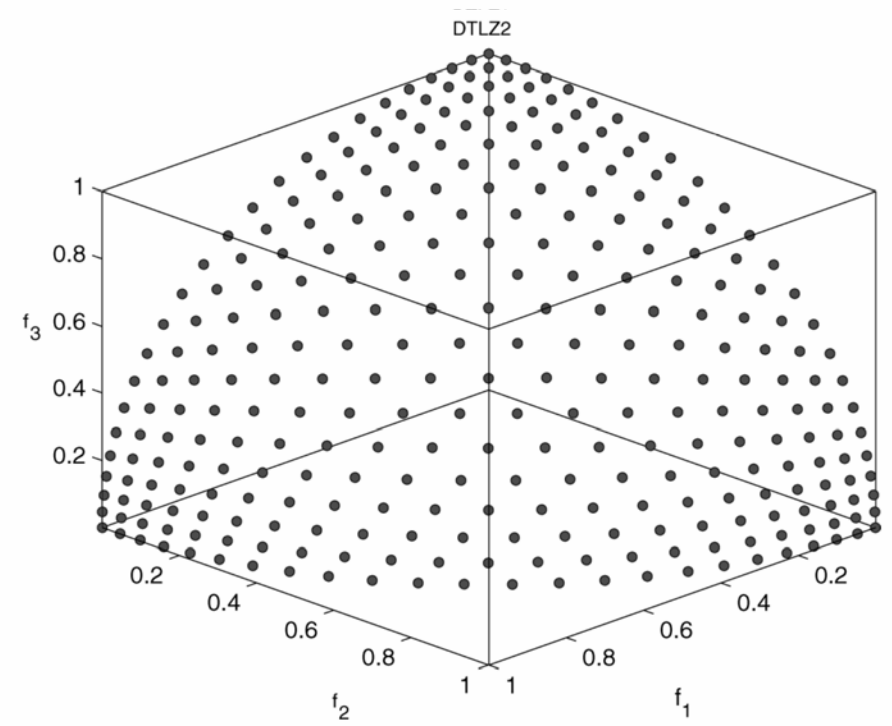


Step 2

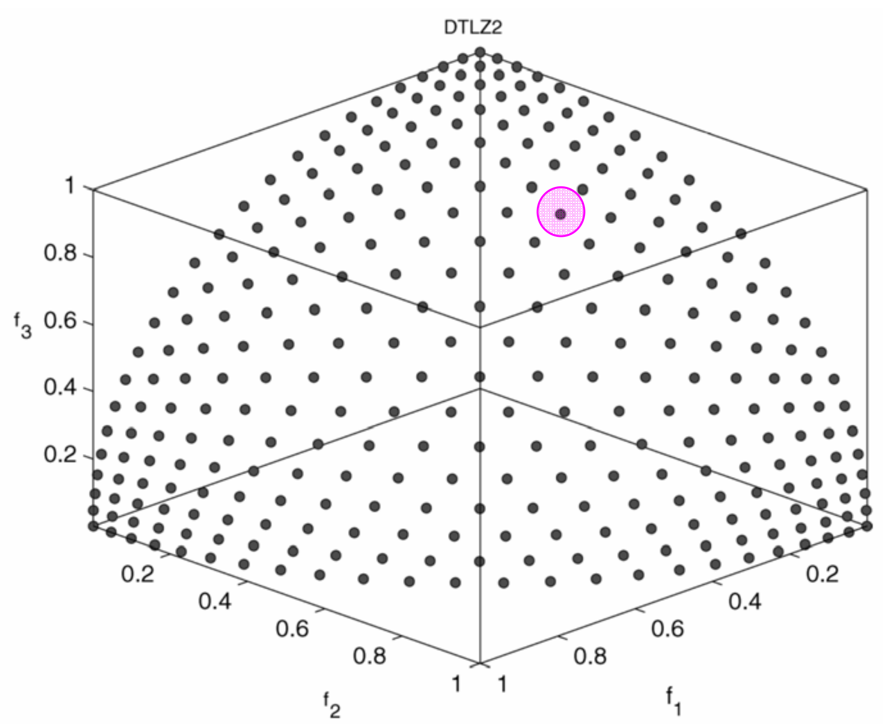
EMO Approach to Decision Making

Difficulty in Step 1: It is not always easy to find a set of non-dominate solutions that covers the entire Pareto front.

Difficulty in Step 2: It is not always easy for the DM to choose a single solution form a large number of alternatives.



Step 1



Step 2

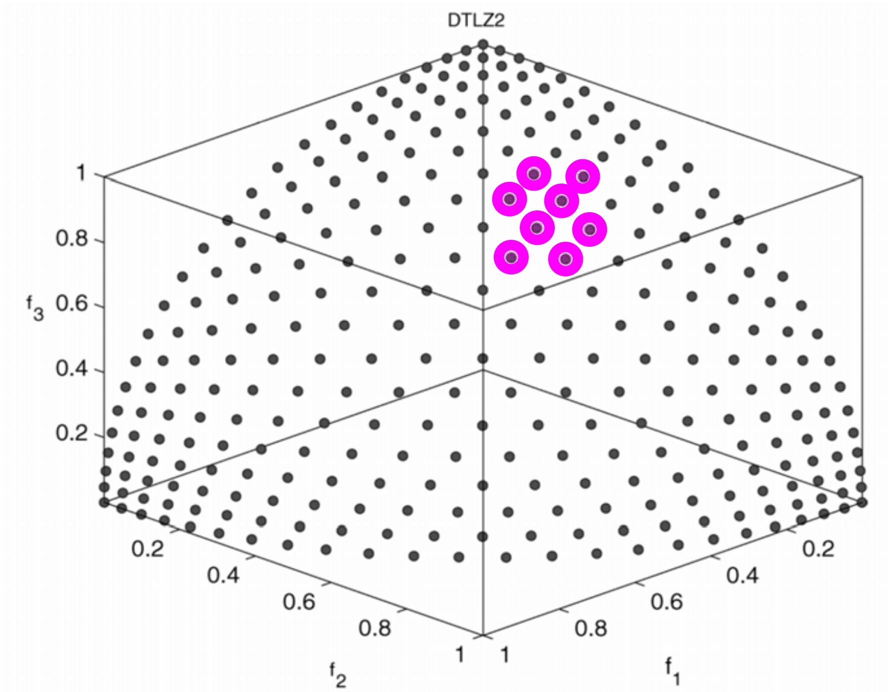
EMO Approach to Decision Making

Difficulty in Step 1: **It is not always easy to find a set of non-dominate solutions that covers the entire Pareto front.**

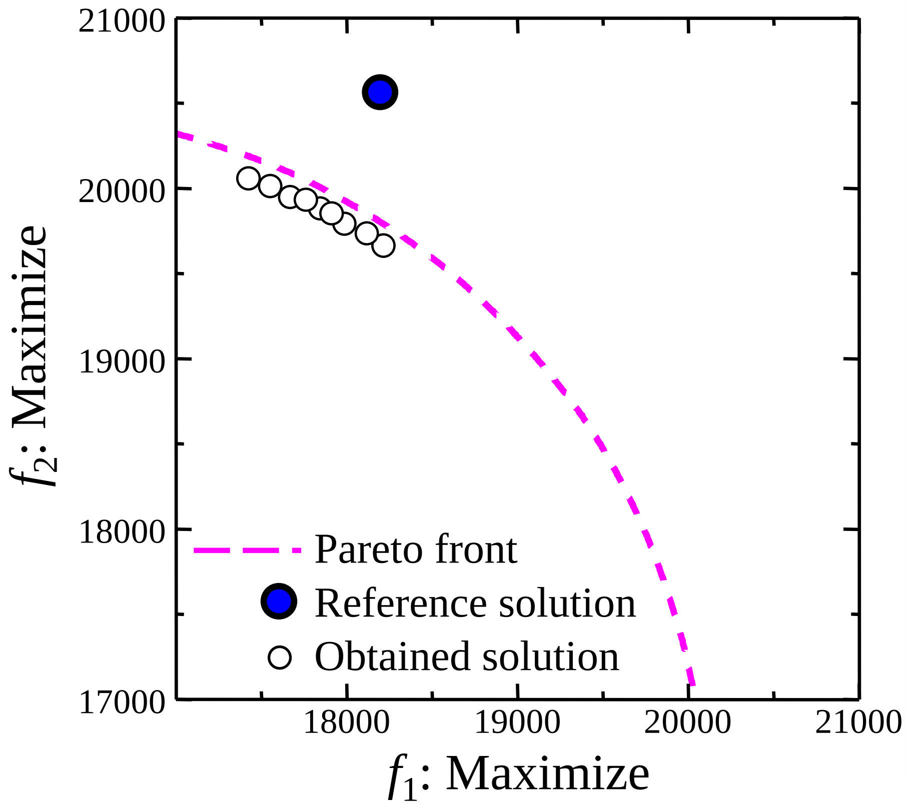
Difficulty in Step 2: **It is not always easy for the DM to choose a single solution form a large number of alternatives.**

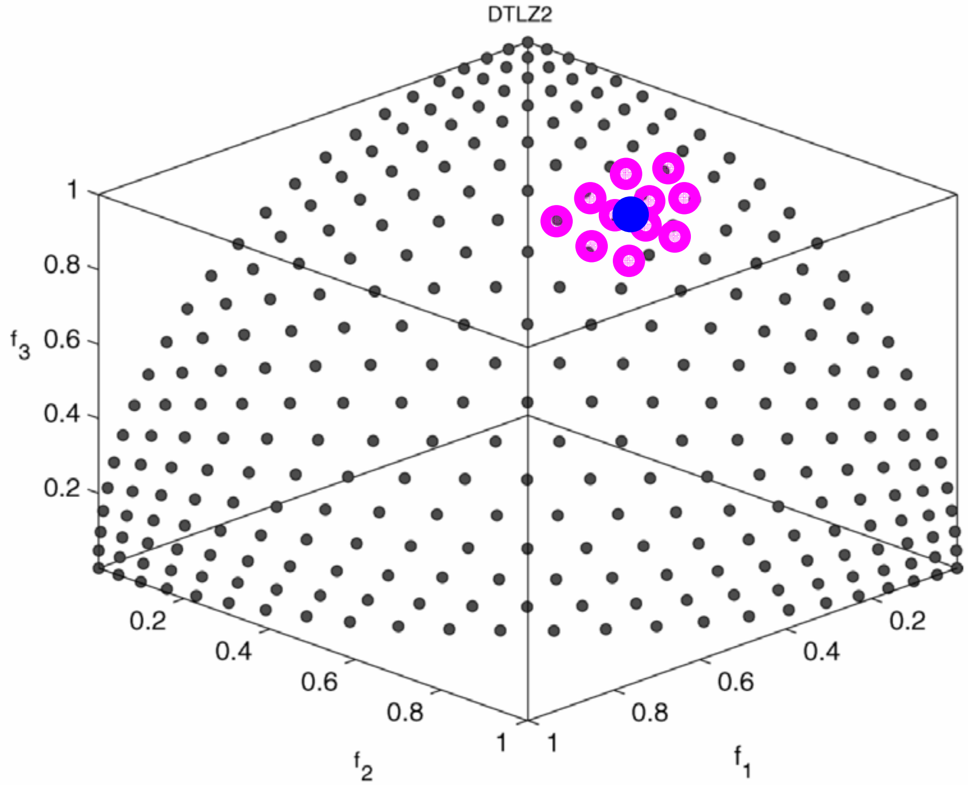
One idea to tackle these two difficulties:

To search for a small number of non-dominate solutions.



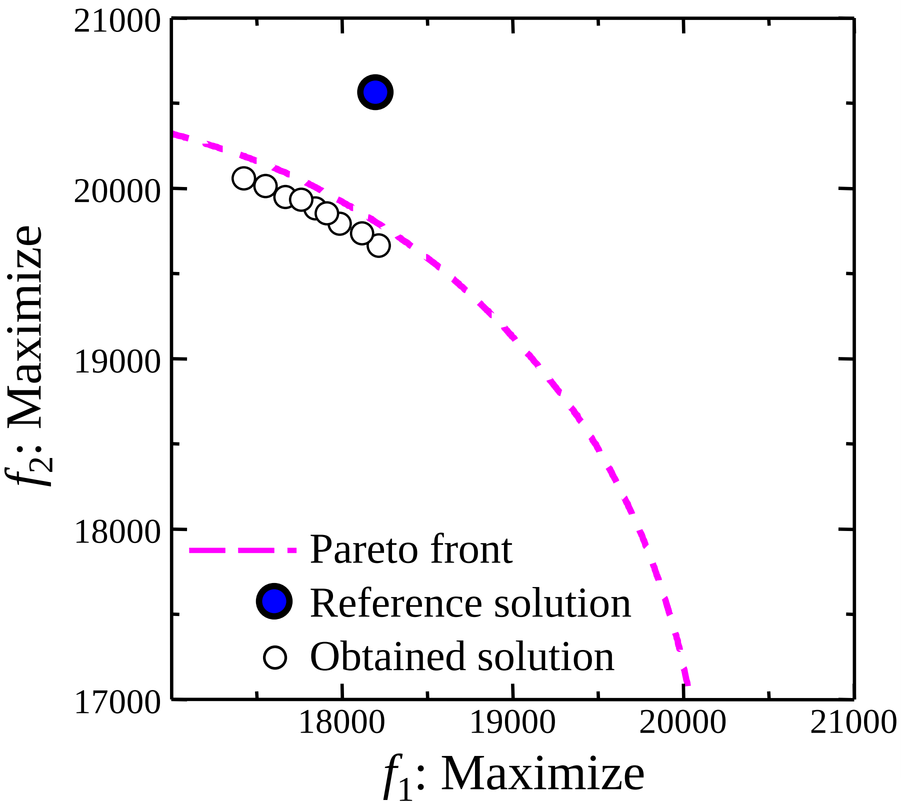
Utilization of Preference Information

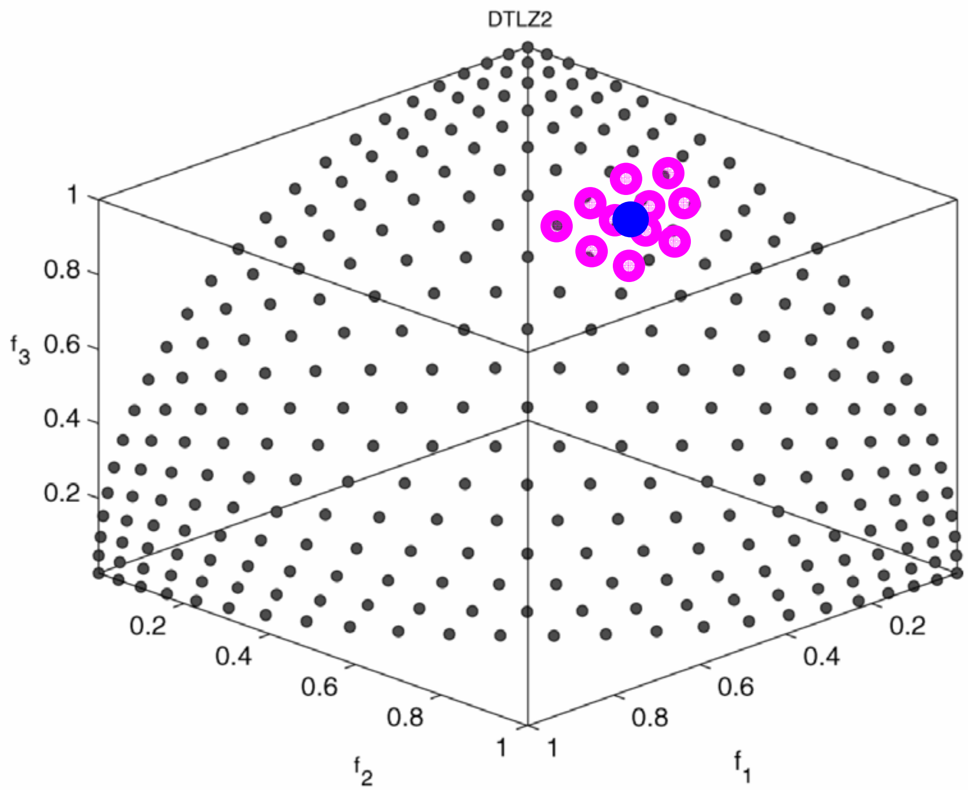




Basic Idea: Concentration on the preferred region of the Pareto front. The decision maker is not always interested in all the Pareto-front.

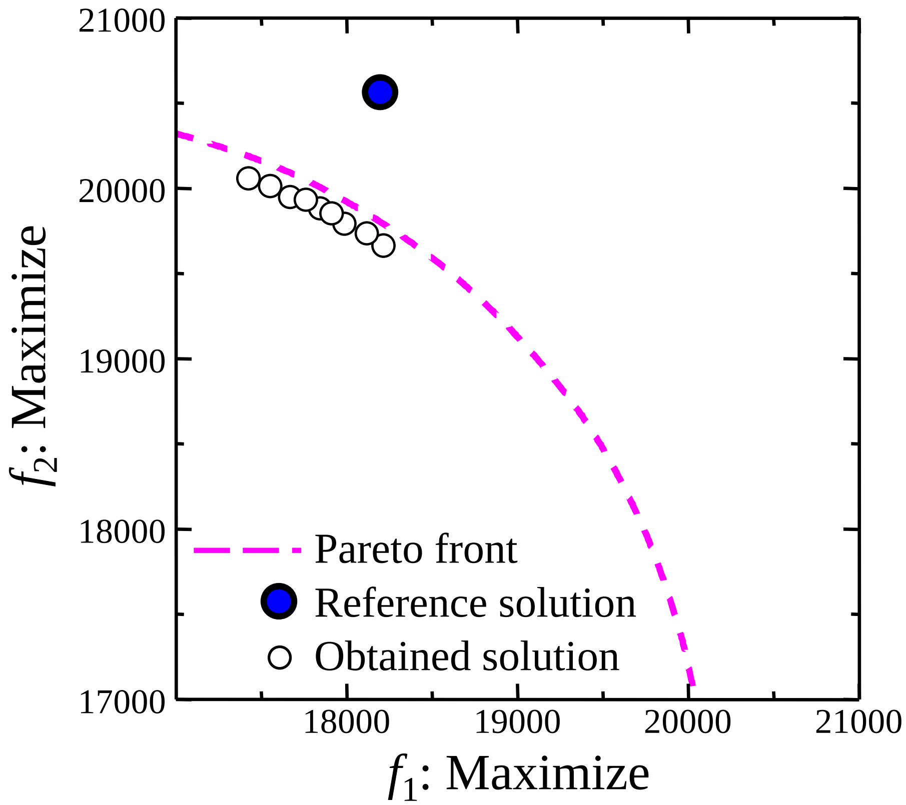
Utilization of Preference Information





Difficulty: **It is not easy to extract preference information from the decision maker (DM). It may be much simpler to compare different solutions. ==> Interactive Approaches.**

Extraction of Preference Information



Preference Extraction

(1) Relatively Easy Case

- Number of Objectives: Two
- Pareto Front: Known
- The DM knows the problem

(2) Very Difficult Case

- Number of Objectives: Many
- Pareto Front: Unknown
- The DM does not know the problem very well.

Example: Flight Tickets (Cost, # of Stops, Total Time)

Case 1: You are planning to buy a ticket to your home town.

Case 2: You are planning to buy a ticket to Easter Island.

Another Hot Issue:
Evolutionary Many-Objective Optimization

Why are many-objective problems difficult?

1. **Many Objectives: Difficulty in Multiobjective Search**

   Selection pressure toward the Pareto front becomes very weak since almost all solutions are non-dominated.

2. **Many Solutions: Difficulty in Approximation**

   A large number of non-dominated solutions are needed to approximate the entire Pareto front.

3. **Many Solutions with Many Objectives: Presentation**

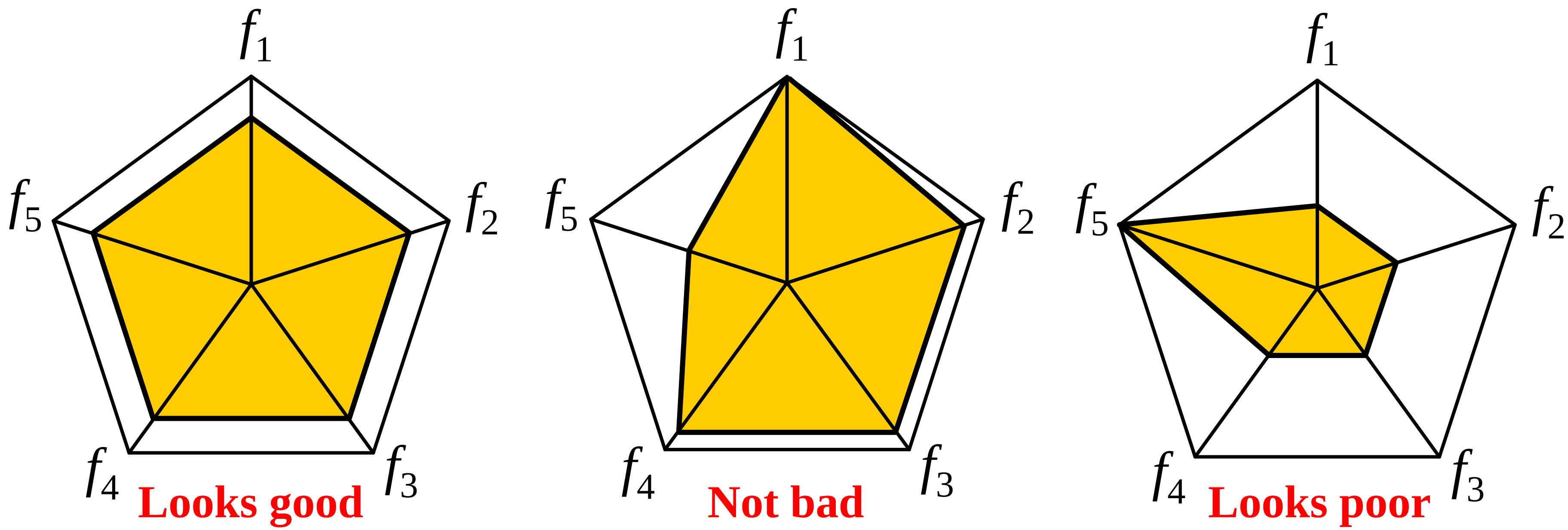
   It is very difficult to present a large number of obtained solutions in the high-dimensional object space to the decision maker in a visually understandable manner.

Difficulties in Many-Objective Optimization

Q. Why are many-objective problems hard for EMO ?

A. Solutions with many objectives are usually non-dominated with each other. This means very low selection pressure toward the Pareto front in Pareto dominance-based EMO.

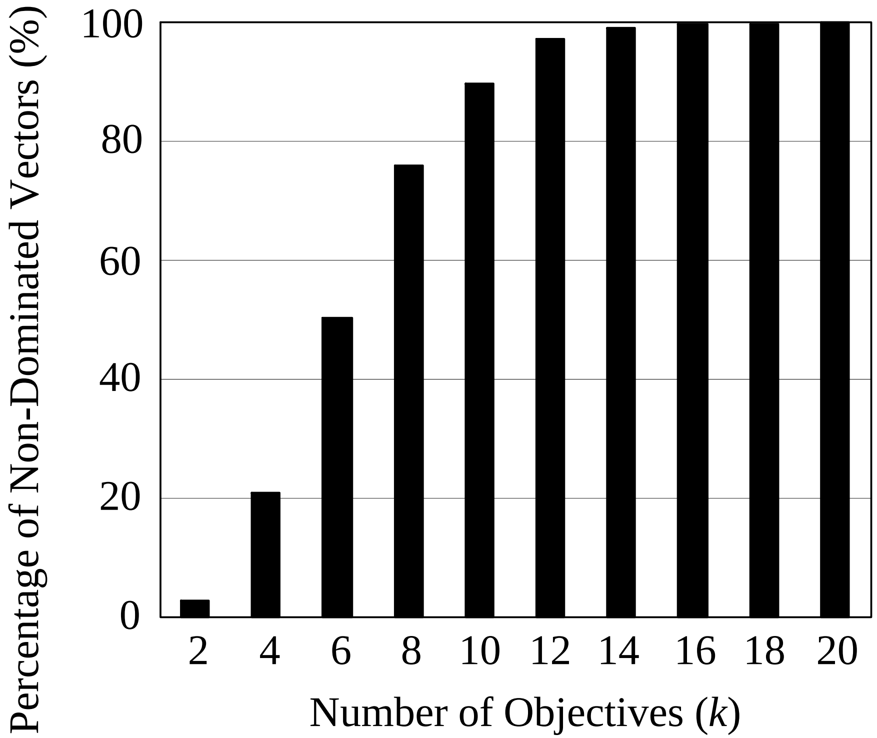
Five-Objective Maximization Example (Non-dominated Vectors)



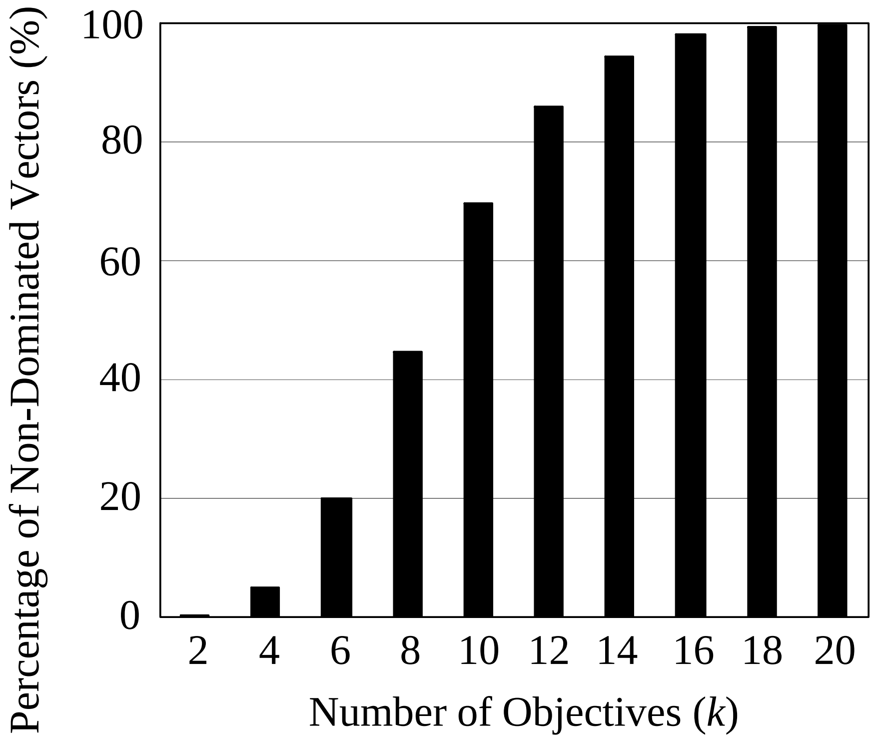
Difficulties in Many-Objective Optimization

Percentage of Non-Dominated Vectors

We randomly generate vectors in a *k*-dimensional space.



(1) Among 200 vectors.



(2) Among 2,000 vectors

Experimental Results of NSGA-II

Standard Implementation of NSGA-II

Generation Update: (100 + 100) ES

Current Population: 100 Individuals

Offspring Population: 100 Individuals

Next Population: The best 100 individuals from the current population and the offspring population.

Fitness Evaluation:

1st Criterion: Pareto Dominance

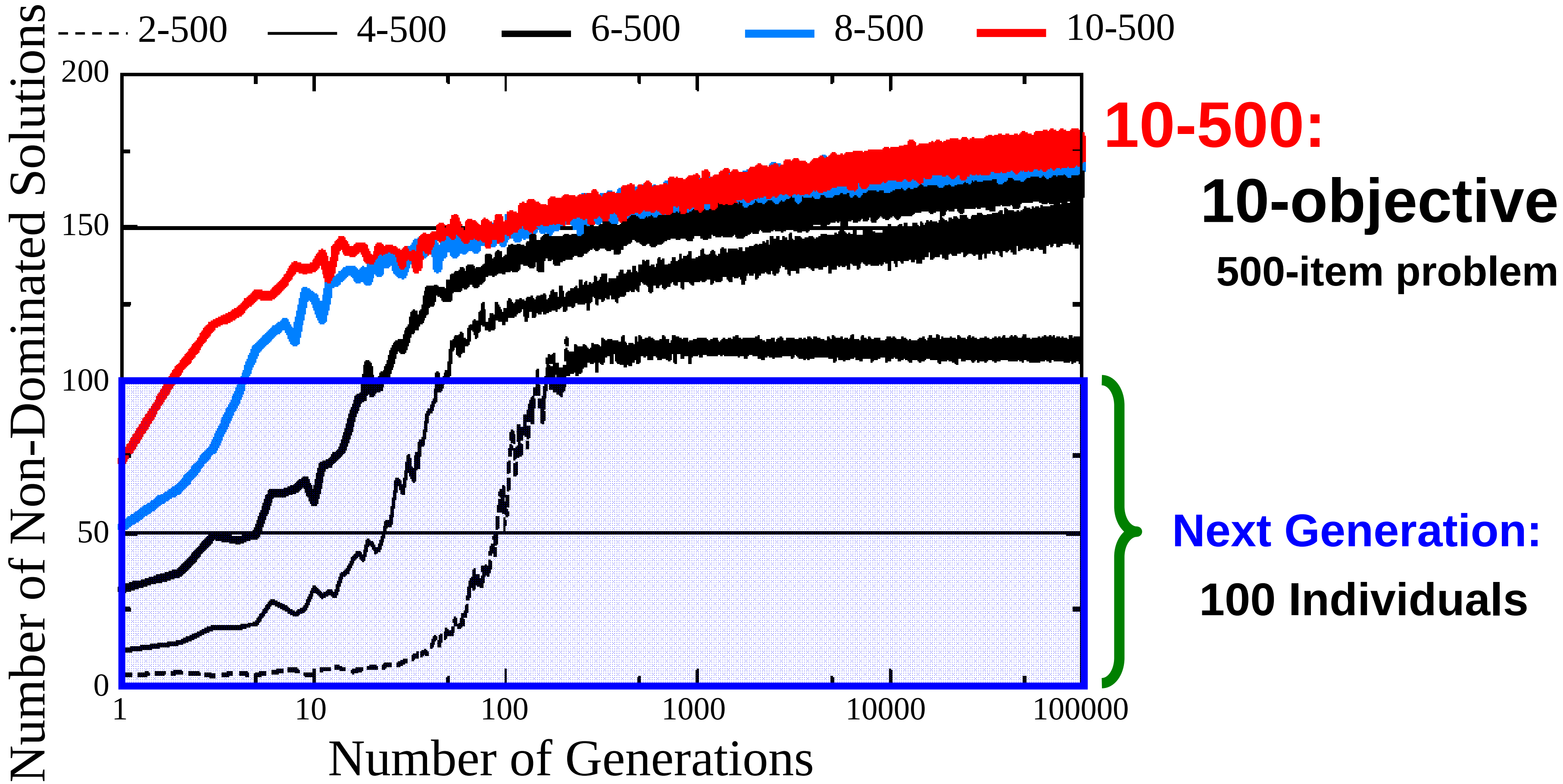
2nd Criterion: Crowding Distance

Test Problems

k-objective 500-item knapsack problems (k-500 problem)

k = 2, 4, 6, 8, 10

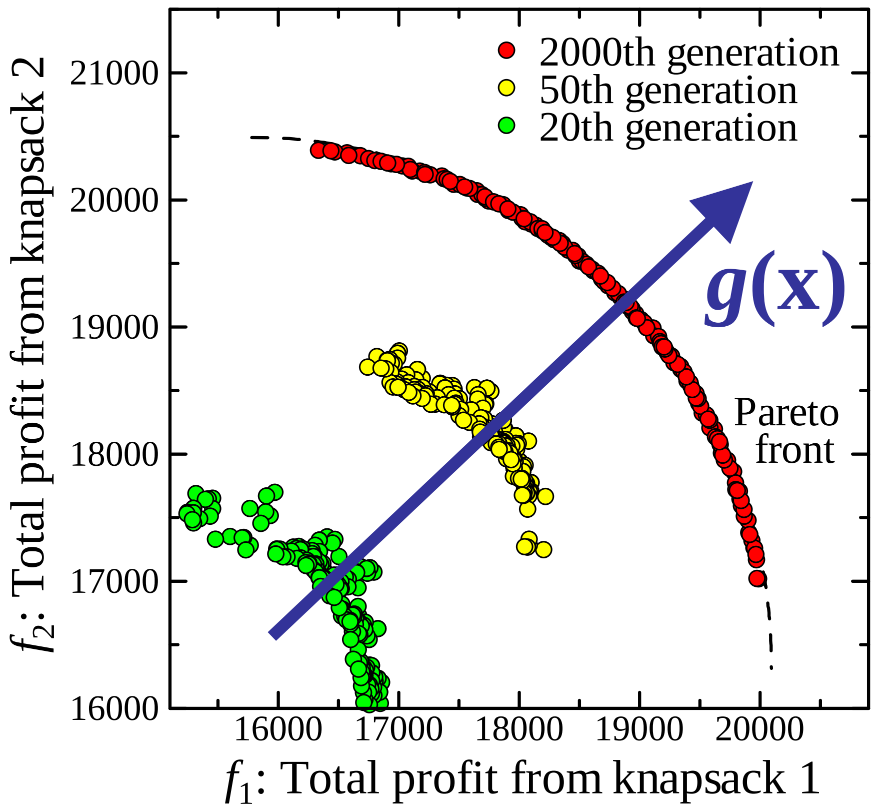
Number of Non-Dominated Solutions
(Among 200 solutions before the generation update in NSGA-II)



All individuals are non-dominated solutions after a few generations (10-500 problem) and after about 200 generations (2-500 problem).

Very Simple Measure of Convergence

The sum of the given objectives: $g(\mathbf{x}) = f_1(\mathbf{x}) + f_2(\mathbf{x})$

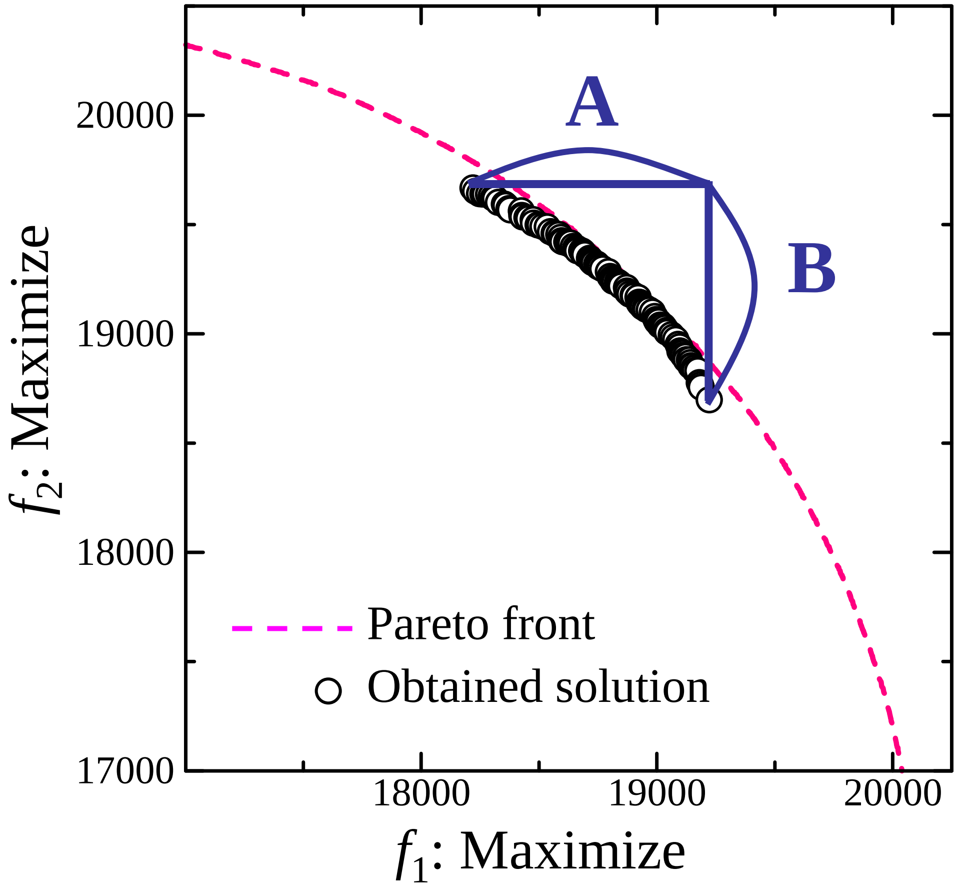


MaxSum

$= \text{Max}\ \{g(\mathbf{x})\}$

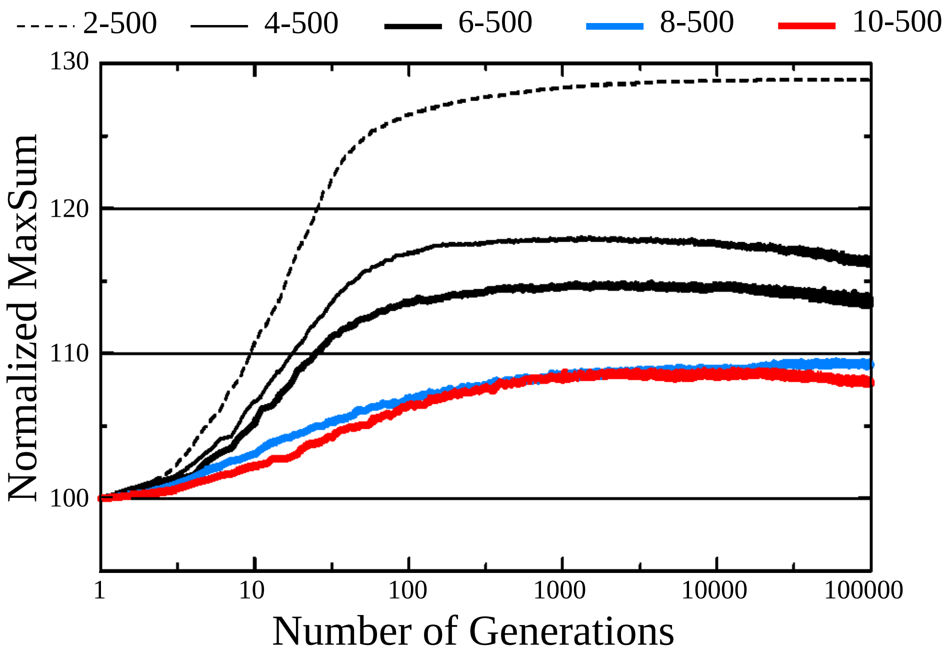
Very Simple Measure of Diversity

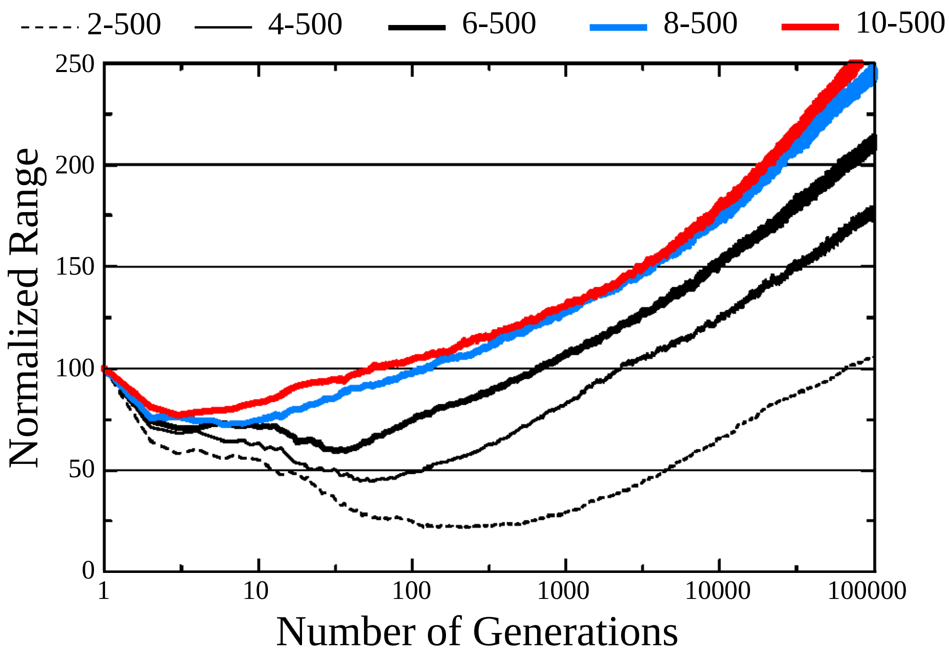
Range Measure



Range = A + B

Experimental Results of NSGA-II





MaxSum: Convergence

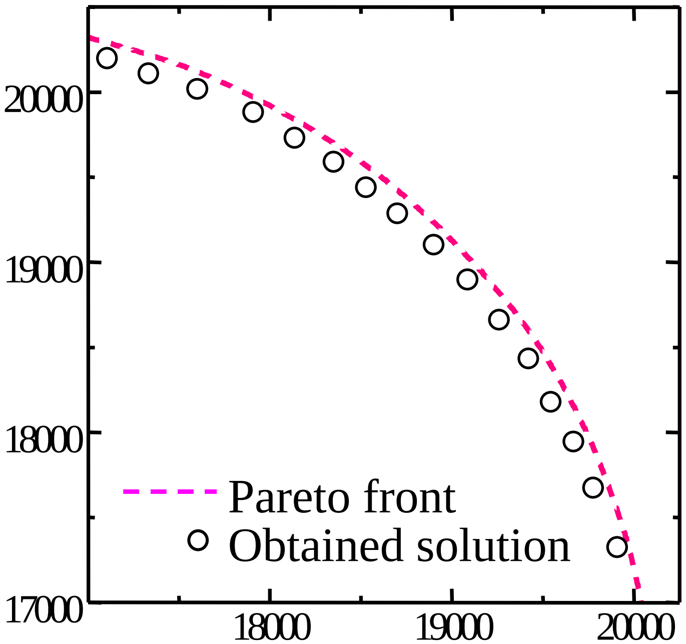
Range: Diversity of solutions

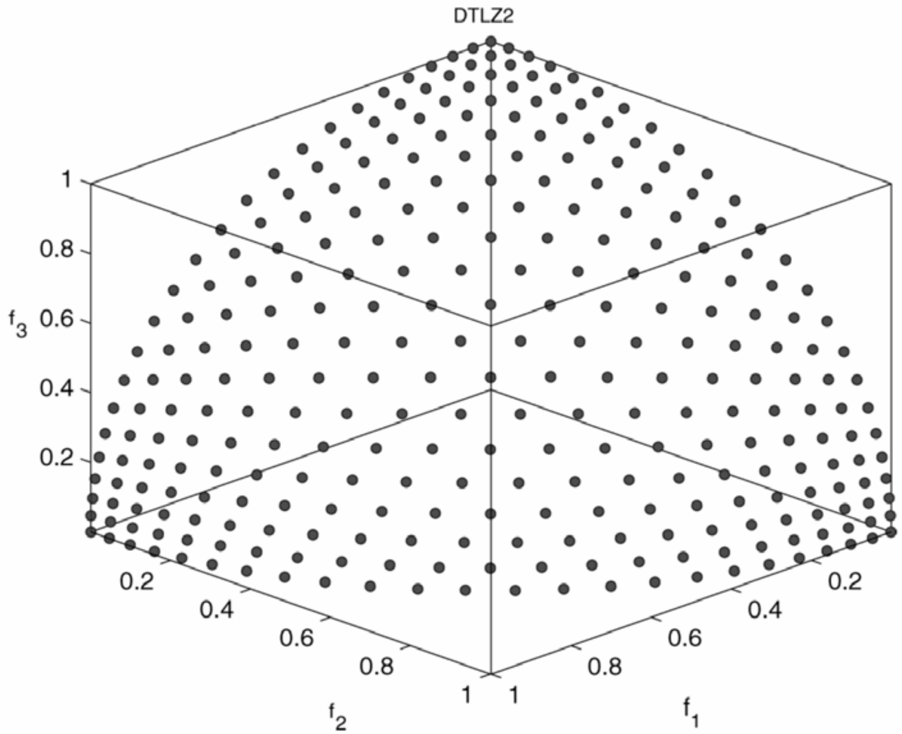
Observation: Only the convergence was improved in the early generations. After that, only the diversity was improved.

Approximation of the Pareto Front

Q: **How many non-dominated solutions are needed to approximate the entire Pareto-front of the *k*-objective problem? (*k* = 2, 3, 4, ...)**

A: Huge when *k* is large (It exponentially increases with *k*)





***k* = 2**

***k* = 3**

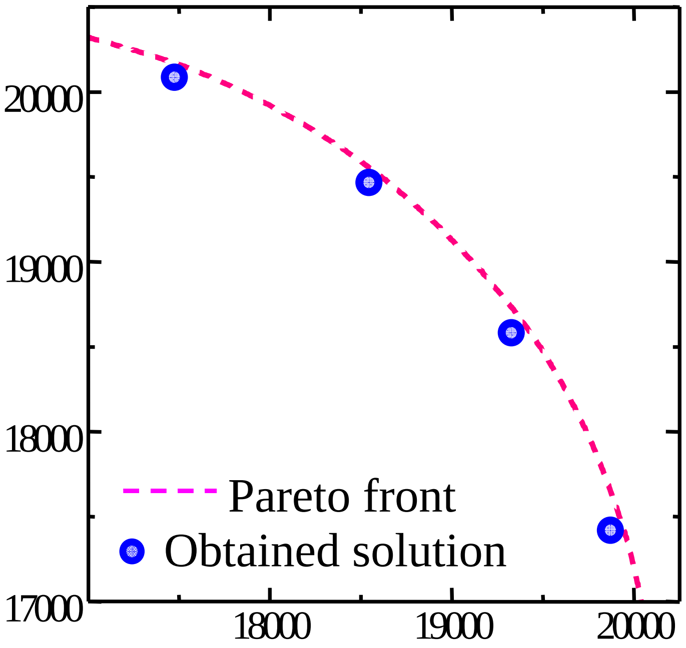
Approximation with Finite Solutions

Two Strategies for Many-Objective Problems

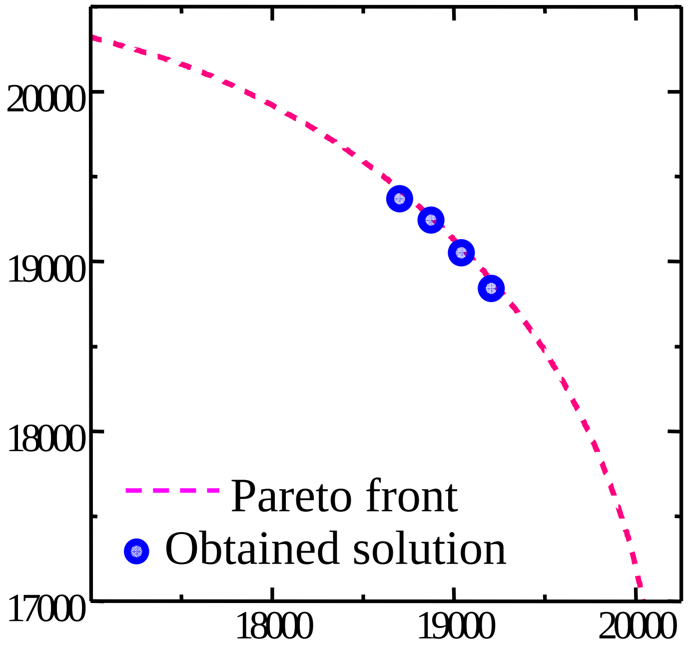
(1) Sparse approximation of the entire Pareto front.

(2) Dense approximation of only a part of the Pareto front.

Dense approximation of the entire Pareto front is impossible in the case of many objectives.



(1) Sparse Approximation



(2) Dense Approximation

Approximation with Finite Solutions

Two Strategies for Many-Objective Problems

(1) Sparse approximation of the entire Pareto front.

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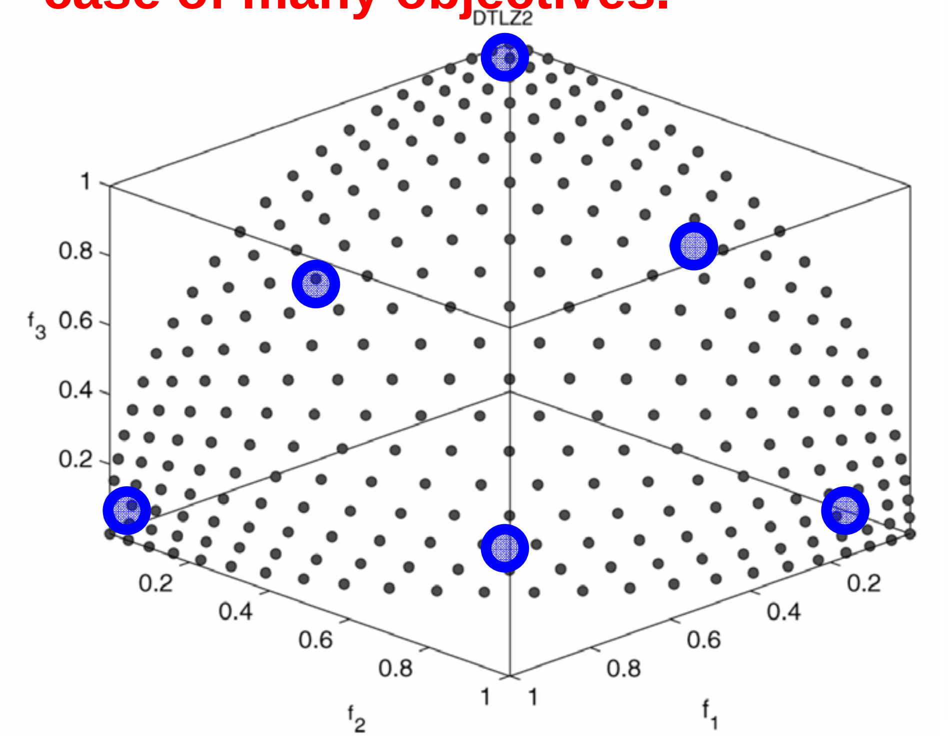
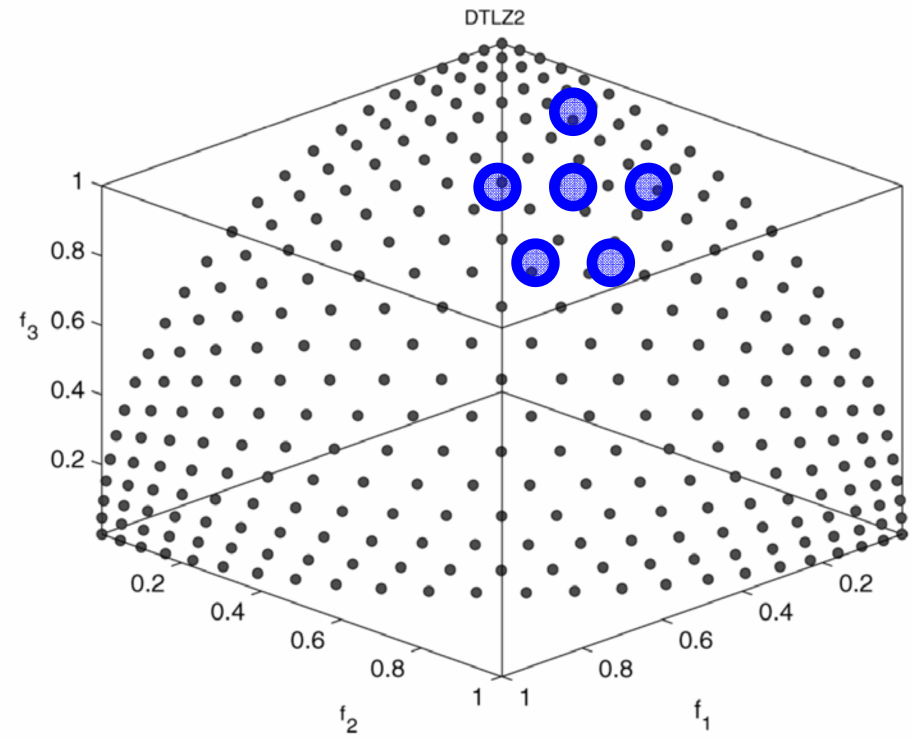


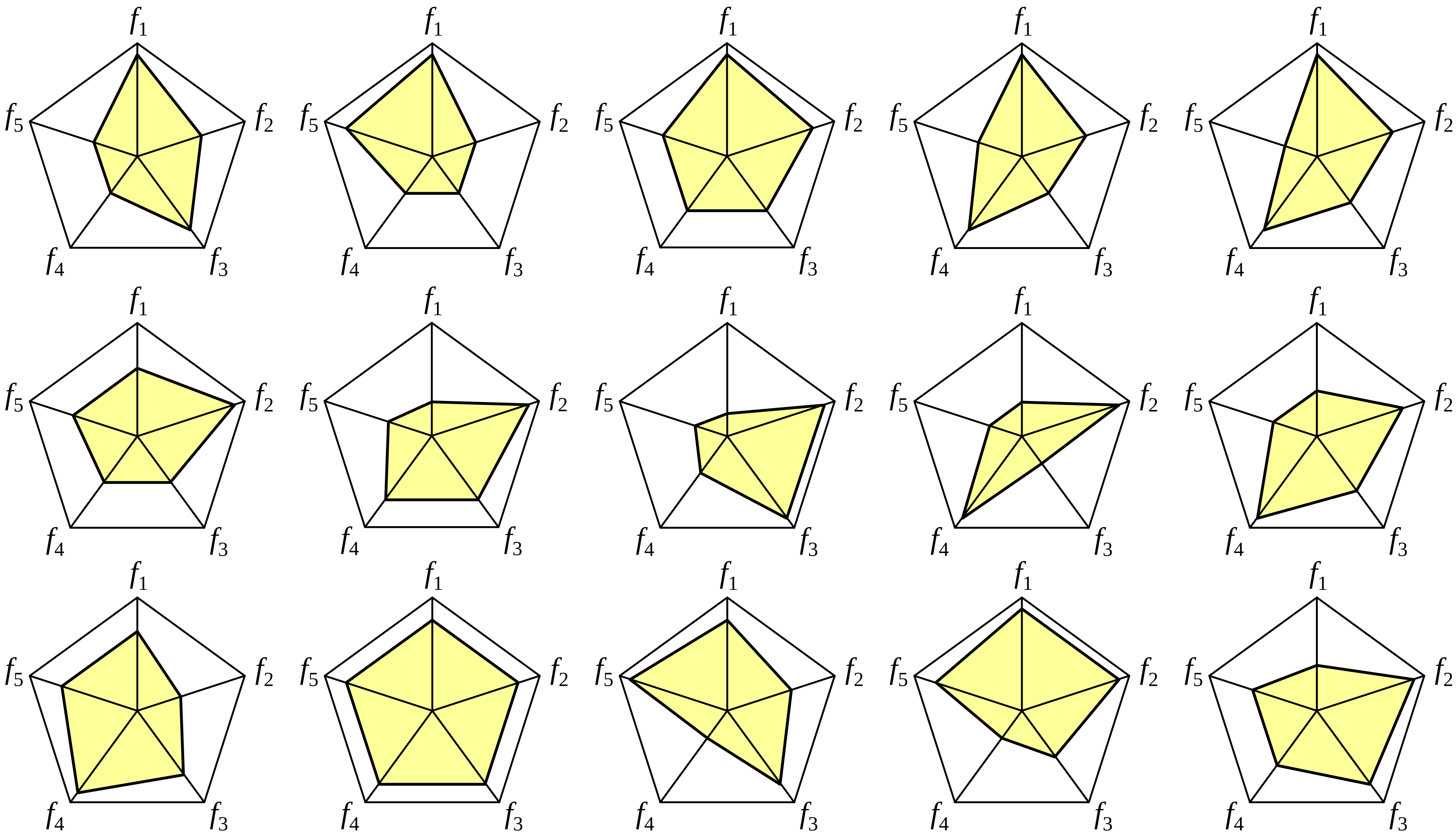
(1) Sparse Approximation



(2) Dense Approximation

Handling of Obtained Solutions

Difficulty: **How to show a large number of non-dominated solutions.**



Another Hot Issue: Hybridization
Multiobjective Memetic Algorithm (MOMA)

Powerful Approach to Single-Objective Optimization: MA

Evolutionary algorithm	+	Local search	=	Memetic algorithm

Multiobjective Memetic Algorithm: MOMA

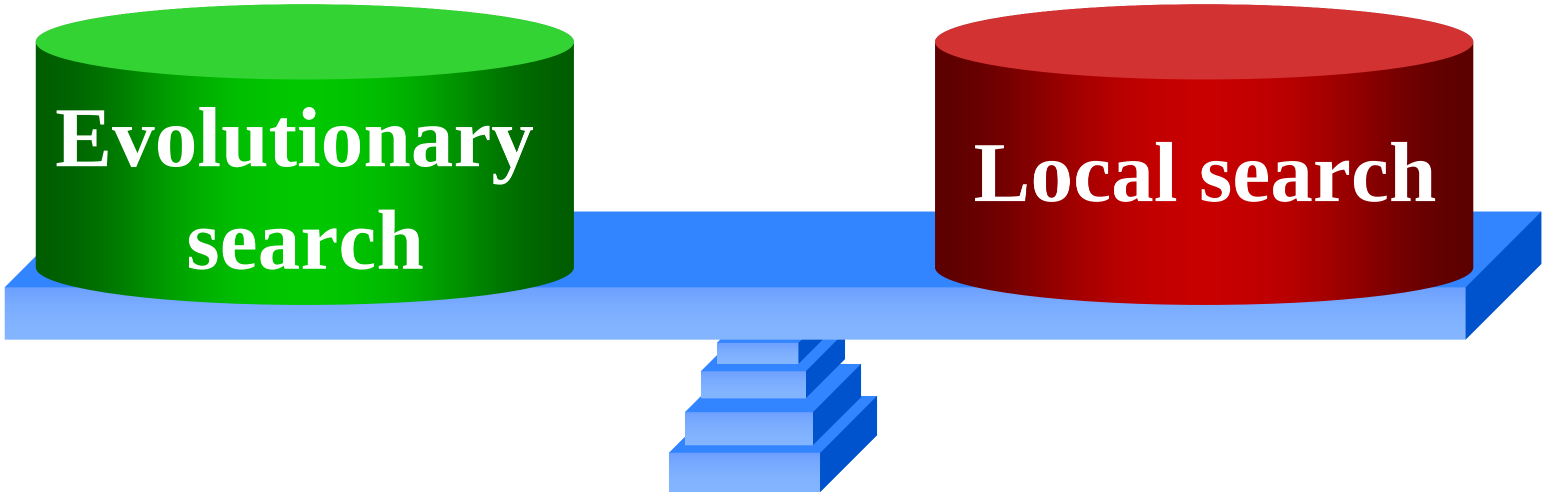
Multiobjective evolutionary algorithm	+	Multiobjective local search	=	Multiobjective memetic algorithm

Design of MA and MOMA

One important implementation issue:

Specification of the balance between evolutionary search and local search (or its dynamic adaptation).

Ishibuchi H, Yoshida T, Murata T (2003) Balance between genetic search and local search in memetic algorithms for multiobjective permutation flowshop scheduling. *IEEE Trans. on Evolutionary Computation.*

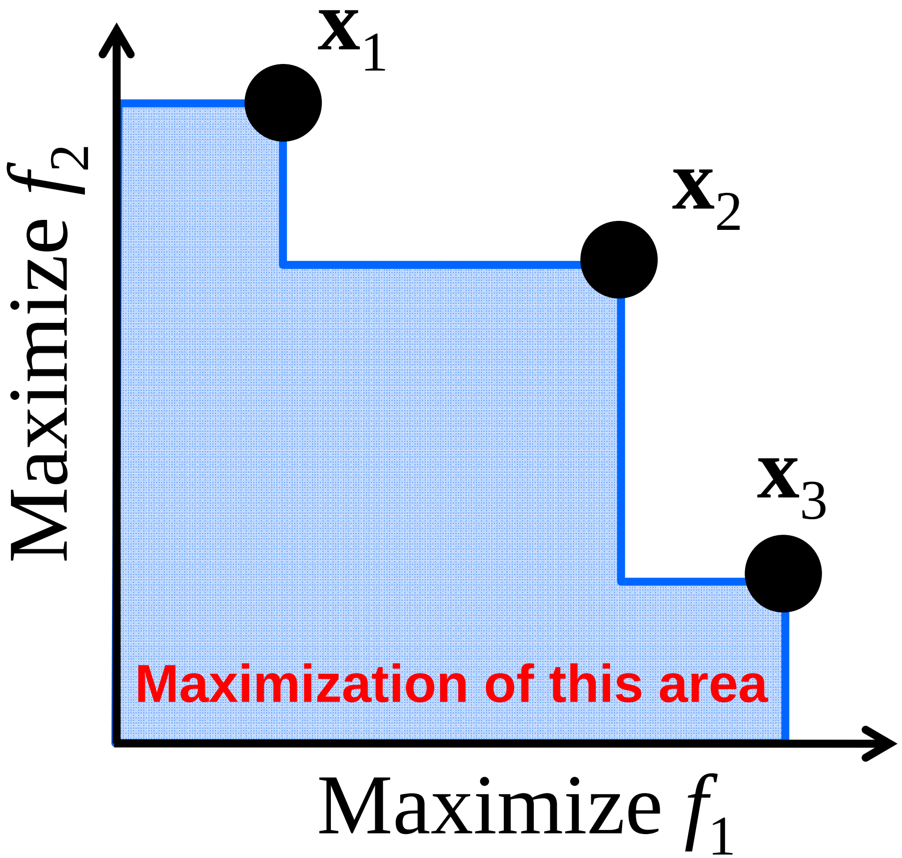


New Trend in EMO Algorithm Design

IBEA: Indicator-Based Evolutionary Algorithm

Basic Idea

To maximize a performance indicator of a solution set (not a solution): Hypervolume is often used.



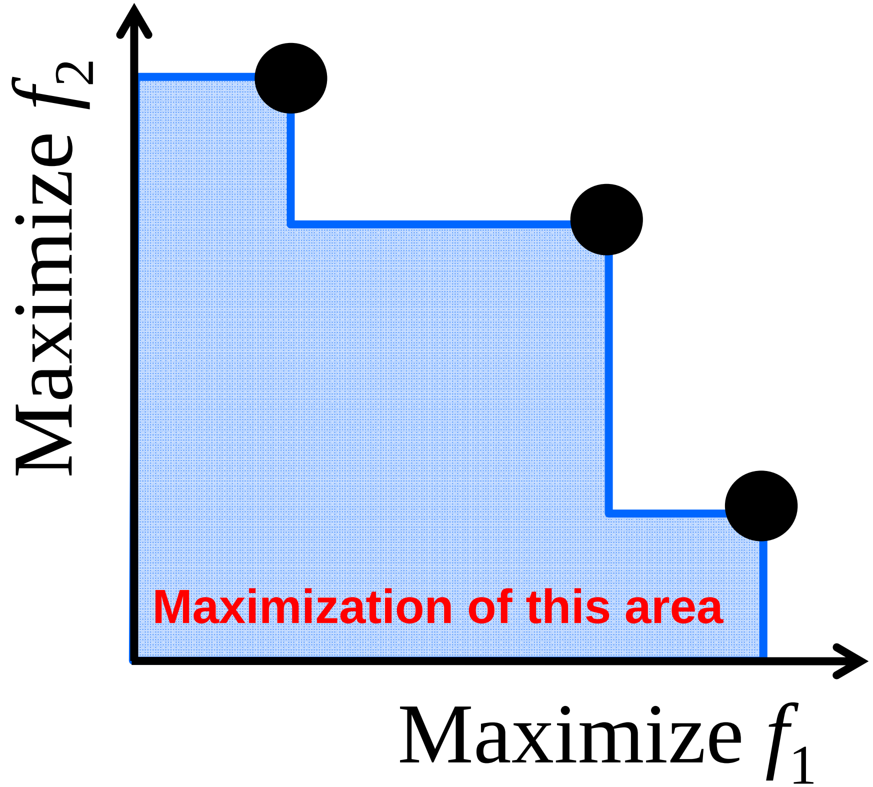
New Trend in EMO Algorithm Design

IBEA: Indicator-Based Evolutionary Algorithm

$$\text{Maximize } \boldsymbol{I}(\boldsymbol{S})$$ **(Maximization of an Indicator Function)**

$$\text{subject to } |\boldsymbol{S}| \le \boldsymbol{N} \quad \text{where } \boldsymbol{S} \subset \{\mathbf{x} \mid \mathbf{x} \in \mathbf{X}\}$$

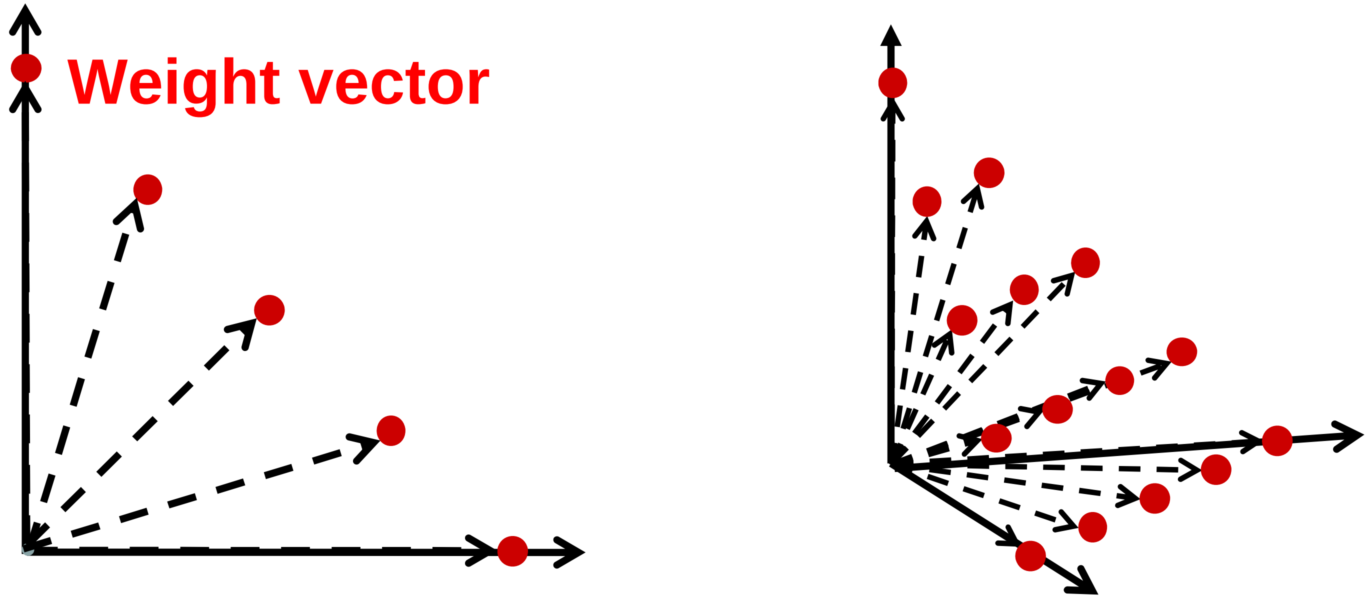
- $\boldsymbol{S}$ **:** **A set of solutions**
- $\boldsymbol{N}$ **:** **A pre-specified number of required solutions**
- $\mathbf{X}$ **:** **A feasible region**



New Trend in EMO Algorithm Design
MOEA/D: Use of Scalarizing Functions

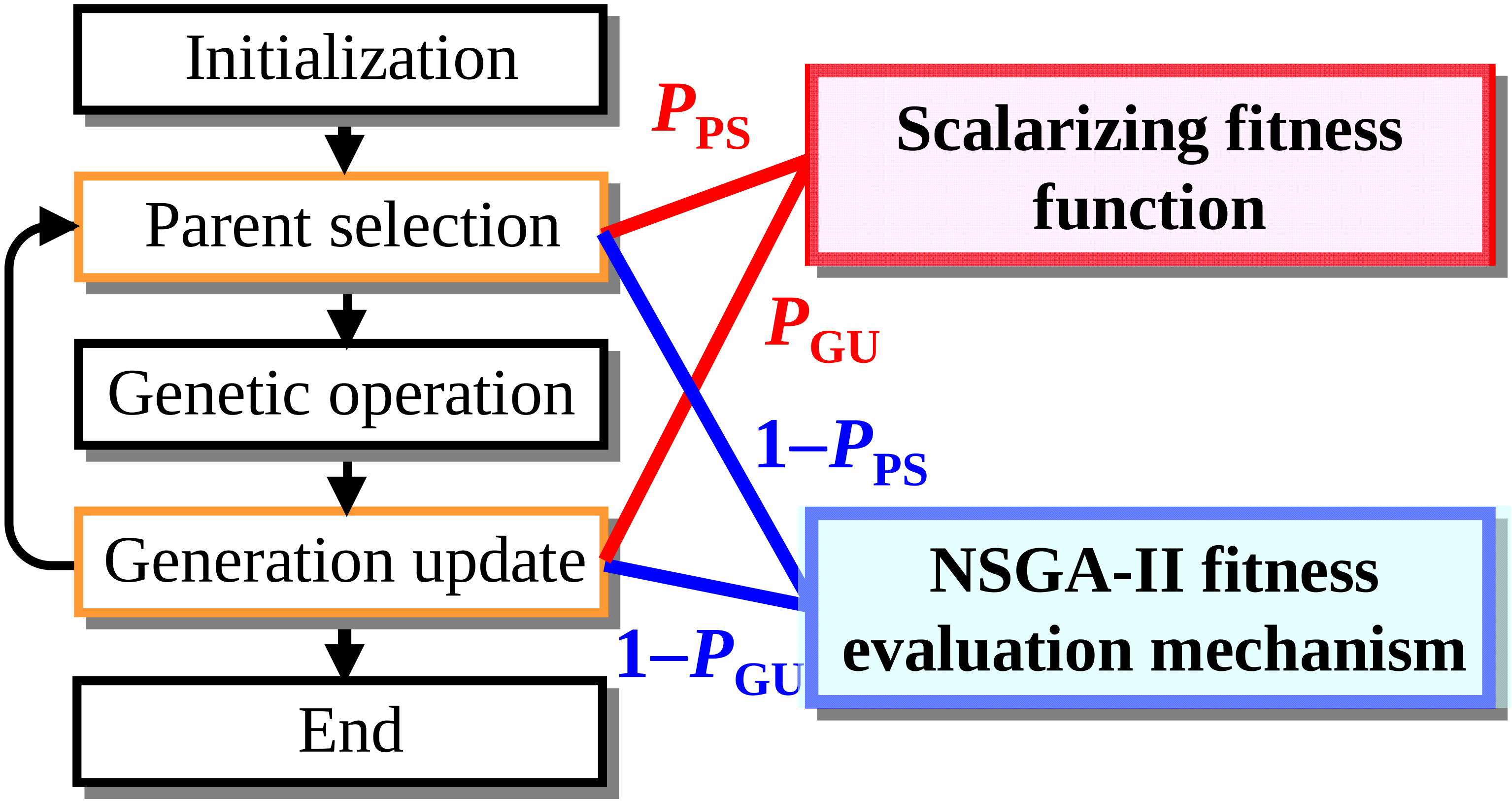
MOEA/D: Multi-objective evolutionary algorithm based on decomposition by Zhang and Li (IEEE TEC 2007)

Its Basic Idea (Decomposition): A multi-objective problem is handled as a set of scalarizing function optimization problems with different weight vectors.



(a) Two-objective case (b) Three-objective case

New Trend in EMO Algorithm Design
Hybrid Method: Use of Scalarizing Functions



Probability for scalarizing fitness functions:

Parent selection: P_{PS} **Generation update: P_{GU}**

Ishibuchi et al. (PPSN 2006)

New Trend in EMO Algorithm Design
Use of Other Meta-Heuristics (PSO, ACO, etc.)

Highly Cited Papers

[1] Coello CAC, Pulido GT, Lechuga MS (2004) Handling Multiple Objectives with Particle Swarm Optimization, IEEE TEC

[2] McMullen PR (2001) An Ant Colony Optimization Approach to Addressing a JIT Sequencing Problem with Multiple Objectives, Artificial Intelligence in Engineering

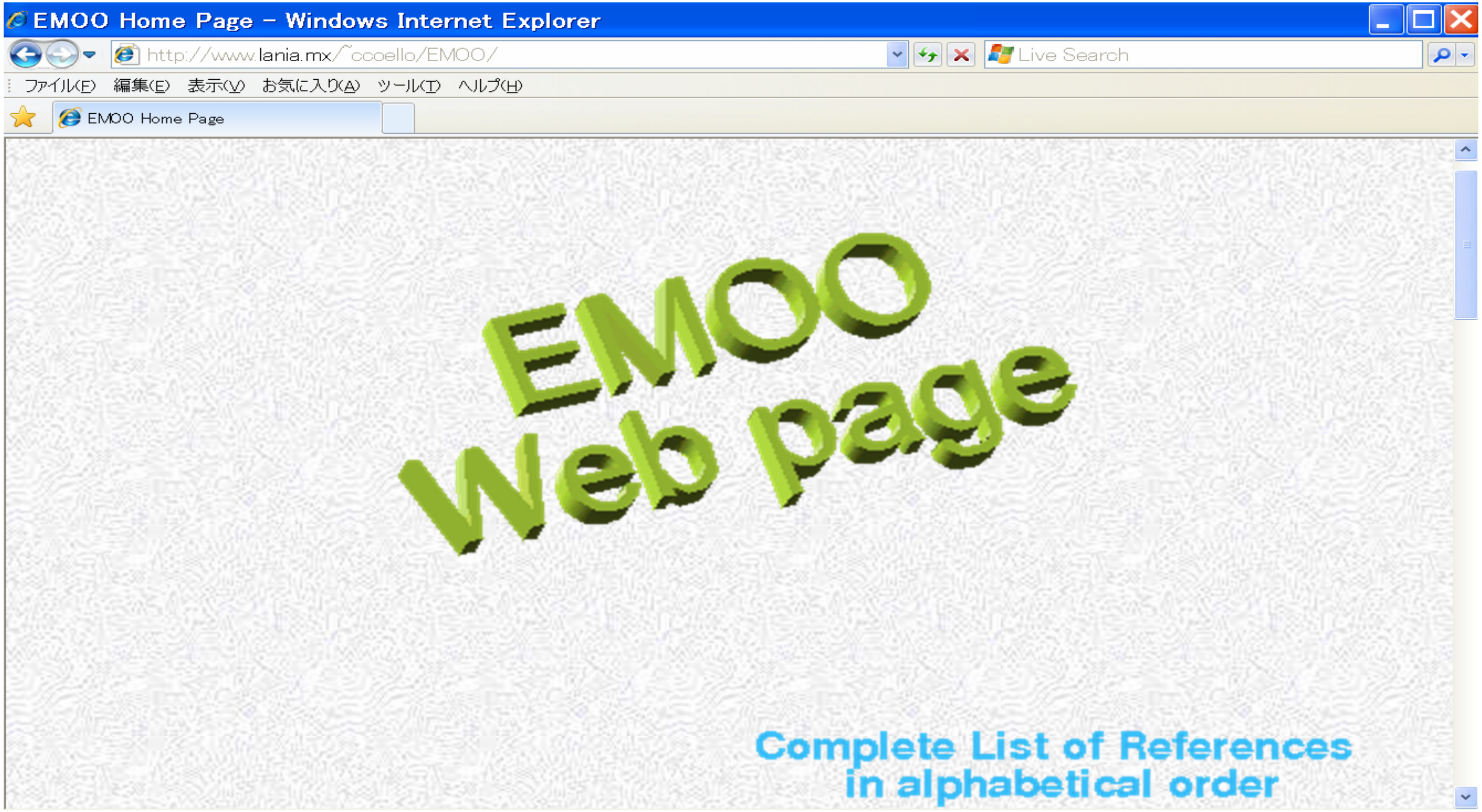
[3] Ray T, Liew KM (2002) A Swarm Metaphor for Multiobjective Design Optimization, Engineering Optimization

[4] Li XD (2003) A Non-Dominated Sorting Particle Swarm Optimizer for Multiobjective Optimization, GECCO 2003.

[5] Ho SL et al. (2005) A Particle Swarm Optimization-Based Method for Multiobjective Design Optimizations, IEEE Trans. on Magnetics

For More Information

Webpage for EMO Papers: EMOO



http://www.lania.mx/~ccoello/EMOO/

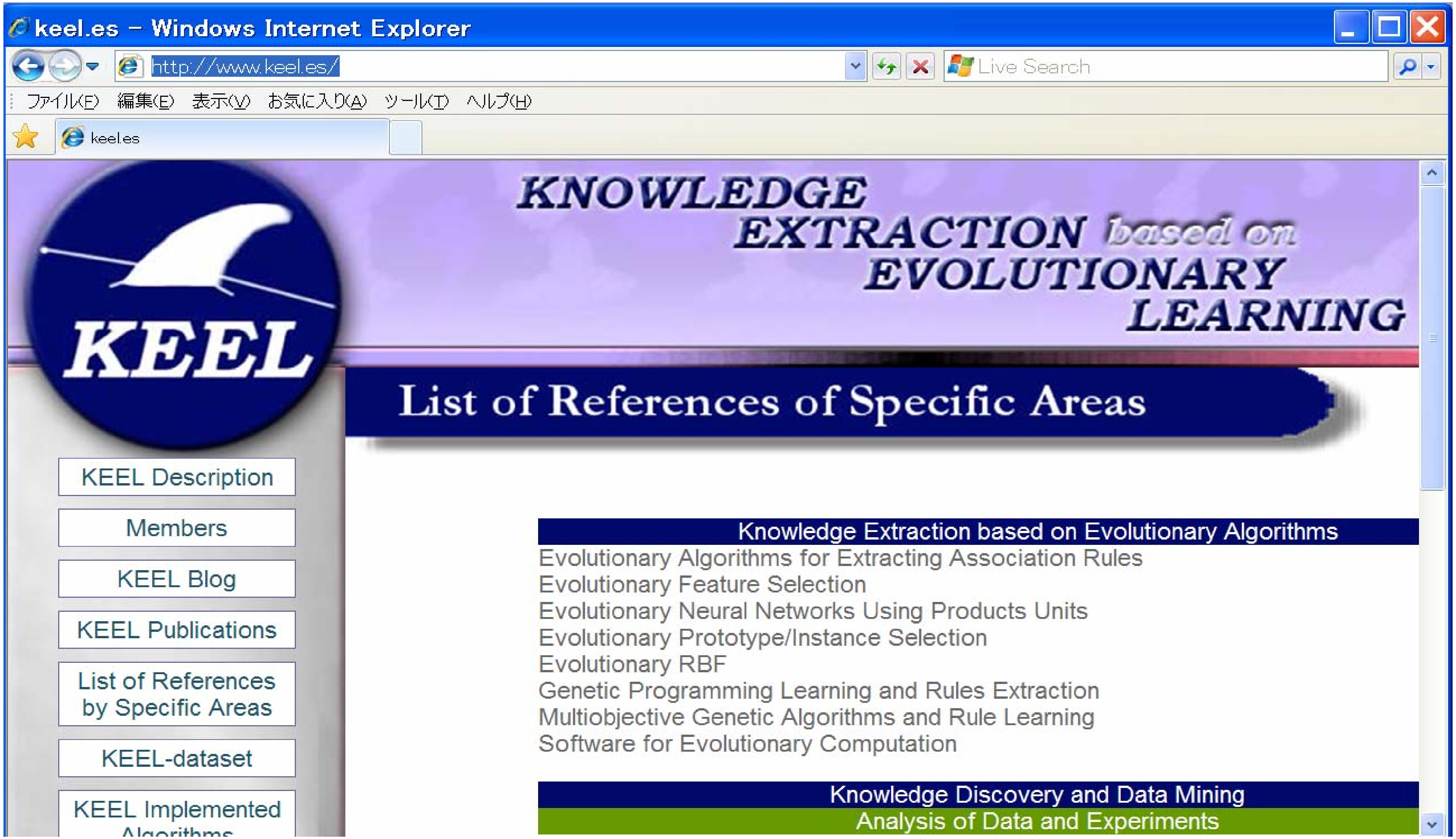
For More Information

Webpage for EMO Algorithms and Problems: PISA

http://www.tik.ee.ethz.ch/sop/pisa/

Webpage for Evolutionary Learning: KEEL

Maintained by Granada University, Spain



http://www.keel.es/

Contents of This Tutorial

1. Introduction
   - Overview of Fuzzy System Design from Numerical Data
2. Evolutionary Multiobjective Optimization (EMO)
   - Some Basic Concepts in Multiobjective Optimization
   - Framework of Evolutionary Multiobjective Optimization
3. Genetic Fuzzy Systems (GFS)
   - Introduction to Genetic Fuzzy System Research
   - Current State of Genetic Fuzzy Systems
4. Interpretability-Accuracy Tradeoff of Fuzzy Systems
   - Interpretability Issues in Fuzzy System Design
   - Some Examples on the Tuning of Fuzzy Systems
5. Multiobjective Genetic Fuzzy Systems (MoGFS)
   - Overview of MoGFS Research
   - New Research Directions in MoGFS

Introduction to genetic fuzzy systems

- **Brief Introduction**
- **Taxonomy of Genetic Fuzzy Systems**
- **Why do we use GAs?**
- **The birth, GFSs roadmap, current state and most cited papers**

Introduction to genetic fuzzy systems

- **Brief Introduction**
- **Taxonomy of Genetic Fuzzy Systems**
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- **The birth, GFSs roadmap, current state and most cited papers**

Introduction to genetic fuzzy systems
Brief Introduction

- The use of genetic/evolutionary algorithms (GAs) to design fuzzy systems constitutes one of the branches of the Soft Computing paradigm: genetic fuzzy systems (GFSs)

- The most known approach is that of genetic fuzzy rule-based systems, where some components of a fuzzy rule-based system (FRBS) are derived (adapted or learnt) using a GA

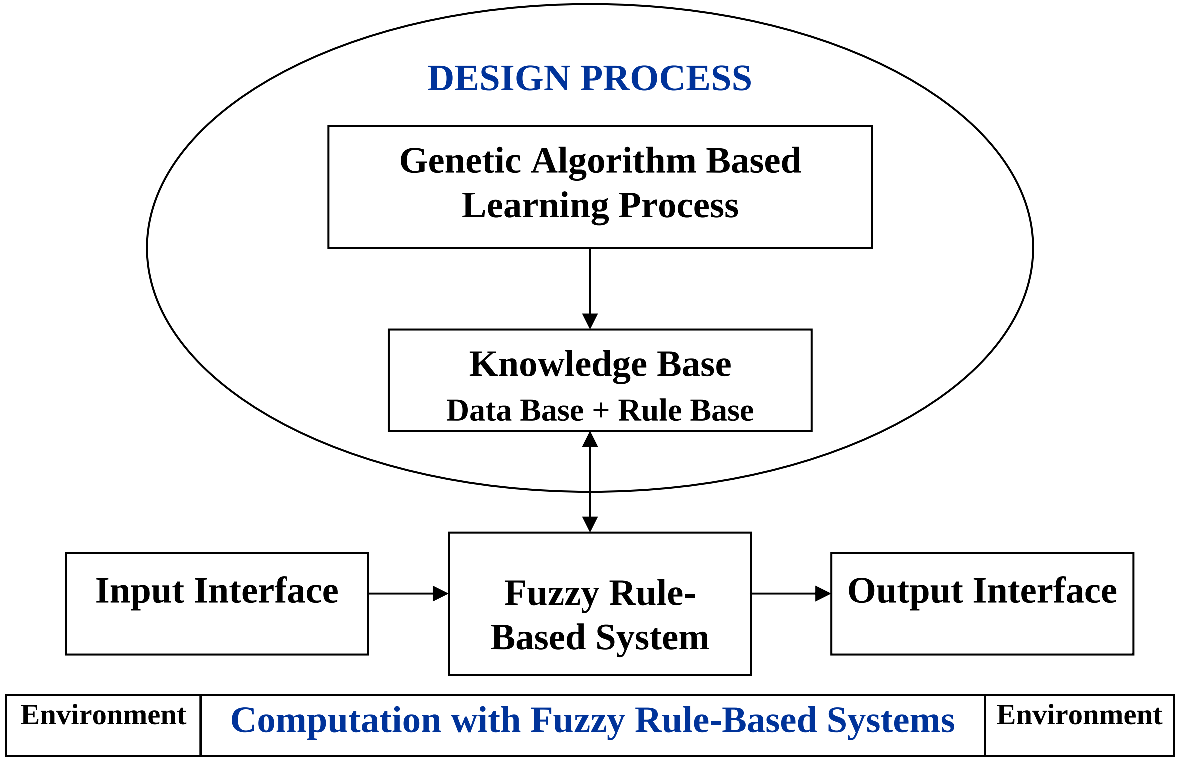
- Some other approaches include genetic fuzzy neural networks and genetic fuzzy clustering, among others

Introduction to genetic fuzzy systems
Brief Introduction

Evolutionary algorithms and machine learning:

- Evolutionary algorithms were not specifically designed as machine learning techniques, like other approaches like neural networks
- However, it is well known that a learning task can be modelled as an optimization problem, and thus solved through evolution
- Their powerful search in complex, ill-defined problem spaces has permitted applying evolutionary algorithms successfully to a huge variety of machine learning and knowledge discovery tasks
- Their flexibility and capability to incorporate existing knowledge are also very interesting characteristics for the problem solving.

Genetic Fuzzy Rule-Based Systems:



Introduction to genetic fuzzy systems
Brief Introduction

Design of fuzzy rule-based systems:

- An FRBS (regardless it is a fuzzy model, a fuzzy logic controller or a fuzzy classifier), is comprised by two main components:
  - The Knowledge Base (KB), storing the available problem knowledge in the form of fuzzy rules
  - The Inference System, applying a fuzzy reasoning method on the inputs and the KB rules to give a system output
- Both must be designed to build an FRBS for a specific application:
  - The KB is obtained from expert knowledge or by machine learning methods
  - The Inference System is set up by choosing the fuzzy operator for each component (conjunction, implication, defuzzifier, etc.)

  Sometimes, the latter operators are also parametric and can be tuned using automatic methods

Introduction to genetic fuzzy systems
Brief Introduction

The KB design involves two subproblems, related to its two subcomponents:

- **Definition of the Data Base (DB):**
  - **Variable universes of discourse**
  - **Scaling factors or functions**
  - **Granularity (number of linguistic terms/labels) per variable**
  - **Membership functions associated to the labels**
- **Derivation of the Rule Base (RB): fuzzy rule composition**

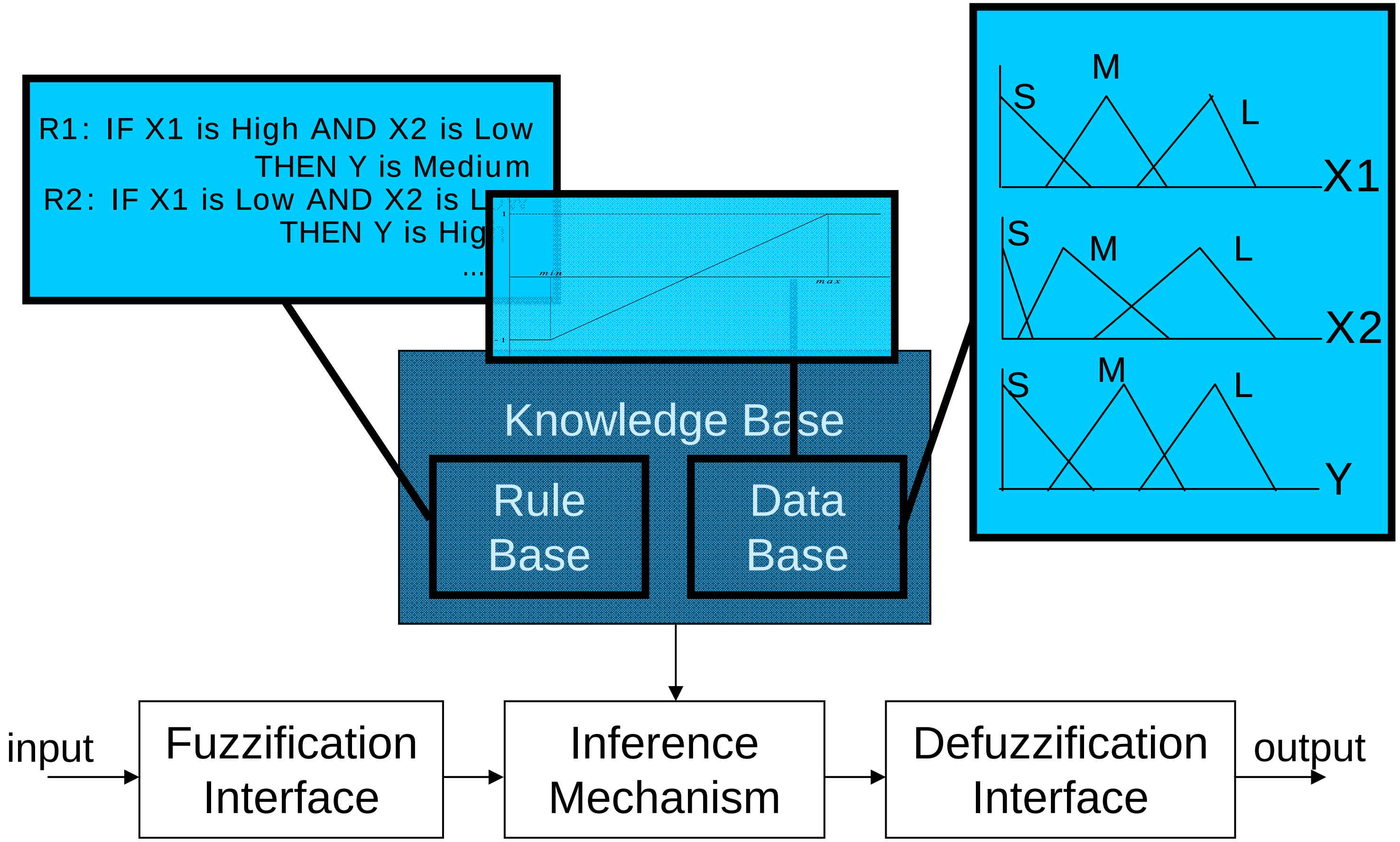
Introduction to genetic fuzzy systems
Brief Introduction

As said, there are two different ways to design the KB:

- **From human expert information**
- **By means of machine learning methods guided by the existing numerical information (fuzzy modeling and classification) or by a model of the system being controlled**

Introduction to genetic fuzzy systems

Brief Introduction

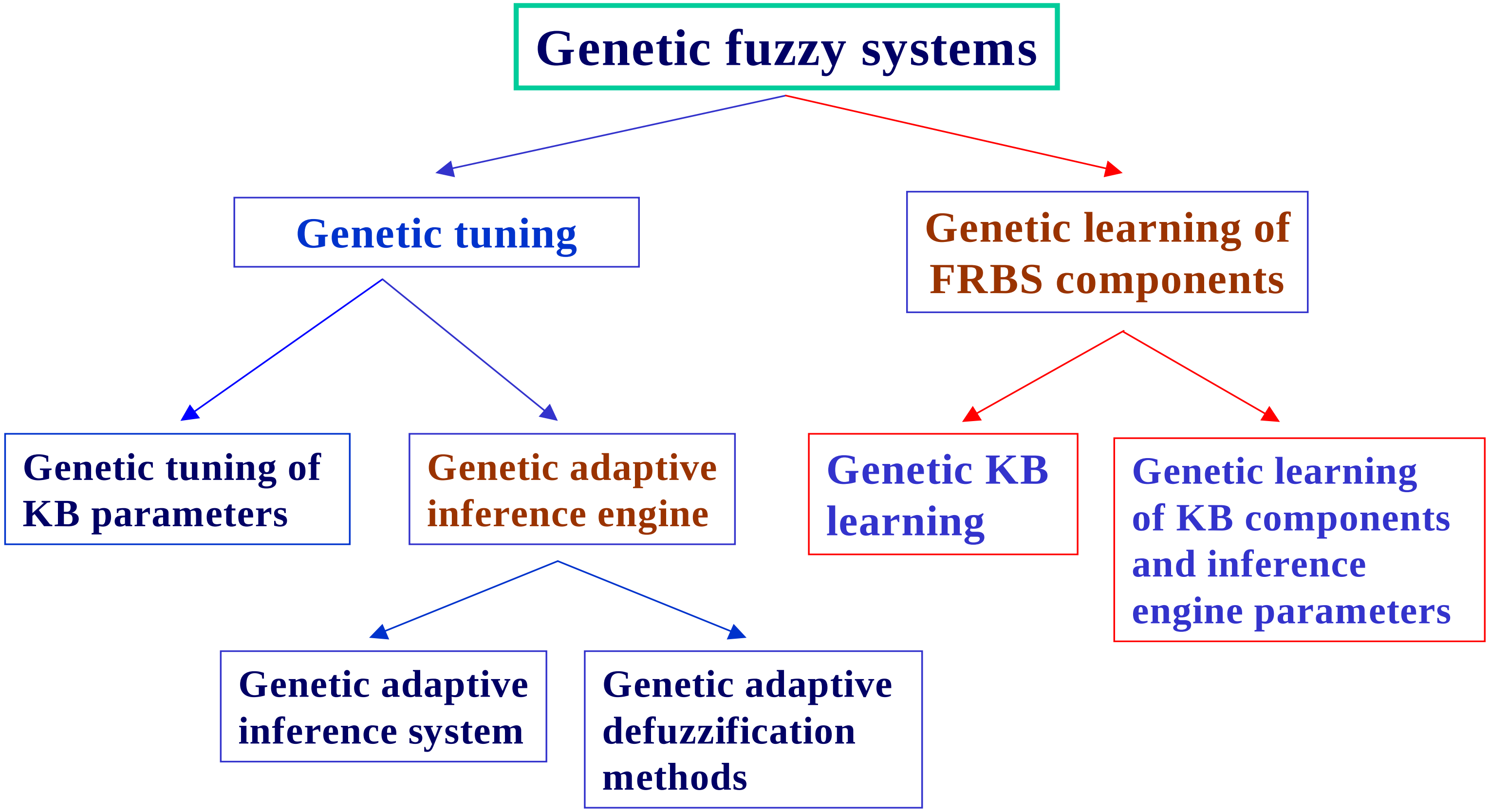


Fuzzy rule-based system

Introduction to genetic fuzzy systems

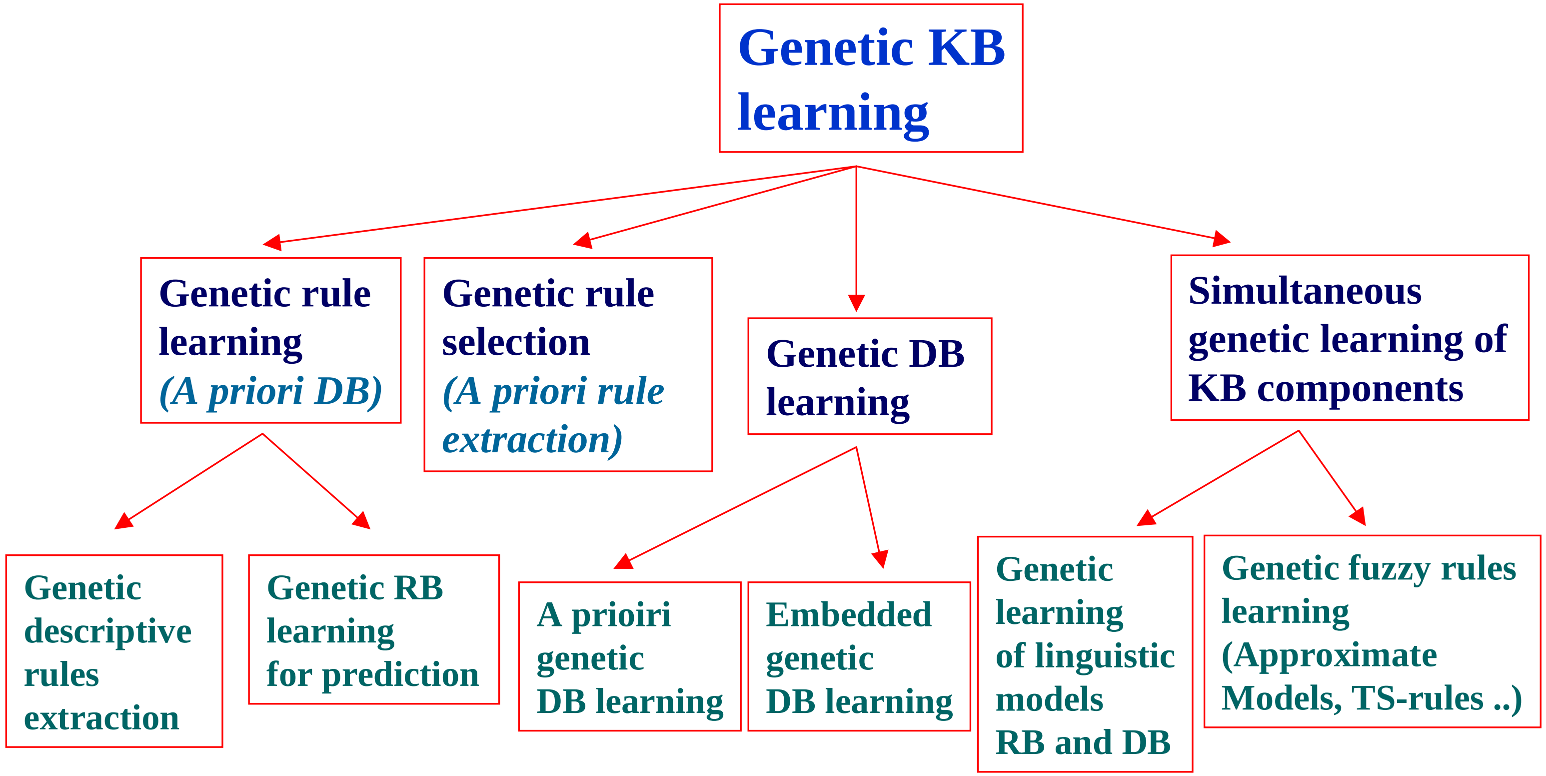
- **Brief Introduction**
- **Taxonomy of Genetic Fuzzy Systems**
- **Why do we use GAs?**
- **The birth, GFSs roadmap, current state and most cited papers**

Introduction to genetic fuzzy systems
Taxonomy of Genetic Fuzzy Systems



Introduction to genetic fuzzy systems

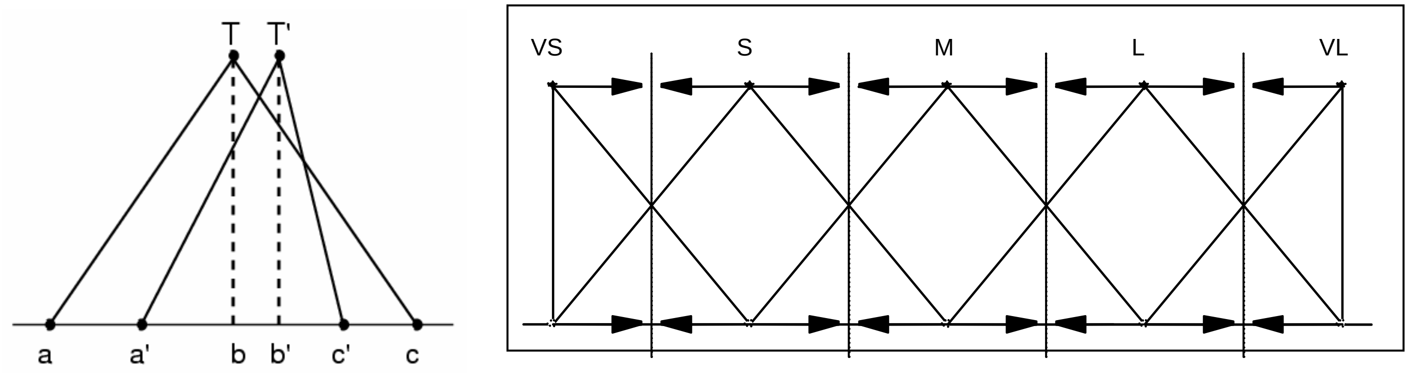
Taxonomy of Genetic Fuzzy Systems



Introduction to genetic fuzzy systems
1. Genetic Tuning

Classically:

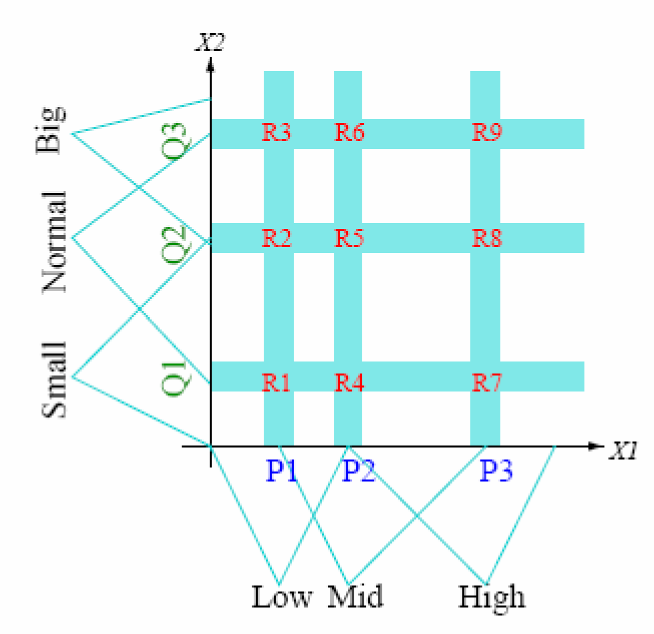
- **performed on a predefined DB definition**
- **tuning of the membership function shapes by a GA**

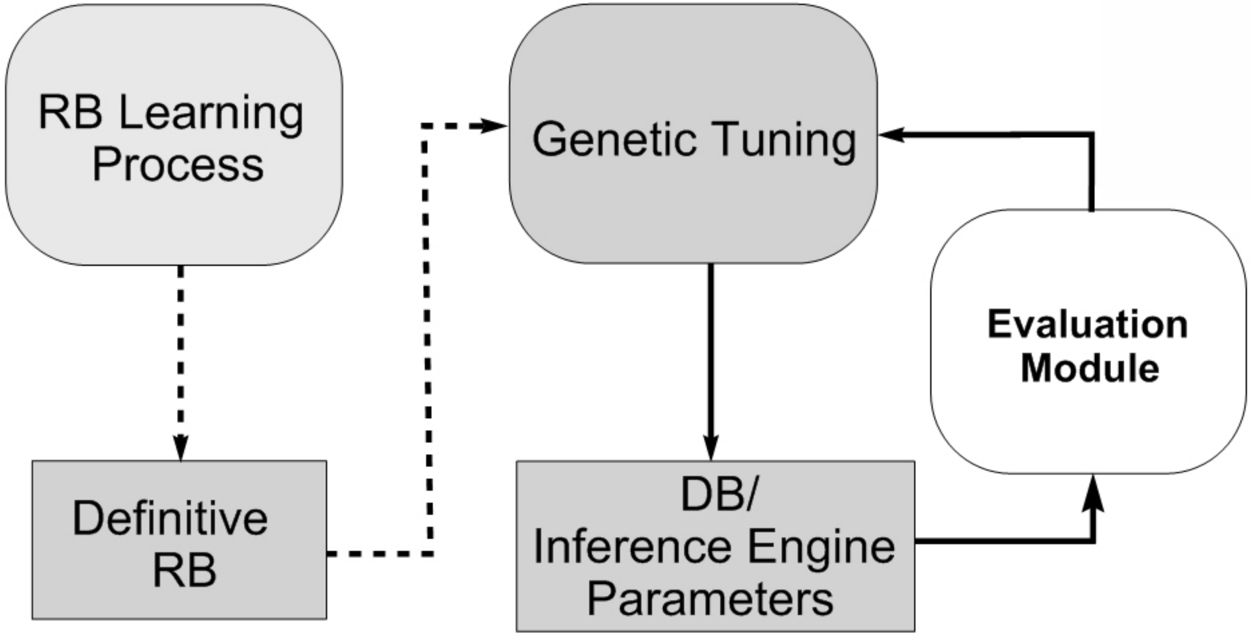


- **tuning of the inference parameters**

Introduction to genetic fuzzy systems

1. Genetic Tuning



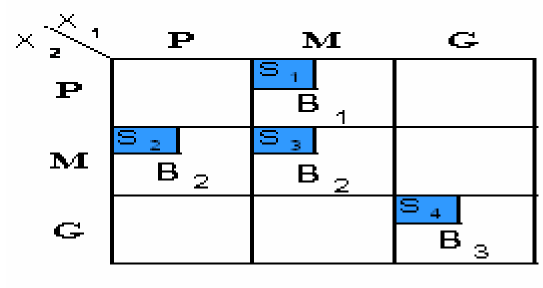


Introduction to genetic fuzzy systems

2. Genetic Rule Learning

A predefined Data Base definition is assumed

– **The fuzzy rules** **(usually Mamdani-type)** **are derived by a GA**



Rule Base

R_1 = IF X_1 is **M** and X_2 is **P** THEN Y is $\mathbf{B_1}$

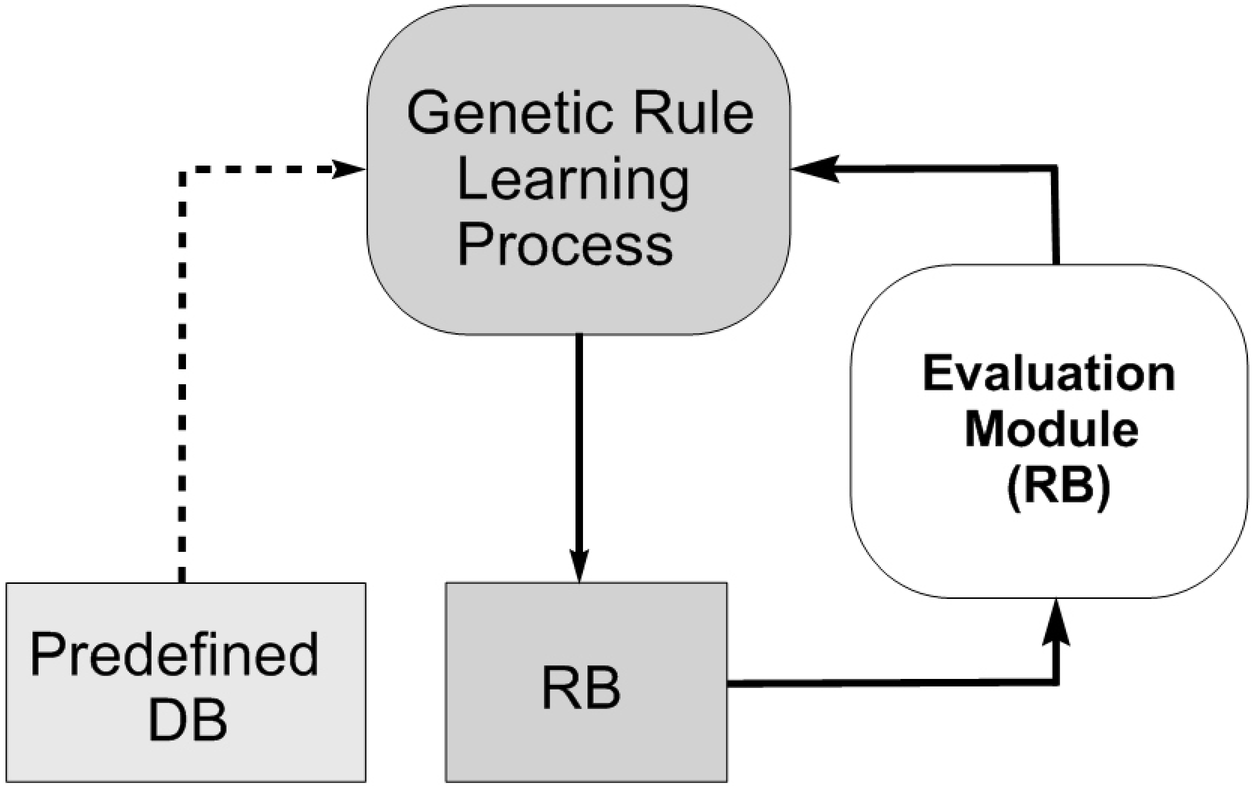
R_2 = IF X_1 is **P** and X_2 is **M** THEN Y is $\mathbf{B_2}$

R_3 = IF X_1 is **M** and X_2 is **M** THEN Y is $\mathbf{B_2}$

R_4 = IF X_1 is **G** and X_2 is **G** THEN Y is $\mathbf{B_3}$

Introduction to genetic fuzzy systems

2. Genetic Rule Learning



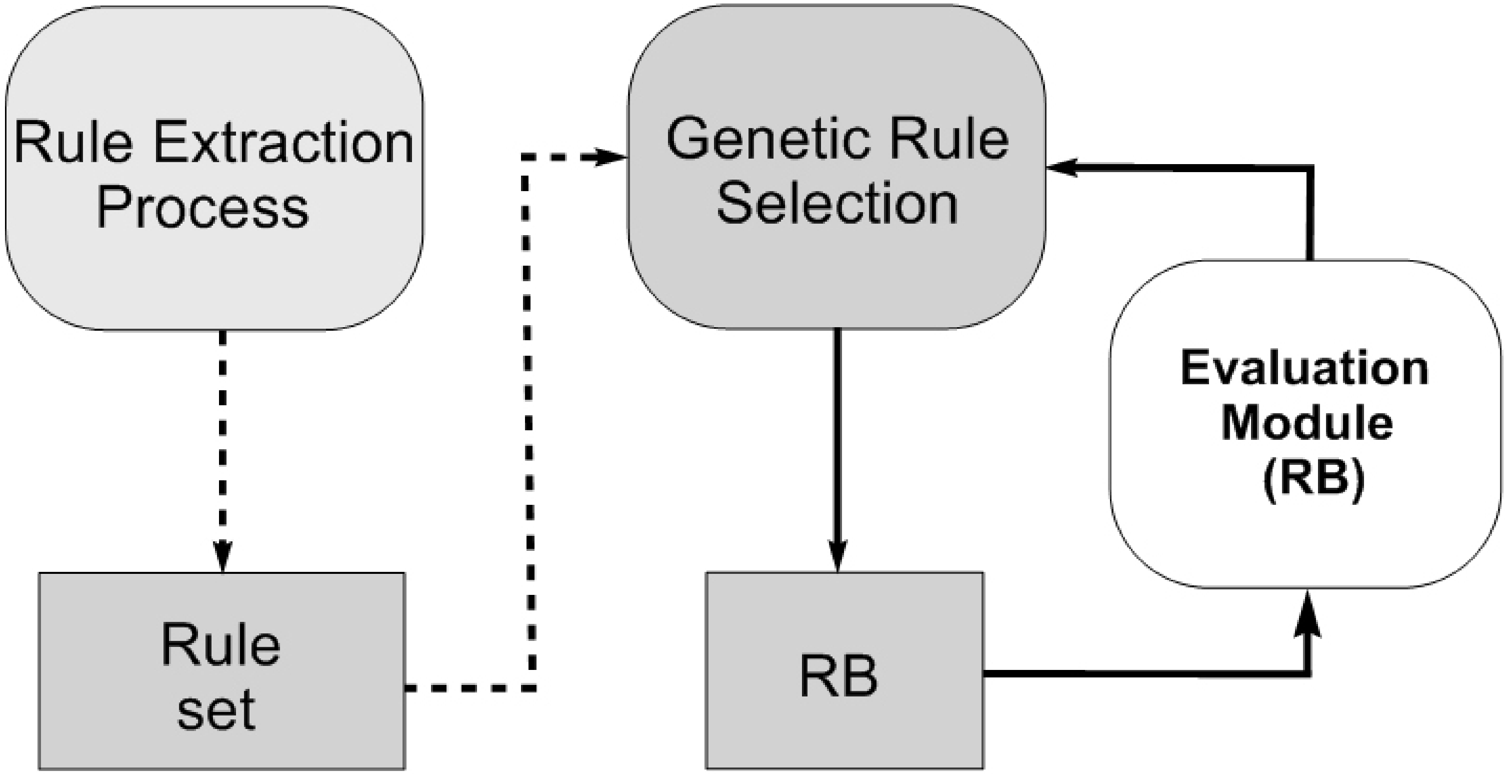
Introduction to genetic fuzzy systems

3. Genetic Rule Selection

- **A predefined Rule Bases definition is assumed**
- **The fuzzy rules are selection by a GA for getting a compact rule base (more interpretable, more precise)**

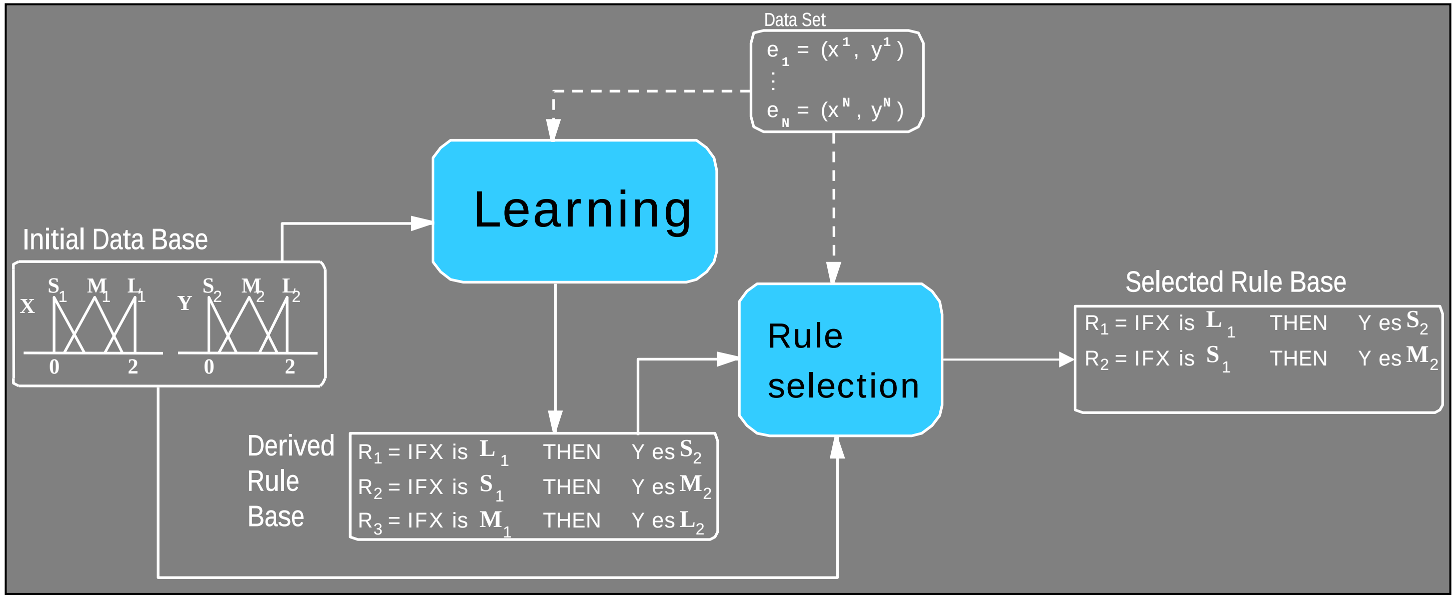
Introduction to genetic fuzzy systems

3. Genetic Rule Selection



Introduction to genetic fuzzy systems

3. Genetic Rule Selection

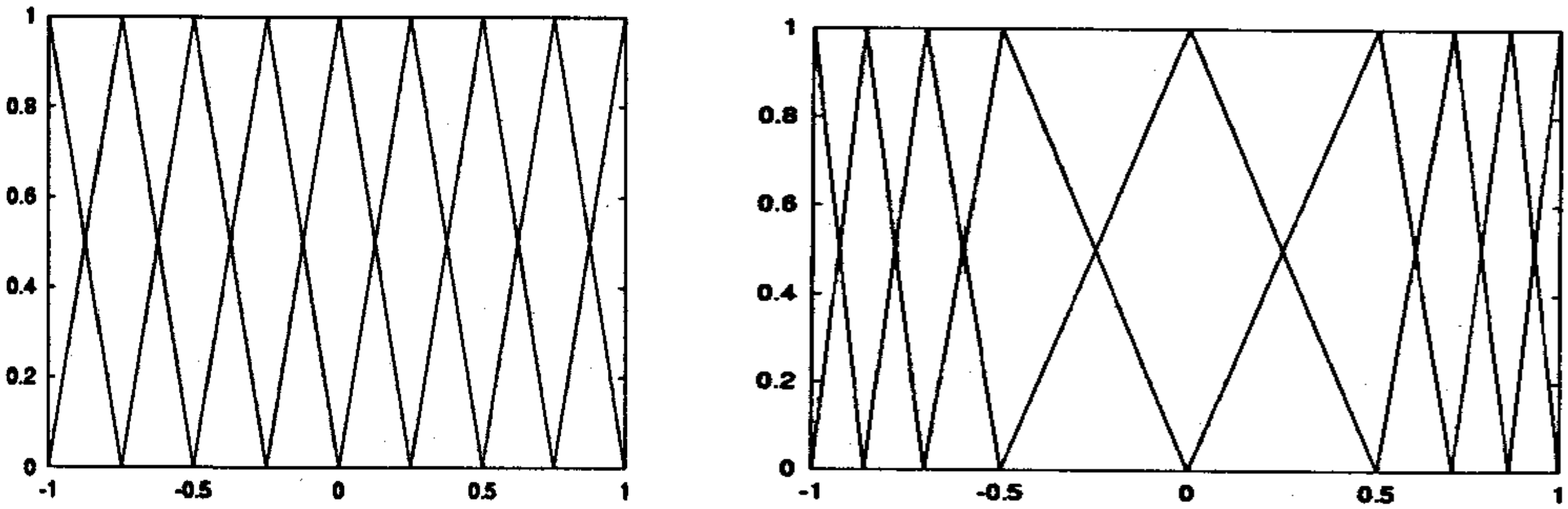


Example of genetic rule selection

Introduction to genetic fuzzy systems

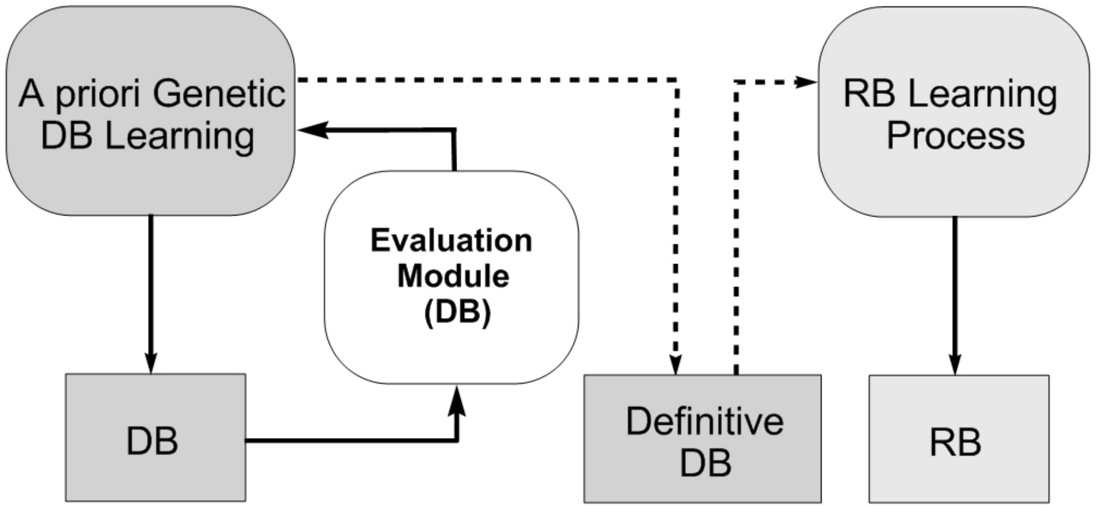
4. Genetic DB Learning

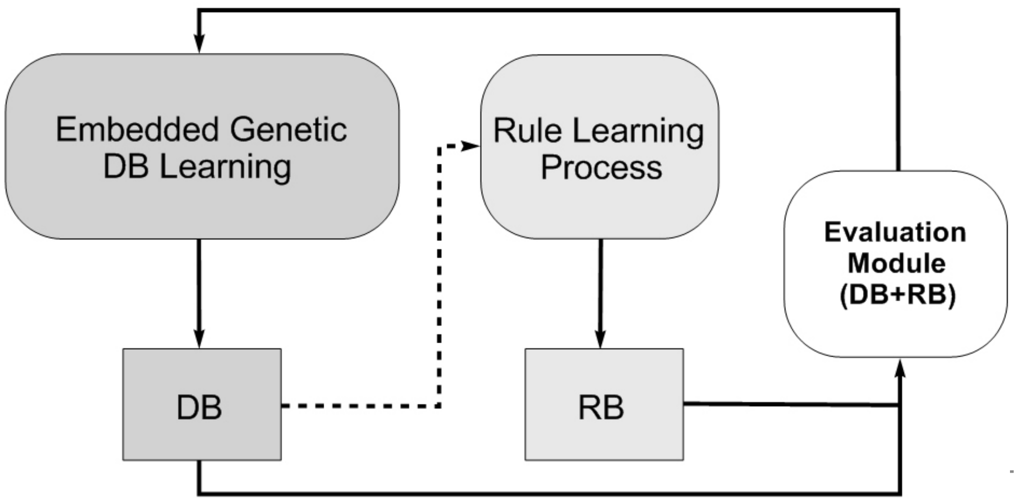
– **Learning of the membership function shapes by a GA**



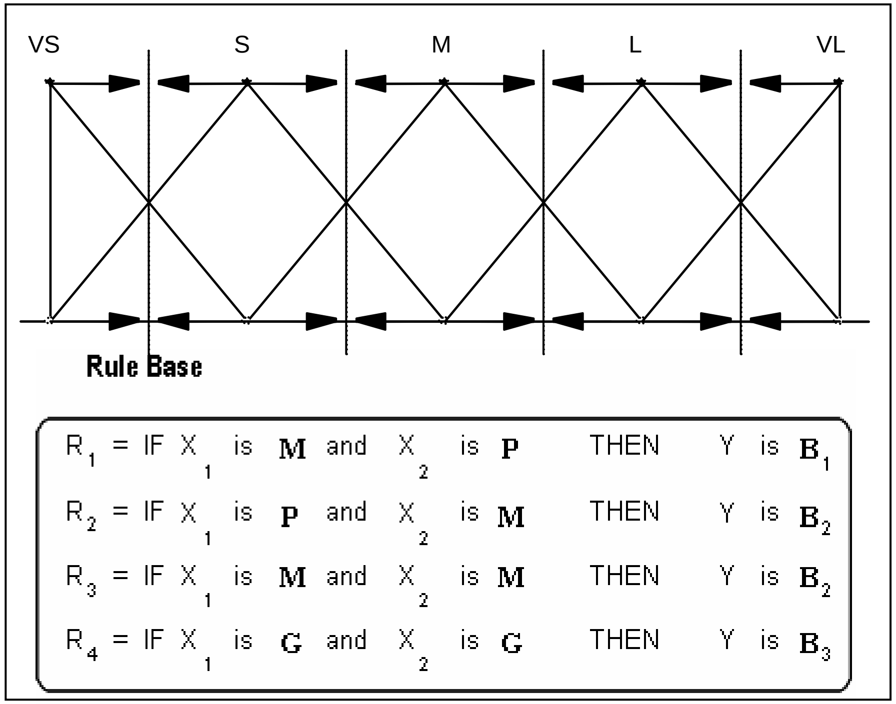
Introduction to genetic fuzzy systems

4. Genetic DB Learning

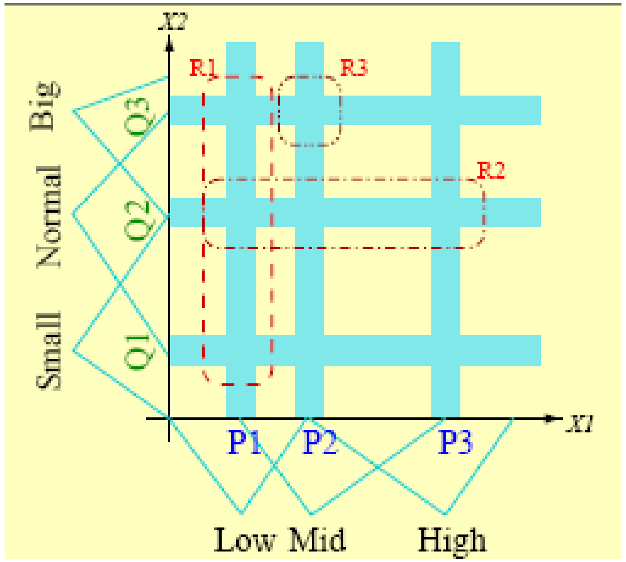


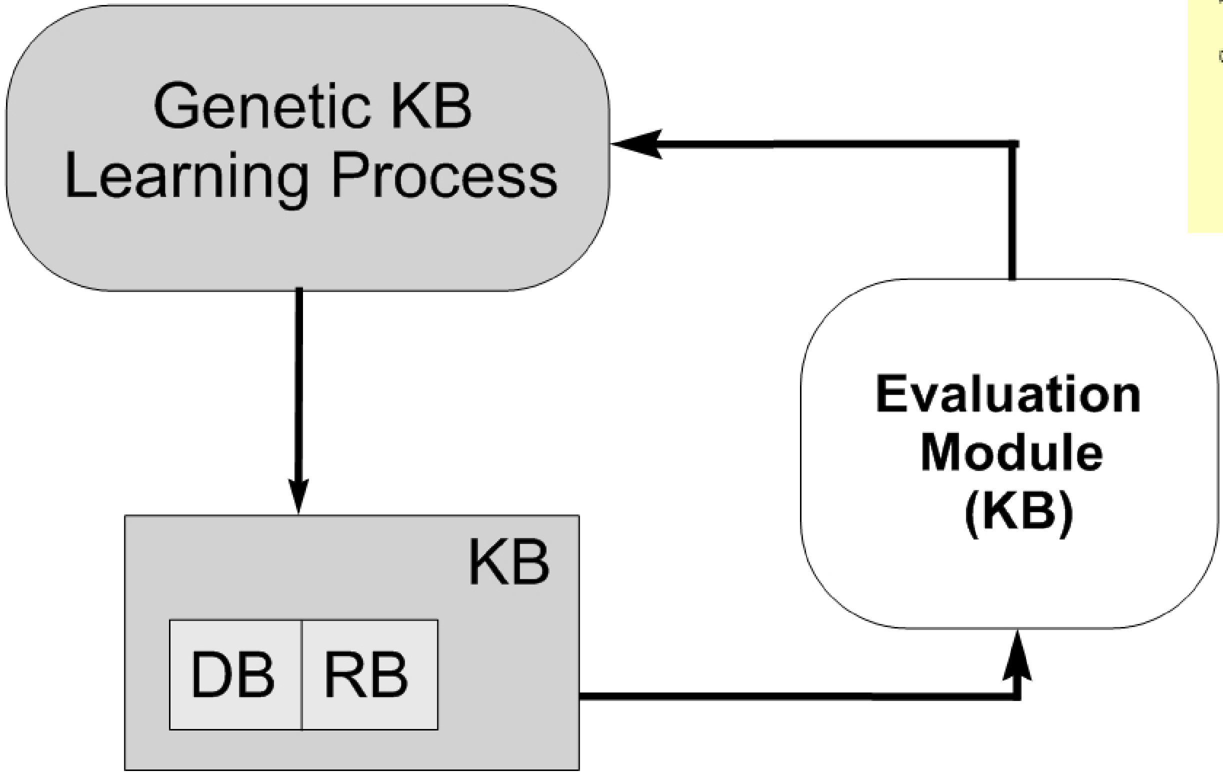


The simultaneous derivation properly addresses the strong dependency existing between the RB and the DB



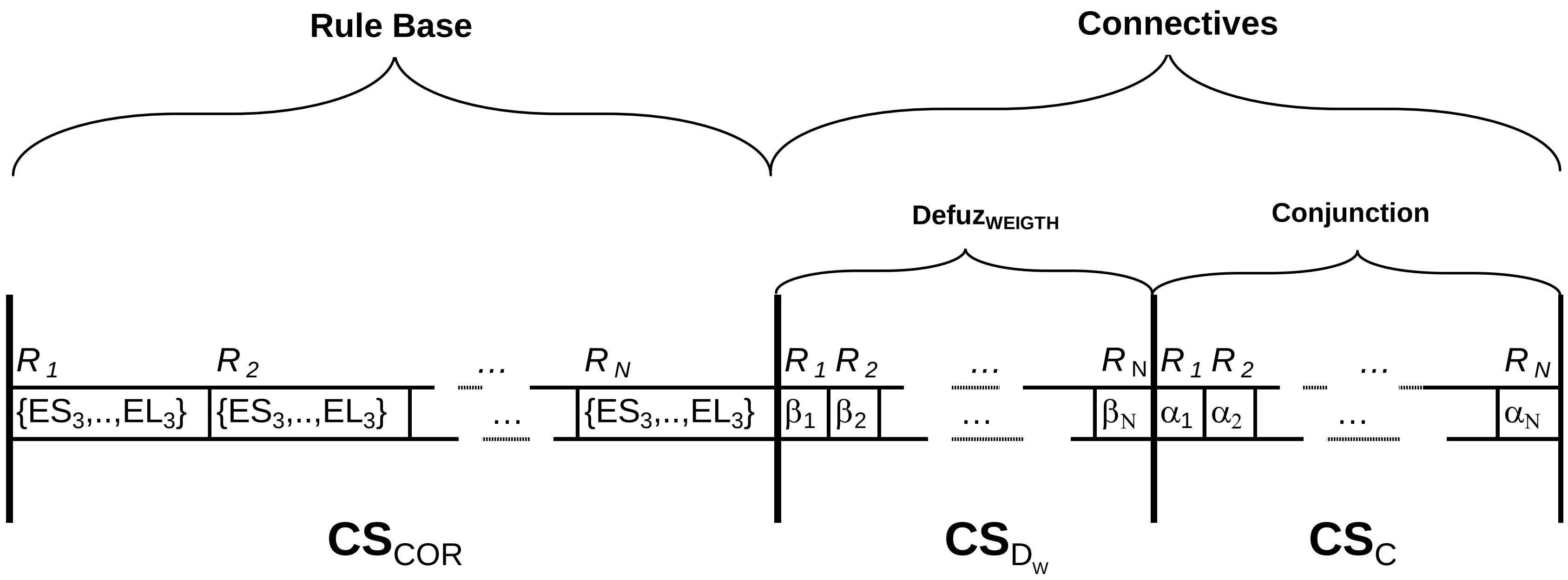
Introduction to genetic fuzzy systems
5. Simultaneous Genetic Learning of KB Components





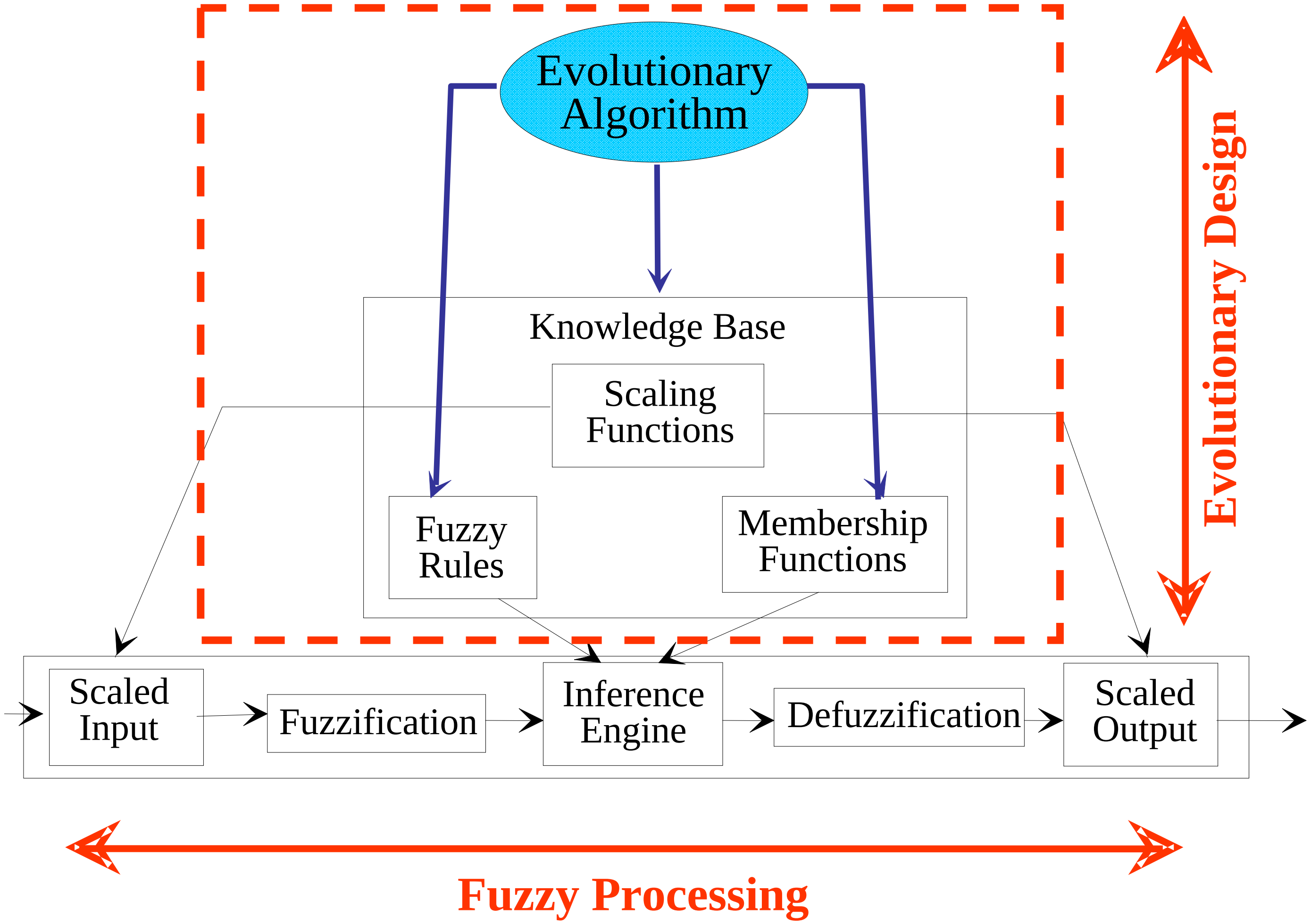
Introduction to genetic fuzzy systems

6. Genetic Learning of KB Components and Inference Engine Parameters



Example of the coding scheme for learning an RB and the inference connective parameters

Introduction to genetic fuzzy systems



Introduction to genetic fuzzy systems

- Brief Introduction
- Taxonomy of Genetic Fuzzy Systems
- Why do we use GAs?
- The birth, GFSs roadmap, current state and most cited papers

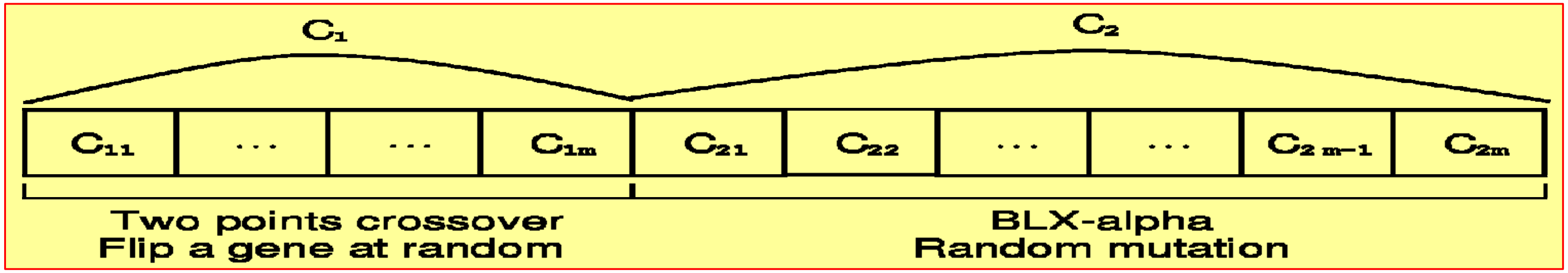
Introduction to genetic fuzzy systems

Why do we use GAs?

Advantages of the Genetic Fuzzy Systems

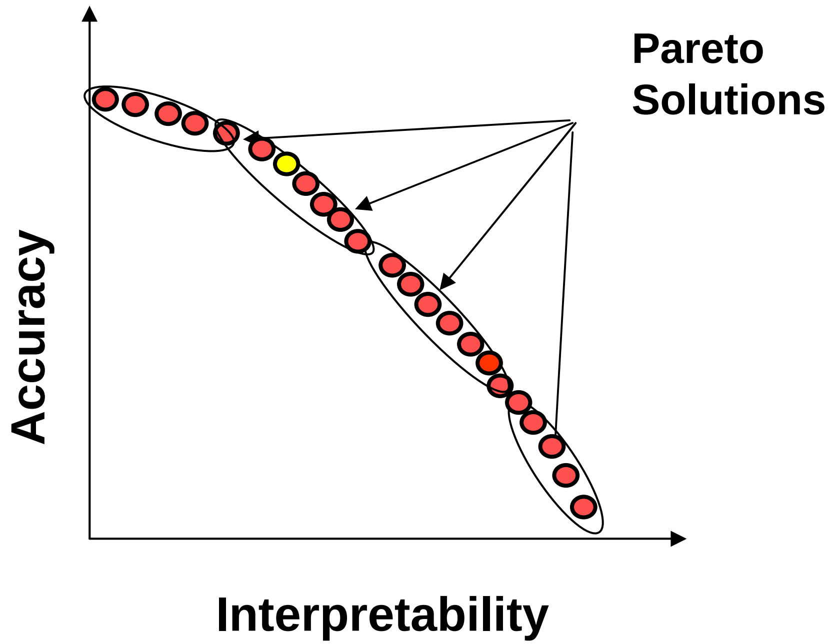
- **We can code different FS components in a chromosome:**
  - **Identify relevant inputs**
  - **Scaling factors**
  - **Membership functions, shape functions, optimal shape of membership funct., granularity (number of labels per variable)**
  - **Fuzzy rules, Any inference parameter, ...**

We can define different mechanism for managing them (combining genetic operators, coevolution,...)



Advantages of the Genetic Fuzzy Systems

- **We can consider multiple objectives in the learning model (interpretability, precision, ….)**



Introduction to genetic fuzzy systems

- **Brief Introduction**
- **Taxonomy of Genetic Fuzzy Systems**
- **Why do we use GAs?**
- **The birth, GFSs roadmap, current state and most cited papers**

Introduction to genetic fuzzy systems

The birth, GFSs roadmap, current status and most cited papers

The birth of GFSs: 1991

- Thrift's ICGA91 paper (Mamdani-type Rule Base Learning. Pittsburgh approach)
- Thrift P (1991) **Fuzzy logic synthesis with genetic algorithms.** *In: Proc. of 4th International Conference on Genetic Algorithms (ICGA'91), pp 509-513*
- Valenzuela-Rendón's PPSN-I paper (Scatter Mamdani-type KB Learning. Michigan approach)
- Valenzuela-Rendon M (1991) **The fuzzy classifier system: A classifier system for continuously varying variables**. *In: Proc. of 4th International Conference on Genetic Algorithms (ICGA'91), pp 346-353*
- Pham and Karaboga's Journal of Systems Engineering paper (Relational matrix-based FRBS learning. Pittsburgh approach)
- Pham DT, Karaboga D (1991) **Optimum design of fuzzy logic controllers using genetic algorithms**. *Journal of Systems Engineering 1:114-118*).
- Karr's AI Expert paper (Mamdani-type Data Base Tuning)
- Karr C (1991) **Genetic algorithms for fuzzy controllers**. *AI Expert 6(2):26-33*.

Almost the whole basis of the area were established in the first year!

Thrift's GFS:

P. Thrift, Fuzzy logic synthesis with genetic algorithms, Proc. Fourth Intl. Conf. on Genetic Algorithms (ICGA'91), San Diego, USA, 1991, pp. 509–513

- **Classical approach: Pittsburgh – the decision table is encoded in a rule consequent array**
- **The output variable linguistic terms are numbered from 1 to n and comprise the array values. The value 0 represents the rule absence, thus making the GA able to learn the optimal number of rules**
- **The ordered structure allows the GA to use simple genetic operators**

X_2 \ X_1	S	M	L
S	R_1 B	R_2 —	R_3 M
M	R_4 —	R_5 M	R_6 —
L	R_7 M	R_8 —	R_9 A

1 2 3

Y → {B, M, A}

1	0	2	0	2	0	2	0	3

GFSs roadmap

1991-1996/7: INITIAL GFS SETTING: KB LEARNING:

- Establishment of the three classical learning approaches in the GFS field: Michigan, Pittsburgh, and IRL
- Different FRBS types: Mamdani, Mamdani DNF, Scatter Mamdani, TSK
- Generic applications: Classification, Modeling, and Control

1995-...: FUZZY SYSTEM TUNING:

- First: Membership function parameter tuning
- Later: other DB components adaptation: scaling factors, context adaptation (scaling functions), linguistic hedges, ...
- Recently: interpretability consideration

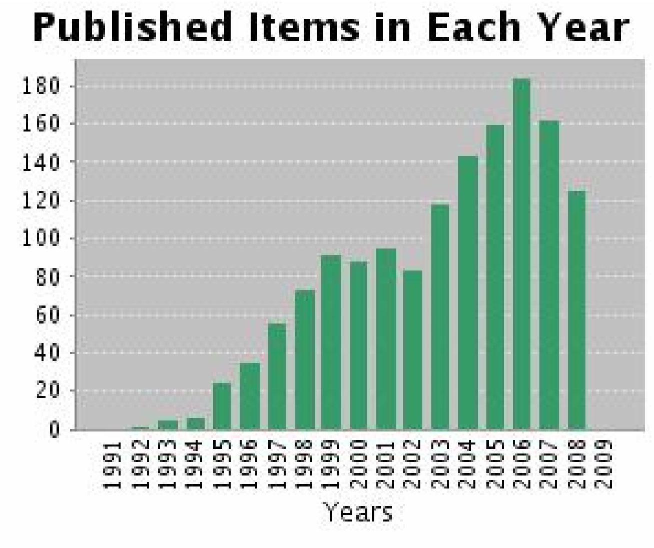
GFSs roadmap

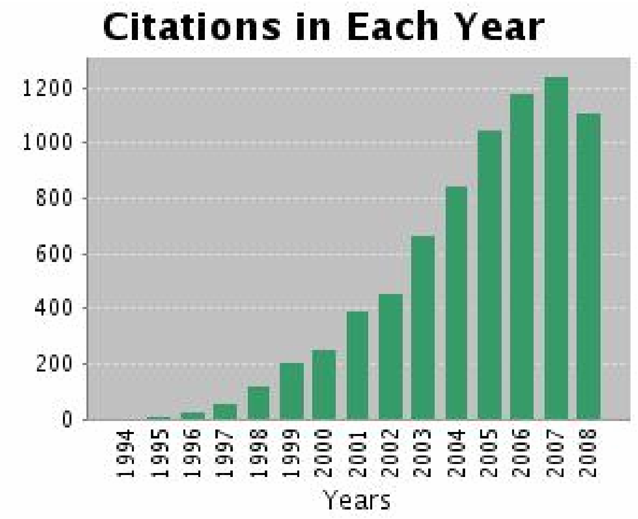
1998-...: APPROACHING TO MATURITY?
NEW GFS LEARNING APPROACHES:

- New EAs: Bacterial genetics, DNA coding, Virus-EA, genetic local search (memetic algorithms), ...
- Hybrid learning approaches: a priori DB learning, GFNNs, Michigan-Pitt hybrids, ...
- Multiobjective evolutionary algorithms
- Interpretability-accuracy trade-off consideration
- Course of dimensionality (handling large data sets and complex problems):
  - Rule selection (1995-...)
  - Feature selection at global level and fuzzy rule level
  - Hierarchical fuzzy modeling
- “Incremental” learning

Introduction to genetic fuzzy systems
Current state of the GFS area

Number of papers on GFSs published in JCR journals





Source: **The Thomson Corporation ISI Web of Knowledge**

Query: **(TS = (("GA-" OR "GA based" OR evolutionary OR "genetic algorithm*" OR "genetic programming" OR "evolution strate*" OR "genetic learning" OR "particle swarm" OR "differential evolutio*" OR "ant system*" OR "ant colony" OR "genetic optimi*" OR "estimation of distribution algorithm*") AND ("fuzzy rule*" OR "fuzzy system*" OR "fuzzy neural" OR "neuro-fuzzy" OR "fuzzy control*" OR "fuzzy logic cont*" OR "fuzzy class*" OR "fuzzy if" OR "fuzzy model*" OR "fuzzy association rule*" OR "fuzzy regression"))**

Number of papers: **1459** Number of citations: **5,237,630**

Average citations per paper: **5.23**

Introduction to genetic fuzzy systems

Current state of the GFS area

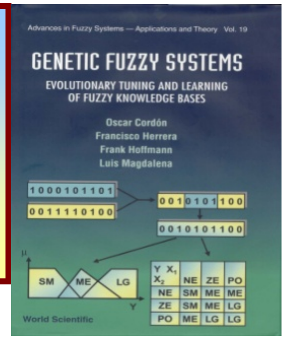
Most cited papers on GFSs

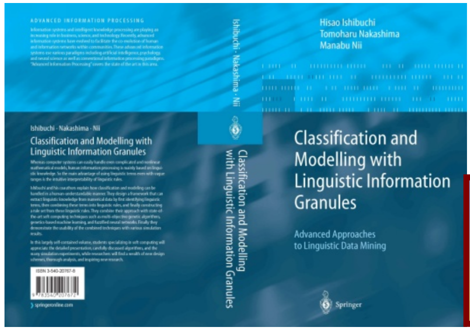
1. **Homaifar, A., McCormick, E., Simultaneous Design of Membership Functions and rule sets for fuzzy controllers using genetic algorithms, IEEE TFS 3 (2) (1995) 129-139.** Citations: 184
2. **Ishibuchi, H., Nozaki, K., Yamamoto, N., Tanaka, H., Selecting fuzzy if-then rules for classification problems using genetic algorithms, IEEE TFS 3 (3) (1995) 260-270.** Citations: 164
3. **Setnes, M., Roubos, H., GA-fuzzy modeling and classification: complexity and performance, IEEE TFS 8 (5) (2000) 509-522 .** Citations: 101
4. **Ishibuchi, H., Nakashima, T., Murata, T., Performance evaluation of fuzzy classifier systems for multidimensional pattern classification problems, IEEE TSMC B 29 (5) (1999) 601-618.** Citations: 93
5. **Park, D., Kandel, A., Langholz, G., Genetic-based new fuzzy reasoning models with application to fuzzy control, IEEE TSMC B 24 (1) (1994) 39-47.** Citations: 86
6. **Herrera, F., Lozano, M., Verdegay, J.L., Tuning fuzzy-logic controllers by genetic algorithms, IJAR 12 (3-4) (1995) 299-315.** Citations: 71
7. **Shi, Y.H., Eberhart, R., Chen, Y.B., Implementation of evolutionary fuzzy systems, IEEE TFS 7 (2) (1999) 109-119.** Citations: 63
8. **Carse B., Fogarty, TC., Munro, A., Evolving fuzzy rule based controllers using genetic algorithms, FSS 80 (3) (1996) 273-293.** Citations: 63
9. **Juang, C.F., Lin, J.Y., Lin, C.T., Genetic reinforcement learning through symbiotic evolution for fuzzy controller design, IEEE TSMC B 30 (2) (2000) 290-302.** Citations: 59
10. **Cordon, O., Herrera, F., A three-stage evolutionary process for learning descriptive and approximate fuzzy-logic-controller knowledge bases from examples, IJAR 17 (4) (1997) 369-407.** Citations: 58

Introduction to genetic fuzzy systems

Some References

GENETIC FUZZY SYSTEMS
Evolutionary Tuning and Learning of Fuzzy Knowledge Bases.
O. Cordón, F. Herrera, F. Hoffmann, L. Magdalena
World Scientific, July 2001





H. Ishibuchi, T. Nakashima, M. Nii, Classification and Modeling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer (2005)

– F. Herrera, Genetic Fuzzy Systems: Taxonomy, Current Research Trends and Prospects. *Evolutionary Intelligence 1 (2008) 27-46 doi: 10.1007/s12065-007-0001-5,*

– F. Herrera, Genetic Fuzzy Systems: Status, Critical Considerations and Future Directions, International Journal of Computational Intelligence Research 1 (1) (2005) 59-67

– O. Cordón, F. Gomide, F. Herrera, F. Hoffmann, L. Magdalena, Ten Years of Genetic Fuzzy Systems: Current Framework and New Trends, FSS 141 (1) (2004) 5-31

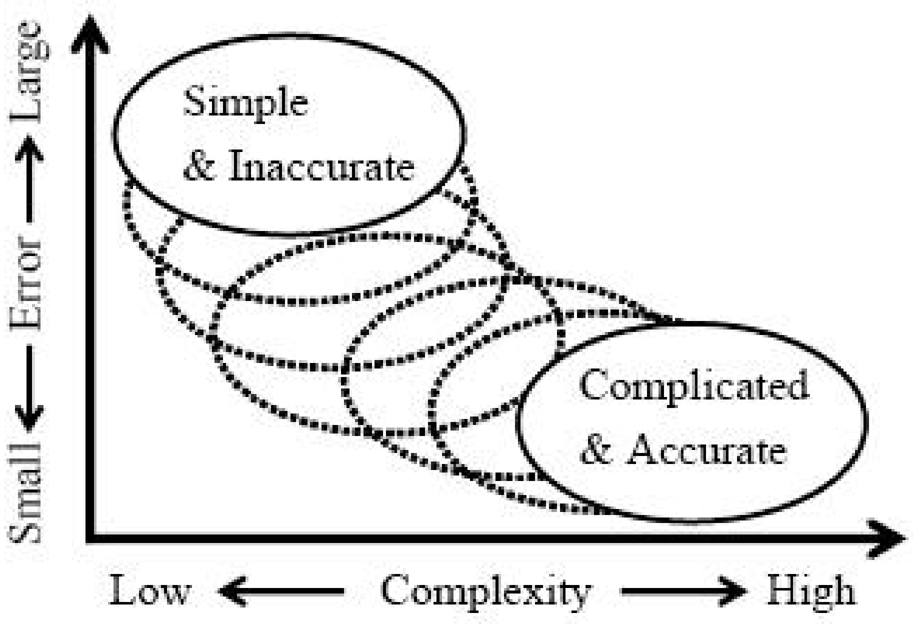
– F. Hoffmann, Evolutionary Algorithms for Fuzzy Control System Design, Proceedings of the IEEE 89 (9) (2001) 1318-1333

Contents of This Tutorial

1. Introduction
   - Overview of Fuzzy System Design from Numerical Data
2. Evolutionary Multiobjective Optimization (EMO)
   - Some Basic Concepts in Multiobjective Optimization
   - Framework of Evolutionary Multiobjective Optimization
3. Genetic Fuzzy Systems (GFS)
   - Introduction to Genetic Fuzzy System Research
   - Current State of Genetic Fuzzy Systems
4. Interpretability-Accuracy Tradeoff of Fuzzy Systems
   - Interpretability Issues in Fuzzy System Design
   - Some Examples on the Tuning of Fuzzy Systems
5. Multiobjective Genetic Fuzzy Systems (MoGFS)
   - Overview of MoGFS Research
   - New Research Directions in MoGFS

Interpretability Issues in Fuzzy System Design
Complexity Criteria

- **Highly used criteria: Complexity criteria in the learning of FRBSs.**



Number of variables, labels, rules, conditions ...

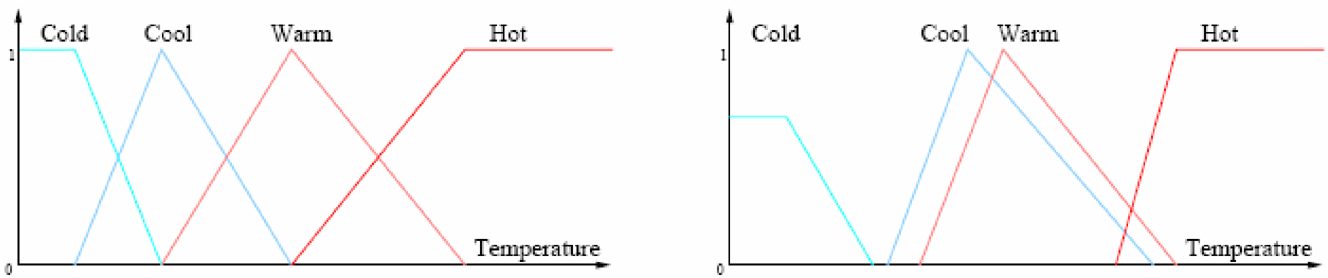
Interpretability Issues in Fuzzy System Design
Semantic Criteria

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

Interpretability considerations: semantic criteria

Semantics: the study of meanings

- Distinguishability: Each linguistic label has semantic meaning
- Number of elements: Compatible with human capabilities
- Coverage: Any element belongs to at least one fuzzy set
- Normalization: At least one element has unitary membership
- Complementarity: For each element, the sum of memberships is one



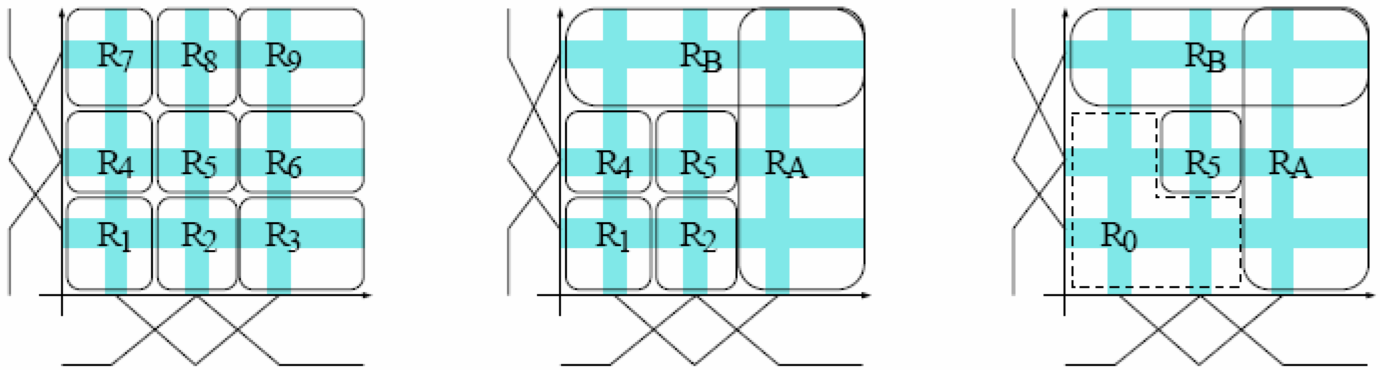
Interpretability Issues in Fuzzy System Design
Syntactic Criteria

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

Interpretability considerations: syntactic criteria

Syntax: the way in which linguistic elements are put together

- Completeness: for any input, at least one rule must fire
- Rule−base simplicity: Set of rules as small as possible
- Rule readability: small number of conditions in rule antecedents
- Consistency: rules firing simultaneously must have similar consequents

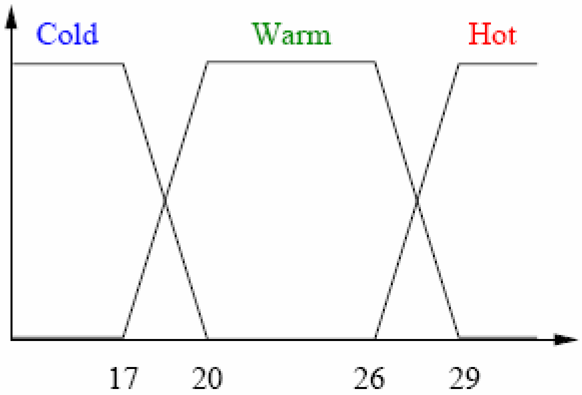


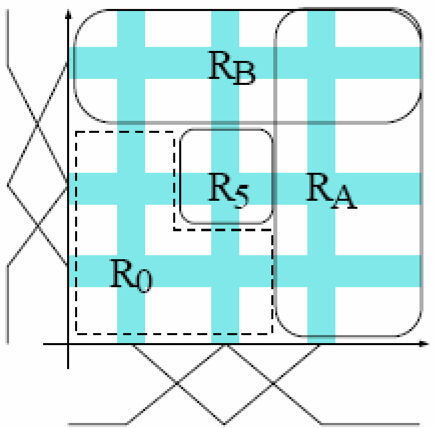
Interpretability Issues in Fuzzy System Design
Strategies to Satisfy Interpretability

- **Interpretability quality:** associated to the meaning of the labels and the size of the rule base

Strategies to satisfy interpretability criteria

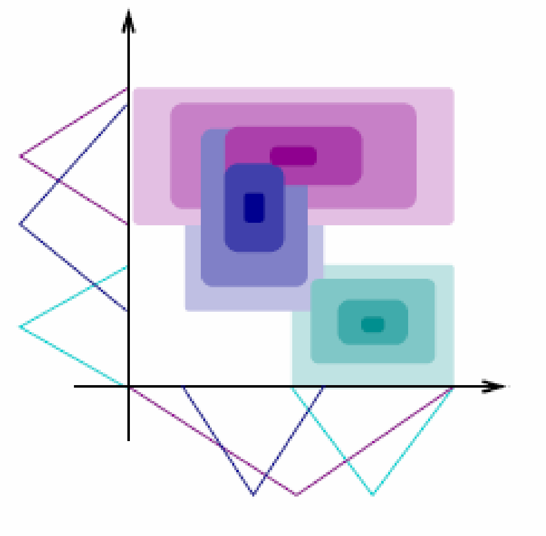
- Linguistic labels shared by all rules
- Normal, orthogonal membership functions
- Don’t care conditions

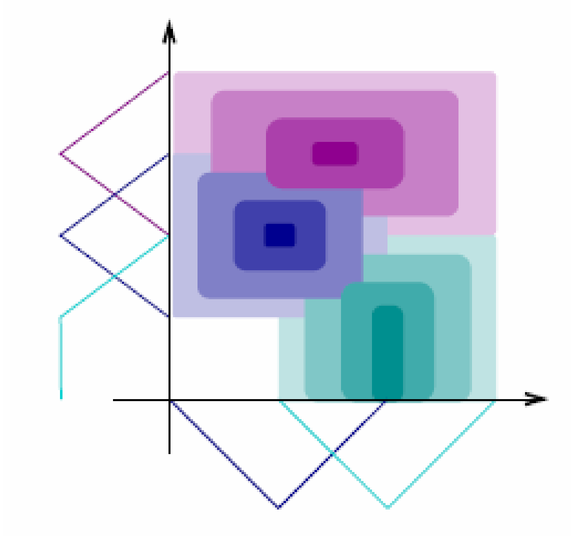




Interpretability Issues in Fuzzy System Design
Still not Clear Concepts

- **Interpretability quality:**





What is the most interpretable rule base?

Some Approaches

Some Models to improve the Trade-Off:

- **Advanced RB Generation** +
- **Linguistic Modifiers Use**
- **Using Double Consequent Rules**
- **Tuning of Membership Functions**
- **Using Weighted Rules**
- **Using Multiple Consequent Rules** -

Interpretability

Evolutionary Tuning of FRBSs

Tuning of Membership Functions

- **A genetic tuning process that slightly adjusts the shapes of the membership functions of a preliminary DB definition**

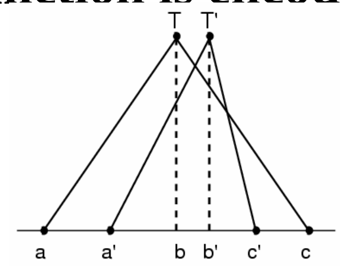
- **Each chromosome encodes a whole DB definition by joining the partial coding of the different membership functions involved**

- **The coding scheme depends on:**
  - **The kind of membership function considered (triangular, trapezoidal, bell-shaped, …) $\rightarrow$ different real-coded definition parameters**
  - **The kind of FRBS:**
    - **Grid-based: Each linguistic term in the fuzzy partition has a single fuzzy set definition associated**
    - **Non grid-based (free semantics, scatter partitions, fuzzy graphs): each variable in each rule has a different membership function definition**

Evolutionary Tuning of FRBSs
Tuning of Membership Functions

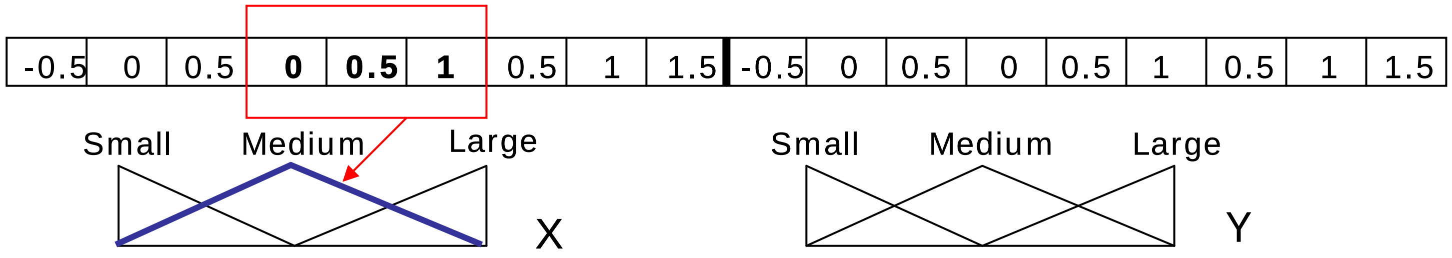
- **Example: Tuning of the triangular membership functions of a grid-based SISO Mamdani-type FRBS, with three linguistic terms for each variable fuzzy partition**

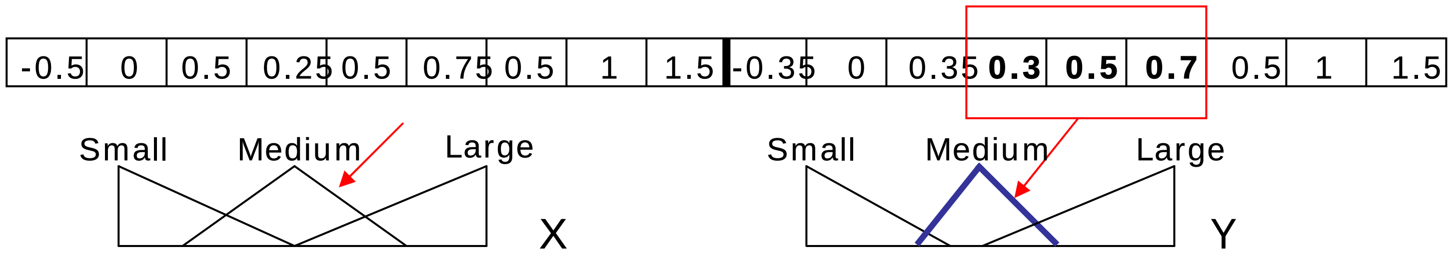
- **Each chrosome encodes a different DB definition:**
  - **2 (variables) · 3 (linguistic labels) = 6 membership functions**
  - **Each triangular membership function is encoded by 3 real values (the three definition points):**
  - **So, the chromosome length is 6 · 3 = 18 real-coded genes (binary coding can be used but but is not desirable)**



- **Either definition intervals have to be defined for each gene and/or appropriate genetic operators in order to obtain meaningful membership functions**

Evolutionary Tuning of FRBSs
Tuning of Membership Functions





The RB remains unchanged!

R1: IF X1 is Small THEN Y is Large
R2: IF X1 is Medium THEN Y is Med
. . .

Evolutionary Tuning of FRBSs

References

- C. Karr, Genetic algorithms for fuzzy controllers, AI Expert 6 (2) (1991) 26–33
- C. Karr, E.J. Gentry, Fuzzy control of pH using genetic algorithms, IEEE TFSs 1 (1) (1993) 46–53
- J. Kinzel, F. Klawonn, R. Kruse, Modifications of genetic algorithms for designing and optimizing fuzzy controllers, Proc. First IEEE Conf. on Evolutionary Computation (ICEC'94), Orlando, FL, USA, 1994, pp. 28–33
- D. Park, A. Kandel, G. Langholz, Genetic-based new fuzzy reasoning models with application to fuzzy control, IEEE TSMC 24 (1) (1994) 39–47
- F. Herrera, M. Lozano, J.L. Verdegay, Tuning fuzzy controllers by genetic algorithms, IJAR 12 (1995) 299–315
- P.P. Bonissone, P.S. Khedkar, Y. Chen, Genetic algorithms for automated tuning of fuzzy controllers: a transportation application, in Proc. Fifth IEEE Int. Conf. on Fuzzy Systems (FUZZ-IEEE'96), New Orleans, USA, 1996, pp. 674–680
- O. Cordón, F. Herrera, A three-stage evolutionary process for learning descriptive and approximate fuzzy logic controller knowledge bases from examples, IJAR 17 (4) (1997) 369–407
- H.B. Gurocak, A genetic-algorithm-based method for tuning fuzzy logic controllers, FSS 108 (1) (1999) 39–47

Evolutionary Tuning of FRBSs

Genetic tuning of DB and RB using linguistic edges

J. Casillas, O. Cordón, M.J. del Jesus, F. Herrera, Genetic tuning of fuzzy rule deep structures preserving interpretability and its interaction with fuzzy rule set reduction, IEEE TFS 13 (1) (2005) 13-29

Genetic tuning process that refines a preliminary KB working at two different levels:

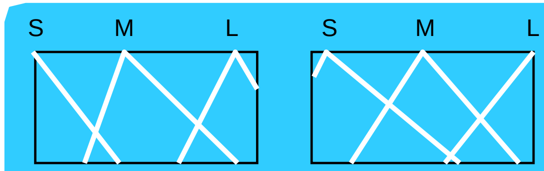
- **DB level: Linearly or non-linearly adjusting the membership function shapes**
- **RB level: Extending the fuzzy rule structure using automatically learnt linguistic hedges**

Evolutionary Tuning of FRBSs

Genetic tuning of DB and RB using linguistic edges

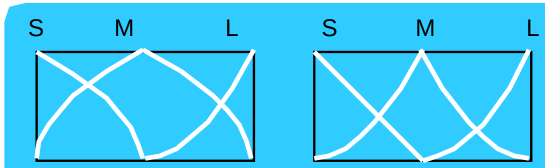
- **Tuning of the DB:**

Linear tuning



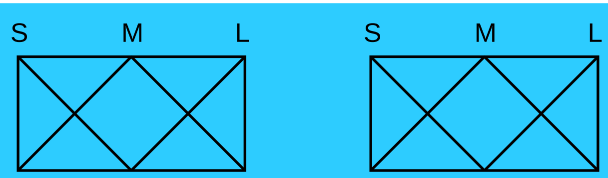
R_1 = IF X is S THEN Y es M
R_2 = IF X Is M THEN Y es L
R_3 = IF X Is L THEN Y es S

Non-linear tuning



R_1 = IF X is S THEN Y es M
R_2 = IF X Is M THEN Y es L
R_3 = IF X Is L THEN Y es S

- **Tuning of the RB: linguistic hedges 'very' and 'more-*or-less*'**



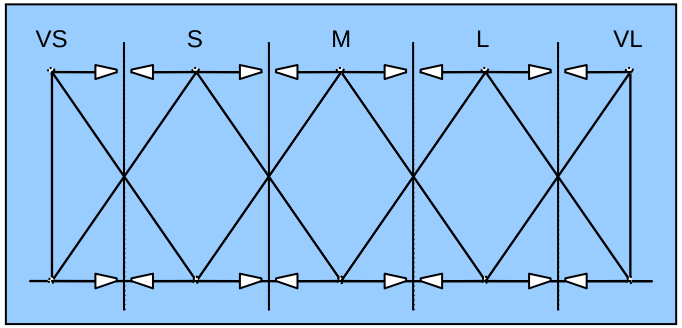
R_1 = IF X is **more-or-less** S THEN Y is M
R_2 = IF X is **very** M THEN Y is **more-or-less** L
R_3 = IF X is **very** L THEN Y is **very** S

Evolutionary Tuning of FRBSs

Genetic tuning of DB and RB using linguistic edges

Triple coding scheme:

– **Membership function parameters (P) (DB linear tuning): real coding**



– **Alpha values (A) (DB non linear tuning): real coding**

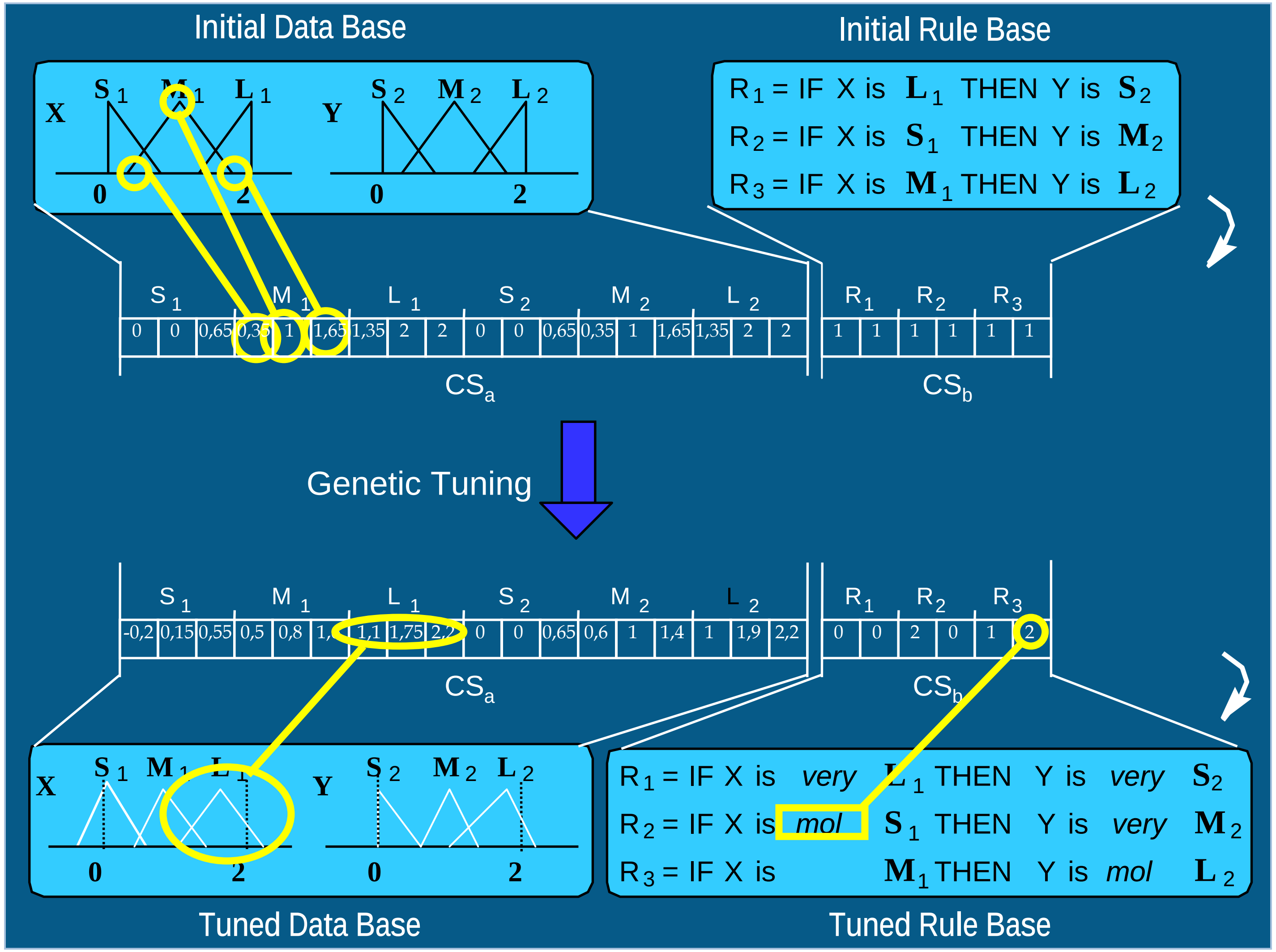
$$\alpha = \begin{cases} 1 + c_{ij}^{A}, & \text{si } c_{ij}^{A} \in [-1,0] \\ 1 + 4 \cdot c_{ij}^{A}, & \text{si } c_{ij}^{A} \in]0,1] \end{cases}$$

– **Linguistic hedges (L) (RB tuning): integer coding**

$$\begin{aligned} c_{ij} &= 0 \quad \leftrightarrow \quad \text{'very'} \\ c_{ij} &= 1 \quad \leftrightarrow \quad \text{no hedge} \\ c_{ij} &= 2 \quad \leftrightarrow \quad \text{'more-or-less'} \end{aligned}$$

Evolutionary Tuning of FRBSs

Genetic tuning of DB and RB using linguistic edges



Evolutionary Tuning of FRBSs

Experimental Study on Electrical Line Problems

- Learning method considered: Wang-Mendel

- Tuning method variants:

TUNING PROCESSES CONSIDERED IN THIS EXPERIMENTAL STUDY

Method	Basic m.f. parameters	α m.f. parameter	Surface structure with linguistic hedges
P-tun	✓		
A-tun		✓	
L-tun			✓
PA-tun	✓	✓	
PL-tun	✓		✓
AL-tun		✓	✓
PAL-tun	✓	✓	✓

- Evaluation methodology: 5 random training-test partitions 80-20% (5-fold cross validation) $\times$ 6 runs = 30 runs per algorithm

Evolutionary Tuning of FRBSs

Experimental Study on Electrical Line Problems

Maintenance cost estimation for *low* and *medium* voltage lines in Spain: O. Cordón, F. Herrera, L. Sánchez, Solving electrical distribution problems using hybrid evolutionary data analysis techniques, Appl. Intell. 10 (1999) 5-24

- Spain's electrical market (before 1998): Electrical companies shared a business, Red Eléctrica Española, receiving all the client fees and distributing them among the partners
- The payment distribution was done according to some complex criteria that the government decided to change
- One of them was related to the maintenance costs of the power line belonging to each company
- The different producers were in trouble to compute them since:
  - As low voltage lines are installed in small villages, there were no actual measurement of their length
  - The goverment wanted the maintenance costs of the optimal medium voltage lines installation and not of the real one, built incrementally

Evolutionary Tuning of FRBSs

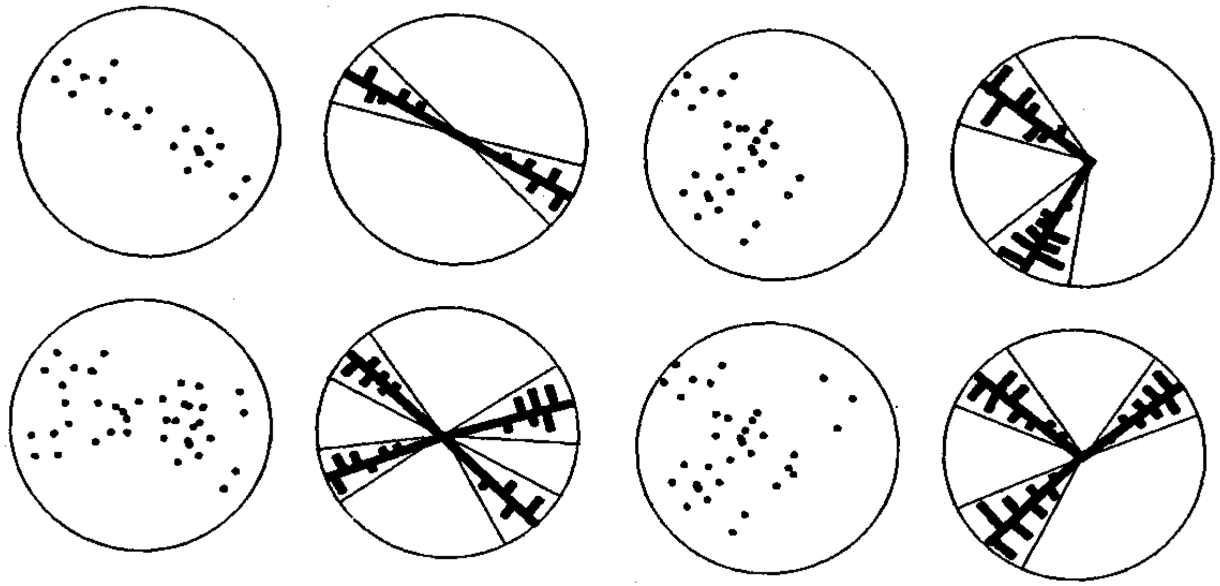
Low Voltage Line Maintenance Cost Estimation

- Goal: estimation of the low voltage electrical line length installed in 1000 rural towns in Asturias
- Two input variables: number of inhabitants and radius of village
- Output variable: length of low voltage line
- Data set composed of 495 rural nuclei, manually measured and affected by noise
- 396 (80%) examples for training and 99 (20%) examples for test randomly selected
- Seven linguistic terms for each linguistic variable

Evolutionary Tuning of FRBSs

Low Voltage Line Maintenance Cost Estimation

- Classical solution: numerical regression on different models of the line installation in the villages



Evolutionary Tuning of FRBSs

Medium Voltage Line Maintenance Cost Estimation

- Goal: estimation of the maintenance cost of the optimal medium voltage electrical line installed in the Asturias' towns
- Four input variables: street length, total area, total area occupied by buildings, and supplied energy
- Output variable: medium voltage line maintenance costs
- Data set composed of 1059 simulated cities
- 847 (80%) examples for training and 212 (20%) examples for test randomly selected
- Five linguistic terms for each linguistic variable

Evolutionary Tuning of FRBSs

Obtained Results for the Medium Voltage Line Problem

Tuning methods:

	Electrical Problem									
	$\bar{x}$				$\sigma_{\bar{x}_i}$			$\sigma_{\bar{x}_i}$		
Method	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**h:m:s**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**
WM	**65**	56,135	56,359	0:00:00	**0.0**	1,498	4,685	—	—	—
WM+P-tun	65	18,395	22,136	**0:22:41**	0.0	778	3,200	—	1,110	1,988
WM+A-tun	65	37,243	38,837	0:33:58	0.0	455	1,816	—	**125**	**572**
WM+L-tun	65	20,967	23,420	0:25:16	0.0	632	3,207	—	336	1,439
WM+PA-tun	65	17,967	21,377	0:38:02	0.0	1,078	1,625	—	2,133	2,628
WM+PL-tun	65	**9,617**	**13,519**	0:25:33	0.0	**263**	3,153	—	694	1,509
WM+AL-tun	65	20,544	23,207	0:34:55	0.0	834	2,701	—	797	1,430
WM+PAL-tun	65	11,222	14,741	0:38:12	0.0	380	**1,315**	—	801	2,136

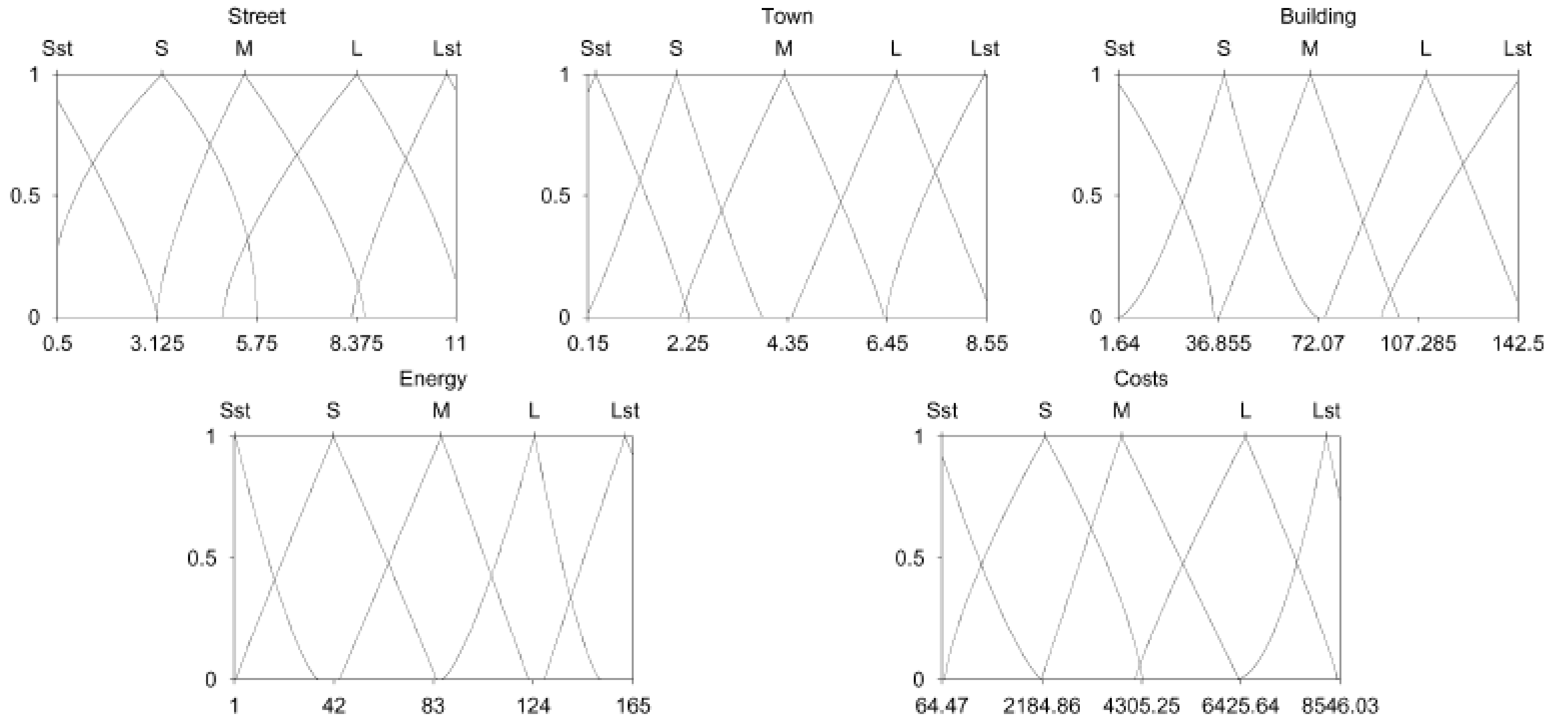
Other fuzzy modeling techniques and GFS:

	Electrical Problem									
	$\bar{x}$				$\sigma_{\bar{x}_i}$			$\sigma_{\bar{x}_i}$		
Method	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**h:m:s**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**	**#R**	**MSE$_{tra}$**	**MSE$_{tst}$**
Nozaki [5]	**532**	**26,705**	**27,710**	**0:00:00**	**0.0**	**764**	**2,906**	—	—	—
Thrift [38]	565.3	31,228	37,579	3:13:25	2.6	1,018	7,279	6.1	**2,110**	**3,609**
Liska [45]	624.9	49,263	56,089	7:13:34	0.1	2,356	4,628	**0.1**	7,522	11,191

Evolutionary Tuning of FRBSs

Obtained Results for the Medium Voltage Line Problem

Example of one KB derived from the WM+PAL-tun method:



R_{33}	M	L	L	M	L

Before tuning: $MSE_{tra/test}$ = 58032 / 55150

After tuning: $MSE_{tra/test}$ = 11395 / 14465

Interpretability-Accuracy Trade-Off
Some Effective Approaches for FRBS Tuning

New coding schemes: 2- and 3-tuples:

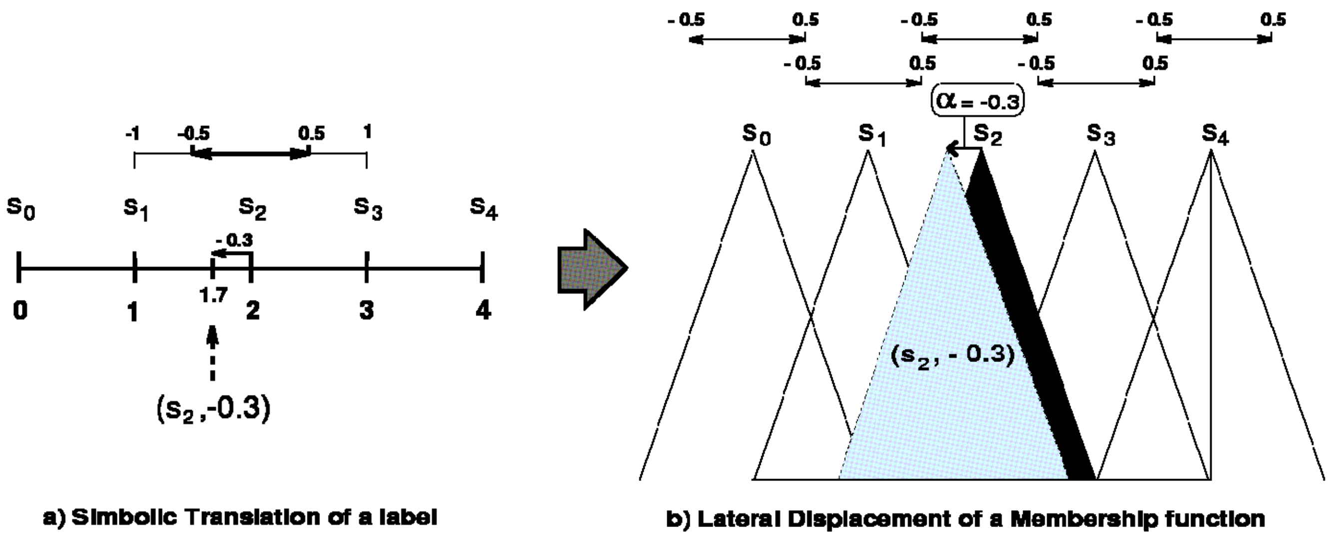
IDEA: New fuzzy rule representation model allowing a more flexible definition of the fuzzy sets of the linguistic labels

- R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Rule base reduction and genetic tuning of fuzzy systems based on the linguistic 3-tuples representation, Soft Computing 11 (5) (2007) 401-419
- R. Alcalá, J. Alcalá-Fdez, F. Herrera, A proposal for the genetic lateral tuning of linguistic fuzzy systems and its interaction with rule selection, IEEE Transactions on Fuzzy Systems 15:4 (2007) 616-635

New coding schemes: 2- and 3-tuples

IDEA: New fuzzy rule representation model allowing a more flexible definition of the fuzzy sets of the linguistic labels

– 2-tuples: **label id.** i **and a displacement parameter** $\alpha_i \in$ **[-0.5,0.5]**

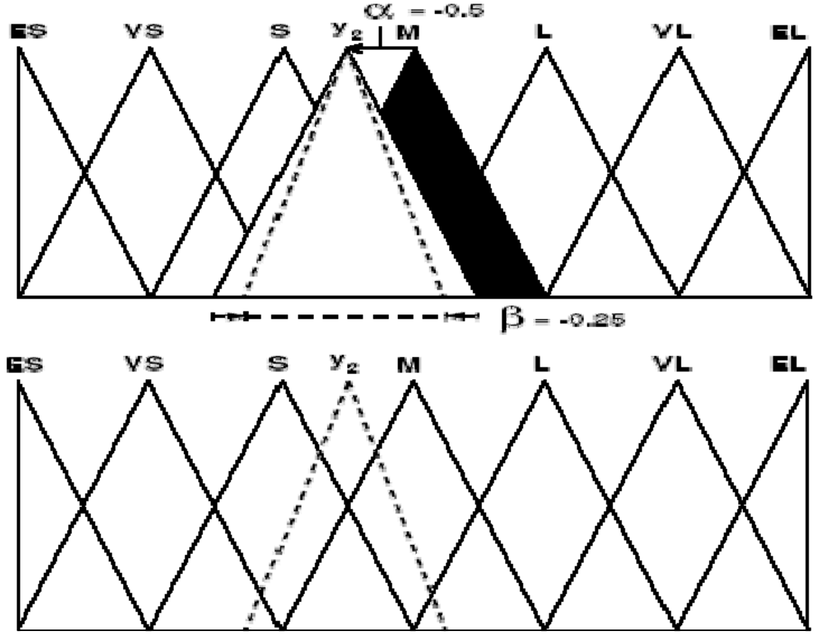


– **New rule structure:**

IF X_1 IS (S^1_i, α_1) AND … AND X_n IS (S^n_i, α_n) THEN Y IS (S^y_i, α_y)

Interpretability-Accuracy Trade-Off
Some Effective Approaches for FRBS Tuning

– 3-tuples: **label id. i, a displacement parameter $\alpha_i \in [-0.5,0.5]$, and a width parameter $\beta_i \in [-0.5,0.5]$**



– **New rule structure:**

IF X_1 IS (S^1_i,α_1,β_1) AND … AND X_n IS (S^n_i,α_n,β_n) THEN Y IS (S^y_i,α_y,β_y)

Interpretability-Accuracy Trade-Off
Some Effective Approaches for FRBS Tuning

New coding schemes: 2- and 3-tuples

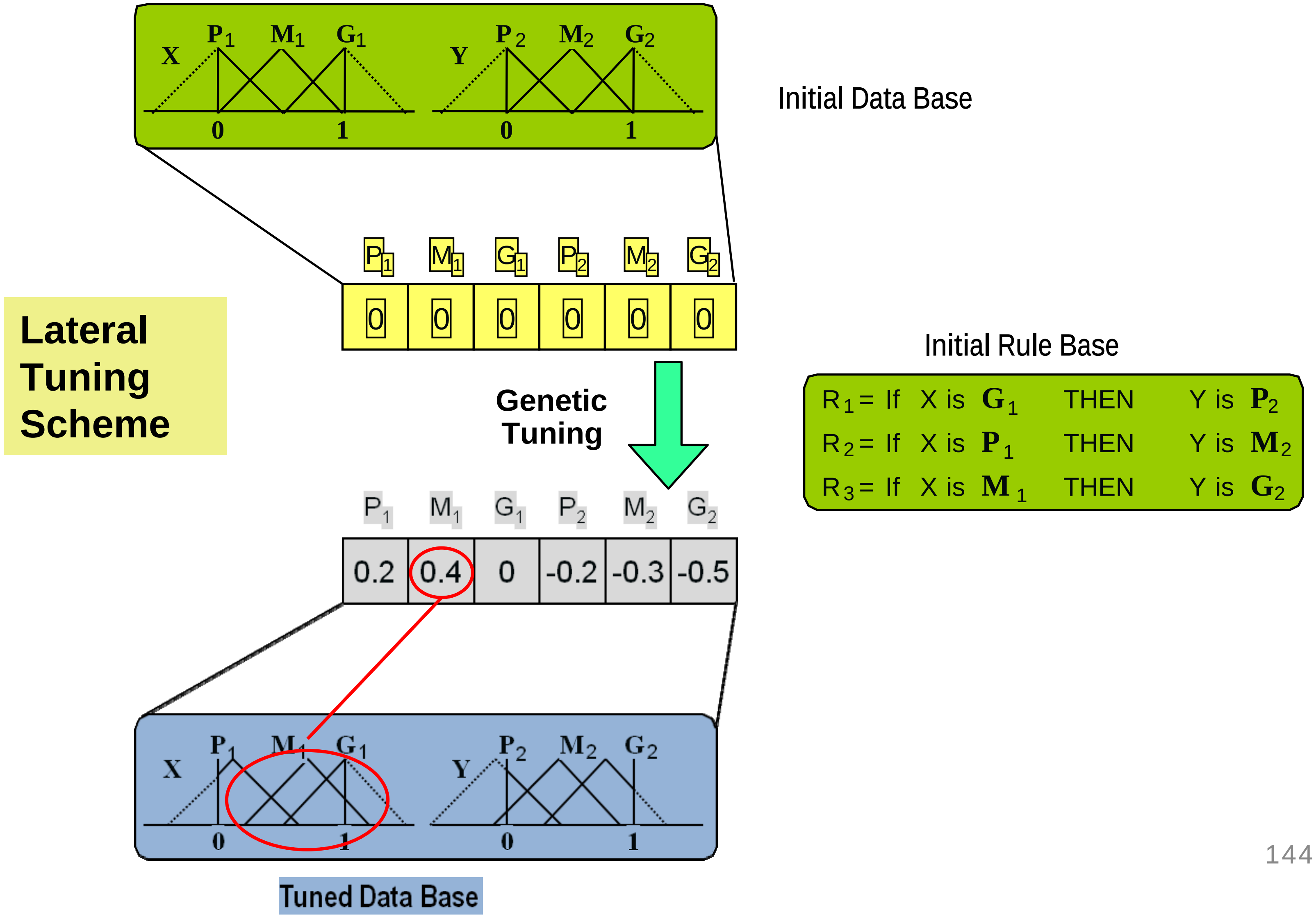
COLATERAL ADVANTAGE: Both structures decreases the KB learning/tuning large scale problem, since the fuzzy sets are encoded using a lower number of parameters

Existing proposals:

- **Genetic 2-tuple/3-tuple DB global tuning: adjustment of the global fuzzy sets → full interpretability (usual fuzzy partitions)**
- **Genetic 2-tuple/3-tuple DB tuning at rule level→ lower interpretability, higher flexibility (like scatter Mamdani FRBSs)**
- **Genetic 2-tuple/3-tuple DB tuning + rule selection**

Interpretability-Accuracy Trade-Off

Some Effective Approaches for FRBS Tuning



Interpretability-Accuracy Trade-Off

Some Effective Approaches for FRBS Tuning

Medium voltage electrical network in towns

Genetic 2-tuple tuning + rule selection method:

WM	Wang and Mendel Learning Method
S	Rule Selection Method
GL	Global Lateral Tuning
LL	Local Lateral Tuning
T	Classical Genetic Tuning
P A L	Tuning of: Parameters, Domains, and Linguistic Modifiers

Five labels per linguistic variable

50000 Evaluations per run

Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
WM	65	57605	2841	+	57934	4733	+
S	40.8	41086	1322	+	59942	4931	+
T	65	18602	1211	+	22666	3386	+
PAL	65	10545	279	+	13973	1688	+
T+S	41.9	14987	391	+	18973	3772	+
PAL+S	57.4	12851	362	+	16854	1463	+
GL	65	23064	1479	+	25654	2611	+
LL	65	3664	390	*	5858	1798	*
GL+S	49.1	18801	2669	+	22586	3550	+
LL+S	58.0	3821	385	■	6339	2164	■

5 data partitions 80% - 20%
6 runs per data partition
Averaged results from 30 runs
t-student Test with $\alpha = 0.05$

Interpretability-Accuracy Trade-Off
Some Effective Approaches for FRBS Tuning

Obtained results for the low voltage line problem:

Genetic 2-tuple tuning + rule selection method:

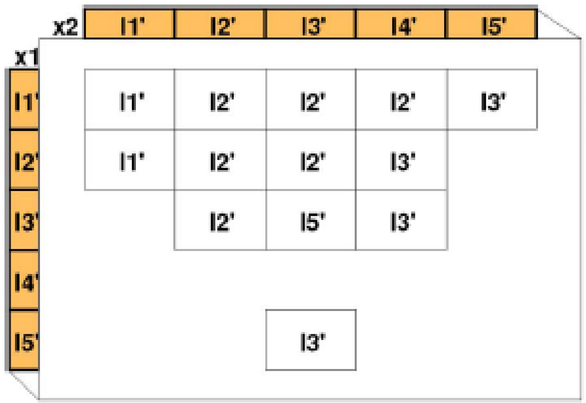
Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
Approaches without tuning							
WM	12.4	234712	32073	+	242147	24473	+
S	10.0	226135	19875	+	241883	19410	+
Approaches with global semantics							
T	12.4	158662	6495	+	221613	29986	+
T+S	8.9	156313	2967	+	193477	49912	=
GL_{dd}	12.4	166674	11480	+	189216	14743	=
GL_{dd}+S	9.0	160081	7316	+	189844	22448	=
Approaches with local semantics							
PAL	12.4	141638	4340	+	189279	19523	=
PAL+S	10.6	145712	5444	+	191922	16987	=
LL_{dd}	12.4	**139189**	3155	⋆	191604	18243	=
LL_{dd}+S	10.5	141446	3444	=	**186746**	15762	⋆

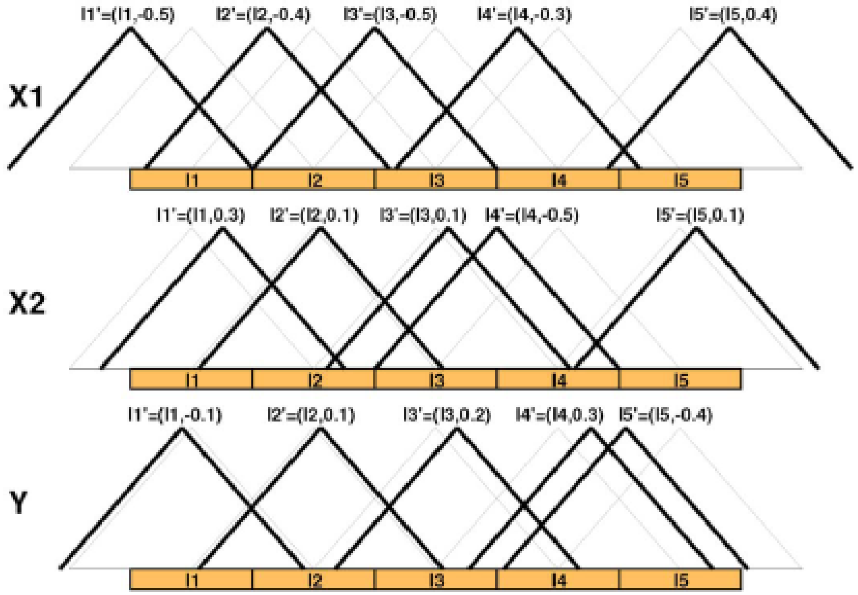
- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

Interpretability-Accuracy Trade-Off
Some Effective Approaches for FRBS Tuning

Obtained results for the low voltage line problem:

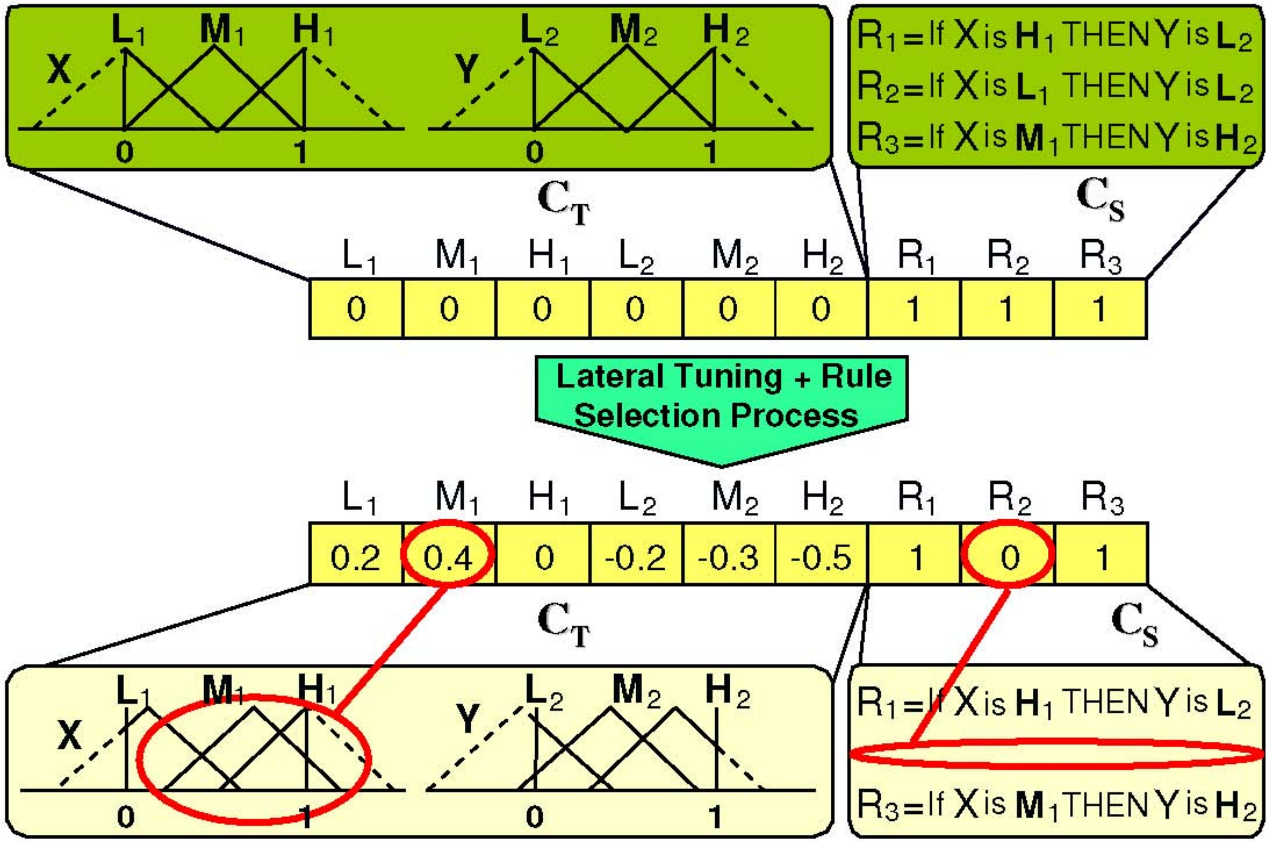
Example of one KB derived from the global tuning method:





After tuning+rule selection: #R=13; $MSE_{tra/test}$ = 187494 / 176581

Interpretability-Accuracy Trade-Off
Some Effective Approaches for FRBS Tuning



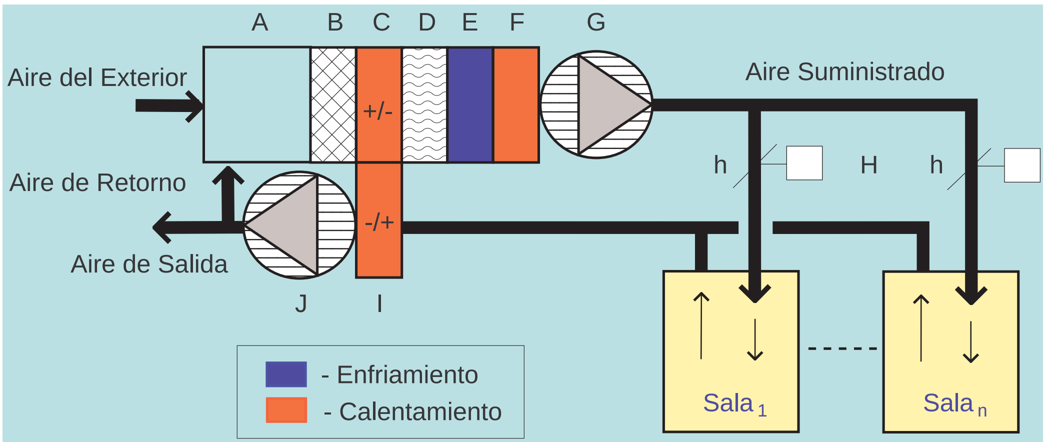
Example of genetic lateral tuning and rule selection

GENETIC FUZZY SYSTEMS (Acc/Int Trade-Off): APPLICATION TO A HVAC PROBLEM

Heating Ventilating and Air Conditioning Systems: Problem

GENESYS

JOULE-THERMIE JOE-CT98-0090



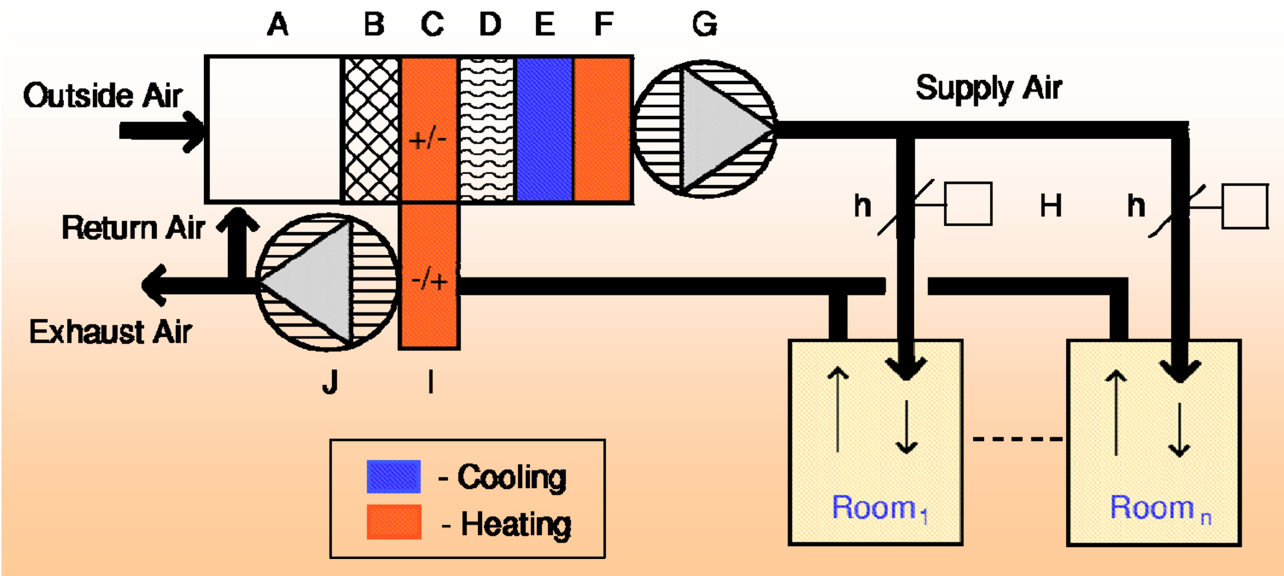
Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

Heating Ventilating and Air Conditioning Systems: Problem

- Energy consumption in buildings is the 40% of the total and more than a half is for indoor climate conditions
- The use of specific technologies can save up to a 20% of the energy consumption
- The use of appropriate automatic control strategies could result in energy savings ranging 15-85 %
- Moreover, in current systems, several criteria are considered and optimized independently without a global strategy

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

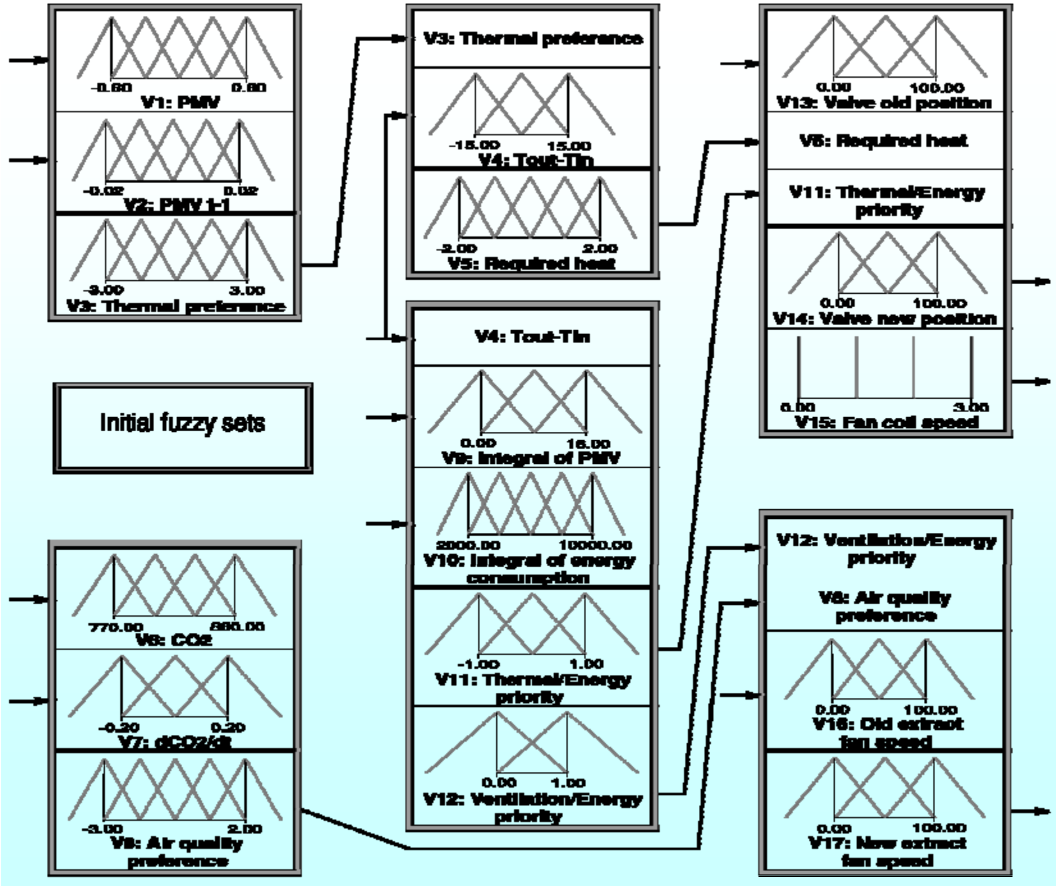
Generic Structure of an Office Building HVAC System



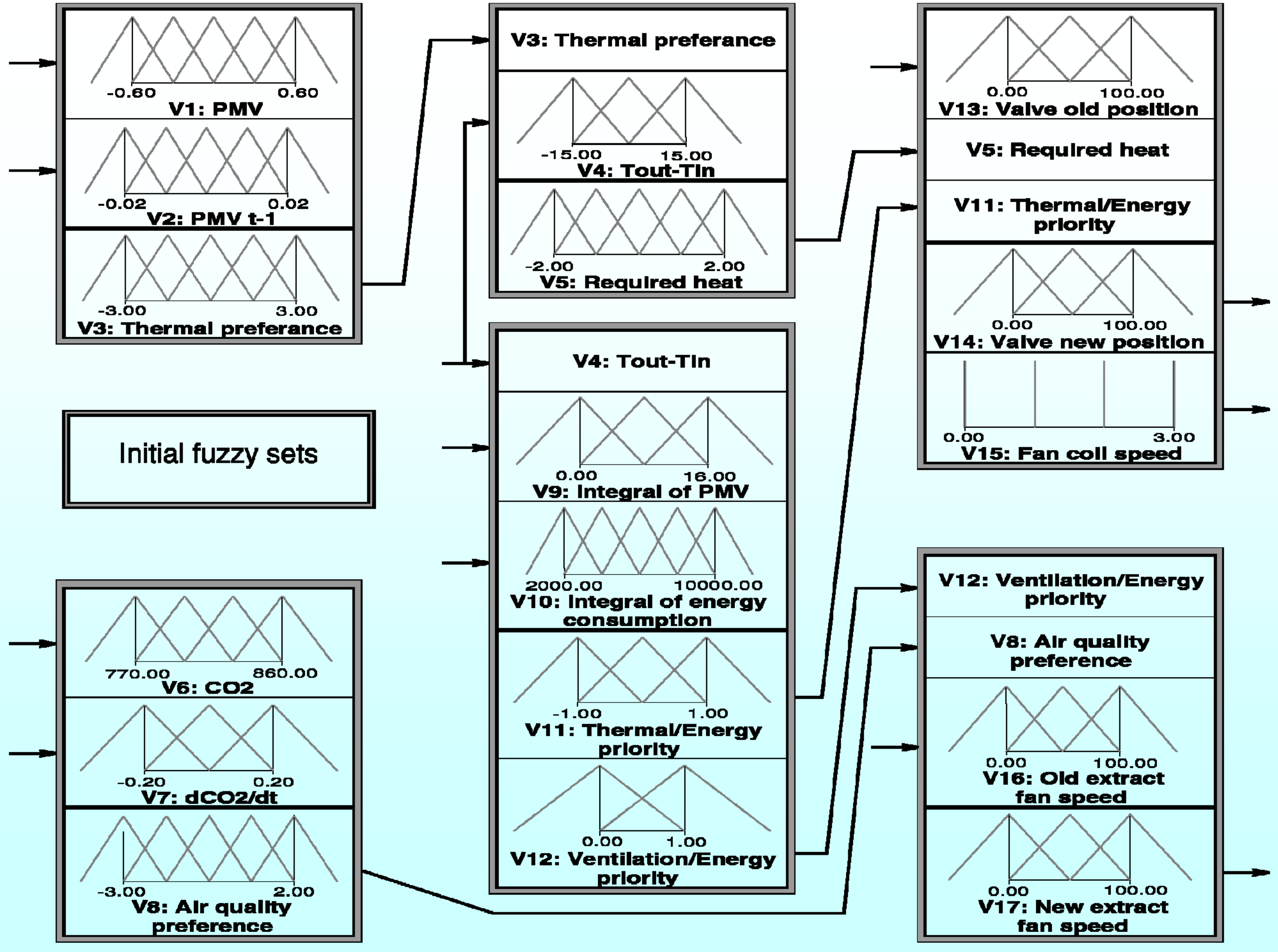
- It maintain a good thermal quality in summer and winter
- It dilutes and removes emissions from people, equipment and activities and supplies clean air

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

Initial Data Base



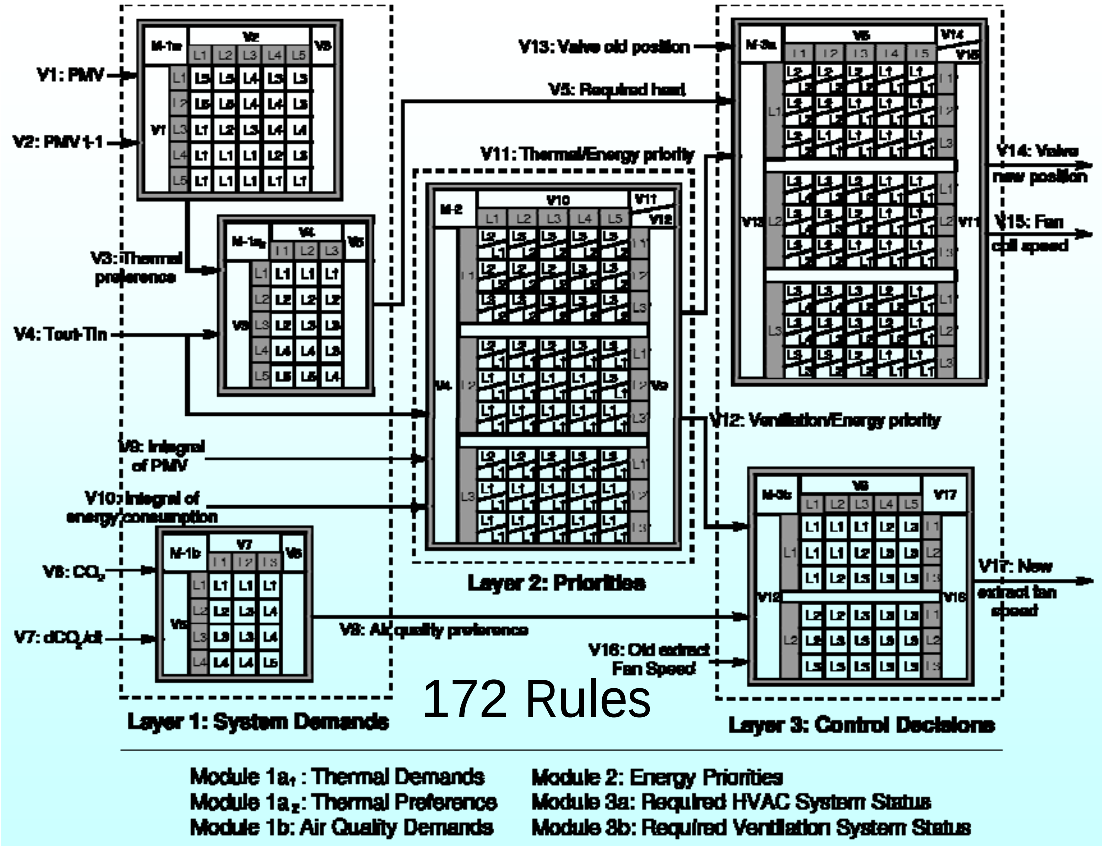
17 Variables



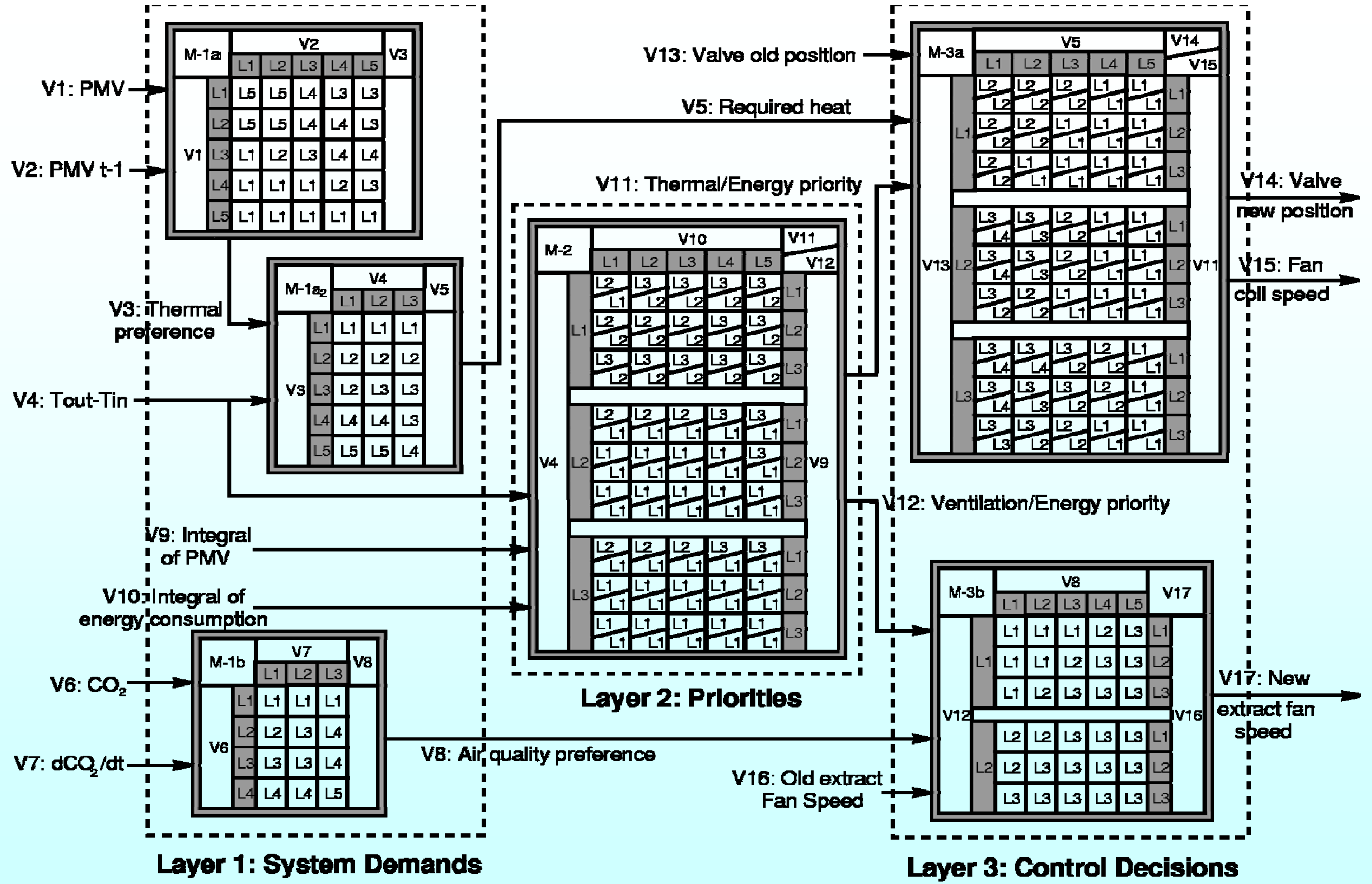
-0.60
0.60
V1: PMV
-0.02
0.02
V2: PMV t-1
-3.00
3.00
V3: Thermal preferance
V3: Thermal preferance
-15.00
15.00
V4: Tout-Tin
-2.00
2.00
V5: Required heat
0.00
100.00
V13: Valve old position
V5: Required heat
V11: Thermal/Energy priority
0.00
100.00
V14: Valve new position
0.00
3.00
V15: Fan coil speed
V4: Tout-Tin
0.00
16.00
V9: Integral of PMV
2000.00
10000.00
V10: Integral of energy consumption
-1.00
1.00
V11: Thermal/Energy priority
0.00
1.00
V12: Ventilation/Energy priority
Initial fuzzy sets
770.00
860.00
V6: CO2
-0.20
0.20
V7: dCO2/dt
-3.00
2.00
V8: Air quality preference
V12: Ventilation/Energy priority
V8: Air quality preference
0.00
100.00
V16: Old extract fan speed
0.00
100.00
V17: New extract fan speed

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

Initial Rule Base and FLC Structure



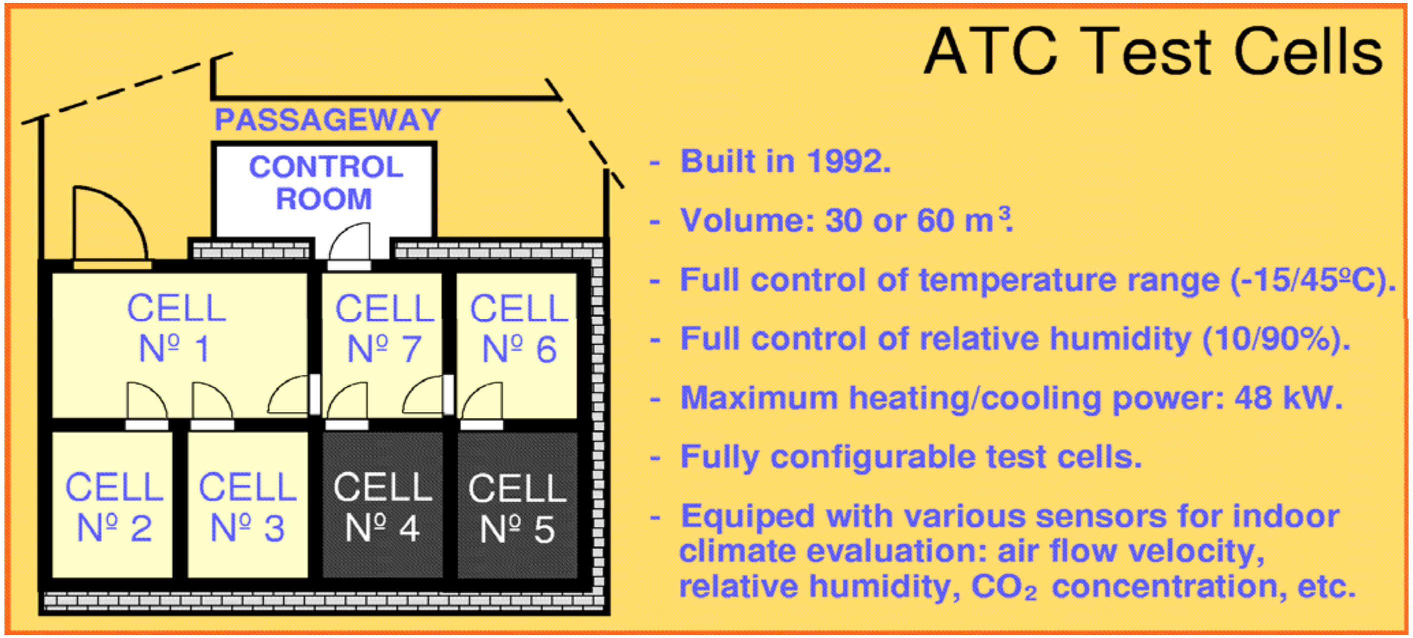
172 Rules



Module $1a_1$: Thermal Demands	Module 2: Energy Priorities
Module $1a_2$: Thermal Preference	Module 3a: Required HVAC System Status
Module 1b: Air Quality Demands	Module 3b: Required Ventilation System Status

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

Representation of the Test Cells



- **Two adjacent twin cells were available**

- **A calibrated and validated model of this site was developed to evaluate each FLC**

Fuzzy Logic Controllers for Energy Efficiency Consumption in Buildings

- **Goal: multi-criteria optimization of an expert FLC for an HVAC system: reduction of the energy consumption but maintaining the required indoor comfort levels**

O_1 Upper thermal comfort limit [3]: $if\ PMV > 0.5, O_1 = O_1 + (PMV - 0.5)$.

O_2 Lower thermal comfort limit: $if\ PMV < -0.5, O_2 = O_2 + (-PMV - 0.5)$.

O_3 IAQ requirement: $if\ CO_2\ conc. > 800ppm, O_3 = O_3 + (CO_2 - 800)$.

O_4 Energy consumption: $O_4 = O_4+$ Power at time t.

O_5 System stability: $C_5 = C_5+$ System change from time t to $(t-1)$.

- **INITIAL RESULTS**

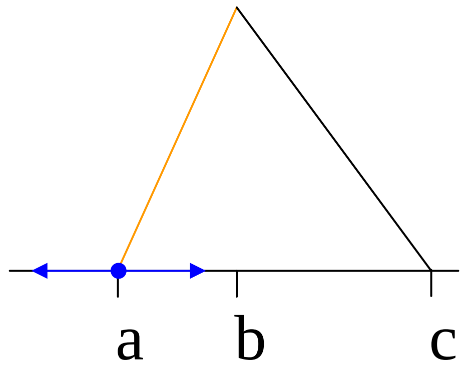
MODELS	#R	PMV>0.5		PMV<-0.5		$C0_2$		ENERGY		STABILITY	
		0_1		0_2		0_3		0_4	%	0_5	%
ON-OFF	-	0,0		0		0		3206400	-	1136	-
FLC	172	0,0		0		0		2901686	9,50	1505	-32,48

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Improving the FLC Performance

The main objective was the reduction of the energy consumption (10%), improving the stability of the controller, maintaining the required indoor comfort levels

- **Genetic tuning of the Data Base**
  - **Local modification of the membership function definition points**



GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Objectives (to be minimized)

O_1 Upper thermal comfort limit [3]: $if\ PMV > 0.5, O_1 = O_1 + (PMV - 0.5)$.

O_2 Lower thermal comfort limit: $if\ PMV < -0.5, O_2 = O_2 + (-PMV - 0.5)$.

O_3 IAQ requirement: $if\ CO_2\ conc. > 800ppm, O_3 = O_3 + (CO_2 - 800)$.

O_4 Energy consumption: $O_4 = O_4 +$ Power at time t.

O_5 System stability: $C_5 = C_5 +$ System change from time t to $(t-1)$.

- Expert knowledge as objective weights:

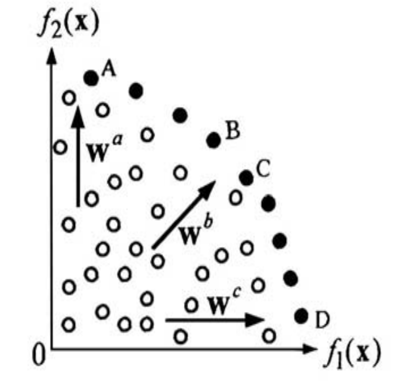
$$w_1^o = w_2^o = 0.0041511\ ;\ w_3^o = 0.0000022833$$

$$w_4^o = 0.0000017832\ ;\ w_5^o = 0.000761667$$

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Problem Restrictions

- **Different-Criteria-Based Evaluation**
  - Multiple Criteria Algorithms:
    - Multi-objective approach
    - Aggregation approach



$$F(x) = w_1 \cdot f_1(x) + \ldots + w_n \cdot f_n(x)$$

$$\sum w_i = 1, \qquad 0 \le w_i \le 1, \qquad i = \{1, \ldots, n\}$$

Since trusted weights exist:

- The problem solving is easier
- Quicker algorithms can be designed

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Problem Restrictions

- **The controller accuracy is assessed by means of simulations which approximately take 3-4 minutes**

  - Efficient tuning methodologies:

    - Local adjustment of each tuned parameter
    - Steady-State Genetic Algorithms: quick convergence

      2000 evaluations $\Rightarrow$ 1 run takes approximately 4 days

    - Considering a small population (31 individuals)

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Genetic Tuning of the Data Base

- **A real coded steady-state genetic algorithm for local tuning of the membership function definition points.**
  - **Two individuals are selected to be crossed and four descendents are obtained**
  - **The two best offspring are included in the population replacing the two worst individuals if they are better adapted than the latter**
  - **A restarting approach is considered if the population converges**

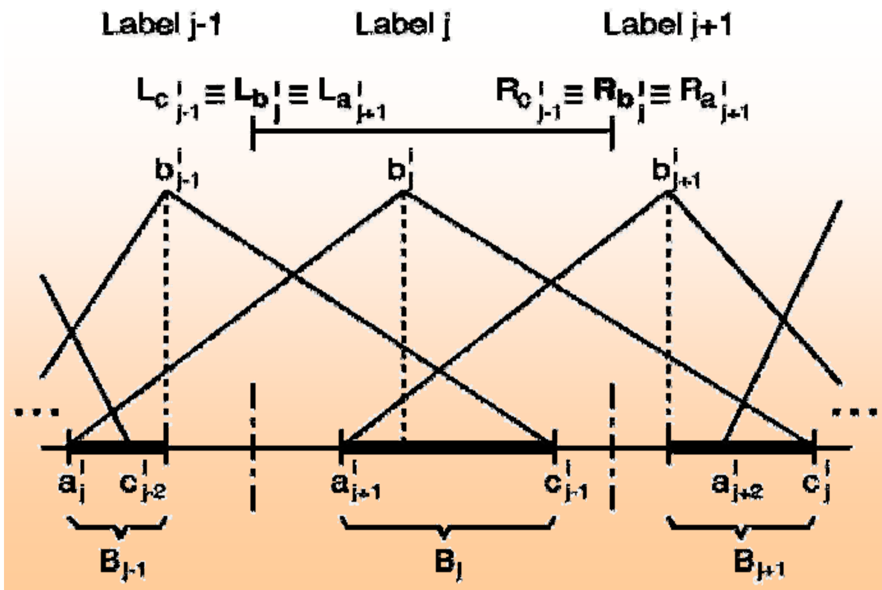
GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Data Base Tuning: Algorithm (1)

- **Coding Scheme** (with *n* variables and L_i labels)**:**

$$C_i = \left(a_1^i, b_1^i, c_1^i, \ldots, a_{L_i}^i, b_{L_i}^i, c_{L_i}^i\right),\ i = 1, \ldots, n$$

$$C = C_1 C_2 \ldots C_n$$



GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

Data Base Tuning: Algorithm (2)

- **Genetic operators:**
  - The max-min-arithmetical crossover. From parents C^v and C^w, four offspring are obtained:

$$
\begin{aligned}
C^v &= (c_1, \ldots, c_k, \ldots, c_H) \\
C^w &= (c'_1, \ldots, c'_k, \ldots, c'_H)
\end{aligned}
$$

$$
\begin{aligned}
&C^{1'} = aC^w + (1-a)C^v \\
&C^{2'} = aC^v + (1-a)C^w \\
&C^{3'} \text{ with } c_{3k} = \min\{c_k, c'_k\} \\
&C^{4'} \text{ with } c_{4k} = \max\{c_k, c'_k\}
\end{aligned}
$$

  - Michalewicz's non-uniform mutation.

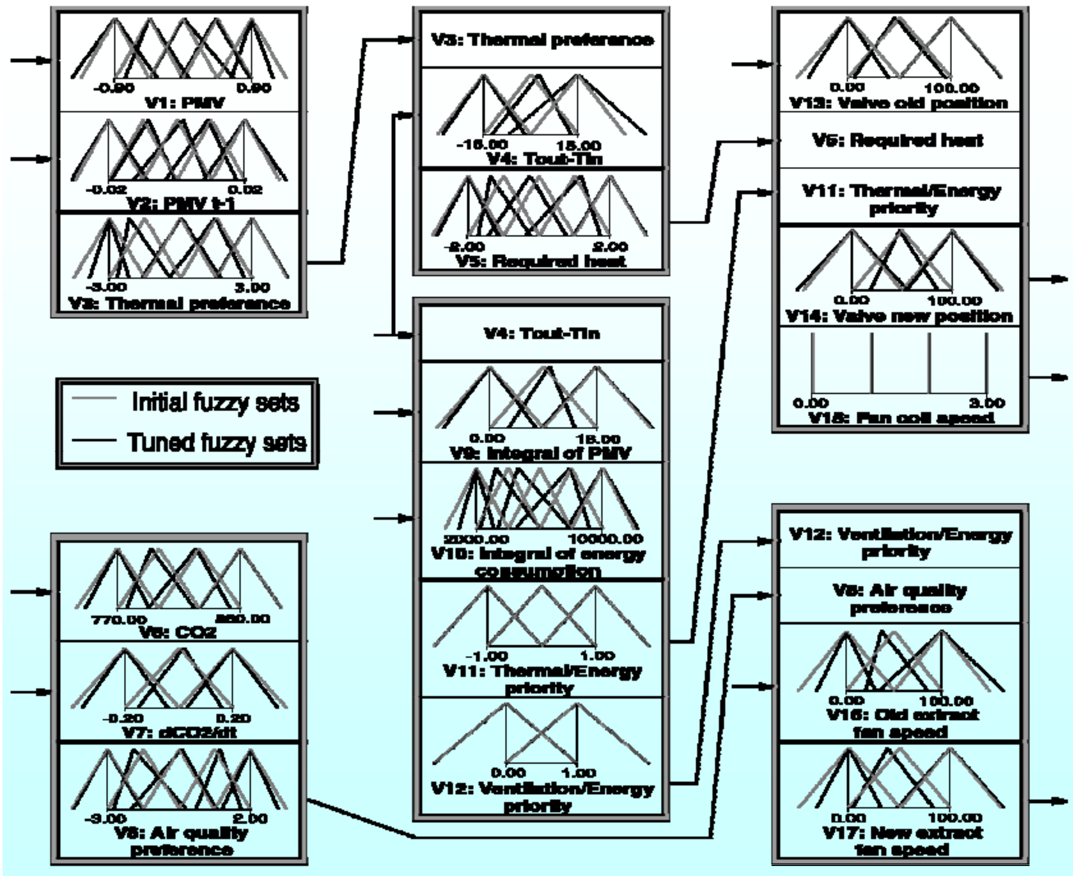
GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

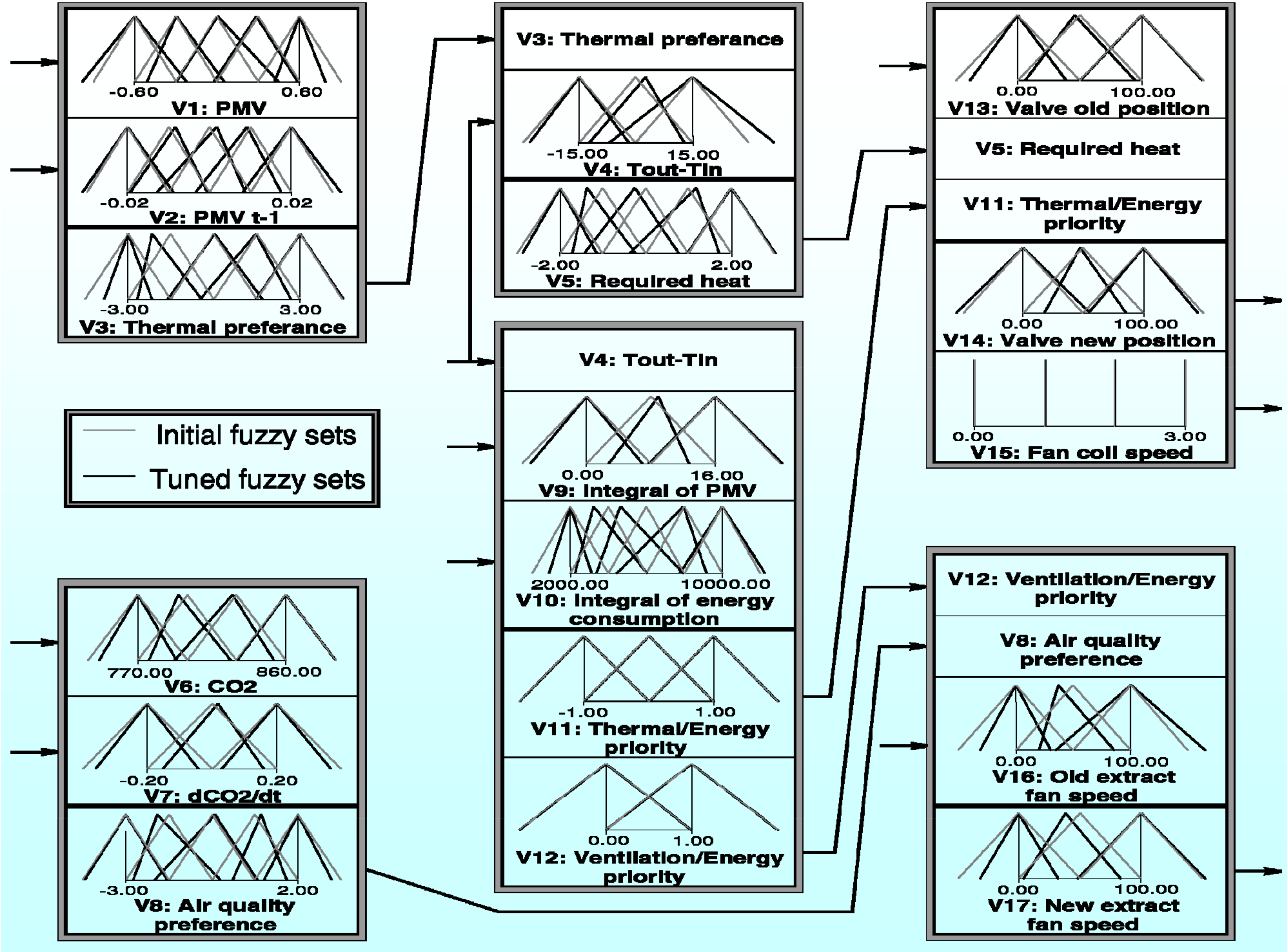
MODELS	#R	PMV>0.5 0_1	PMV<-0.5 0_2	$C0_2$ 0_3	ENERGY 0_4	%	STABILITY 0_5	%
CLASSICAL ON-OFF	-	0,0	0	0	3206400	-	1136	-
FLC	172	0,0	0	0	2901686	9,50	1505	-32,48
FLC TUNING	172	0,0	0	0	2596875	19,01	1051	7,48

R. Alcalá, J.M. Benítez, J. Casillas, O. Cordón, R. Perez, Fuzzy control of HVAC systems optimised by genetic algorithms, Appl. Intell. 18 (2003) 155–177

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning

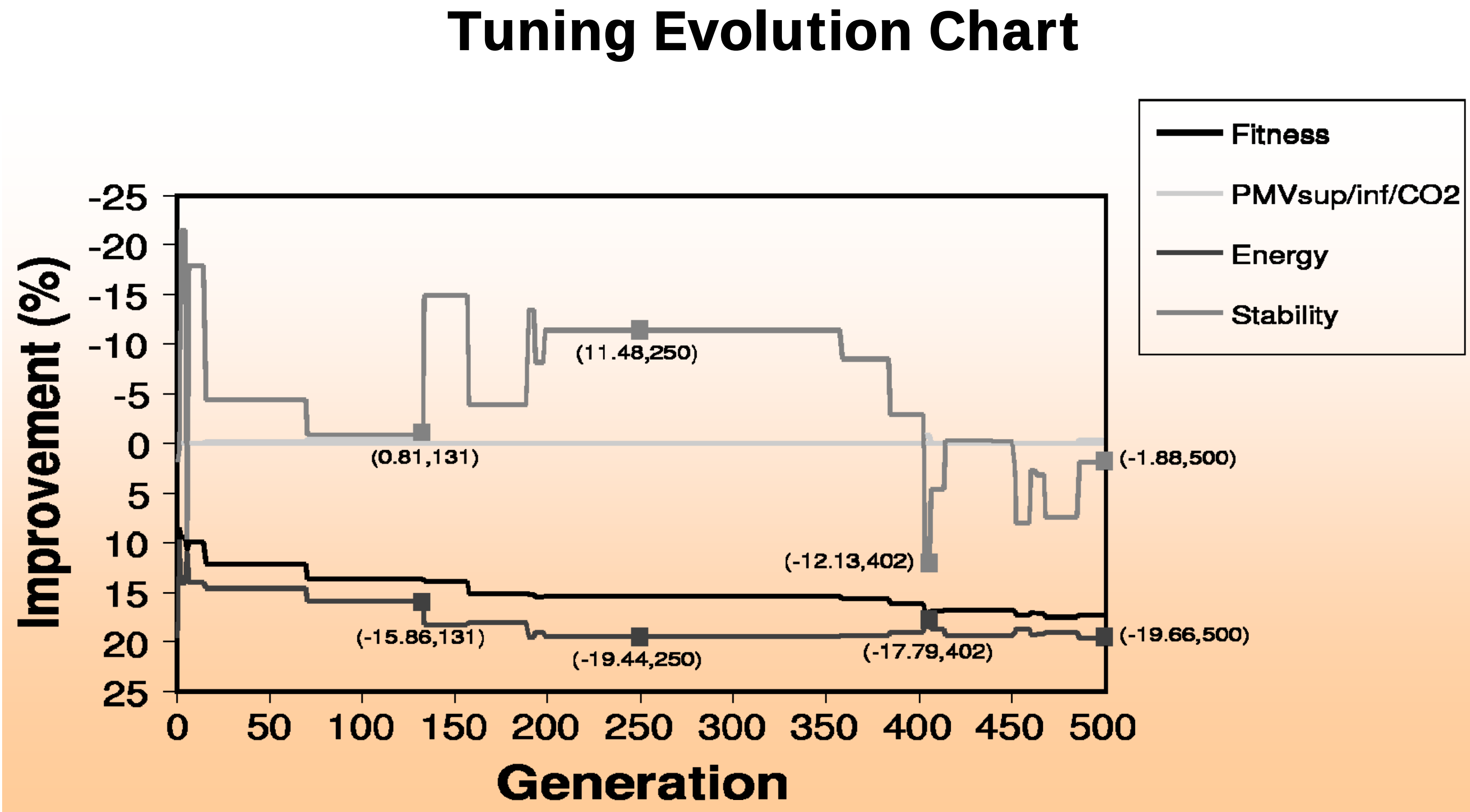
Tuned Data Base





-0.60 0.60
V1: PMV
-0.02 0.02
V2: PMV t-1
-3.00 3.00
V3: Thermal preferance
V3: Thermal preferance
-15.00 15.00
V4: Tout-Tin
-2.00 2.00
V5: Required heat
0.00 100.00
V13: Valve old position
V5: Required heat
V11: Thermal/Energy priority
0.00 100.00
V14: Valve new position
0.00 3.00
V15: Fan coil speed
Initial fuzzy sets
Tuned fuzzy sets
V4: Tout-Tin
0.00 16.00
V9: Integral of PMV
2000.00 10000.00
V10: Integral of energy consumption
-1.00 1.00
V11: Thermal/Energy priority
0.00 1.00
V12: Ventilation/Energy priority
770.00 860.00
V6: CO2
-0.20 0.20
V7: dCO2/dt
-3.00 2.00
V8: Air quality preference
V12: Ventilation/Energy priority
V8: Air quality preference
0.00 100.00
V16: Old extract fan speed
0.00 100.00
V17: New extract fan speed

GFS Models for Fuzzy Control of HVAC Systems: Genetic Tuning



GFS Models for Fuzzy Control of HVAC Systems: Genetic Rule Selection with Weights

GENETIC RULE WEIGHT DERIVATION AND RULE SELECTION

OBJECTIVE OF GETTING:

- **a subset of rules presenting good cooperation**
- **the weights associated to rules**

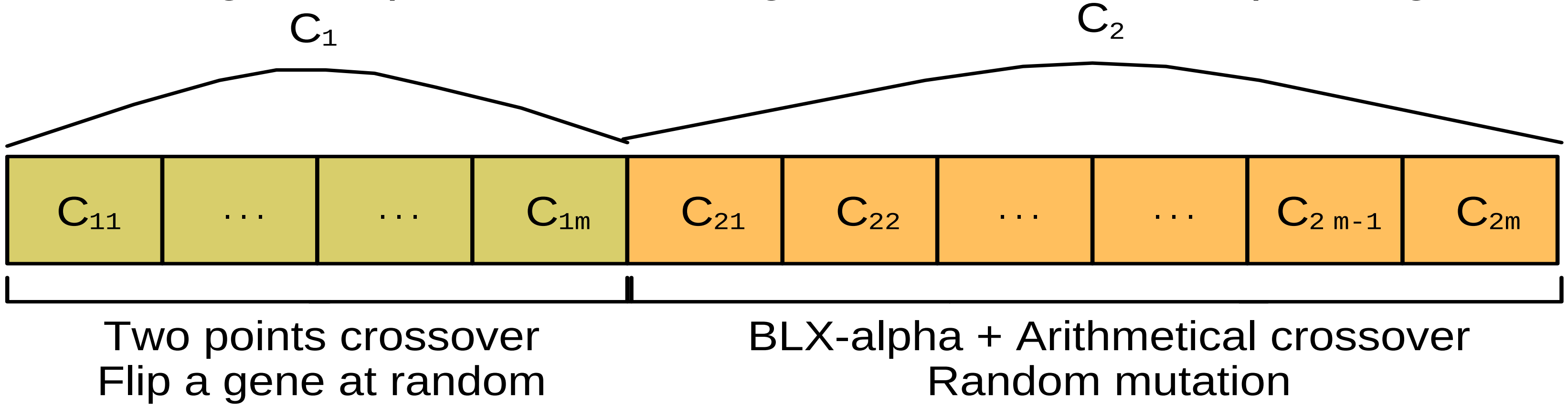
IF X_1 is A_1 and … and X_n is A_n THEM Y is B with [w], $w \in [0,1]$

We use a steady-state genetic algorithm with a double coding scheme.

GFS Models for Fuzzy Control of HVAC Systems: Genetic Rule Selection with Weights

Weight Learning: Algorithm

- **A double coding scheme ($C=C_1+C_2$):**
  - C_1: The coding scheme generates binary-coded strings of length m (number of single rules in the previously derived rule set):
  - C_2: The coding scheme generates real-coded strings of length m. Each gene represents the weight used in the corresponding rule



GFS Models for Fuzzy Control of HVAC Systems: Genetic Rule Selection with Weights

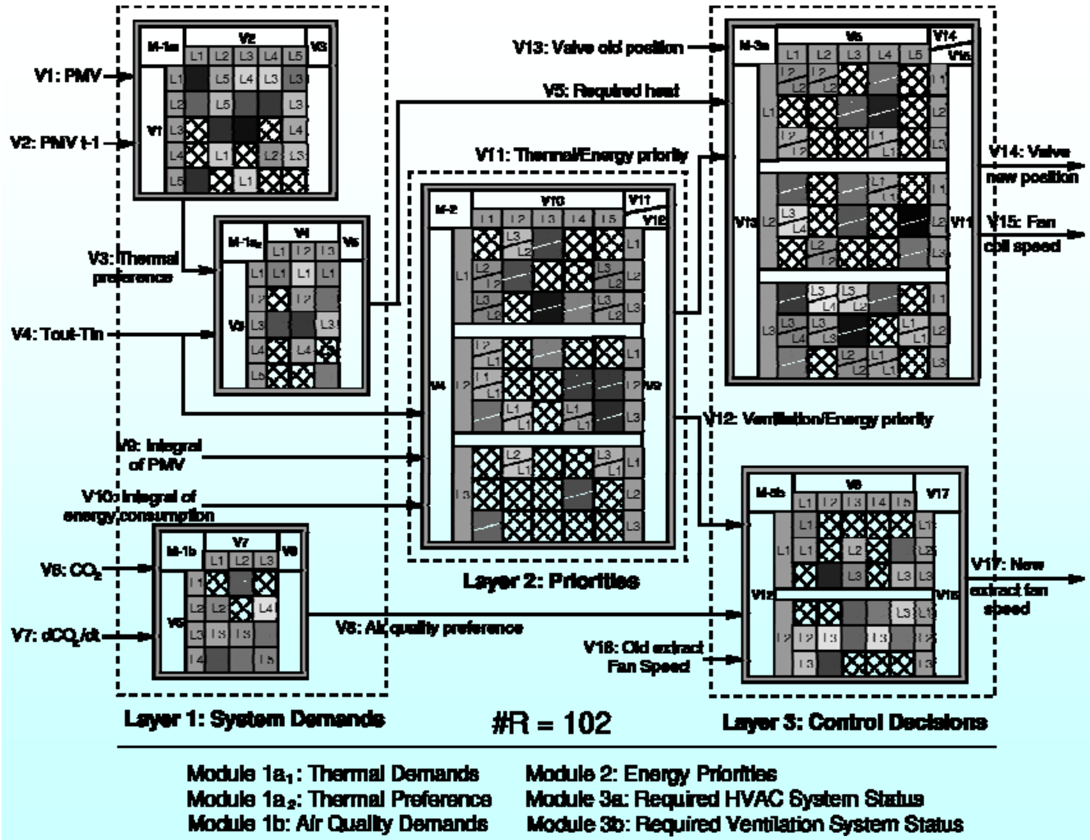
Obtained Results

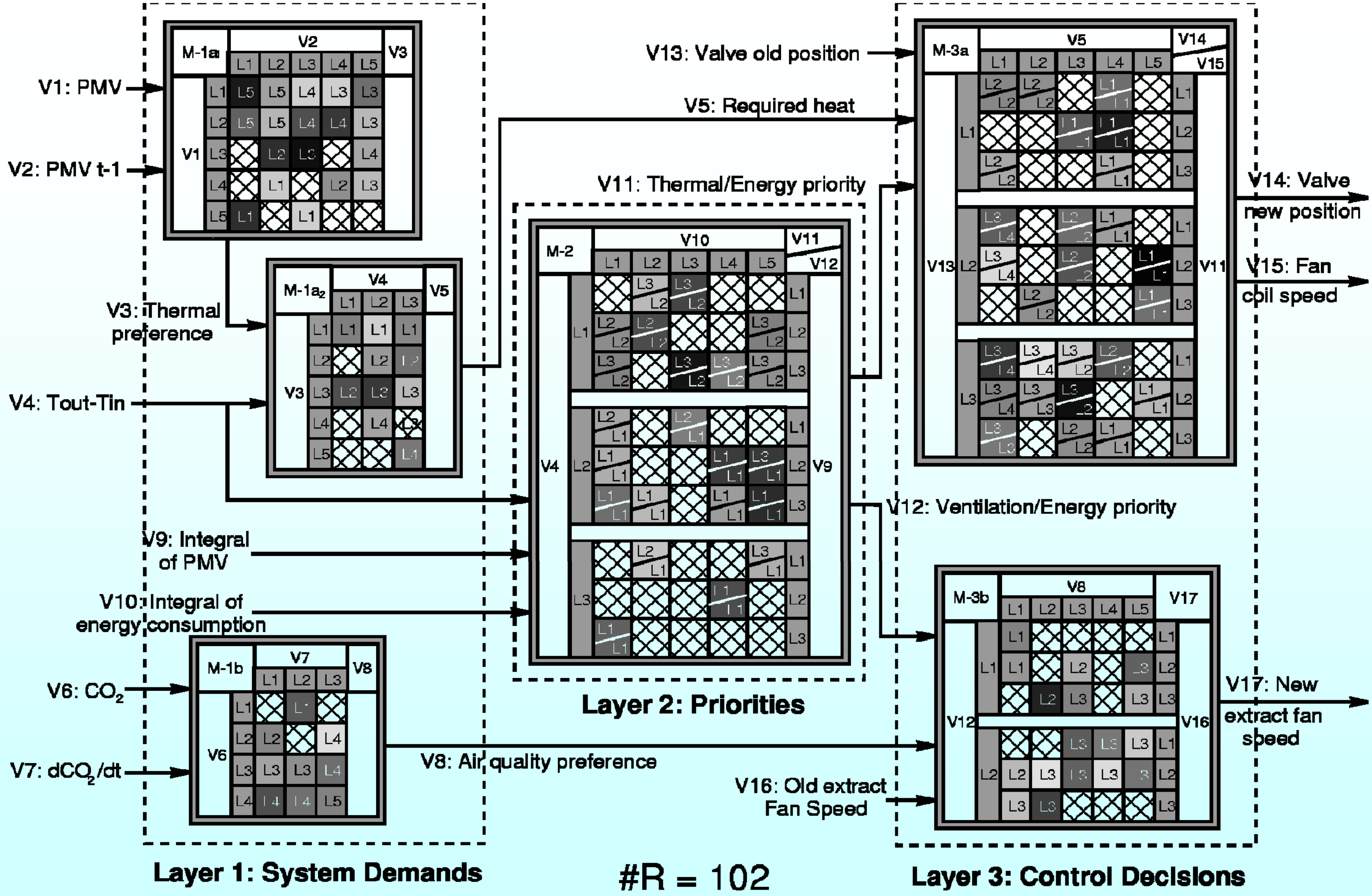
MODELS	#R	PMV>0.5 0_1	PMV<-0.5 0_2	$C0_2$ 0_3	ENERGY 0_4	ENERGY %	STABILITY 0_5	STABILITY %
ON-OFF	-	0,0	0	0	3206400	-	1136	-
FLC	172	0,0	0	0	2901686	9,50	1505	-32,48
TUNING	172	0,0	0	0	2596875	19,01	1051	7,48
SELECTION	147	0,2	0	0	2867692	10,56	991	12,76
SEL. + TUNING	109	0,1	0	0	2492462	22,27	989	12,94
SEL + WEIGTS	102	0,7	0	0	2731798	14,80	942	17,08

R. Alcalá, J. Casillas, O. Cordón, A. González, F. Herrera, **A Genetic Rule Weighting and Selection Process for Fuzzy Control of Heating, Ventilating and Air Conditioning Systems**. *Engineering Applications of Artificial Intelligence 18:3 (2005) 279-296*

GFS Models for Fuzzy Control of HVAC Systems: Genetic Rule Selection with Weights

Weighted Rule Base





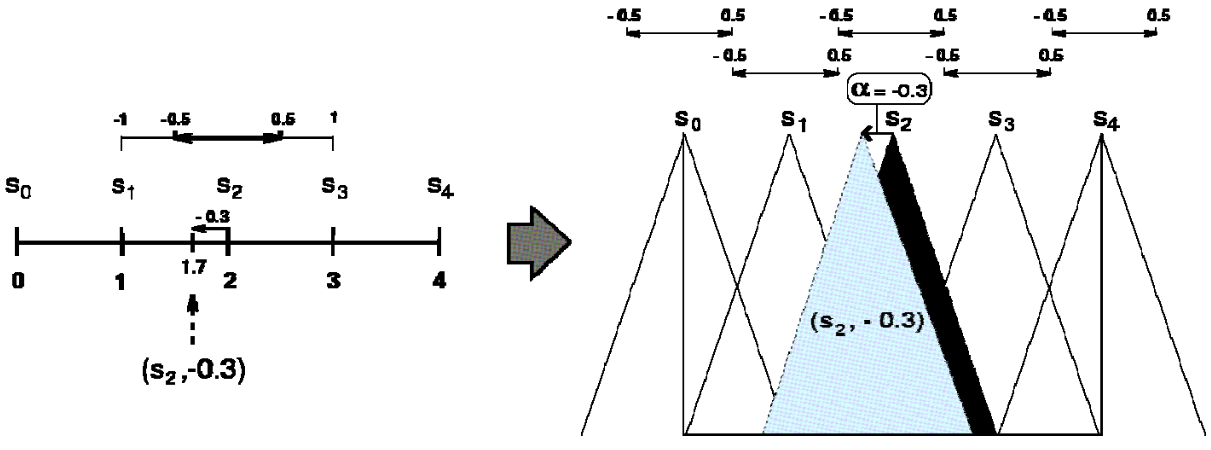
Module $1a_1$: Thermal Demands
Module $1a_2$: Thermal Preference
Module 1b: Air Quality Demands
Module 2: Energy Priorities
Module 3a: Required HVAC System Status
Module 3b: Required Ventilation System Status

GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

New coding schemes: 2- and 3-tuples:

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Improving Fuzzy Logic Controllers Obtained by Experts: A Case Study in HVAC Systems. *Applied Intelligence, doi:10.1007/s10489-007-0107-6, 31:1 (2009) 10-35*.

– 2-tuples: **label id.** i **and a displacement parameter** $\alpha_i \in$**[-0.5,0.5]**



a) Simbolic Translation of a label

b) Lateral Displacement of a Membership function

GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

GENETIC LATERAL TUNING

MODELS	#R	PMV>0.5 O	PMV<-0.5 O	$C0_2$ O	ENERGY O	ENERGY %	ESTABILITY O	ESTABILITY %
ON-OFF	-	0,0	0	0	3206400	-	1136	-
FLC	172	0,0	0	0	2901686	9,50	1505	-32,48
TUNING	172	0,0	0	0	2596875	19,01	1051	7,48
SELECTION	147	0,2	0	0	2867692	10,56	991	12,76
SELEC. +	109	0,1	0	0	2492462	22,27	989	12,94
SEL +	102	0,7	0	0	2731798	14,80	942	17,08
GL 1	172	0,7	0	0	2378784	25,81	1069	5,90
GL 2	172	1,0	0	0	2327806	27,40	1066	6,16
GL 3	172	0,9	0	0	2268689	29,25	1080	4,93
LL 1	172	0,9	0	0	2386033	25,59	896	21,13
LL 2	172	0,8	0	0	2343409	26,92	943	16,99
LL 3	172	0,3	0	0	2377596	25,85	938	17,43

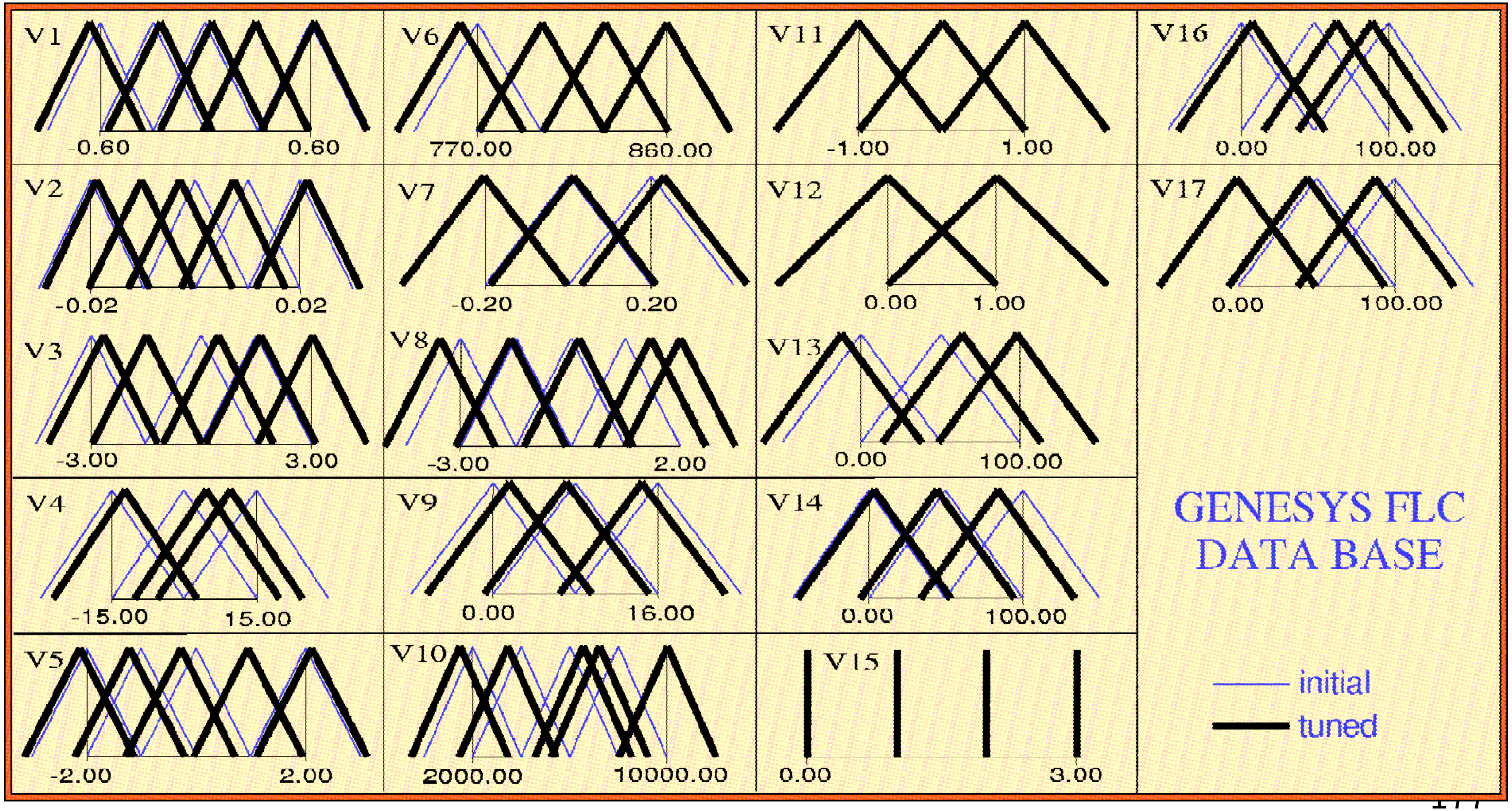
GFS Models for Fuzzy Control of HVAC Systems:

Genetic Lateral Tuning and Rule Selection

MODELS	#R	PMV>0.5 0_1	PMV<-0.5 0_2	$C0_2$ 0_3	ENERGY 0_4	%	ESTABILITY 0_5	%
ON-OFF	-	0,0	0	0	3206400	-	1136	-
FLC	172	0,0	0	0	2901686	9,50	1505	-32,48
TUNING	172	0,0	0	0	2596875	19,01	1051	7,48
SELECTION	147	0,2	0	0	2867692	10,56	991	12,76
SEL +	109	0,1	0	0	2492462	22,27	989	12,94
SEL +	102	0,7	0	0	2731798	14,80	942	17,08
GL 2	172	0,9	0	0	2268689	29,25	1080	4,93
LL 1	172	0,9	0	0	2386033	25,59	896	21,13
GL - S 1	105	1,0	0	0	2218598	30,81	710	37,50
GL - S 2	115	0,4	0	0	2358405	26,45	818	27,99
GL - S 3	118	0,8	0	0	2286976	28,68	872	23,24
LL – S 1	133	0,5	0	0	2311986	27,90	788	30,63
LL – S 2	104	0,6	0	0	2388470	25,51	595	47,62
LL – S 3	93	0,5	0	0	2277807	28,96	1028	9,51

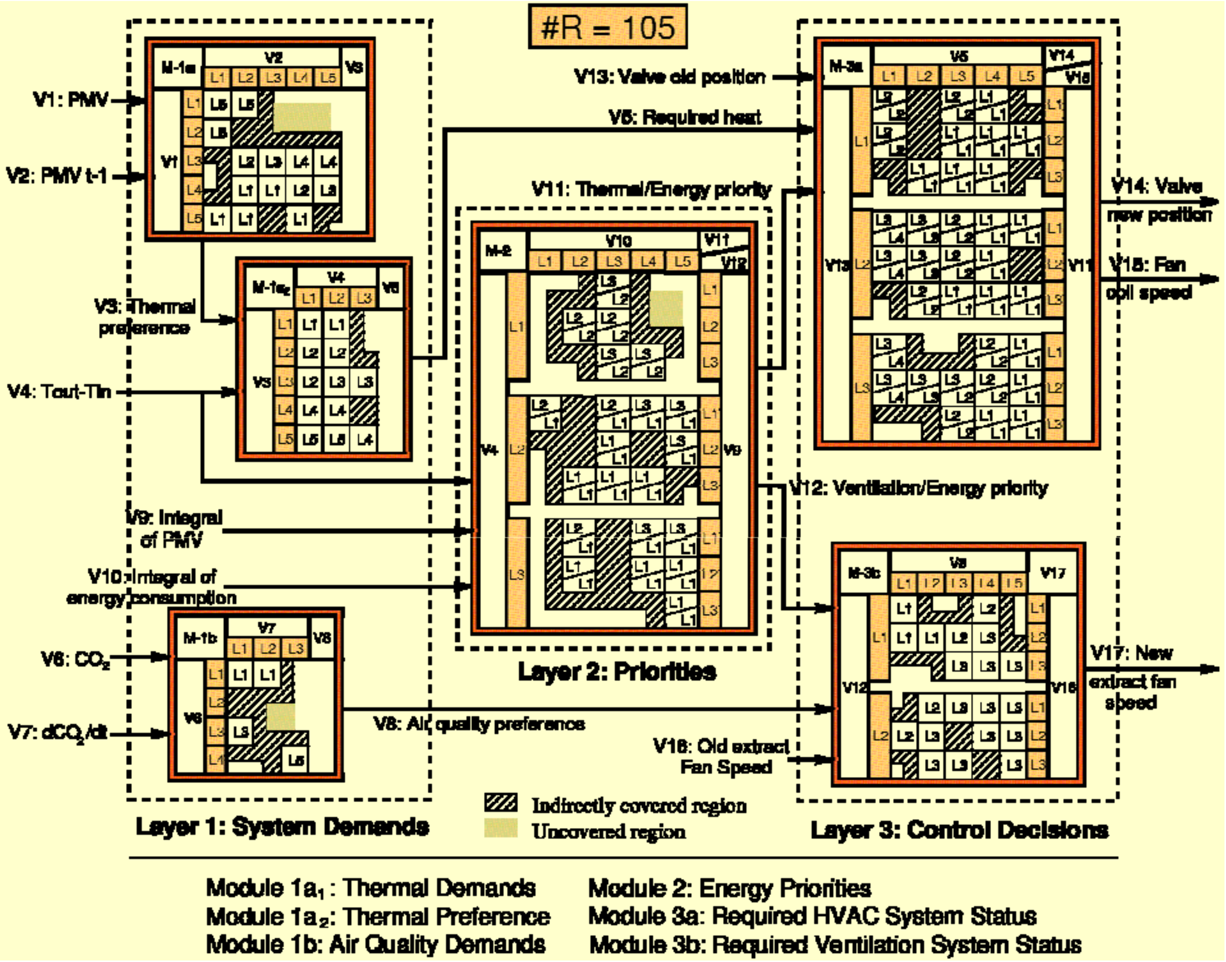
GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

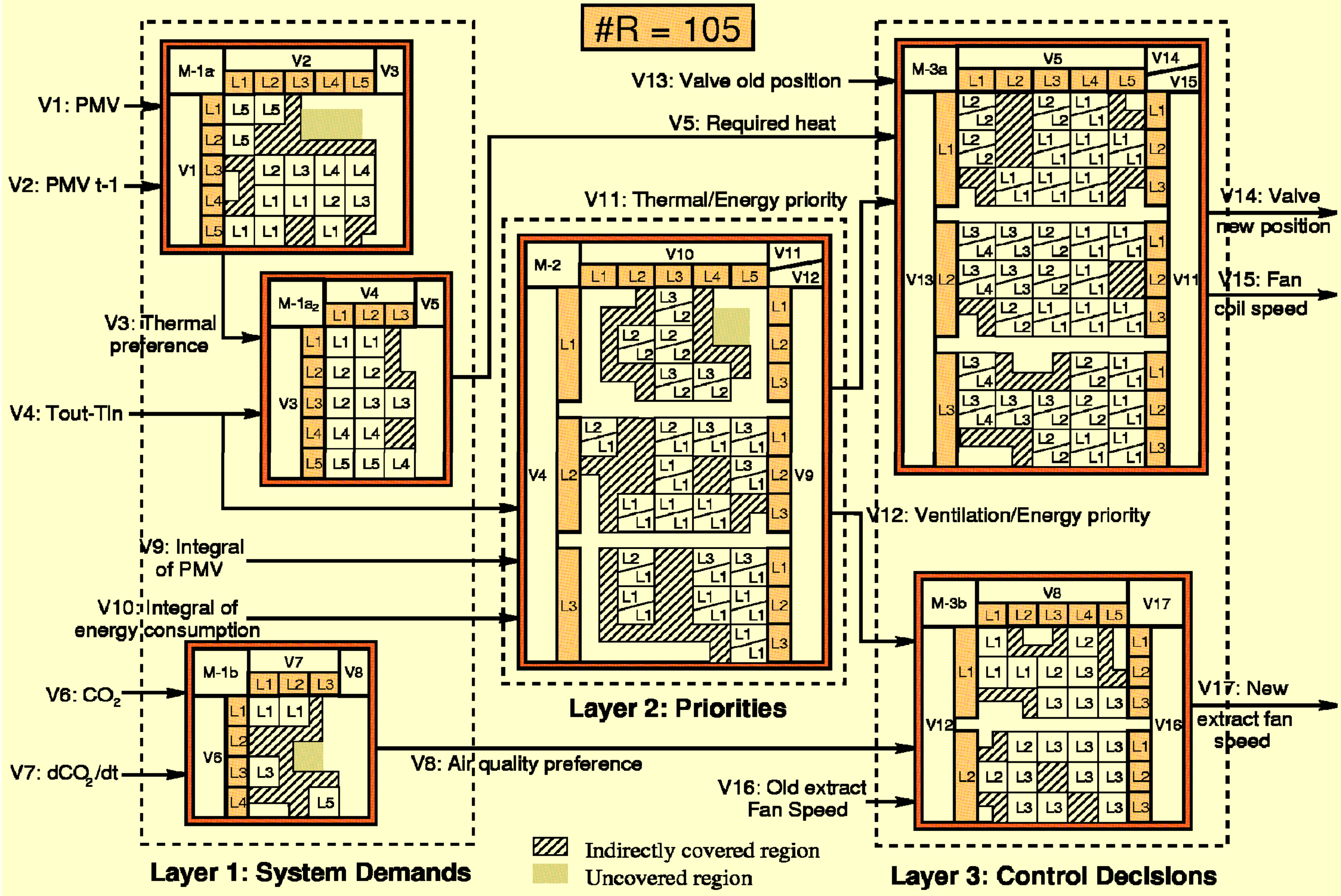
Tuned Data Base (GL-S_1):



GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

Selected Rule Base (GL-S$_1$):





Module $1a_1$: Thermal Demands

Module $1a_2$: Thermal Preference

Module 1b: Air Quality Demands

Module 2: Energy Priorities

Module 3a: Required HVAC System Status

Module 3b: Required Ventilation System Status

GFS Models for Fuzzy Control of HVAC Systems: Genetic Lateral Tuning and Rule Selection

⇨**The combination of lateral tuning (global and local) and rules selection allow us to eliminate redundant rules, tuning the parameters, and geting and high behaviour reducting the energy comsuption and with good stability.**

⇨ **¿What is the reason of the good behavior?**

The SBRDs tuning for an HVAC system is a large scale problem wiht 17 variables and a lot of parameters, and the use of 1 parameter per label allows us to reduce the search space, allowing to get a better optimal local than using 3 parameters per label.

GFS Models for Fuzzy Control of HVAC Systems

Bibliography

R. Alcalá, J.M. Benítez, J. Casillas, O. Cordón, R. Pérez, Fuzzy Control of HVAC Systems Optimized by Genetic Algorithms. ***Applied Intelligence 18:2 (2003) 155-177*****.**

R. Alcalá, J. Casillas, O. Cordón, A. González, F. Herrera, A Genetic Rule Weighting and Selection Process for Fuzzy Control of Heating, Ventilating and Air Conditioning Systems. ***Engineering Applications of Artificial Intelligence 18:3 (2005) 279-296*****.**

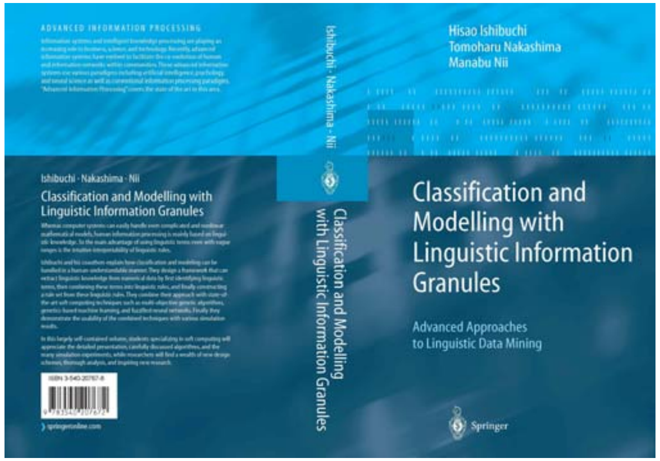
R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, Improving Fuzzy Logic Controllers Obtained by Experts: A Case Study in HVAC Systems. ***Applied Intelligence 31:1 (2009) 10-35*****.**

Contents of This Tutorial

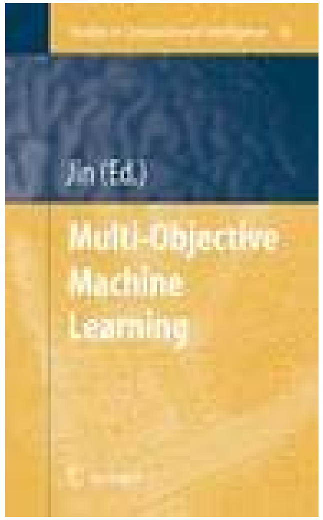
1. Introduction
    - Overview of Fuzzy System Design from Numerical Data
2. Evolutionary Multiobjective Optimization (EMO)
    - Some Basic Concepts in Multiobjective Optimization
    - Framework of Evolutionary Multiobjective Optimization
3. Genetic Fuzzy Systems (GFS)
    - Introduction to Genetic Fuzzy System Research
    - Current State of Genetic Fuzzy Systems
4. Interpretability-Accuracy Tradeoff of Fuzzy Systems
    - Interpretability Issues in Fuzzy System Design
    - Some Examples on the Tuning of Fuzzy Systems
5. Multiobjective Genetic Fuzzy Systems (MoGFS)
    - Overview of MoGFS Research
    - New Research Directions in MoGFS

Multiobjective Genetic Fuzzy Systems

Bibliography



H. Ishibuchi, T. Nakashima, M. Hii. Classification and Modelling with Linguistic Information Granules. Advanced Approaches to Linguistic Data Mining. Springer-Verlag, 2004.



Jin, Yaochu (Ed.)
Multi-Objective Machine Learning
Springer-Verlag, 2006

Literature (http://www.keel.es)

Multiobjective Genetic Algorithms and Rule Learning

http://sci2s.ugr.es/keel/specific.php?area=44

Different Models of Multiobjective GFSs

Bibliography on Interpretability/Accuracy

Year	Ref.	Problem	MOEA/Gen.	#Objs.	RS	FS	RL	LP	
			MAMDANI LINGUISTIC MODELS						
1995/7/8	[12]–[14]	Class.	MOGA/1^{st}	2	$\surd$	$\surd$	-	-	Ishibuchi et all.
2001	[11]	Class.	MOGA/1^{st}	2	-	$\surd$	$\surd$	-	
2001	[15], [16]	Class.	MOGA/1^{st}	3	$\surd$	$\surd$	$\surd$	-	
2004	[18]	Class.	MOGLS/1^{st}	3	$\surd$	$\surd$	-	-	
2005	[20]	Class.	NSGA-II*/2^{nd}	3	$\surd$	$\surd$	-	-	
2007	[19]	Class.	NSGA-II*/2^{nd}	3	-	$\surd$	$\surd$	-	
2008	[21]	Class.	NSGA-II*/2^{nd}	3	-	$\surd$	$\surd$	$\surd$	Koivisto, Pulkkinen
2008	[22]	Class.	NSGA-II*/2^{nd}	3	-	$\surd$	$\surd$	$\surd$	
2003	[17]	Regr.	MOGA/1^{st}	3	$\surd$	$\surd$	$\surd$	-	Ishibuchi et all
2007	[24]	Regr.	PAES*/2^{nd}	2	-	$\surd$	$\surd$	-	Marcelloni et all
2007/2009	[26], [27]	Regr.	SPEA2*/2^{nd}	2	$\surd$	-	-	$\surd$	Alcalá et all
2009	[28]	Regr.	NSGA-II*/2^{nd}	2	-	-	-	$\surd$	Marcelloni et all
-	In press	Regr.	PAES*/2^{nd}	2	-	$\surd$	$\surd$	$\surd$	Alcalá – Marcelloni et all
			TAKAGI-SUGENO MODELS						
2001	[38], [39]	Regr.	Specific/1^{st}	3	-	-	$\surd$	$\surd$	
2005	[40]	Regr.	MOGA*/1^{st}	5	$\surd$	$\surd$	$\surd$	$\surd$	
2005	[41]	Regr.	NSGA-II*/2^{nd}	5	$\surd$	$\surd$	$\surd$	$\surd$	

RS = Rule Selection, **FS** = Feature Selection, **RL** = Rule Learning, **LP** = Learning/Tuning of parameters; Class./Regr.: Classification/Regression; $1^{st}/2^{nd}$: First/second generation of MOEAs; * based on that algorithm.

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

MULTIOBJECTIVE RULE SELECTION (CLASSIFICATION)

H. Ishibuchi, T. Yamamoto, Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining, *Fuzzy Sets and Systems, Vol. 141, pp. 59-88 (2004)*

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Two-Stage Approach

1. Heuristic Rule Extraction

   A pre-specified number of candidate fuzzy rules are extracted from numerical data using a heuristic rule evaluation criterion (data mining).

2. Multiobjective Genetic Fuzzy Rule Selection

   A small number of fuzzy rules are selected from the extracted candidate rules using a multi-objective genetic algorithm (evolutionary optimization).

H. Ishibuchi and T. Yamamoto, "Fuzzy rule selection by multi-objective genetic local search algorithms and rule evaluation measures in data mining," ***Fuzzy Sets and Systems***, Vol. 141, pp. 59-88 (2004).

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Fuzzy Rules for *n*-dimensional Problems

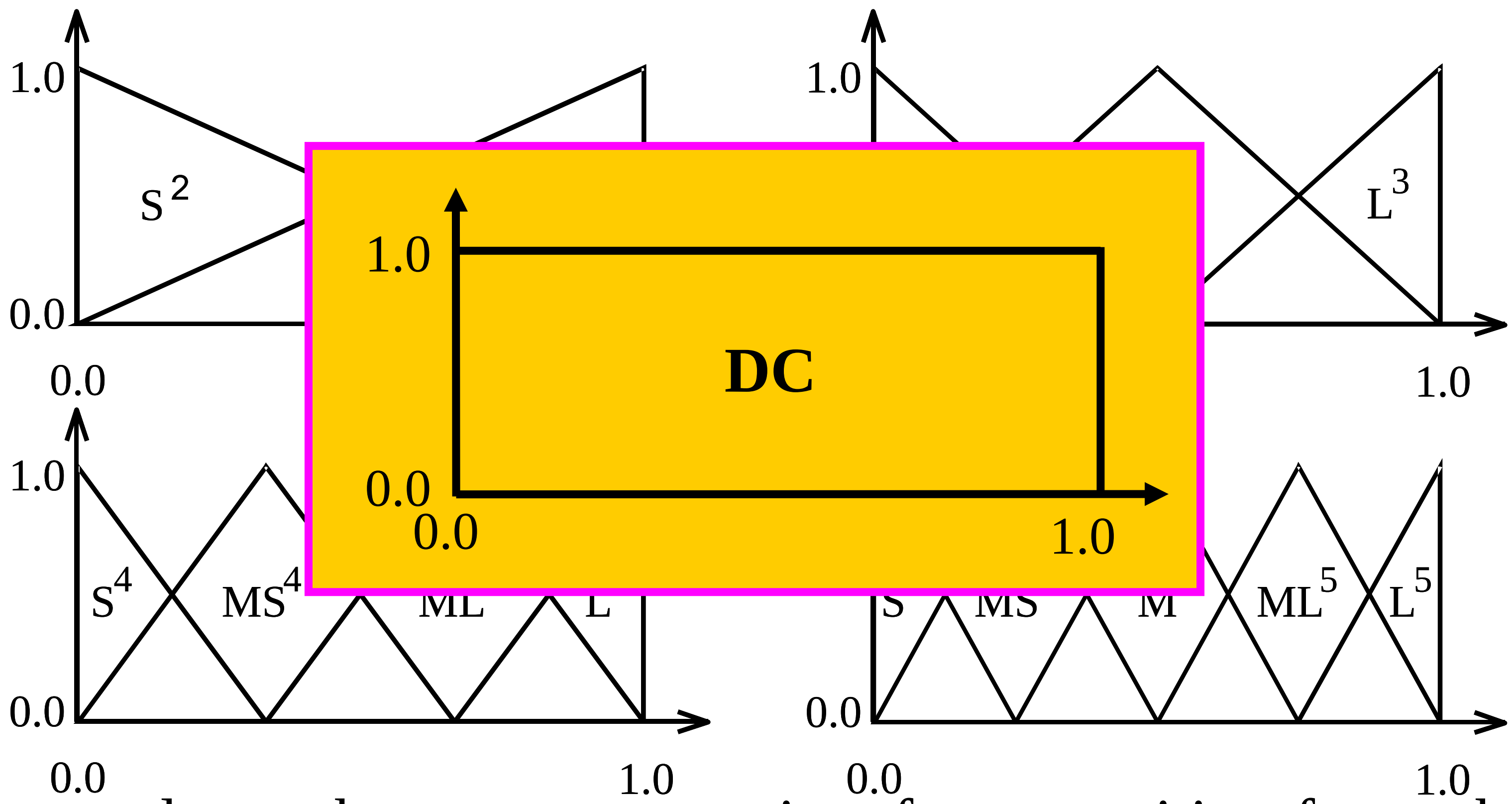
If x_1 is A_1 and … and x_n is A_n

then Class C with CF

A_i :	Antecedent fuzzy set
Class C :	Consequent class
CF :	Rule weight (Certainty factor)

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

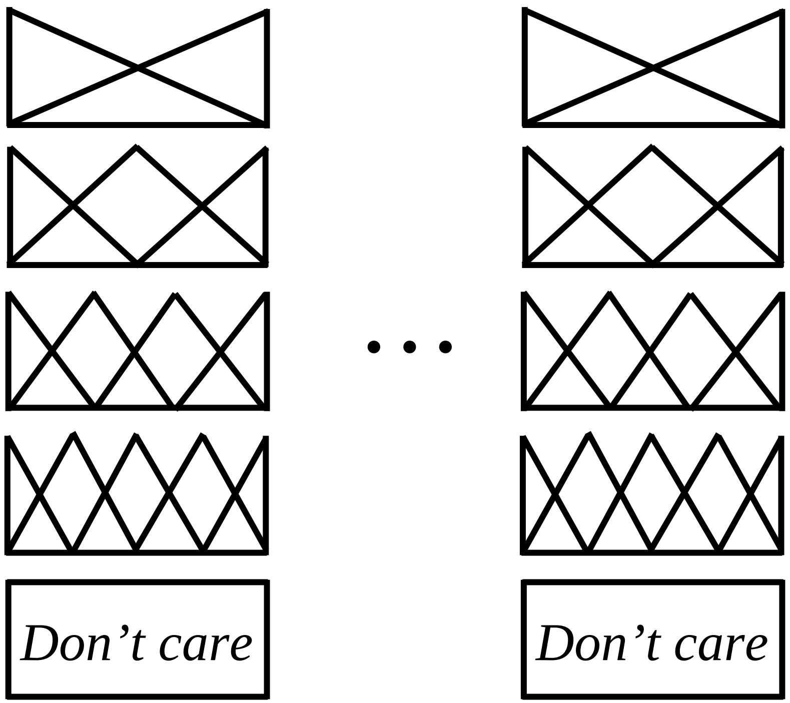
Antecedent Fuzzy Sets (Multiple Partitions)



Usually we do not know an appropriate fuzzy partition for each input variable.

Possible Fuzzy Rules

Total number of possible fuzzy rules



$$x_1 \qquad\qquad x_n$$

$$(14+1)\times \;\cdots\; \times(14+1)=(15)^n$$

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Examined Fuzzy Rules

They only examine short fuzzy rules with only a few antecedent conditions.

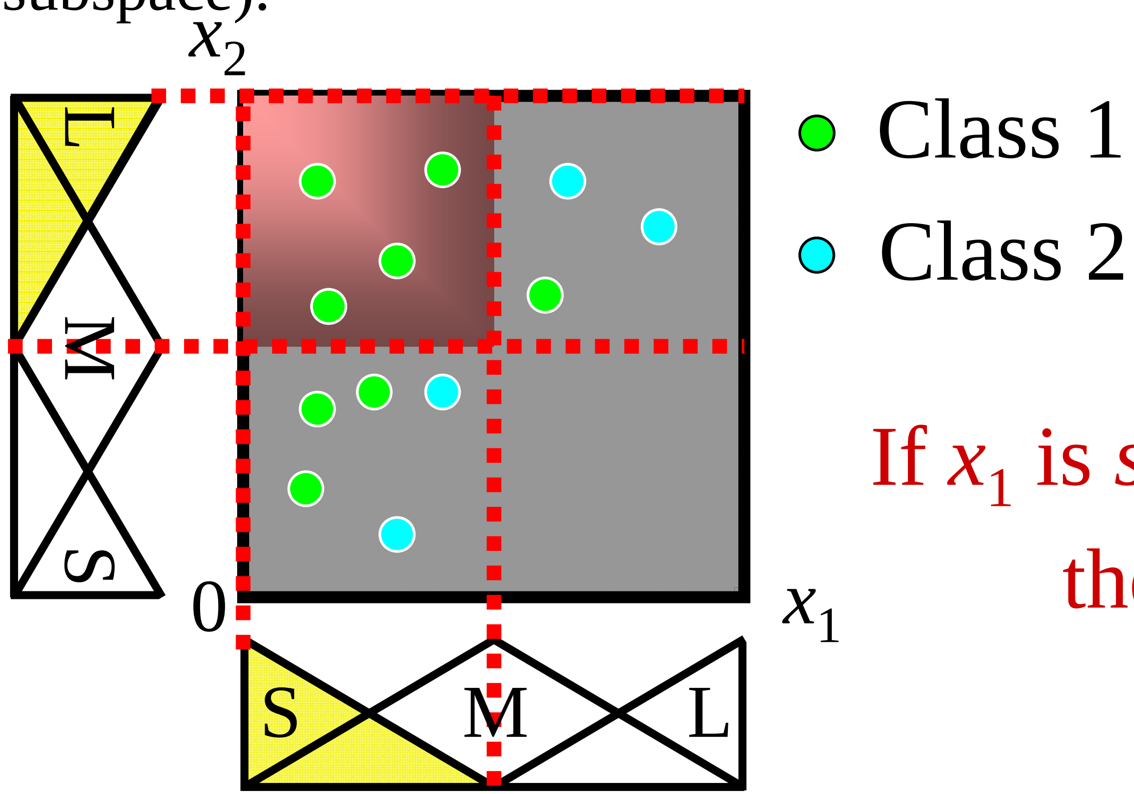
If x_1 is *small* and x_{48} is *large*

then Class 1 with 0.58

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Consequent Class

The consequent class of each fuzzy rule is determined by compatible training patterns (i.e., the dominant class in the corresponding fuzzy subspace).

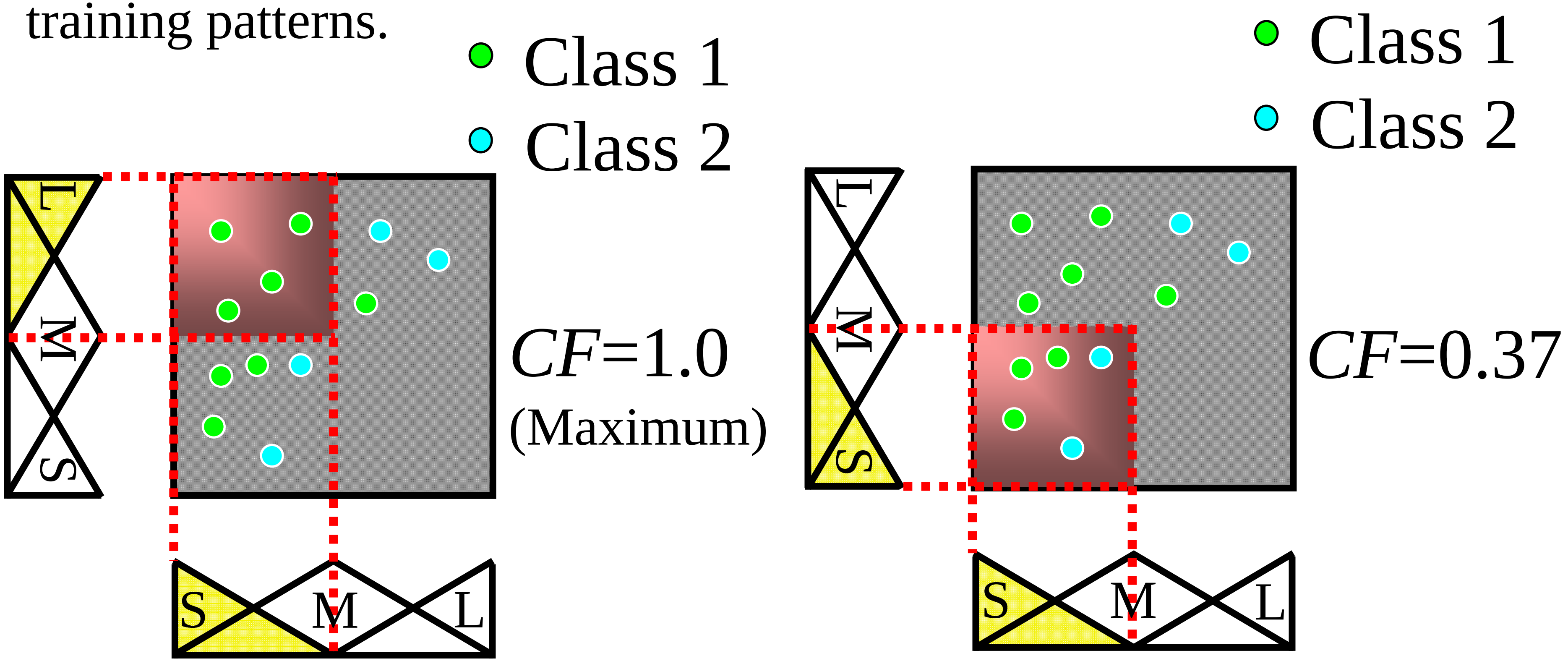


If x_1 is *small* and x_2 is *large* then Class 1 with 1.0

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Rule Weight **(Certainty Factor)**

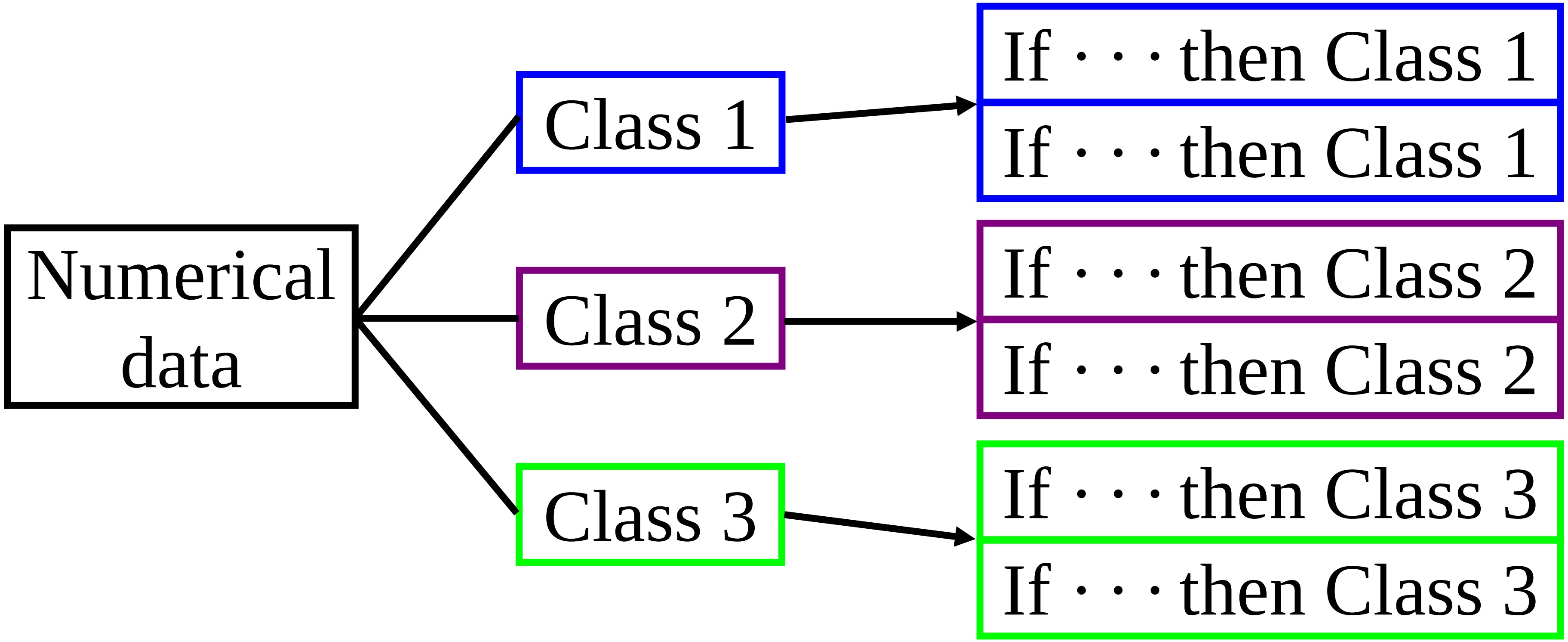
The rule weight *CF* of each fuzzy rule is calculated from compatible training patterns.



Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Heuristic Rule Extraction

They extract a pre-specified number of the best fuzzy rules with respect to a pre-specified heuristic rule evaluation criterion.



Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Heuristic Rule Extraction

Possible fuzzy rules: $(15)^n$ rules

↓

Restriction on the rule length :

Only short fuzzy rules

↓

Rule evaluation criterion:

The best rules for each class

300 fuzzy rules for each class

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Two-Stage Approach

1. Heuristic Rule Extraction

   A pre-specified number of candidate fuzzy rules are extracted from numerical data using a heuristic rule evaluation criterion (data mining).

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Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Implementation of Multiobjective approach

Coding: $S = s_1 s_2 \cdots s_N$

N: Total number of candidate rules

s_j={0, 1}: Inclusion or exclusion of the j-th rule

Objectives: $f_1(S)$, $f_2(S)$, $f_3(S)$

$f_1(S)$: Number of correctly classified patterns by S

$f_2(S)$: Number of selected rules in S

$f_3(S)$: Total number of antecedent conditions in S

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Comparison of Four Approaches

(1) Two-objective approach

Maximize $f_1(S)$ and minimize $f_2(S)$

(2) Weighted sum of the two objectives

Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S)$

(3) Three-objective approach

Maximize $f_1(S)$ and minimize $f_2(S)$, $f_3(S)$

(4) Weighted sum of the three objectives

Maximize $w_1 \cdot f_1(S) - w_2 \cdot f_2(S) - w_3 \cdot f_3(S)$

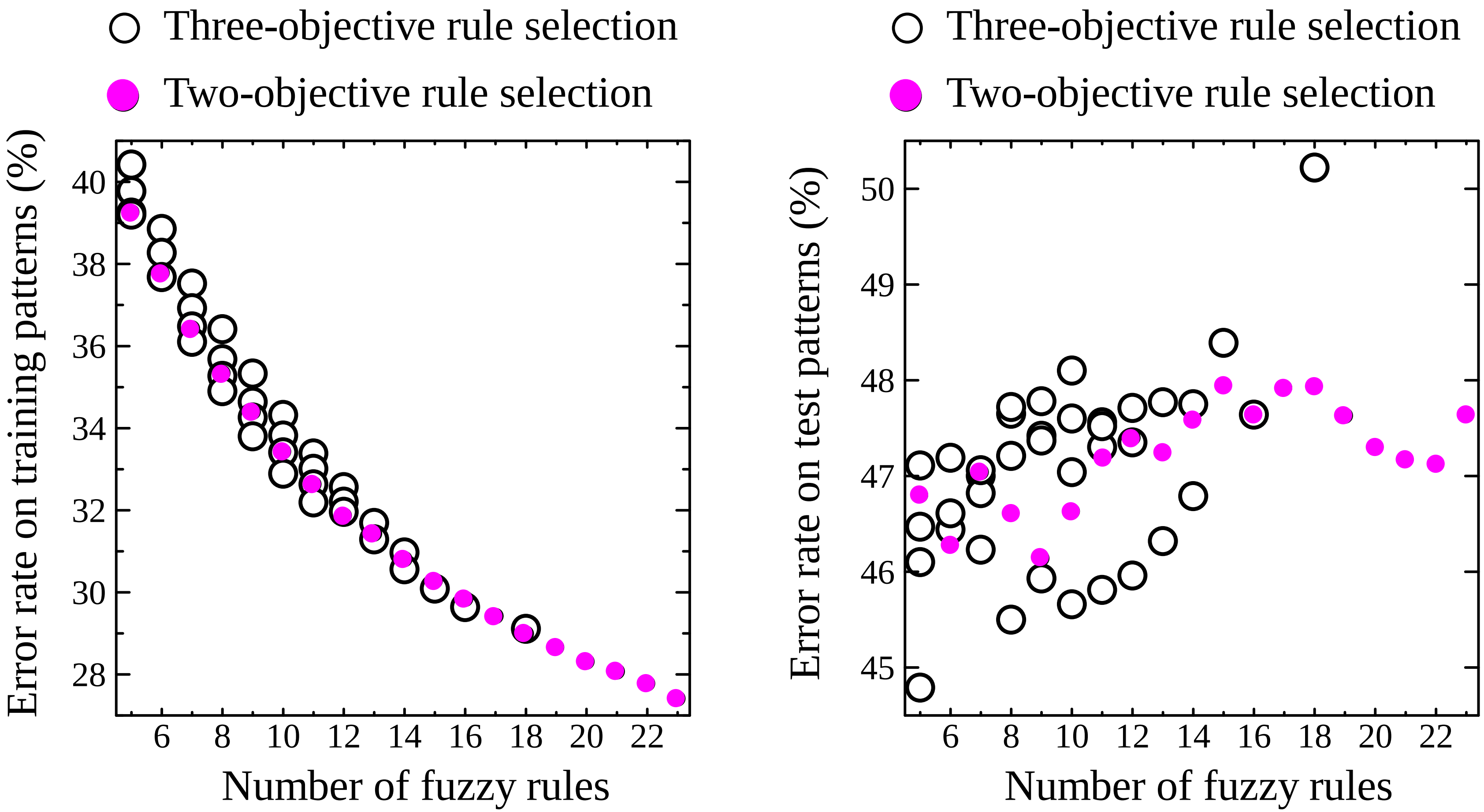
Data Sets

Data set	Attributes	Patterns	Classes	Length
Breast W	9	683*	2	3
Diabetes	8	768	2	3
Glass	9	214	6	3
Heart C	13	297*	5	3
Iris	4	150	3	3
Sonar	60	208	2	2
Wine	13	178	3	3

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Experimental Results (Cleveland Heart)



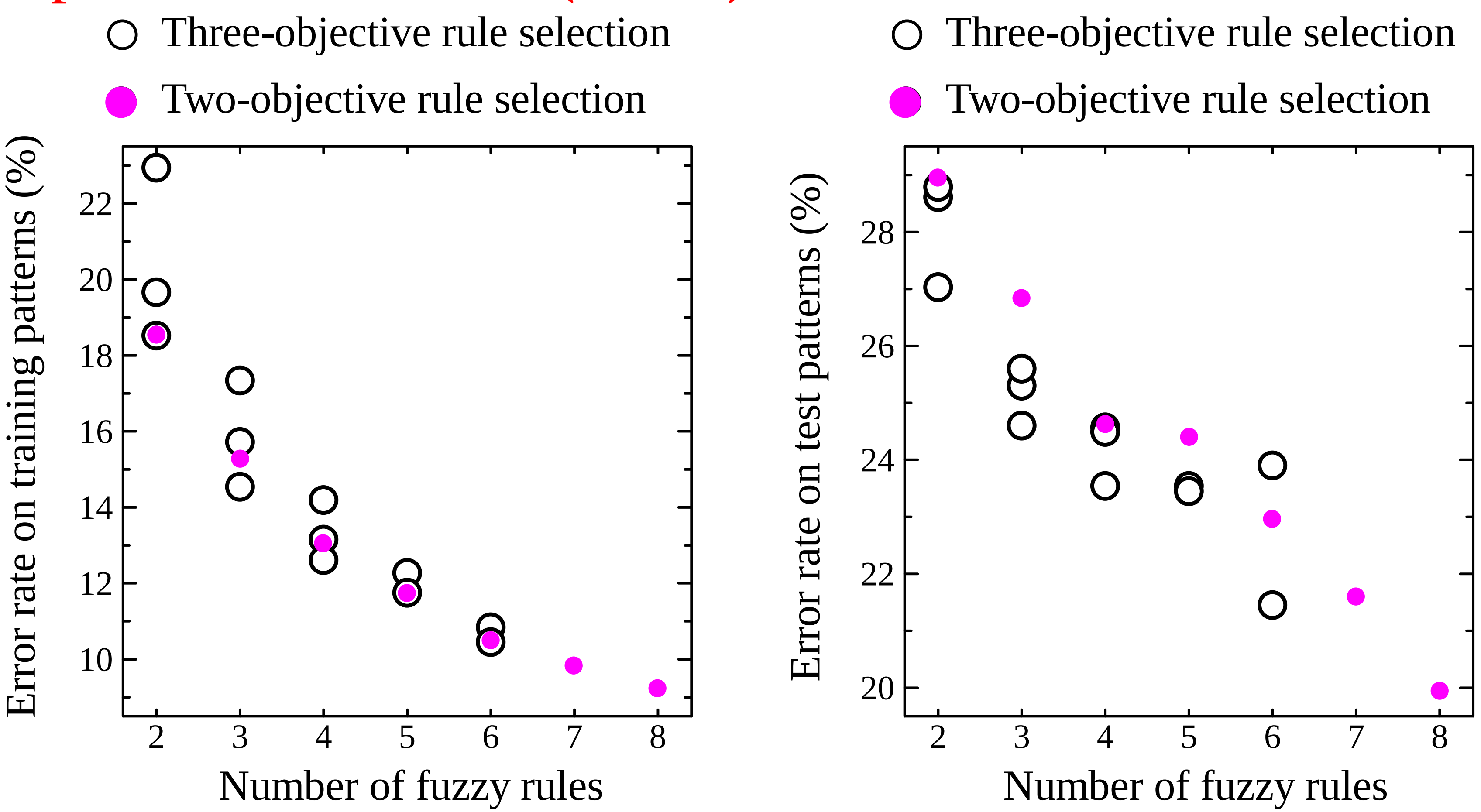
(a) Error rates on **training data** (b) Error rates on **test data**

We can observe the overfitting due to the increase in the number of fuzzy rules.

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Experimental Results (Sonar)



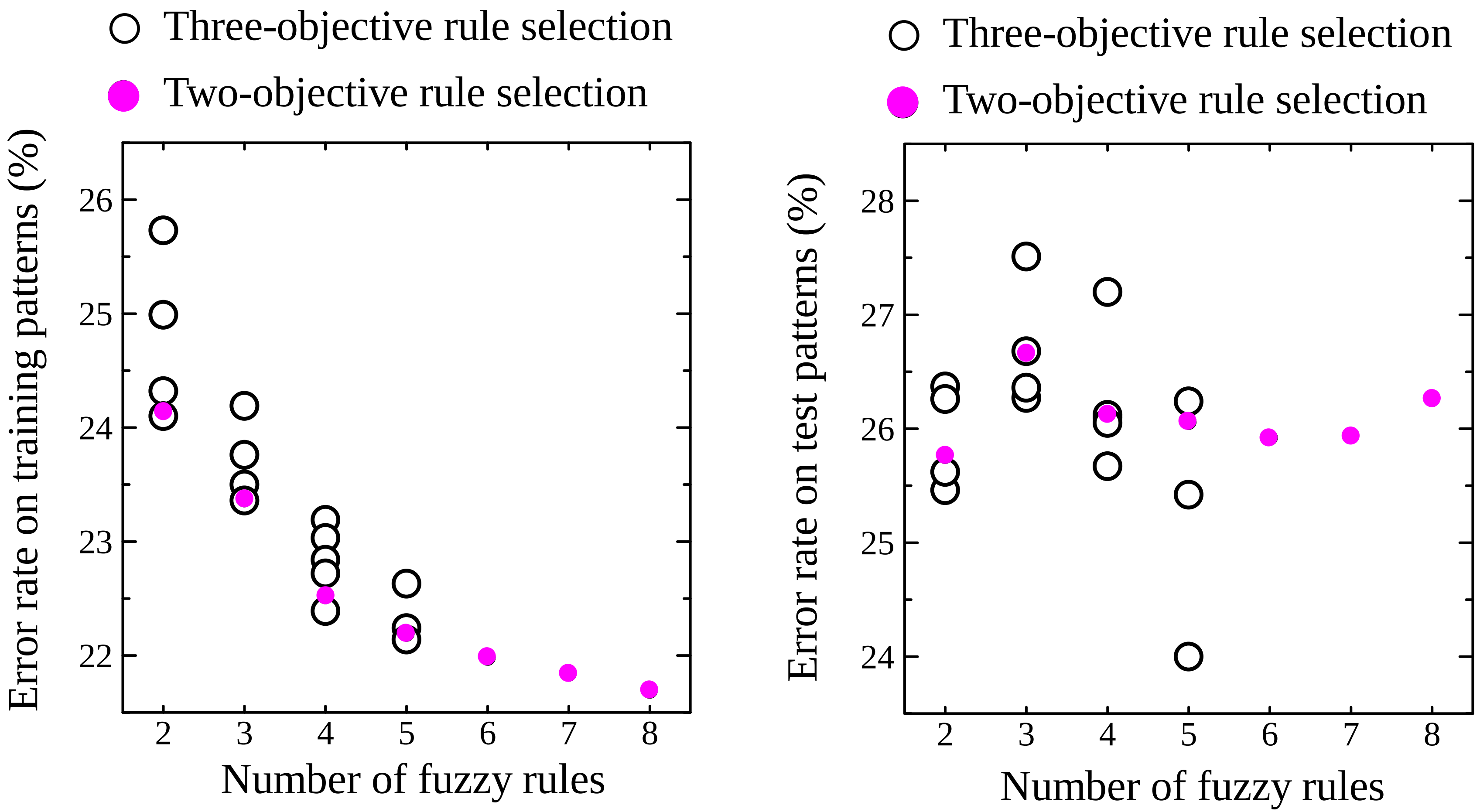
(a) Error rates on **training data** (b) Error rates on **test data**

The generalization ability is increased by increasing the number of fuzzy rules (i.e., the overfitting is not observed).

Different Models of Multiobjective GFSs

MODEL 1: Multiobjective Rule Selection

Experimental Results (Diabetes)

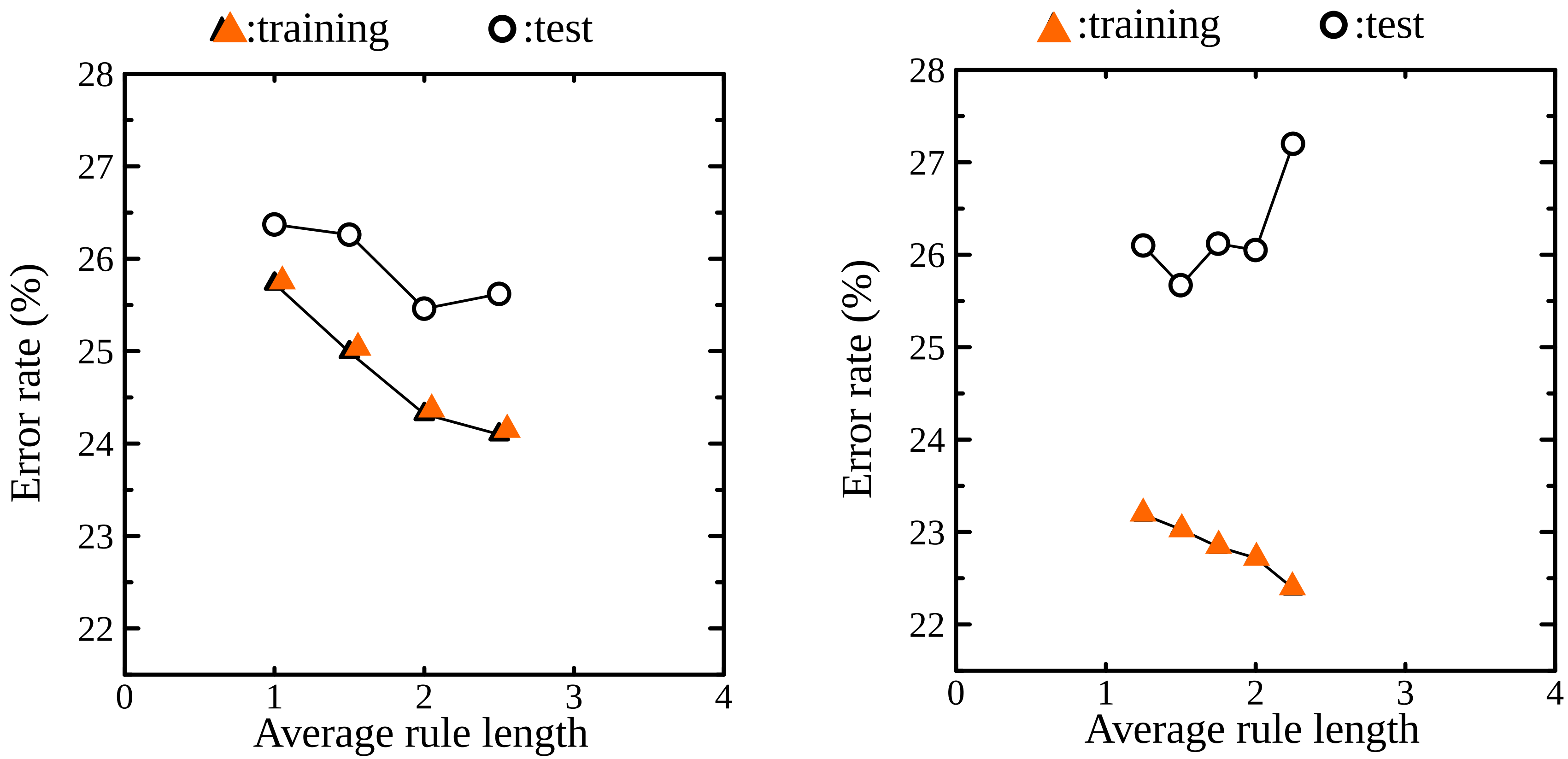


(a) Error rates on **training data** (b) Error rates on **test data**

The effect of the increase in the number of fuzzy rules is not clear.

Different Models of Multiobjective GFSs
MODEL 1: Multiobjective Rule Selection

Experimental Results (Diabetes)



(a) Rule sets with two rules **(b) Rule sets with four rules**

We can observe the overfitting due to the increase in the rule length in the right figure for rule sets with four fuzzy rules.

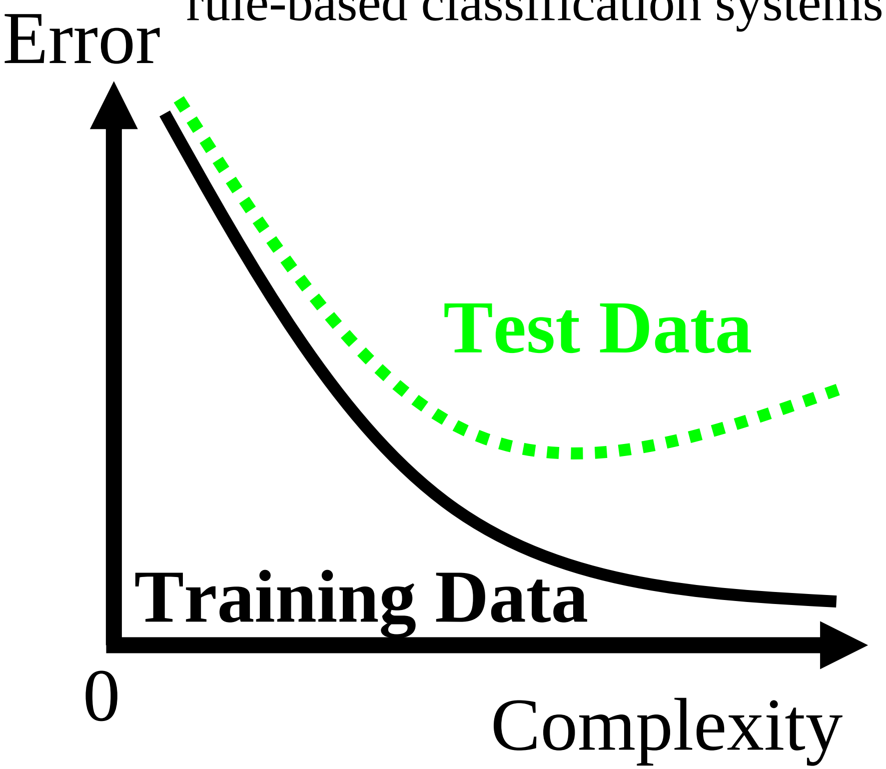
Different Models of Multiobjective GFSs

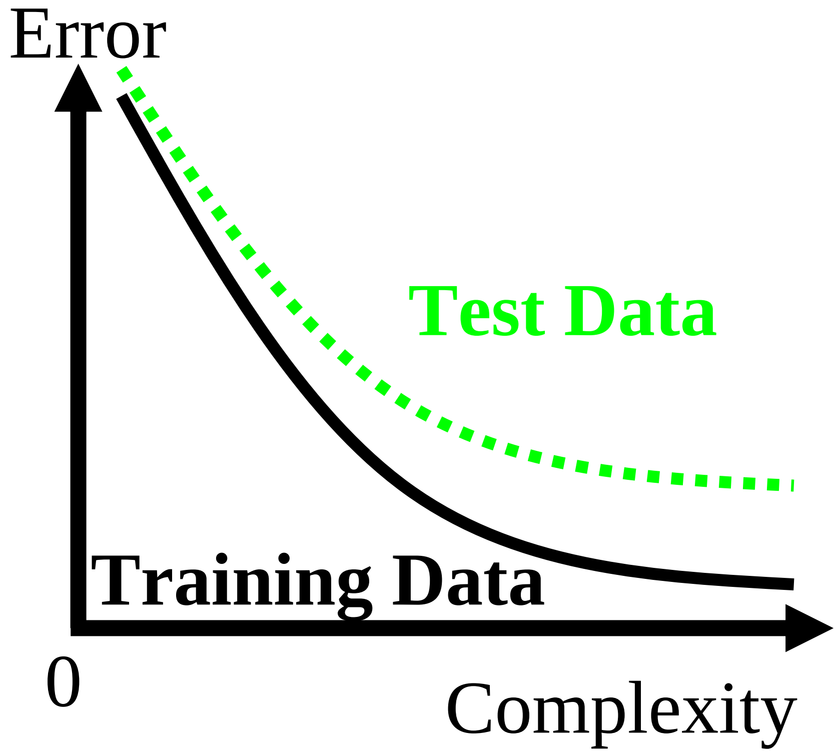
MODEL 1: Multiobjective Rule Selection

Observation

(1) Experimental results showed that each test problem has a different tradeoff structure.

(2) Knowledge on the tradeoff structure is useful in the design of fuzzy rule-based classification systems.





MULTIOBJECTIVE TUNING AND RULE SELECTION IN REGRESSION PROBLEMS

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15:5 (2007) 539–557*

Different Models of Multiobjective GFSs
MODEL 2: Multiobjective Tuning and Rule Selection

R. Alcalá, J. Alcalá-Fdez, M.J. Gacto, F. Herrera, A multi-objective genetic algorithm for tuning and rule selection to obtain accurate and compact linguistic fuzzy rule-based systems, *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, 15:5 (2007) 539–557*,

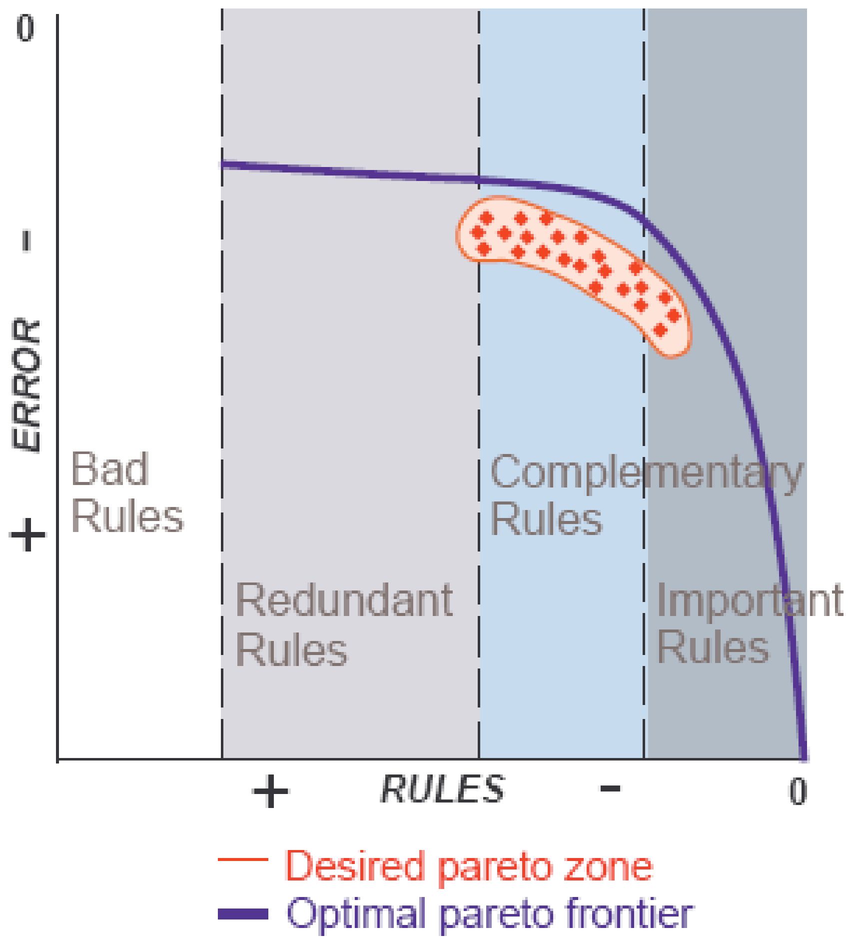
Multi-objective EAs are powerful tools to generate GFSs but they are based on getting a large, well distributed and spread off, Pareto set of solutions

- **The two criteria to optimize in GFSs are accuracy and interpretability. The former is more important than the latter, so many solutions in the Pareto set are not useful**

- **Solution: Inject knowledge through the MOEA run to bias the algorithm to generate the desired Pareto front part**

Different Models of Multiobjective GFSs
MODEL 2: Multiobjective Tuning and Rule Selection

Pareto front classification in an interpretability-accuracy GFSs:



- Bad rules zone: solutions with bad performance rules. Removing them improves the accuracy, so no Pareto solutions are located here
- Redundant rules zone: solutions with irrelevant rules. Removing them does not affect the accuracy and improves the interpretability
- Complementary rules zone: solutions with neither bad nor irrelevant rules. Removing them slightly decreases the accuracy
- Important rules zone: solutions with essential rules. Removing them significantly decreases the accuracy

Different Models of Multiobjective GFSs
MODEL 2: Multiobjective Tuning and Rule Selection

Accuracy-oriented modifications performed:

- **Restart the genetic population at the middle of the run time, keeping the individual with the highest accuracy as the only one in the external population and generating all the new individuals with the same number of rules it has**

- **In each MOGA step, the number of chromosomes in the external population considered for the binary tournament is decreased, focusing the selection on the higher accuracy individuals**

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Obtained results for the medium voltage line problem:

Multi-objective genetic tuning + rule selection method:

Method	#R	MSE_{tra}	σ_{tra}	t-test	MSE_{tst}	σ_{tst}	t-test
WM	65	57605	2841	+	57934	4733	+
WM+T	65	18602	1211	+	22666	3386	+
WM+S	40.8	41086	1322	+	59942	4931	+
WM+TS	41.9	14987	391	+	18973	3772	+
NSGAII	41.0	14488	965	+	18419	3054	+
$NSGAII_{ACC}$	48.1	16321	1636	+	20423	3138	+
SPEA2	**33**	13272	1265	+	17533	3226	+
$SPEA2_{ACC}$	34.5	**11081**	1186	*	**14161**	2191	*

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

STUDY ON SEVERAL ALTERNATIVE APPROACHES AND IMPROVEMENTS

M.J. Gacto, R. Alcalá, F. Herrera, Adaptation and Application of Multi-Objective Evolutionary Algorithms for Rule Reduction and Parameter Tuning of Fuzzy Rule-Based Systems. ***Soft Computing 13:5 (2009) 419-436***

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

- **To perform the study we have applied six different approaches based on the two most known and successful MOEAs:**
  - **Ap**
  - **Tw ... GA-II$_A$ and NSGA-II$_U$**
  - **Tw ... $_{Acc}$ and SPEA2$_{Acc2}$**
- **Two obj ... mber of Rules**
- **Proper c**
- **The de ... size becomes an importa ... GA-II**

Method	Description
WM	Wang & Mendel algorithm
T	Tuning of Parameters
S	Rule Selection
TS	Tuning & Selection
Application of standard MOEAs for general use	
TS-SPEA2	Tuning & Selection by SPEA2
TS-NSGA-II	Tuning & Selection by NSGA-II
TS-NSGA-II$_A$	Tuning & Selection by NSGA-II$_{angle}$
TS-NSGA-II$_U$	Tuning & Selection by NSGA-II$_{utility}$
Extended MOEAs for specific application	
TS-SPEA2$_{Acc}$	Accuracy-Oriented SPEA2
TS-SPEA2$_{Acc^2}$	Extension of SPEA2$_{Acc}$

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

NSGA-II FOR FINDING KNEES

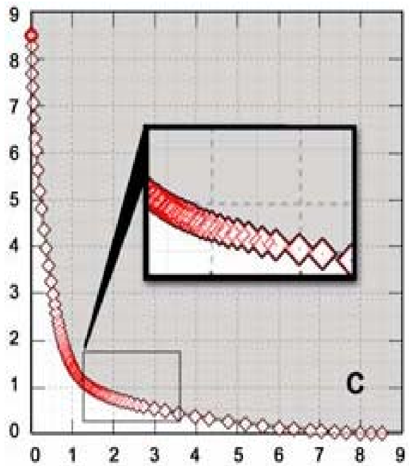
J. Branke, K. Deb, H. Dierolf, and M. Osswald, “Finding Knees in Multi-objective Optimization,” Proc. Parallel Problem Solving from Nature Conf. - PPSN VIII, LNCS 3242, (Birmingham, UK, 2004) 722–731.

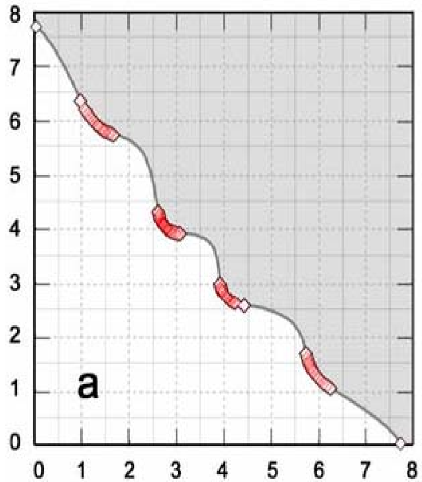
- **A variation of NSGAII in order to find knees in the Pareto front by replacing the crowding measure by either an angle-based measure or an utility-based measure**

Two different approaches

Angle Based Approach

Utility Based Approach





- **In our case, a knee could represent the best compromise between accuracy and number of rules.**

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Extension of $SPEA2_{Acc}$ ($SPEA2_{Acc2}$)

A New Crossover Operator for the Rule Part

- **Objective:** to improve the search with a more intelligent operator replacing the HUX crossover in $SPEA2_{ACC}$
- Once BLX is applied a normalized euclidean distance is calculated between the centric point of the MFs used by each rule of the offpring and each parent
- **The closer parent determines if this rule is selected or not** for this offpring
- Whit this crossover operator, **mutation can be particularly used to remove rules**

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Obtained results for the medium voltage line problem:

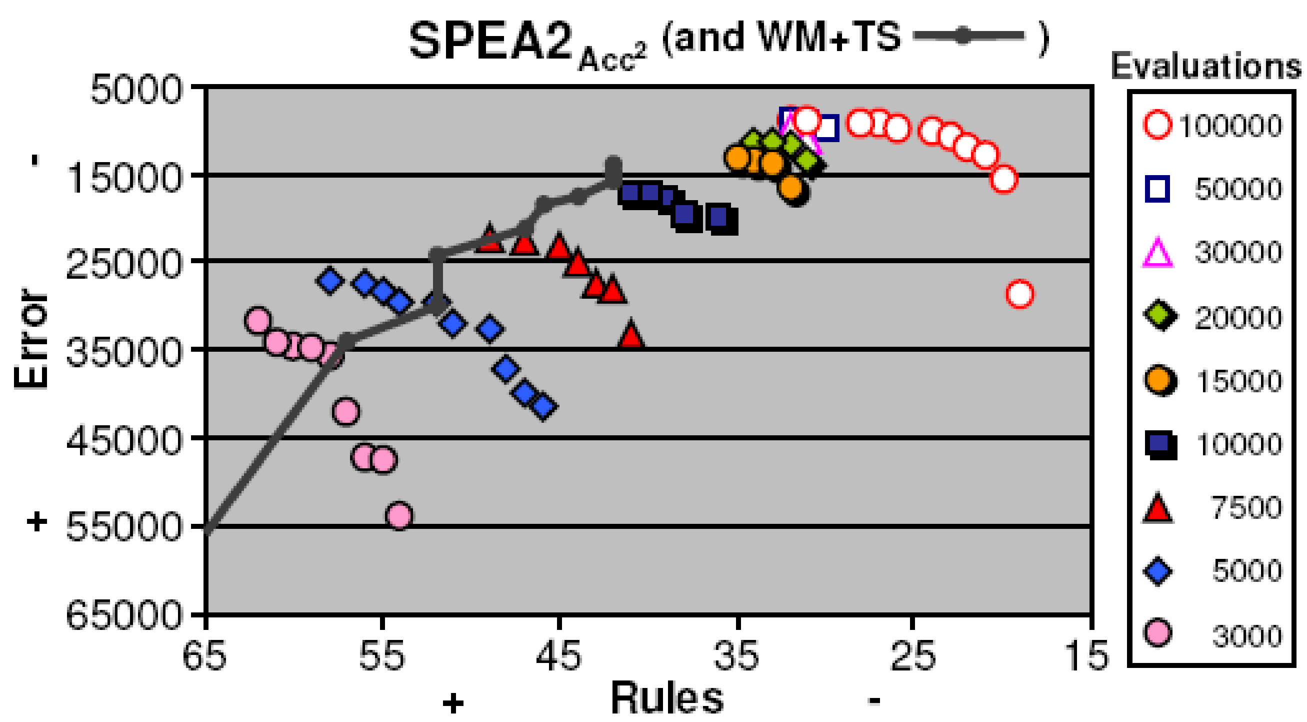
Method	#R	MSE_{tra}	σ_{tra}	t	MSE_{tst}	σ_{tst}	t
			100,000 evaluations				
WM	**65.0**	**57605**	**2841**	+	**57934**	**4733**	**+**
T	**65.0**	**17020**	**1893**	**+**	**21027**	**4225**	**+**
S	**40.9**	**41158**	**1167**	**+**	**42988**	**4441**	**+**
TS	**41.3**	**13387**	**1153**	**+**	**17784**	**3344**	**+**
TS-SPEA2	**28.9**	**11630**	**1283**	+	**15387**	**3108**	**+**
TS-NSGA-II	**31.4**	**11826**	**1354**	**+**	**16047**	**4070**	**+**
TS-NSGA-II_A	**29.7**	**11798**	**1615**	**+**	**16156**	**4091**	**+**
TS-NSGA-II_U	**30.7**	**11954**	**1768**	**+**	**15879**	**4866**	**+**
TS-$SPEA2_{Acc}$	**32.3**	**10714**	**1392**	=	**14252**	**3181**	=
TS-$SPEA2_{Acc2}$	**29.8**	**10325**	**1121**	*	**13935**	**2759**	*

- 5-fold cross validation × 6 runs = 30 runs per algorithm
- T-student test with 95% confidence

Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

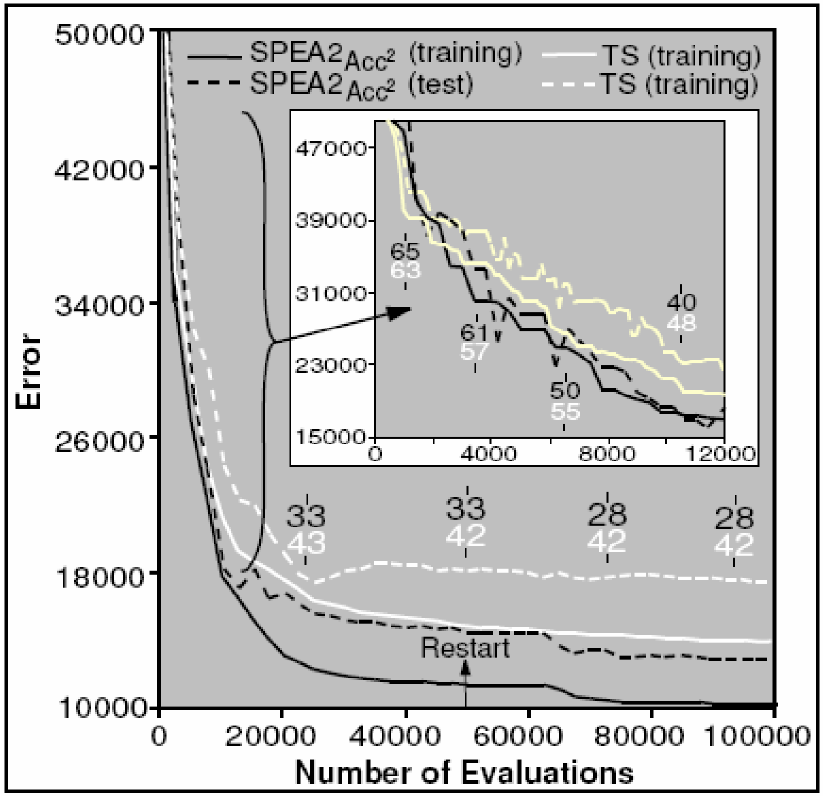
Comparison of the SPEA2acc² and classical GA for for the medium voltage line problem:

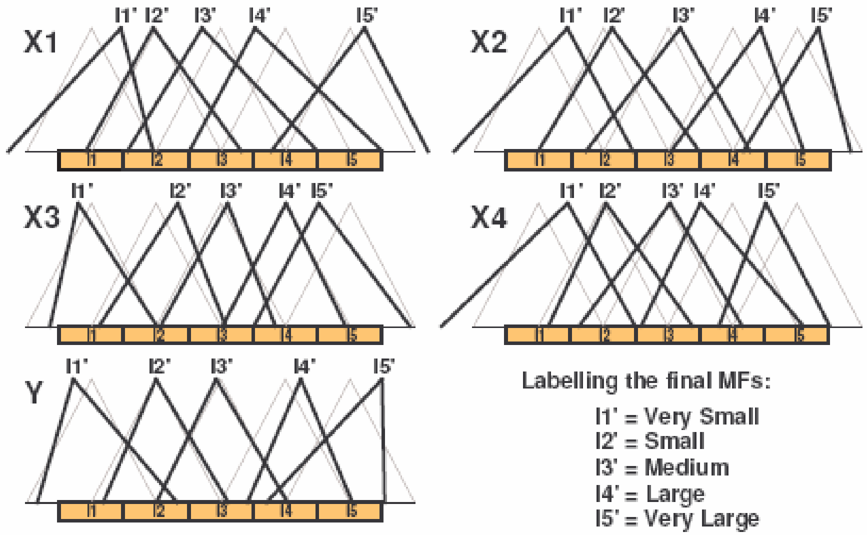


Different Models of Multiobjective GFSs

MODEL 2: Multiobjective Tuning and Rule Selection

Convergence and an example model





#R: 28 *MSE-tra: 8232* *MSE-tst: 14670*

X1	X2	X3	X4	Y
l1'	l1'	l1'	l1'	l1'
l1'	l1'	l1'	l2'	l2'
l2'	l1'	l1'	l1'	l1'
l2'	l1'	l1'	l2'	l2'
l2'	l1'	l2'	l2'	l2'
l2'	l2'	l2'	l1'	l2'
l2'	l3'	l3'	l1'	l3'
l3'	l2'	l1'	l1'	l1'
l3'	l2'	l1'	l2'	l2'

X1	X2	X3	X4	Y
l3'	l2'	l1'	l3'	l2'
l3'	l2'	l2'	l3'	l3'
l3'	l3'	l2'	l2'	l2'
l3'	l3'	l3'	l2'	l3'
l3'	l4'	l3'	l3'	l3'
l4'	l2'	l2'	l2'	l2'
l4'	l3'	l2'	l1'	l2'
l4'	l3'	l2'	l3'	l3'
l4'	l3'	l2'	l4'	l3'

X1	X2	X3	X4	Y
l4'	l3'	l3'	l2'	l3'
l4'	l3'	l3'	l3'	l4'
l4'	l4'	l3'	l2'	l3'
l4'	l4'	l4'	l2'	l4'
l4'	l4'	l4'	l4'	l5'
l4'	l5'	l4'	l2'	l3'
l4'	l5'	l5'	l3'	l5'
l5'	l2'	l2'	l5'	l4'
l5'	l2'	l3'	l2'	l3'
l5'	l4'	l3'	l5'	l5'

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

MULTIOBJECTIVE LEARNING OF DB AND RB (REGRESSION)

R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, F. Marcelloni, A Multi-Objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems, *IEEE Transactions on Fuzzy Systems*, doi:10.1109/TFUZZ.2009.2023113, *in press (2009)*

Different Models of Multiobjective GFSs

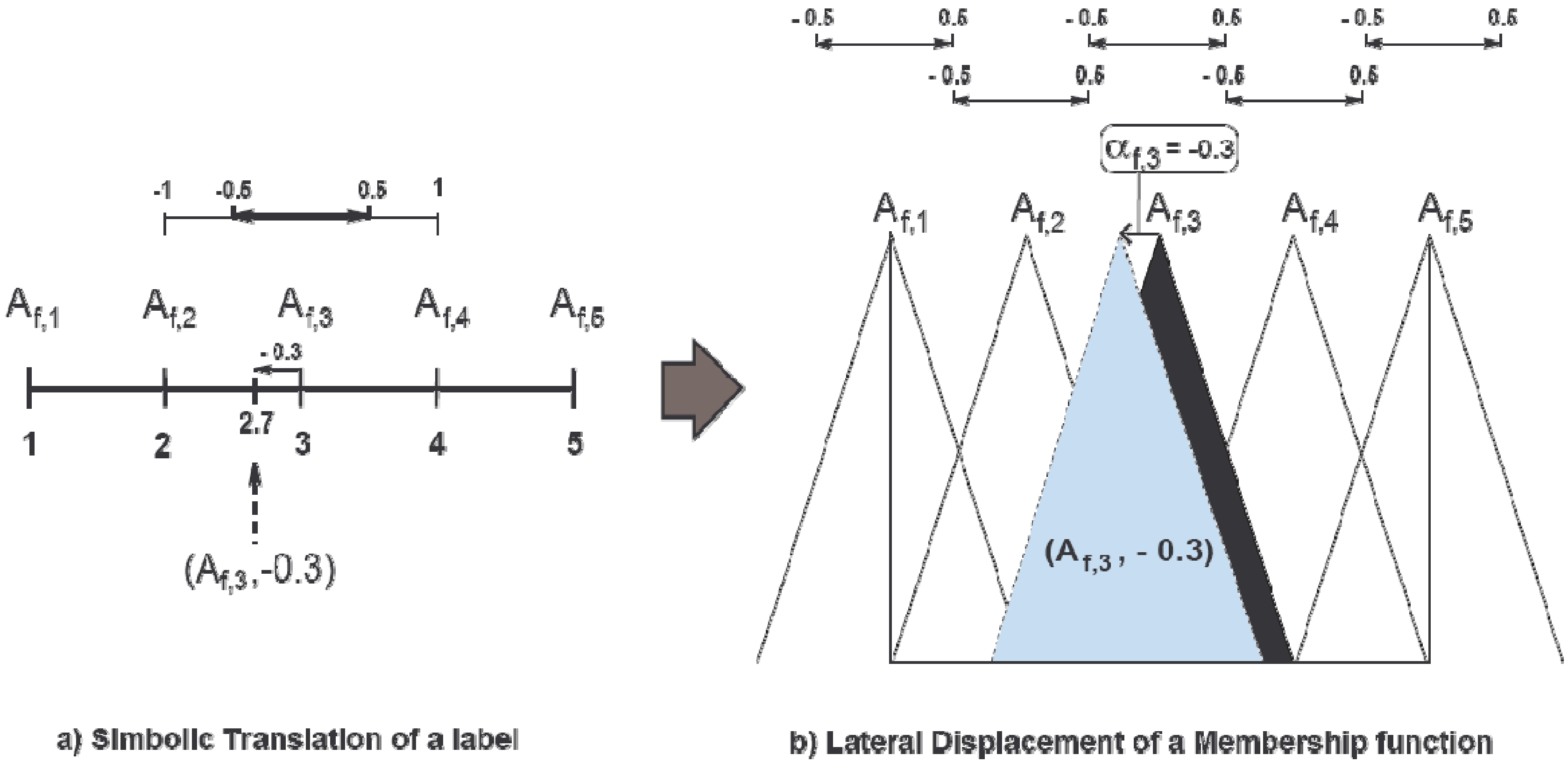
MODEL 3: Multiobjective Learning of DB and RB

R. Alcalá, P. Ducange, F. Herrera, B. Lazzerini, F. Marcelloni, A Multi-Objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems, *IEEE Transactions on Fuzzy Systems, doi:10.1109/TFUZZ.2009.2023113, in press (2009)*,

- **Rule bases and parameters of the membership functions of the associated linguistic labels are learnt concurrently.**
- **Accuracy and interpretability are measured in terms of approximation error (MSE) and rule base complexity (#Conditions), respectively.**
- **To manage the size of the search space, the linguistic 2-tuple representation model, which allows the symbolic translation of a label by only considering one parameter, has been exploited**

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB



This proposal decreases the tuning complexity, since the 3 parameters per label of the classical tuning are reduced to only 1 translation parameter (the tuning is applied to the level of linguistic partitions)

Different Models of Multiobjective GFSs
MODEL 3: Multiobjective Learning of DB and RB

Coding Scheme and Operators

- **A double coding scheme ($C = C_{RB} + C_{DB}$)**

$$\underbrace{\overbrace{j_1^1 \ \cdots \ j_F^1 \ j_{F+1}^1}^{R^1} \ \cdots \ \overbrace{j_1^M \ \cdots \ j_F^M \ j_{F+1}^M}^{R^M}}_{C_{RB}\ \textit{(integer coding)}} \ \underbrace{\overbrace{\alpha_{1,1} \ \cdots \ \alpha_{1,T_1}}^{P_1} \ \cdots \ \overbrace{\alpha_{F+1,1} \ \cdots \ \alpha_{F+1,T_{F+1}}}^{P_{F+1}}}_{C_{DB}\ \textit{(real coding)}}$$

- **Crossover operator: one point + BLX-α crossovers (2 offsprings)**

- **Mutation operators:**
  - **Rule Adding: It adds γ random rules to the RB, where γ is randomly chosen in $[1, \gamma_{max}]$**

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Operators and Selection Schemes

- **Modify RB:** **It randomly changes δ elements of the RB part. The number δ is randomly generated in [1, δ_{max}]**

- **Modify DB:** **It changes a gene value at random in the DB part**

PAES, NSGA-II and SOGA were applied using this representation and crossover

```
[p1, p2] = selection(archive/population);

if (rand() < Pcross)
        [s1, s2] = crossover(p1,p2);
        PmRB = 0.01;
else
        s1 = p1;
        s2 = p2;
        PmRB = 1;
endif
Loop i=1,2
        if (rand() < PmRB)
                if (rand <PmAdd)
                        si = add_rule();
                else
                        si = modify_rule_base();
                endif
        endif
        if (rand() < PmDB)
                si = mutate_DB();
        endif
endLoop
```

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Analysed Methods

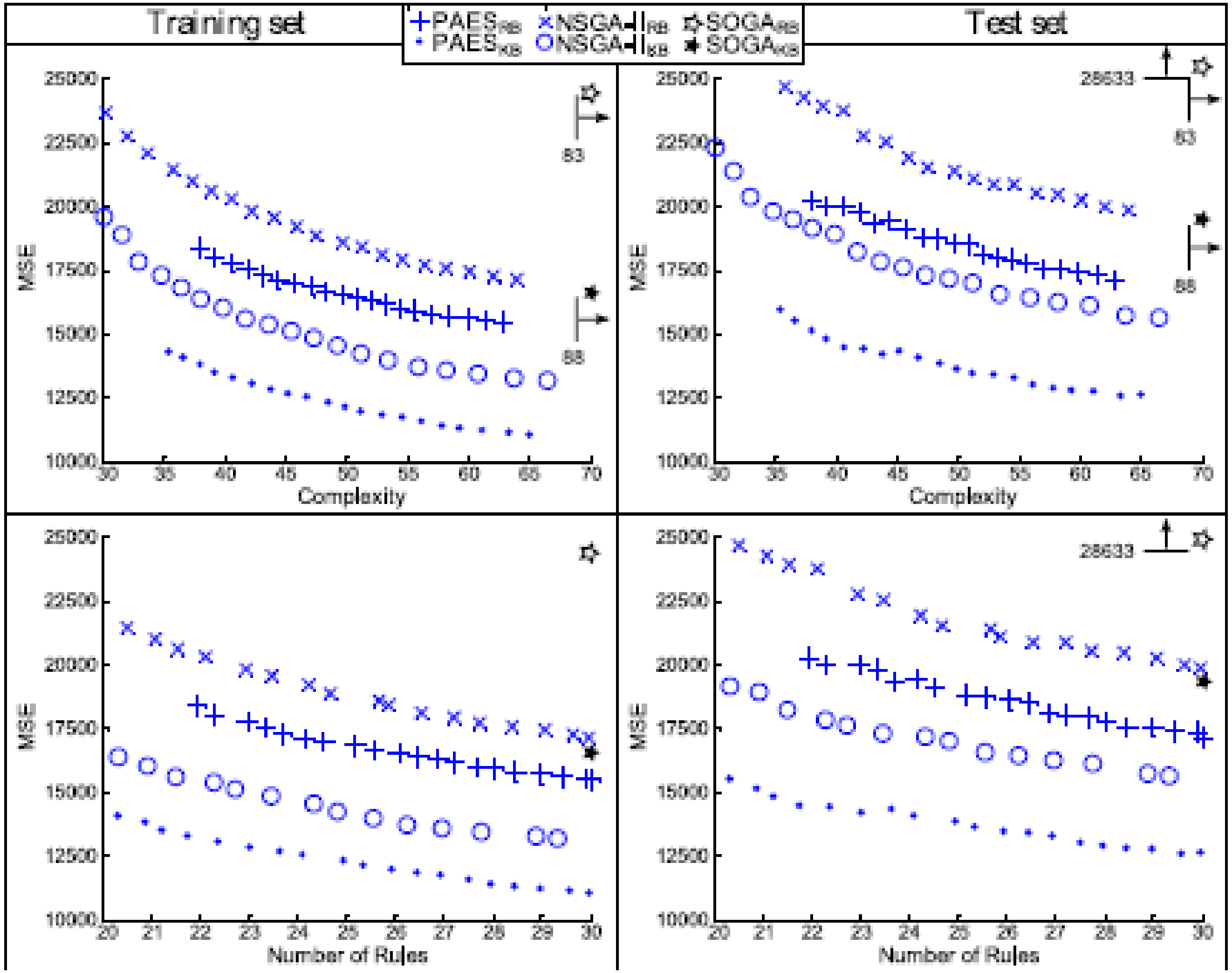
Method	Description	Pop. size
$SOGA_{RB}$	**Rule Base learning with SOGA**	**64**
$NSGA\text{-}II_{RB}$	**Rule Base learning with SOGA**	**64**
$PAES_{RB}$	**Rule Base learning with SOGA**	**64**
$SOGA_{KB}$	**(Rule Base + Data Base) learning with SOGA**	**64**
$NSGA\text{-}II_{KB}$	**(Rule Base + Data Base) learning with SOGA**	**64**
$PAES_{KB}$	**(Rule Base + Data Base) learning with SOGA**	**64**

- **Different population sizes were probed for these MOEAs showing better results when the population used for parent selection has similar sizes than those considered by single objective oriented algorithms.**
- **300,000 evaluations to allow complete convergence in all the algorithms**

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Average Pareto Fronts and average solution by SOGA
(medium voltage lines problem)



5 Data partitions 80% - 20%
6 Runs per partition
A total of 30 Runs
Test t-student α = 0.05

1. **Most accurate solution is selected from each Pareto**
2. **Average values are computed and represented**
3. **These solutions are no more used**
4. **Repeat to extract the desired avarage Pareto**

Only the first 20 solutions are considered

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Statistical Analysis

Statistical comparison among MOEAs

Method	Using the Pareto most accurate solution (FIRST)							Using the Pareto median solution (MEDIAN)							Using the Pareto simplest solution (LAST)						
	# R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t	# R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t	# R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t
NSGA-II$_{RB}$	30/64	17116	4283	+	19834	4996	+	25/48	18853	4672	+	21533	5149	+	18/30	23649	5852	+	26660	6342	+
PAES$_{RB}$	30/63	15454	3882	+	17135	4234	+	27/51	16378	4112	+	18472	4740	+	22/38	18352	4631	+	20238	5419	+
NSGA-II$_{KB}$	29/67	13137	3378	+	15587	4806	+	23/46	15073	4126	+	17581	5853	+	17/29	21629	12156	+	25716	14722	+
PAES$_{KB}$	30/65	**11044**	**2771**	*	**12607**	**3106**	*	25/50	**12133**	**3380**	*	**13622**	**3353**	*	20/35	**14297**	**4449**	*	**15951**	**4405**	*

Statistical comparison of the best MOEA with SOGA

Method	# R/C	E_{tra}	σ_{tra}	t-t	E_{tst}	σ_{tst}	t-t
SOGA$_{RB}$	30/83	24340	8450	+	28633	11861	+
SOGA$_{KB}$	30/88	16502	5136	⋄	19112	6273	⋄
PAES$_{KB}$ (FIRST)	30/65	**11044**	**2771**	-	**12607**	**3106**	-
PAES$_{KB}$ (MEDIAN)	25/50	12133	3380	-	13622	3353	-
PAES$_{KB}$ (LAST)	20/35	14297	4449	=‡	15951	4405	-

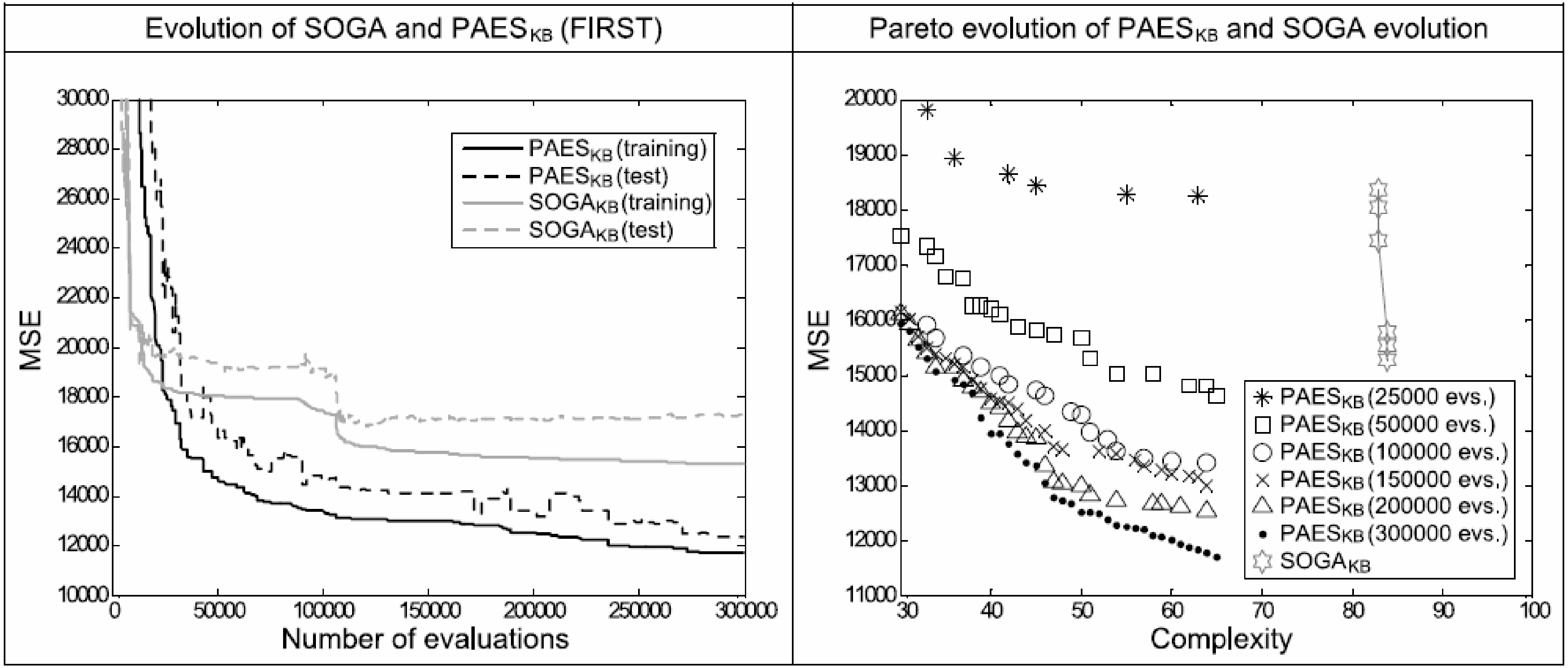
‡ It is (-) with 91% confidence

REMINDER
5 Data partitions 80% - 20%
6 Runs per partition
A total of 30 Runs
Test t-student α = 0.05

Different Models of Multiobjective GFSs

MODEL 3: Multiobjective Learning of DB and RB

Convergence



Different Models of Multiobjective GFSs

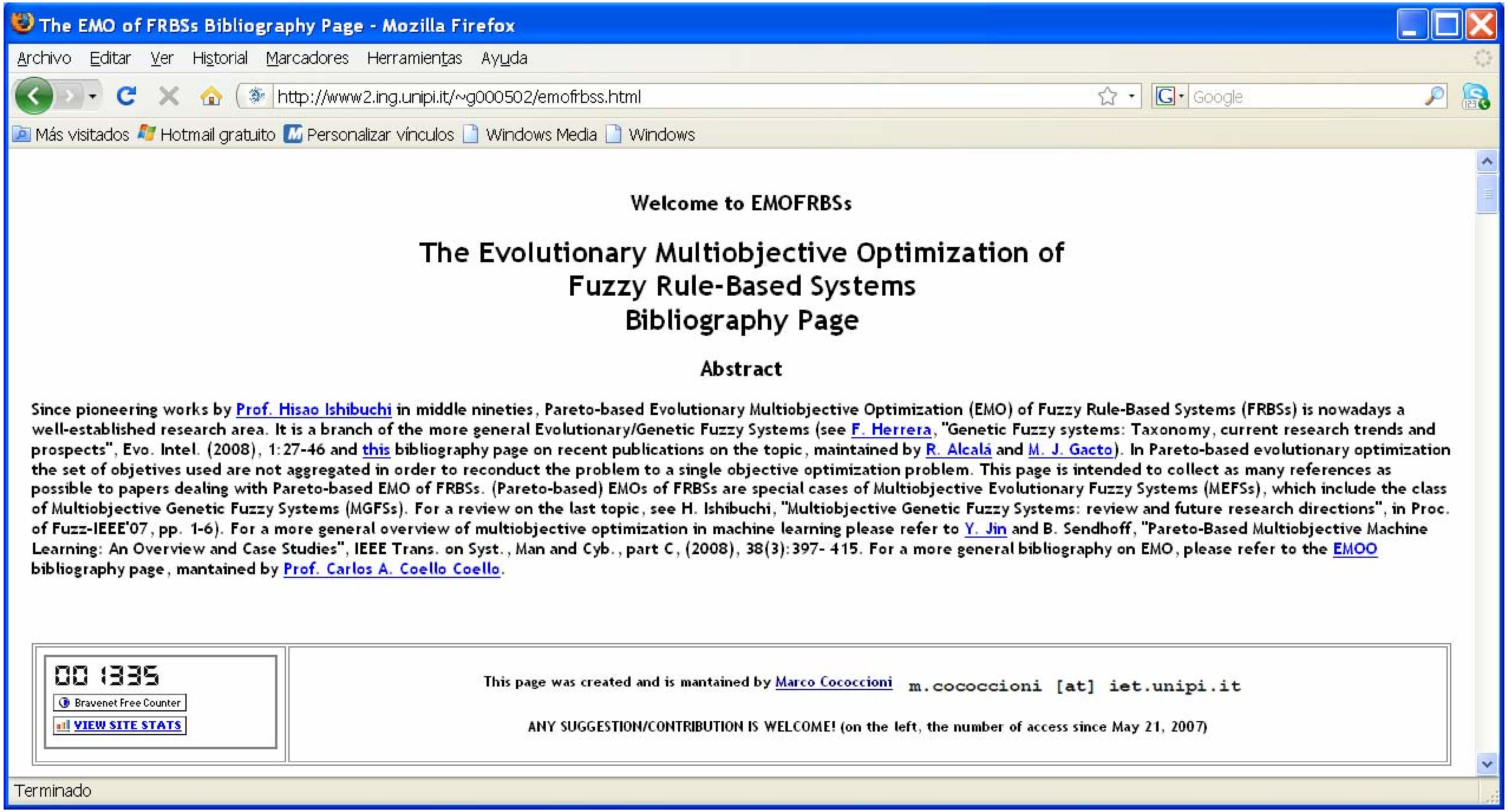
MODEL 3: Multiobjective Learning of DB and RB

- **The models obtained by these new approaches presented a better trade-off than those obtained by only considering performance measures.**

- **Between both multi-objective experimented, namely a modified (2+2)PAES and the classical NSGA-II, the modified (2+2)PAES has shown a better behaviour than NSGA-II.**

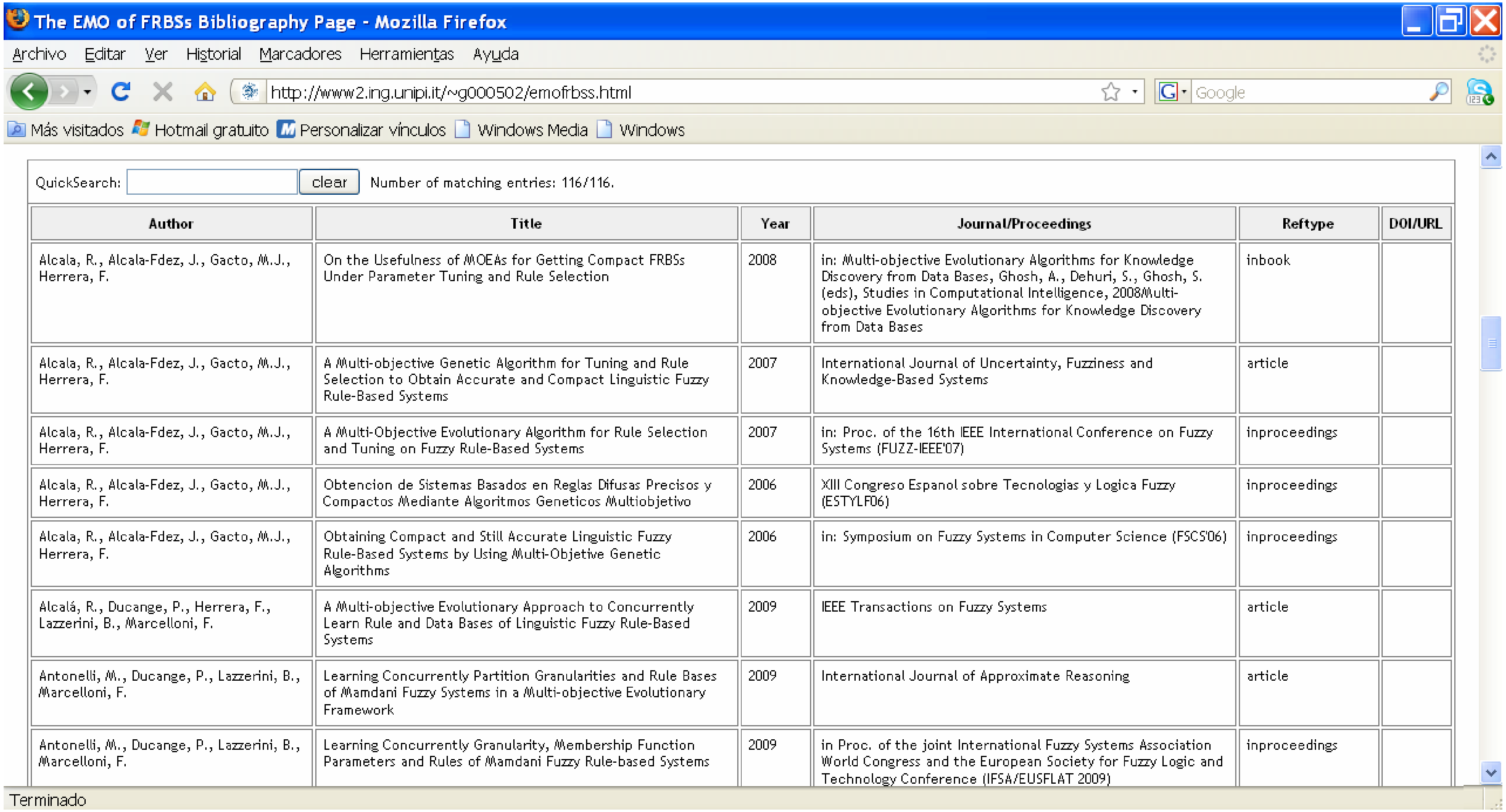
- **Finally, the liguistic 2-tuples representation presented has shown a good positive synergy.**

Webpage of EMOFRBSs



http://www2.ing.unipi.it/~g000502/emofrbss.html

Webpage of EMOFRBSs



The EMO of FRBSs Bibliography Page - Mozilla Firefox

Archivo Editar Ver Historial Marcadores Herramientas Ayuda

http://www2.ing.unipi.it/~g000502/emofrbss.html

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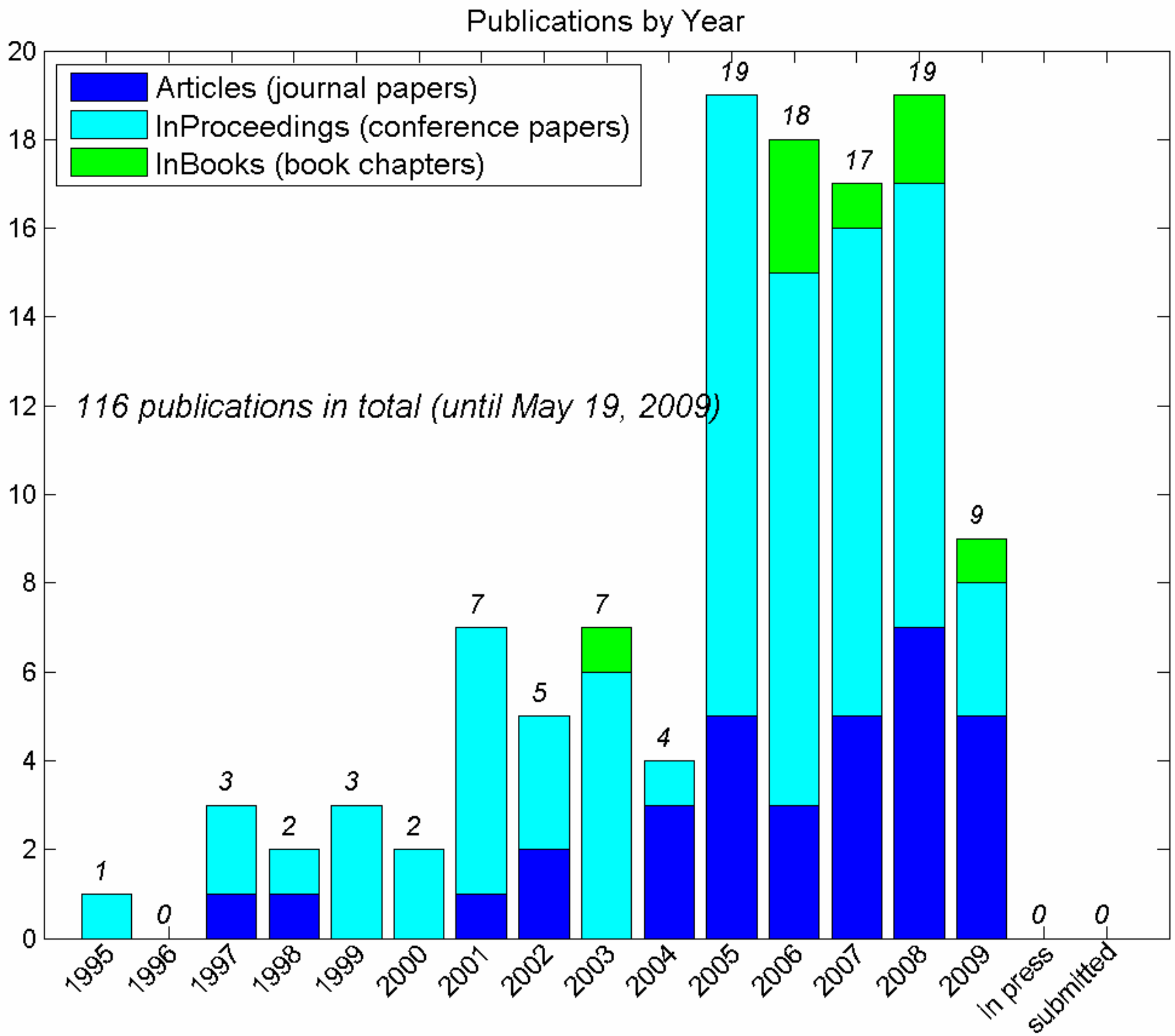
Author	Title	Year	Journal/Proceedings	Reftype	DOI/URL
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	On the Usefulness of MOEAs for Getting Compact FRBSs Under Parameter Tuning and Rule Selection	2008	in: Multi-objective Evolutionary Algorithms for Knowledge Discovery from Data Bases, Ghosh, A., Dehuri, S., Ghosh, S. (eds), Studies in Computational Intelligence, 2008Multi-objective Evolutionary Algorithms for Knowledge Discovery from Data Bases	inbook	
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	A Multi-objective Genetic Algorithm for Tuning and Rule Selection to Obtain Accurate and Compact Linguistic Fuzzy Rule-Based Systems	2007	International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems	article	
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	A Multi-Objective Evolutionary Algorithm for Rule Selection and Tuning on Fuzzy Rule-Based Systems	2007	in: Proc. of the 16th IEEE International Conference on Fuzzy Systems (FUZZ-IEEE'07)	inproceedings	
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	Obtencion de Sistemas Basados en Reglas Difusas Precisos y Compactos Mediante Algoritmos Geneticos Multiobjetivo	2006	XIII Congreso Espanol sobre Tecnologias y Logica Fuzzy (ESTYLF06)	inproceedings	
Alcala, R., Alcala-Fdez, J., Gacto, M.J., Herrera, F.	Obtaining Compact and Still Accurate Linguistic Fuzzy Rule-Based Systems by Using Multi-Objetive Genetic Algorithms	2006	in: Symposium on Fuzzy Systems in Computer Science (FSCS'06)	inproceedings	
Alcalá, R., Ducange, P., Herrera, F., Lazzerini, B., Marcelloni, F.	A Multi-objective Evolutionary Approach to Concurrently Learn Rule and Data Bases of Linguistic Fuzzy Rule-Based Systems	2009	IEEE Transactions on Fuzzy Systems	article	
Antonelli, M., Ducange, P., Lazzerini, B., Marcelloni, F.	Learning Concurrently Partition Granularities and Rule Bases of Mamdani Fuzzy Systems in a Multi-objective Evolutionary Framework	2009	International Journal of Approximate Reasoning	article	
Antonelli, M., Ducange, P., Lazzerini, B., Marcelloni, F.	Learning Concurrently Granularity, Membership Function Parameters and Rules of Mamdani Fuzzy Rule-based Systems	2009	in Proc. of the joint International Fuzzy Systems Association World Congress and the European Society for Fuzzy Logic and Technology Conference (IFSA/EUSFLAT 2009)	inproceedings	

Terminado

http://www2.ing.unipi.it/~g000502/emofrbss.html

List of 116 MGFSs contributions



Future Research Directions in MGFSs

Formulations of the Interpretability

- The number of fuzzy rules
- The number of antecedent conditions in each rule
- The number of input variables
- The separability of adjacent antecedent fuzzy sets

Handling of Large Data Sets

- Design of efficient EMO algorithms
- Subdivision of data sets
- Parallel implementation

Development of Special-Purpose EMO Algorithms

- Handling of many objectives
- Handling of both discrete and continuous variables

Future Research Directions in MGFSs

Development of New MGFS Methods with

- Multiobjective input selection algorithm
- Multiobjective fuzzy partition algorithm
- . . .

Visualization of Pareto-Optimal Fuzzy Systems

- Visualization of a single fuzzy system
- Visualization of multiple fuzzy systems
- Visualization of accuracy-complexity tradeoff

How to compare MGFSs

- A statistical Analysis is needed
- Use of non-parametric statistical tests

Ensemble Classifier Design

- Search for multiple fuzzy systems with a large diversity
- Choice of ensemble members and their combination