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Woeginger's axiomatisation of the h-index and its relation to the g-index, the $h^{(2)}$-index and the R^2-index

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ABSTRACT

Recently Woeginger [Woeginger, G. H. (2008-a). An axiomatic characterization for the Hirsch-index. *Mathematical Social Sciences*. An axiomatic analysis of Egghe's g-index. *Journal of Informetrics*] introduced a set of axioms for scientific impact measures. These lead to a characterization of the *h*-index. In this note we consider a slight generalization and check which of Woeginger's axioms are satisfied by the *g*-index, the $h^{(2)}$-index and the R^2-index.

1. Introduction

Recently Woeginger (2008-a, 2008-b) introduced a set of axioms for scientific impact measures. These lead to a characterization of the *h*-index and the *g*-index. As we consider the idea of studying *h*-type indices in an axiomatic framework one of the most interesting informetric ideas of the year, we think it is worthwhile to check which of Woeginger's original set of axioms (Woeginger, 2008-a) are satisfied by the *g*-index, the $h^{(2)}$-index and the R^2-index. We hope this is a (small) step forward in the axiomatic study of *h*-type indices.

Before beginning our investigations we recall the definitions of the *h*-type indices used here and further on in this article.

The *h*-index is defined as follows: one draws the list of all articles [co-] authored by scientist S, ranked according to the number of citations each of these articles has received. Articles with the same number of citations are given different rankings. Then scientist S' *h*-index is defined as the largest rank such that the first *h* publications received each at least *h* citations.

The *g*-index is calculated as follows: one draws the same list as for the *h*-index; then the *g*-index is defined as the highest rank such that the cumulative sum of the number of citations received is larger than or equal to the square of this rank.

If $X = (x_1, x_2, \ldots, x_n)$, where $x_j \in \mathbb{N}$ ($j = 1, \ldots, n$) denotes the number of citations of the *j*th publication, and publications are ranked according to the number of citations received then The *g*-index is characterized by the inequalities:

$$g^2 \le \sum_{j=1}^{g} x_j \quad \text{and} \quad (g+1)^2 > \sum_{j=1}^{g+1} x_j \tag{1}$$

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For the $h^{(2)}$-index the same list is drawn, but now the $h^{(2)}$-index, which we denote as k, is defined as the highest rank such that the first k publications received each at least k^2 citations. It is characterized by

$$k^2 \leq x_k \quad \text{and} \quad (k+1)^2 > x_{k+1} \tag{2}$$

Finally, the R-index (Jin, Liang, Rousseau, & Egghe, 2007) is equal to the square root of the sum of all citations of articles included in the h-core, i.e. the set of the first h articles:

$$R = \sqrt{\sum_{j=1}^{h} x_j} \tag{3}$$

2. Domination relations and pre-impact indices

Let **P** be the set consisting of the empty set, Ø, and all publication–citation arrays $X = (x_1, x_2, \ldots, x_n)$, where $x_i \in \mathbb{N}$ $(j = 1, \ldots, n)$ denotes the number of citations of the jth publication. Publications are ranked according to the number of citations received so that $x_j \geq x_{j+1}$. The number n is called the dimension of X, in short: $n = \dim X$, where dim(Ø) is considered to be zero.

2.1. Definition: the domination relation (Woeginger, in 2008-a)

For $X, Y \in \mathbf{P}$ array X is dominated by array Y, denoted as $X \ll Y$ if: either $X = \emptyset$, or $\dim(X) \leq \dim(Y)$ and $x_j \leq y_j$ for $1 \leq j \leq \dim(X)$.

2.2. Definition: pre-impact index (Woeginger, in 2008-a)

A function $f : \boldsymbol{P} \to \mathbb{N}$ is a pre-impact index if it satisfies the two natural requirements:

P1a. $f(0, 0, \ldots, 0) = 0$ for all dimensions n.
P1b. $f(\emptyset) = 0$.
P2 $X \ll Y$ implies that $f(X) \leq f(Y)$ (a monotonicity condition).

We note that the term ‘pre-impact index’ is proposed by us: it is not used by Woeginger. Clearly, Hirsch’ h, Egghe’s g, Kosmulski’s $h^{(2)}$ and Jin’s R are pre-impact indices. The A-index (Jin, 2006), defined as the average number of citations of articles belonging to the h-core, is not a pre-impact index, and for this reason is not discussed in this article. An example of this fact is given in Jin et al. (2007). Another example, shown by one of the reviewers, is provided by taking $X = (6, 6, 2)$ and $Y = (6, 6, 3)$. Then $X \ll Y$, $h(X) = 2$ and $h(Y) = 3$, but $A(X) = 6$ and $A(Y) = 5$. The authors of Jin et al. (2007) consider the monotonicity condition as essential and, for that reason, reject – as an acceptable impact function – any function that is not a pre-impact factor.

Now we introduce a stricter requirement.

2.3. Definition: the strict domination relation

For $X, Y \in \mathbf{P}$ array X is strict dominated by array Y, denoted as $X -< Y$ if: either $X = \emptyset$, or $\dim(X) \leq \dim(Y)$ and for each j, $1 \leq j \leq \dim(X)$: $x_j < y_j$.

2.4. Definition: strict pre-impact index

A function f: $f : \boldsymbol{P} \to \mathbb{N}$ is a strict pre-impact index if it satisfies the following two requirements:

S1a = P1a. $f(0, 0, \ldots, 0) = 0$ for all dimensions n.
S1b = P1b. $f(\emptyset) = 0$.
S2. $X -< Y$ implies that $f(X) < f(Y)$.

We show now that neither Hirsch’ h (Hirsch, 2005), nor Egghe’s g (Egghe, 2006a, 2006b) nor Kosmulski’s $h^{(2)}$ (Kosmulski, 2006) satisfies this strict requirement.

Concretely, we prove the following: it is possible that scientist A has written 10 articles, each of which are cited at least 4 times; that scientist B has written 5 articles, each of which are cited at most 4 times, and such that the 4 most cited articles of scientist A are each strictly more cited than the 4 most cited articles of scientist B, and yet $h(A) = h(B)$; $g(A) = g(B)$ and $h^{(2)}(A) = h^{(2)}(B)$.

Example:
$A = (5, 5, 5, 5, 4, 4, 4, 4, 4, 4)$;
$B = (4, 4, 4, 4, 1)$.
In this example $h(A) = h(B) = g(A) = g(B) = 4$; $h^{(2)}(A) = h^{(2)}(B) = 2$.

Note that the R-indices (Jin et al., 2007) of these scientists are not the same: $R(A) \approx 4.47 > R(B) = 4$.

Clearly $B -< A$ but axiom S2 is not satisfied by the first three indices.

The R-index cannot be a (strict) pre-impact index as it does not take values in $\mathbb{N}$. Yet, removing the square root yields R^2, which is an index that takes values in $\mathbb{N}$ (assuming that citations are counted as whole numbers). Then this index is a strict pre-impact index. Indeed, if $X -< Y$ then $x_j < y_j$ for $1 \le j \le \dim(X)$, and hence $h(X) \le h(Y)$. As $R^2(X) = \sum_{j=1}^{h(X)} x_j$ and $R^2(Y) = \sum_{j=1}^{h(Y)} y_j$ the relation $x_j < y_j$ for $1 \le j \le \dim(X)$ implies that $R^2(X) < R^2(Y)$. Note that we do not consider squaring an index a natural operation. In this particular case however, $R^2 = \sum_{j=1}^{h} x_j$ is actually a simpler index than R. Moreover, the author, being one of the 'inventors' of the R-index knows that the main reason why R had been preferred above R^2, was the fact that by taking a square root the R-value of a Hirsch core with the least amount of citations is equal to h (the case that each article in the h-core has exactly h citations). So using R instead of R^2 is just a matter of normalization.

Bringing together these observations we have shown the following proposition.

2.5. Proposition

Among the four h-type indices h, g, $h^{(2)}$ and R^2, only R^2 is a strict pre-impact index.

Note that $X -< Y$ automatically implies $X \ll Y$, hence every pre-impact index satisfies the relation: $X -< Y$ implies $f(X) \le f(Y)$. Further, had we introduced a relation $X -\ll Y$: when either $X = \emptyset$, or $\dim(X) \le \dim(Y)$ and $x_j \le y_j$ for $1 \le j \le \dim(X)$, where at least one of the inequality signs is strict (a suggestion made by one of the reviewers), then also $X -\ll Y$ implies $X \ll Y$, leading to the same conclusion. Moreover, as $X -< Y$ implies $X -\ll Y$, every pair (X, Y) which provides a counterexample that a function f is not a strict pre-impact index provides a counterexample for the relation: $X -\ll Y$ implies $f(X) < f(Y)$.

3. Woeginger's set of axioms for impact indices

After introducing the notion of a pre-impact index Woeginger (2008-a) considers the following five axioms.

- A1. Add to X an article with $f(X)$ citations; this new publication–citation array is denoted as Y; then $f(Y) \le f(X)$.
- A2. Add to X an article with $f(X)+1$ citations, and again call the new array Y; then $f(Y) > f(X)$.
- B. If Y results from X by the increase in the number of citations of only one article then $f(Y) \le f(X)+1$.
- C. Add one citation to each publication in X, and call the new array Y; then $f(Y) \le f(X)+1$.
- D. Add first an article with $f(X)$ citations and then add one citation to each publication then the new array Y satisfies the strict inequality $f(Y) > f(X)$.

One may also consider the following axiom:

D_0: If $X = (m, \ldots, m)$ and $\dim(X) = m$, then $f(X) = m$.

If arrays of the form $X = (m, \ldots, m)$ satisfy axiom D_0 for an impact index f then the inequality described in axiom D holds for this particular type of array. Indeed, knowing that $f(m, \ldots, m) = m$ ($\dim(X) = m$) leads to $Y = (m+1, \ldots, m+1)$ (with $\dim(Y) = m+1$), and then by axiom D_0 $f(Y) = m+1 > m = f(X)$. In this sense one may say that D_0 is 'weaker' than D. Yet, this does not imply that D_0 follows logically from D. Indeed, if $X = (m, \ldots, m)$ then its R^2-index is m^2 ($>m$ if $m \neq 0, 1$). Hence R^2 does not satisfy D_0 and yet it satisfies D (see further). On the other hand, examples can be given of pre-impact indices that satisfy axiom D_0 and not axiom D (see further). This shows that axioms D and D_0 are independent. However, Woeginger shows that axioms B and D together imply axiom D_0. As his proof is somewhat hidden in the proof of another result we add it in the appendix of this article.

We would like to point out that these axioms are mainly proposed because they fit Woeginger's purposes. Considered from the point of view of an h-type index it is neither a bad nor a good property if this index satisfies or does not satisfy one or more of these axioms. For a motivation of these particular axioms we refer to Woeginger (2008-a), who also observes that these axioms are not independent as axiom $A2$ implies axiom D. Moreover, a pre-impact index satisfying axioms $A1$ and $A2$ automatically does not satisfy axiom B.

Woeginger (2008-a) proves that:

- No pre-impact index f can satisfy the five axioms $A1$, $A2$, B, C and D.
- The h-index does not satisfy axiom $A2$, but it satisfies the other four. An example that the h-index does not satisfy axiom $A2$ is the following situation. $X = (1, 1)$ where $h(X) = 1$; then $Y = (2, 1, 1)$ and $h(Y) = 1$, while, according to A2, $h(Y)$ must be strictly larger than $h(X)$.
- The h-index is the only index that satisfies the axioms $A1$, B and D (and then automatically also C).

Next we will check which of the Woeginger axioms, including D_0, are satisfied by g, $h^{(2)}$ and R^2.

4. The *g*-index (Egghe, 2006a)

It is clear that the *g*-index is a pre-impact index. We will now check which axioms are satisfied by the *g*-index. We recall that the *g*-index is characterized by the inequalities:

$$g^2 \leq \sum_{j=1}^{g} x_j \quad \text{and} \quad (g+1)^2 > \sum_{j=1}^{g+1} x_j \tag{1}$$

We also know that $h \leq g$, hence

$$g+1 > x_{g+1} \tag{4}$$

1. The g-index does not satisfy axiom $A1$
   Consider $X = (10, 5, 4, 2, 1)$. Then $g(X) = 4$. However, adding a publication with 4 citations yields the array $Y = (10, 5, 4, 4, 2, 1)$. Its g-index is 5, contradicting the requirements of axiom $A1$.
2. The g-index does not satisfy axiom $A2$
   Consider $X = (9, 5, 4, 1, 1)$. Then $g(X) = 4$. Now, adding a publication with 5 citations yields the array $Y = (9, 5, 5, 4, 1, 1)$. Its g-index is 4, contradicting the requirements of axiom $A2$.
3. The g-index does not satisfy axiom B
   Consider $X = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1)$ of dimension 10; $g(X) = 1$. Adding 20 citations to the first article (or any other one) yields $Y = (21, 1, 1, 1, 1, 1, 1, 1, 1, 1)$. As $g(Y) = 5$ axiom B is not satisfied. Clearly we can obtain by this procedure an array Y with any g-value we would like to attain.
4. The g-index satisfies axiom C
   We know by (1) that $g^2 \leq \sum_{j=1}^{g} x_j$ and $(g+1)^2 > \sum_{j=1}^{g+1} x_j$. Assume that array X's g-index is g. Then we add one citation to each publication, leading to the array Y. We must show that $g(Y) \leq g(X) + 1 = g + 1$. For this we show that $g(Y) < g + 2$. This inequality is true if

$$(g+2)^2 > \sum_{j=2}^{g+2} x_j + g + 2$$

   Now

$$\begin{aligned}(g+2)^2 &= (g+1)^2 + 2g + 3 > \sum_{j=1}^{g+1} x_j + 2g + 3 = \sum_{j=1}^{g+2} x_j - x_{g+2} + 2g + 3 \geq \sum_{j=1}^{g+2} x_j - x_{g+1} \\ &\quad + 2g + 3 > \sum_{j=1}^{g+2} x_j - g - 1 + 2g + 3 = \sum_{j=1}^{g+2} x_j + g + 2\end{aligned}$$

   where the last strict inequality uses (4).
5. The g-index satisfies axiom D
   We have to show now that:

$$(g+1)^2 \leq \left(\sum_{j=1}^{g} x_j + g\right) + g + 1 \tag{5}$$

   Note that we have assumed here that the added article with g citations belongs to the $g+1$ most cited articles (after the addition, and if necessary putting it ahead of other articles with the same number of citations). For this to happen g must be at least equal to x_{g+1}. This is true, as otherwise we would have $g < x_{g+1} < g+1$ (using (4)) which is impossible as x_{g+1} is assumed to be a natural number.
   Now, by (1): $(g+1)^2 = g^2 + 2g + 1 \leq \sum_{j=1}^{g} x_j + 2g + 1$. This proves inequality (5).
6. The g-index satisfies axiom D_0
   If X is the m-dimensional array $(m, \ldots, m)$ then $g(X) = m$.

5. Kosmulski's $h^{(2)}$-index (Kosmulski, 2006)

For simplicity we will denote this index as k and the first k articles will we referred to as the k-core. Kosmulski's $h^{(2)}$-index is characterized by

$$k^2 \leq x_k \quad \text{and} \quad (k+1)^2 > x_{k+1} \tag{2}$$

1. *Kosmulski's $h^{(2)}$-index satisfies axiom A1.*
   As k is a natural number we always have that $k \leq k^2$ with equality only if $k=0$ or $k=1$. Consequently, adding an article with k citations to a list and putting this new article last if there already exist articles with k citations, never brings this article into the (new) k-core.
2. *Kosmulski's $h^{(2)}$-index does not satisfy axiom A2.*
   Consider $X=(2, 1)$, then $k(X)=1$. Adding an article with $k+1=2$ citations leads to $Y=(2, 2, 1)$. As $k(Y)=1$, axiom $A2$ is not satisfied.
3. *Kosmulski's $h^{(2)}$-index satisfies axiom B.*
   We have to show that increasing the citations of one article (to which we refer as the target article) can increase k at most by one.
   Adding citations to one article leads to a new array, denoted as $Y=(y_1, y_2, \ldots, y_n)$, where only one of the y's is different (actually strictly larger) than one of x's (but this may change the original ranking).
   If the target article belongs to the k-core then clearly $k=k(X)=k(Y)$. If the target article does not belong to the k-core, but the new number of citations is at most k^2 then again $k(X)=k(Y)$. If the target article does not belong to the k-core, but its new rank is between rank 1 and rank $k+1$, then $y_{k+2}=x_{k+1}<(k+1)^2<(k+2)^2$, showing that $k(Y) \leq k(X)+1$. Finally, if the article at rank $k+1$ is the target article, then $y_{k+2}=x_{k+2}<(k+2)^2$, showing again that $k(Y) \leq k(X)+1$.
   An illustration: for $X=(10, 10, 8, 8)$ $k(X)=2$. Adding 12 citations to the last one leads to $Y=(20, 10, 10, 8)$ with $k(Y)=3$. Note that, even adding a much larger number of citations, say 100, to the third article of $X'=(9, 8, 1)$ leads to $Y'=(101, 9, 8)$ for which $k(Y)=2=k(X)$.
4. *Kosmulski's $h^{(2)}$-index satisfies axiom C.*
   We have to show that $(k+2)^2>x_{k+2}+k+2$, where $k=k(X)$. Now $(k+2)^2=(k+1)^2+2k+3>x_{k+1}+2k+3>x_{k+2}+k+2$, proving that Kosmulski's $h^{(2)}$-index satisfies axiom C.
5. *Kosmulski's $h^{(2)}$-index does not satisfy axiom D.*
   Let $X=(1, 1, 1)$, then $k(X)=1$. Now $Y=(2, 2, 2, 2)$, and $k(Y)=1$.
6. *Kosmulski's $h^{(2)}$-index does not satisfy axiom D_0.*

The example for part 5 is also an example for part 6.

We note that from the fact that axioms B and D imply axiom D_0 it follows logically that not satisfying axiom D_0 and satisfying axiom B leads to not satisfying axiom D.

6. The R^2-index

1. *The R^2-index does not satisfy axiom A1.*
   Consider $X=(2, 1)$. Its h-index is equal to 1 and $R^2(X)=2$. Adding a publication with 2 citations yields the array $Y=(2, 2, 1)$. Its h-index is 2, and its R^2-index is 4, contradicting the requirements of axiom $A1$.
2. *The R^2-index satisfies axiom A2.*
   If $X=(x_1, x_2, \ldots, x_n)$ then Y is derived from X by adding a publication with $\sum_{i=1}^{h} x_i + 1$ citations. This article becomes the most-cited one of the new array Y. Although it is possible that the $h(Y)=h(X)$, this added publication evidently enters the h-core. If $h(Y)>h(X)$ then certainly $R^2(Y)>R^2(X)$ and if $h(Y)=h(X)$ then an article with h citations is removed from the core and replaced by the added article. Also in this case $R^2(Y)>R^2(X)$.
3. *The R^2-index does not satisfy axiom B.*
   Consider $X=(2, 1)$ and $Y=(6, 1)$, then $R^2(Y)=6>R^2(X)+1$ (=3)
4. *The R^2-index does not satisfy axiom C.*
   If $X=(4, 3, 2, 1)$ then $Y=(5, 4, 3, 2)$. We see that $h(X)=2$ and $h(Y)=3$; and hence $R^2(X)=7$, while $R^2(Y)=12$, which is strictly larger than $R^2(X)+1=8$.
5. *The R^2-index satisfies axiom D.*
   Adding one citation to each article in X already increases $R^2(X)$ by h (if $h>0$). If $h=0$ then adding one citation to each article, leads to $R^2(Y)=1$. So, also in this case $R^2(Y)>R^2(X)$. It is clear that axiom $A2$ always implies axiom D.
6. *The R^2-index does not satisfy axiom D_0.*
   If X is the m-dimensional array $(m, \ldots, m)$ then $R^2(X)=m^2$.

7. An example of a pre-impact index that satisfies D_0 and not D

Let F = floor ((h + g)/2), where floor(s) denotes the largest integer smaller than or equal to s. Then $F(0, \ldots, 0) = 0$ and $X \ll Y$ implies that $F(X) \leq F(Y)$. This shows that F is a pre-impact index. If $\dim(X_0) = m$ and $X_0 = (m, \ldots, m)$ then clearly $F(X_0) = m$. Consider now $X_1 = (7, 7, 7, 2, 2, 1)$. Then $h(X_1) = 3$, $g(X_1) = 5$ and $F(X_1) = 4$. Adding an article with 4 citations leads to $X_2 = (7, 7, 7, 4, 2, 2, 1)$ and adding 1 citation to each article gives $Y = (8, 8, 8, 5, 3, 3, 2)$. Now, $h(Y) = 4$, $g(Y) = 5$ and $F(Y) = 4$.

8. Conclusion and suggestions for further research

Among the four h-type indices h, g, $h^{(2)}$ and R^2, only R^2 is a strict pre-impact index. The g-index is a pre-impact index that only satisfies Woeginger's axioms C, D and D_0, while Kosmulski's $h^{(2)}$-index is a pre-impact index that satisfies Woeginger's axioms $A1$, B and C. Finally R^2 is a strict pre-impact index satisfying only axioms $A2$ and D. It is further shown that Woeginger's axioms D and D_0 are independent.

A reviewer would have preferred a theory where f does not take values in $\mathbb{N}$ but in $\mathbb{R}$. In that case the R-index, and also the AR-index (Rousseau & Jin, in press) could have been incorporated. Although certainly interesting, in particular in combination with Marchant's approach (Marchant, 2008a, 2008b), we do not see how this can be done easily. The Woeginger axioms are expressed using discrete units (expressing properties such as: f is at most one unit larger) and his proofs, see also Appendix of our article, make use of mathematical induction. The same reviewer would have liked to see a characterization of the R-index and possibly the AR-index, or at least a property that makes a clear distinction between these h-type indices. Again, this looks like a non-trivial proposal changing the nature of this article. For this reason we postpone such an investigation to the future.

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Appendix A

Proposition 1. *If a pre-impact index f satisfies axiom B then* $f(X) \leq k$. *For each array* $X = (x_1, x_2, \ldots, x_k)$ *having at most* k (>0) *non-zero components.*

Proof. Consider the array $X_0 = (0, 0, \ldots, 0)$, consisting of k zeros. As f is a pre-impact index $f(X_0) = 0$. If X_1 denotes the array $(x_1, 0, \ldots, 0)$, with $k - 1$ zeros, then we know by axiom B that $f(X_1) \leq f(X_0) + 1 = 1$. Consider now $X_2 = (x_1, x_2, 0, \ldots, 0)$, with $k - 2$ zeros. By axiom B we know that $f(X_2) \leq f(X_1) + 1 \leq 2$. Clearly in k steps we obtain the required inequality $f(X) \leq k$. □

Proposition 2. *If a pre-impact index f satisfies axioms B and D, and if* U_m *denotes the array consisting of m* (>0) *components each equal to m, then* $f(U_m) = m$. *In other words: axioms B and Dimply axiom* D_0.

Proof. We already know from Proposition 1 (which uses axiom B) that $f(U_k) \leq k$ for each natural number k (>0). The other inequality is shown by mathematical induction. As f is a pre-impact index $f(\emptyset) = 0$. Applying axiom D to the empty array leads to $f(1) > 0$. This inequality together with Proposition 1 proves the base case. Assume now that for a natural number n (>1) $f(U_n) = n$ then we will show that $f(U_{n+1}) \leq n + 1$. Indeed, applying axiom D to U_n immediately results in $f(U_{n+1}) > n$. Hence $f(U_{n+1}) = n + 1$.

This shows that axiom D_0 follows from axioms B and D. □

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