Test Suite for the Special Issue of Soft Computing on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems

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May 13, 2010

Abstract

In this document, we provide the description of the 19 test functions (F1-F19*) that should be used for the experimental study for the Special Issue of Soft Computing on Scalability of Evolutionary Algorithms and other Metaheuristics for Large Scale Continuous Optimization Problems.

In Section 1 (page 2), we report the main features and properties of the functions F1-F11 and, in Section 2 (page 10), we explain the way hybrid composition functions F12-F19* are obtained combining two functions belonging to the set F1-F11.

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1 Functions F1-F11

The F1-F11 functions are the following:

- Shifted Unimodal Functions:
  - F1: Shifted Sphere Function
  - F2: Shifted Schwefel's Problem 2.21
- Shifted Multimodal Functions:
  - F3: Shifted Rosenbrock's Function
  - F4: Shifted Rastrigin's Function
  - F5: Shifted Griewank's Function
  - F6: Shifted Ackley's Function
- Other Shifted Unimodal Functions
  - F7: Shifted Schwefel's Problem 2.22
  - F8: Shifted Schwefel's Problem 1.2
  - F9: Shifted Extended f10
  - F10: Shifted Bohachevsky
  - F11: Shifted Schaffer

The definition of these functions are shown in Table 1 and their features are sketched in Table 2. Figures 1-6 show graphically these functions for the case of $D = 2$. Finally, their properties are described in Table 3.

Function	Name	Definition
F_1	Shif. Sphere	$\sum_{i=1}^{D} z_i^2 + f\_bias,\ z = x - o$
F_2	Shif. Schwefel 2.21	$max_i\{\lvert z_i\rvert, 1 \leq i \leq D\} + f\_bias,\ z = x - o$
F_3	Shif. Rosenbrock	$\sum_{i=1}^{D-1} (100(z_i^2 + z_{i+1})^2 + (z_i - 1)^2) + f\_bias,\ z = x - o$
F_4	Shif. Rastrigin	$\sum_{i=1}^{D} (z_i^2 - 10\cos(2\pi z_i) + 10) + f\_bias,\ z = x - o$
F_5	Shif. Griewank	$\sum_{i=1}^{D} \frac{z_i^2}{4000} - \prod_{i=1}^{D} \cos(\frac{z_i}{\sqrt{i}}) + 1 + f\_bias,\ z = x - o$
F_6	Shif. Ackley	$-20\exp(-0.2\sqrt{\frac{1}{D}\sum_{i=1}^{D} z_i^2}) - \exp(\frac{1}{D}\sum_{i=1}^{D}\cos(2\pi z_i))$ $+20 + e + f\_bias$
F_7	Shif. Schwefel 2.22	$\sum_{i=1}^{D} \lvert z_i\rvert + \prod_{i=1}^{D} \lvert z_i\rvert,\ z = x - o$
F_8	Shif. Schwefel. 1.2	$\sum_{i=1}^{D} (\sum_{j=1}^{i} z_j)^2,\ z = x - o$
F_9	Shif. Extended f_{10}	$\left(\sum_{i=1}^{D-1} f_{10}(z_i, z_{i+1})\right) + f_{10}(z_D, z_1),\ z = x - o$ $f_{10} = (x^2 + y^2)^{0.25} \cdot \left(\sin^2(50 \cdot (x^2 + y^2)^{0.1}) + 1\right)$
F_{10}	Shif. Bohachevsky	$\sum_{i=1}^{D} \left(z_i^2 + 2z_{i+1}^2 - 0.3\cos(3\pi z_i) - 0.4\cos(4\pi z_{i+1}) + 0.7\right),\ z = x - o$
F_{11}	Shif. Schaffer	$\sum_{i=1}^{D-1} (z_i^2 + z_{i+1}^2)^{0.25} \left(\sin^2(50 \cdot (z_i^2 + z_{i+1}^2)^{0.1}) + 1\right),\ z = x - o$

Table 1: Functions F1-F11

In Table 3, we can see that there are many non-separable functions that may be easily optimized dimension by dimension. In general, this occurs when the fitness is calculated by the sum operator and:

- Each variable affects to one operator only. Thus, the fitness can be optimized by adjusting each variable. This is the case for functions F1, F4, F6, and F7.
- Each variable affects to two operators of the sum, but it affects to the operators in the same way. Thus, the optimum can be reached making a search dimension by dimension. This is the case of functions F3, F9, and F11.

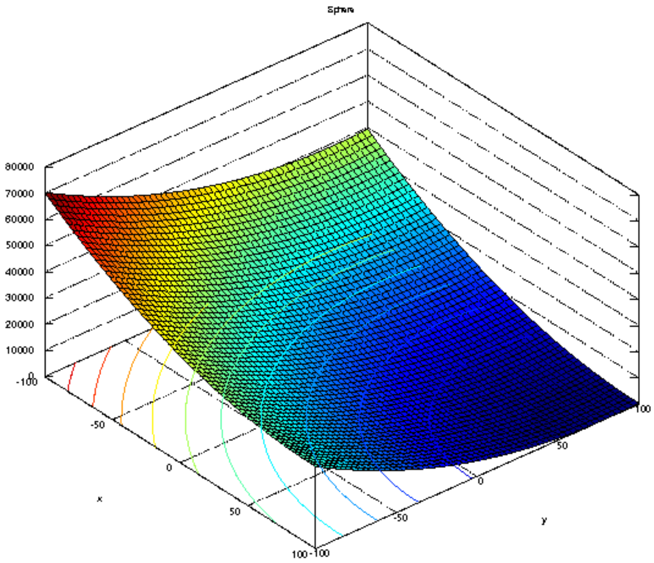
Function	**Name**	**Range**	**Fitness Optimum**
F1	Shifted Sphere Function	$[-100, 100]^D$	-450
F2	Shifted Schwefel's Problem 2.21	$[-100, 100]^D$	-450
F3	Shifted Rosenbrock's Function	$[-100, 100]^D$	390
F4	Shifted Rastrigin's Function	$[-5, 5]^D$	-330
F5	Shifted Griewank's Function	$[-600, 600]^D$	-180
F6	Shifted Ackley's Function	$[-32, 32]^D$	-140
F7	Shifted Schwefel's Problem 2.22	$[-10, 10]^D$	0
F8	Shifted Schwefel's Problem 1.2	$[-65.536, 65.536]^D$	0
F9	Shifted Extended f_{10}	$[-100, 100]^D$	0
F10	Shifted Bohachevsky	$[-15, 15]^D$	0
F11	Shifted Schaffer	$[-100, 100]^D$	0

Table 2: Features of F1-F11

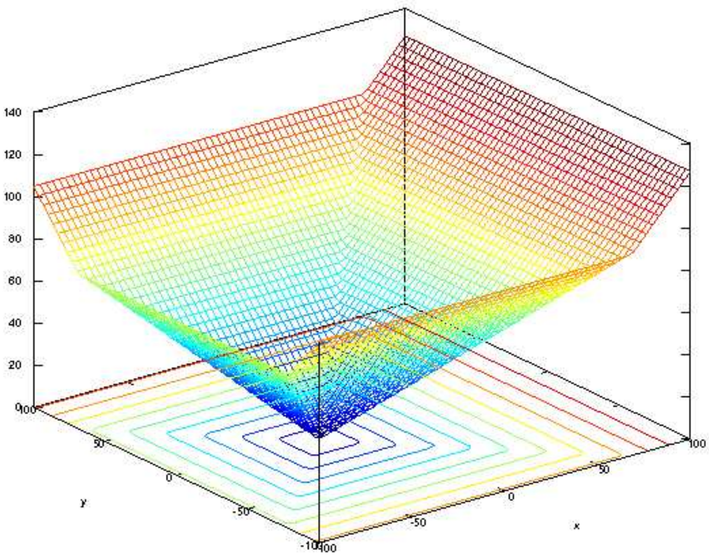
Function	**Unimodal/ Multimodal**	**Shifted**	**Separable**	**Easily optimized dimension by dimension**
F1	U	Y	Y	Y
F2	U	Y	N	N
F3	M	Y	N	Y
F4	M	Y	Y	Y
F5	M	Y	N	N
F6	M	Y	Y	Y
F7	U	Y	Y	Y
F8	U	Y	N	N
F9	U	Y	N	Y
F10	U	Y	N	N
F11	U	Y	N	Y

Table 3: Properties of F1-F11

To sum up, from F1-F11, there are 7 functions that may be easily optimized variable by variable.

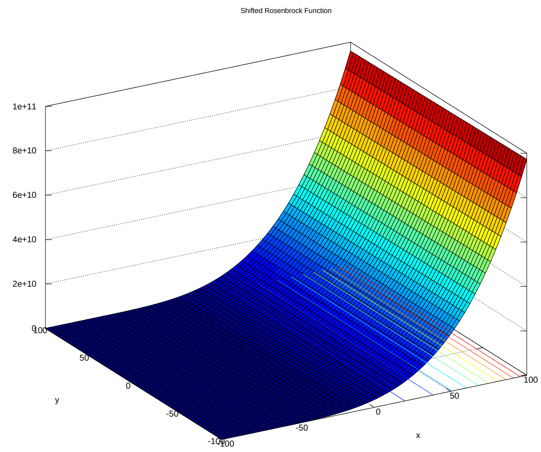


(a) Shifted Sphere Function

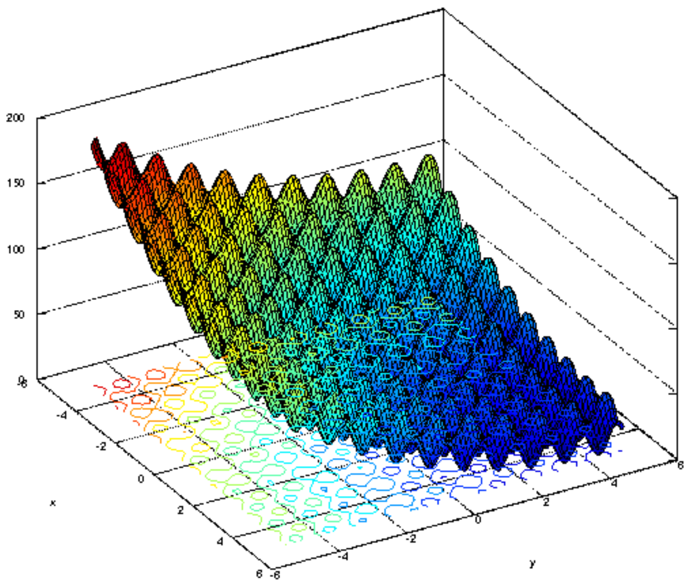


(b) Shifted Schwefel Problem 2.21

Figure 1: Shifted Sphere and Shifted Schwefel

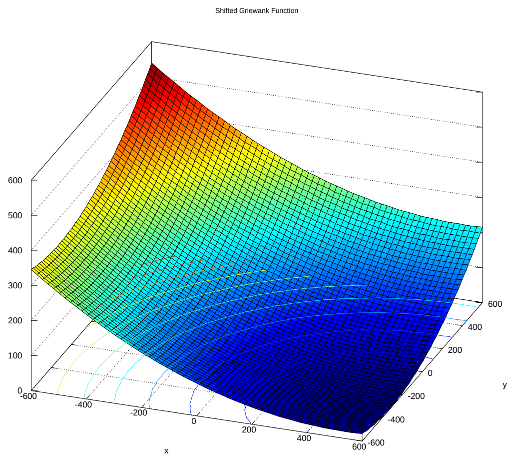


(a) Shifted Rosenbrock Function

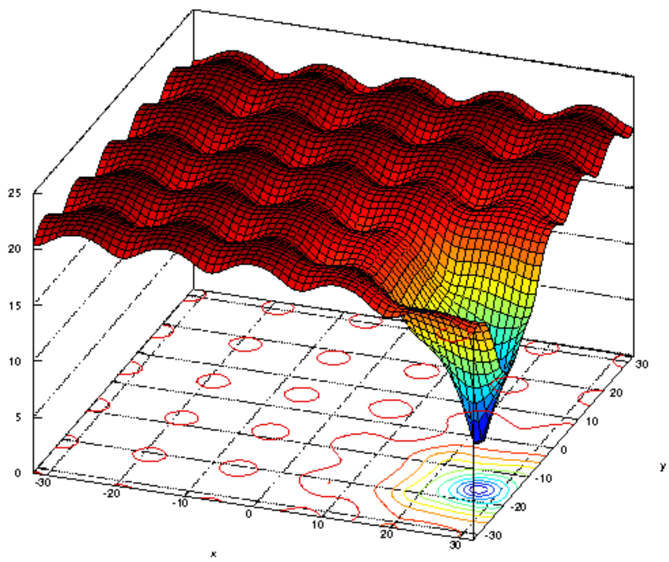


(b) Shifted Rastrigin Function

Figure 2: Shifted Rosenbrock and Shifted Rastrigin

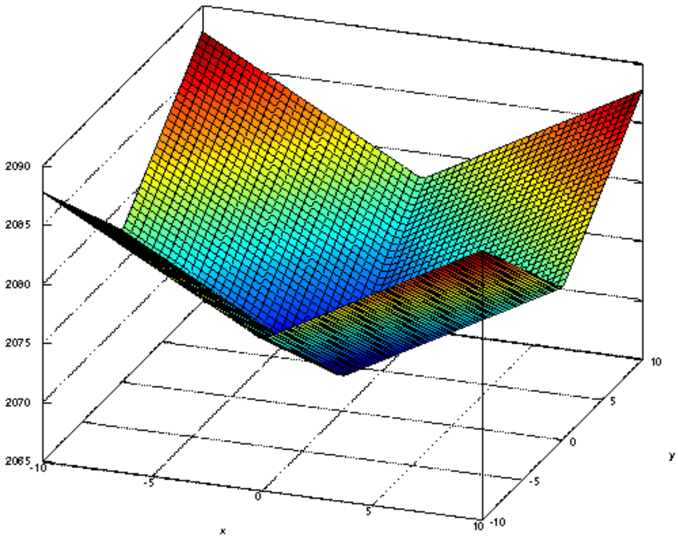


(a) Shifted Griewank Function

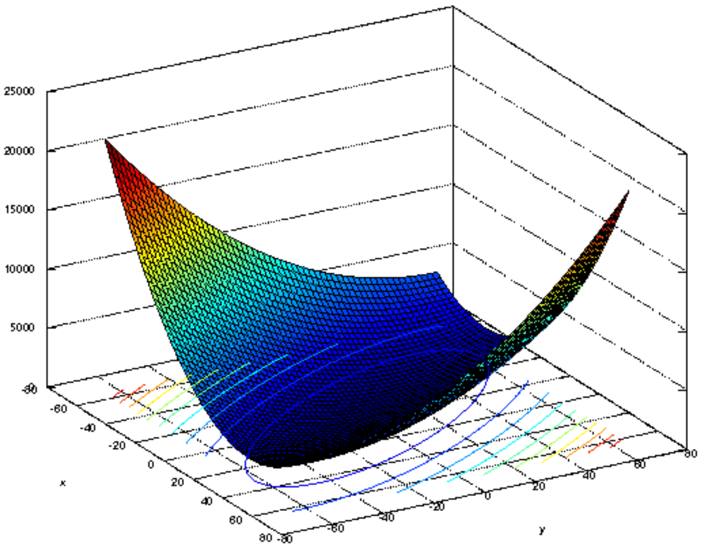


(b) Shifted Ackley Function

Figure 3: Shifted Griewank and Shifted Ackley Figures

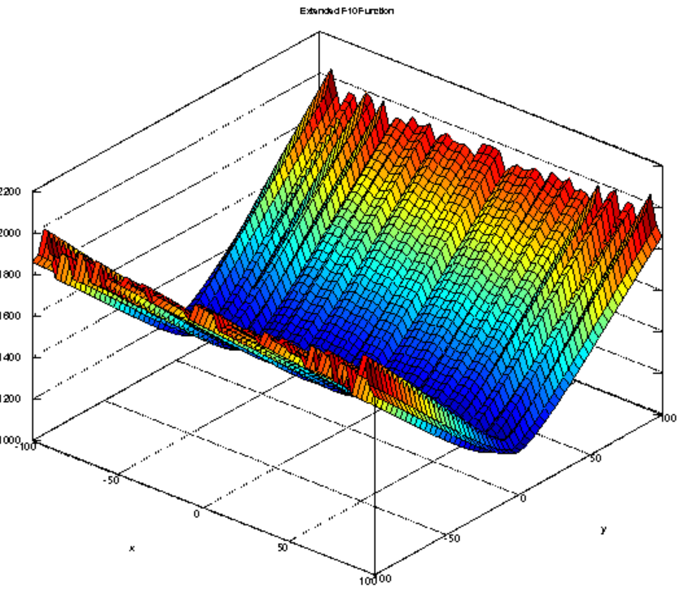


(a) Schwefel Problem 2.22

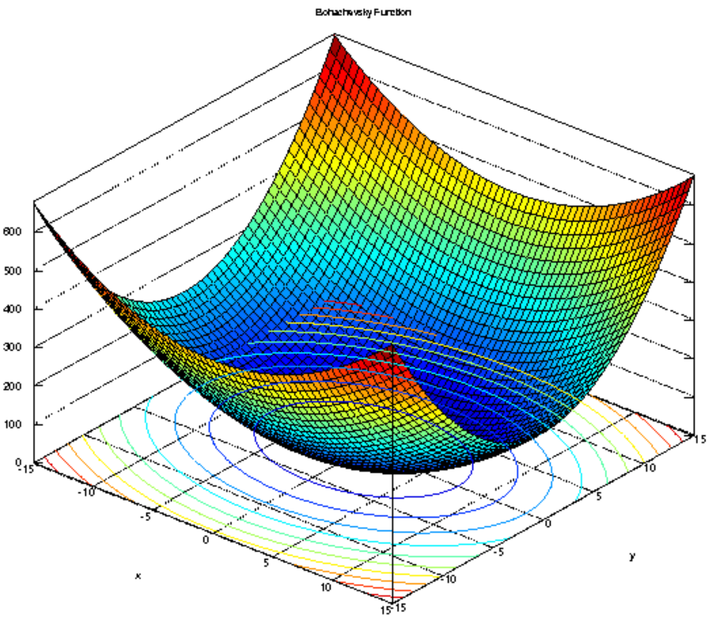


(b) Schwefel Problem 1.2

Figure 4: Shifted Schwefel Problems 2.22 and 1.2



(a) Extended Function 10



(b) Bohachevsky Function

Figure 5: Shifted Extended Function 10 and Bohachevsky Function

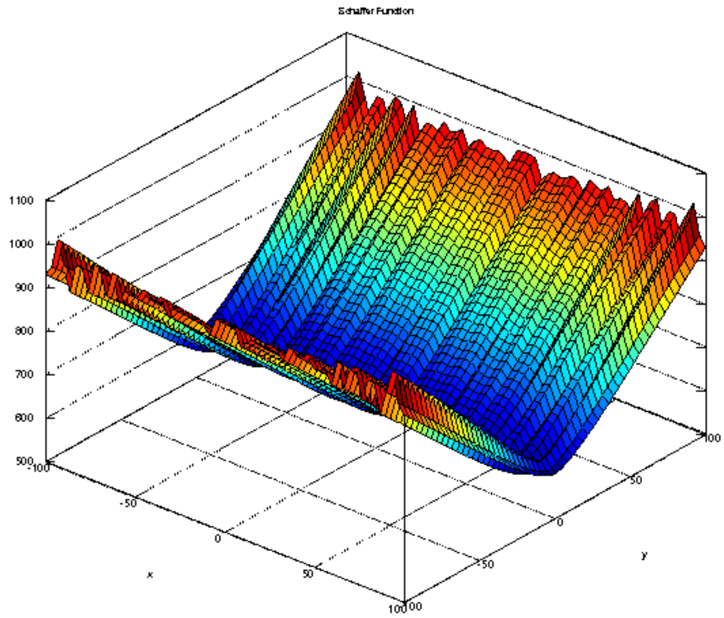


Figure 6: Shifted Schaffer Function

2 Hybrid Composition Functions (F12-F19*)

The *hybrid composition functions*, F12-F19*, are built combining a non-separable function with other function. The considered functions are:

- **Non-Separable Functions**:
  - F3: Shifted Rosenbrock's Function
  - F5: Shifted Griewank's Function
  - NS-F9: Non-shifted Extended $f10$
  - NS-F10: Non-shifted Bohachevsky
- **Other Component Functions**:
  - F1: Shifted Sphere Function
  - F4: Shifted Rastrigin's Function
  - NS-F7: Non-shifted Schwefel's Problem 2.22

The procedure used to hybridize a non-separable function F_{ns} with other function F' (function $F_{ns} \oplus F'$) is shown in Figure 7. Its main steps are: 1) to divide the solution into two parts, 2) to evaluate each one of them with a different function, and 3) to combine their results. The splitting mechanism uses a parameter, m_{ns}, which specifies the ratio of variables that are evaluated by F_{ns}. Using a higher value of m_{ns}, the hybrid function becomes more difficult to optimize dimension by dimension, because there is a greater interrelation between the variables and the fitness. With this procedure, we have defined the instances of hybrid functions shown in Table 4.

Name	F_{ns}	F'	m_{ns}	**Range**	**Fitness Optimum**
F12	NS-F9	F1	0.25	$[-100, 100]^D$	0
F13	NS-F9	F3	0.25	$[-100, 100]^D$	0
F14	NS-F9	F4	0.25	$[-5, 5]^D$	0
F15	NS-F10	NS-F7	0.25	$[-10, 10]^D$	0
F16*	NS-F9	F1	0.5	$[-100, 100]^D$	0
F17*	NS-F9	F3	0.75	$[-100, 100]^D$	0
F18*	NS-F9	F4	0.75	$[-5, 5]^D$	0
F19*	NS-F10	NS-F7	0.75	$[-10, 10]^D$	0

Table 4: Hybrid composition functions

We should point out that the hybrid F15 and F19* functions were shifted.

Function $F_{ns} \oplus F'(S)$

1. S is divided into two parts ($part_1$ and $part_2$):

   - **If** $m_{ns} \leq 0.5$ **then**

     – $part_1$ is composed by the first $D \cdot m_{ns}$ even variables. $(length(part_1) = D \cdot m_{ns})$

     – $part_2$ is composed by the remaining variables. $(length(part_2) = D - length(part_1))$

   - **If** $m_{ns} > 0.5$ **then**

     – $part_2$ is composed by the first $D \cdot (1 - m_{ns})$ odd variables. $(length(part_2) = D \cdot (1 - m_{ns}))$

     – $part_1$ is composed by the remaining variables. $(length(part_1) = D - length(part_2))$

2. **Return** $F_{ns}(part_1) + F'(part_2)$.

Figure 7: Evaluation of a solution S (with D variables) by the hybrid function $F_{ns} \oplus F'$