Benchmarking sep-CMA-ES on the BBOB-2009 Function Testbed

Raymond Ros
Univ. Paris-Sud, LRI
UMR 8623 / INRIA Saclay, projet TAO
F-91405 Orsay, France
raymond.ros@lri.fr

ABSTRACT

A partly time and space linear CMA-ES is benchmarked on the BBOB-2009 noiseless function testbed. This algorithm with a multistart strategy with increasing population size solves 17 functions out of 24 in 20-D.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Evolutionary computation, Covariance matrix adaptation, Evolution strategy

1. INTRODUCTION

The sep-CMA-ES algorithm introduced in [7] is a variant of the covariance matrix adaptation evolution strategy (CMA-ES) [5] that is linear in time and space. This property combined with a faster learning rate makes sep-CMA-ES appropriate for separable function and larger dimensions. A mixed strategy of using sep-CMA-ES and CMA-ES is proposed here and benchmarked on a noiseless function testbed.

2. ALGORITHM PRESENTATION

In its design, the sep-CMA-ES differs from the CMA-ES by two aspects: first, the covariance matrix is constrained to be diagonal at each of its update, second, the learning rate is increased by a factor of $\frac{n+3/2}{3}$, where n is the dimension of the search space [1]. These modifications result in an algorithm that trades model complexity with a time and space scaling that is better than the original CMA-ES. The $(\mu/\mu_{\mathrm{w}}, \lambda)$-sep-CMA-ES has been shown to outperform $(\mu/\mu_{\mathrm{w}}, \lambda)$-CMA-ES on separable functions.

We propose here what would be the best of two worlds: to use sep-CMA-ES for the first few iterations and then switch to CMA-ES. At the time of the switch, all parameters are retained except for the learning rate that is decreased back to its default value. This implies the diagonal covariance matrix acquired using sep-CMA-ES is directly used by CMA-ES. This mixed strategy is therefore expected to be faster than CMA-ES on separable functions. Ongoing work has also shown that for some test functions the first iterations using sep-CMA-ES would not disadvantage the latter use of CMA-ES in any way. In other terms, the cost of initially using sep-CMA-ES would not induce a penalty in the cost of solving the function with CMA-ES afterwards. The author admits some functions could induce such a penalty.

As for the multistart strategy, we use the increasing population size IPOP-CMA-ES [1]. Though this approach has shown its limits [6], independent restart may improve the probability of the algorithm reaching a given target function value.

3. EXPERIMENTAL PROCEDURE

The Matlab implementation of the CMA-ES (version 3.23beta) is used[2]. We use the $(\mu/\mu_{\mathrm{w}}, \lambda)$-IPOP-CMA-ES variant with an initial default population size $\lambda = 4 + \lfloor 3 \ln(n) \rfloor$ increasing twice at each restart. Except the learning rate, all other algorithm parameters are set to their default values. The covariance matrix is constrained to be diagonal only for the first $1 + 100n/\sqrt{\lambda}$ iterations of the *first start*. A maximum of 8 independent restarts is conducted. Restarts occur after $100 + 300n\sqrt{n/\lambda}$ iterations or if any of the default stopping criterion is met. The initial stepsize has been set to 2 and the starting point has been chosen uniformly in $[-4, 4]^n$. The maximum number of function evaluations was set to 10^4 times the dimension. No parameter tuning was done, the CrE [3] is computed to zero.

4. RESULTS AND DISCUSSION

Results from experiments according to [3] on the benchmark functions given in [2, 4] are presented in Figures 1 and 2 and in Table 1. The algorithm solves 17 out of the 24 functions in 20-D. The algorithm performs well on uni-

[1]Please note that the factor for the learning rate is smaller than the one in [7].

[2]Latest version available here:`http://www.lri.fr/~hansen/cmaesintro.html`

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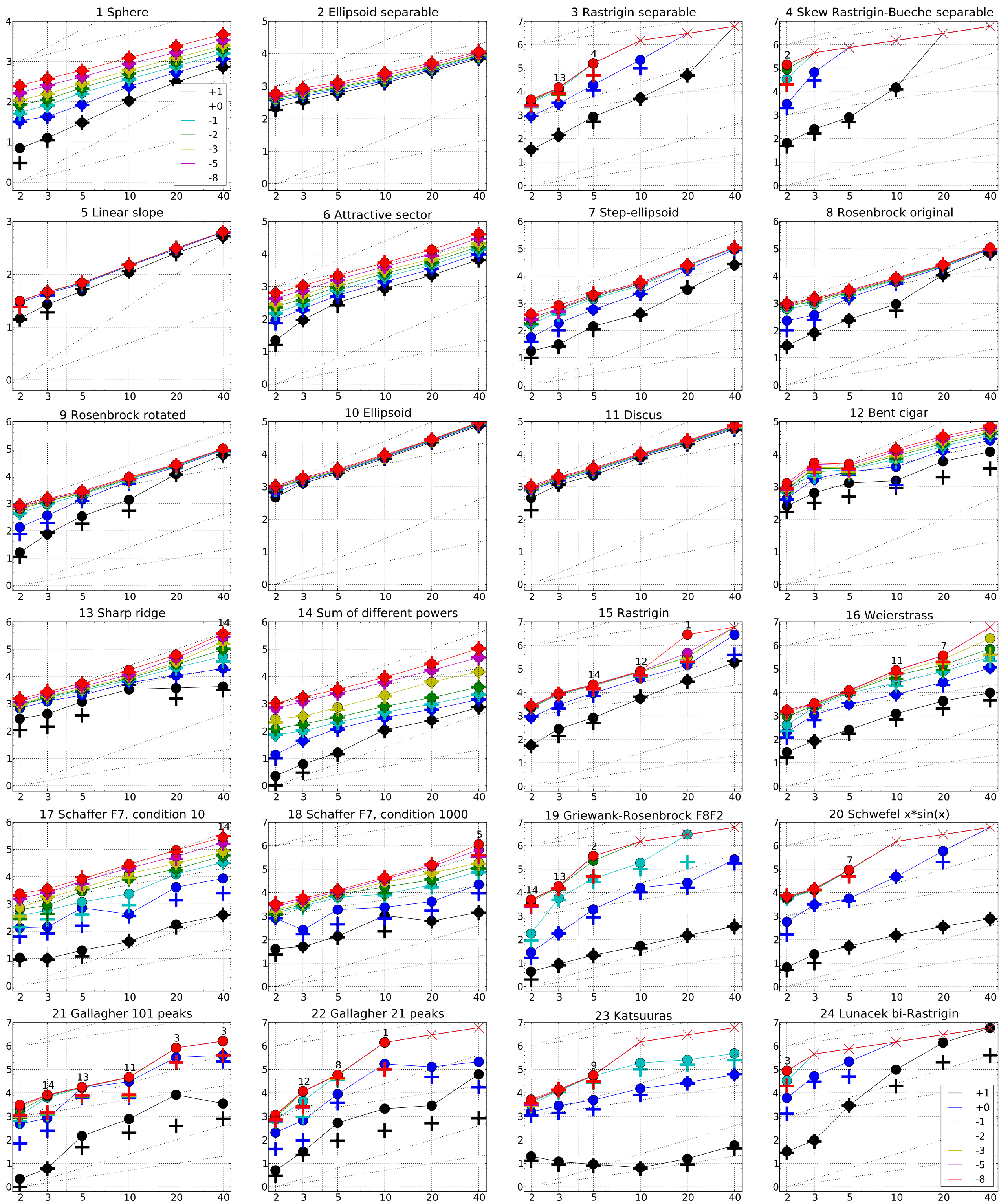

Figure 1: Expected Running Time (ERT, ●) **to reach** $f_{\rm opt} + \Delta f$ **and median number of function evaluations of successful trials** (+), **shown for** $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ **(the exponent is given in the legend of** f_1 **and** f_{24}**) versus dimension in log-log presentation. The** $\mathrm{ERT}(\Delta f)$ **equals to** $\#\mathrm{FEs}(\Delta f)$ **divided by the number of successful trials, where a trial is successful if** $f_{\rm opt} + \Delta f$ **was surpassed during the trial. The** $\#\mathrm{FEs}(\Delta f)$ **are the total number of function evaluations while** $f_{\rm opt} + \Delta f$ **was not surpassed during the trial from all respective trials (successful and unsuccessful), and** $f_{\rm opt}$ **denotes the optimal function value. Crosses (**$\times$**) indicate the total number of function evaluations** $\#\mathrm{FEs}(-\infty)$**. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.**

Δf	f_1 **in 5-D**, N=15, mFE=738					f_1 **in 20-D**, N=15, mFE=2594				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.0e1	2.3e1	3.7e1	3.0e1	15	3.1e2	2.9e2	3.2e2	3.1e2
1	15	8.5e1	7.8e1	9.2e1	8.5e1	15	5.3e2	5.1e2	5.5e2	5.3e2
1e−1	15	1.7e2	1.5e2	1.8e2	1.7e2	15	7.6e2	7.4e2	7.8e2	7.6e2
1e−3	15	2.8e2	2.7e2	3.0e2	2.8e2	15	1.2e3	1.2e3	1.3e3	1.2e3
1e−5	15	4.2e2	4.0e2	4.3e2	4.2e2	15	1.7e3	1.6e3	1.7e3	1.7e3
1e−8	15	6.0e2	5.8e2	6.2e2	6.0e2	15	2.4e3	2.4e3	2.4e3	2.4e3

Δf	f_2 **in 5-D**, N=15, mFE=1554					f_2 **in 20-D**, N=15, mFE=5750				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	6.0e2	5.5e2	6.5e2	6.0e2	15	2.9e3	2.8e3	3.0e3	2.9e3
1	15	7.4e2	7.0e2	7.8e2	7.4e2	15	3.2e3	3.1e3	3.3e3	3.2e3
1e−1	15	8.3e2	7.9e2	8.6e2	8.3e2	15	3.5e3	3.4e3	3.6e3	3.5e3
1e−3	15	9.7e2	9.4e2	1.0e3	9.7e2	15	4.0e3	3.9e3	4.1e3	4.0e3
1e−5	15	1.1e3	1.1e3	1.1e3	1.1e3	15	4.5e3	4.4e3	4.6e3	4.5e3
1e−8	15	1.3e3	1.3e3	1.3e3	1.3e3	15	5.2e3	5.1e3	5.3e3	5.2e3

Δf	f_3 **in 5-D**, N=15, mFE=50422					f_3 **in 20-D**, N=15, mFE=200352				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	8.6e2	5.9e2	1.1e3	8.6e2	15	5.1e4	4.2e4	6.0e4	5.1e4
1	13	1.9e4	1.3e4	2.7e4	1.5e4	0	*60e−1*	*40e−1*	*70e−1*	1.1e5
1e−1	4	1.6e5	9.6e4	3.4e5	3.7e4	.	.	.	.	.
1e−3	4	1.6e5	9.8e4	3.5e5	3.8e4	.	.	.	.	.
1e−5	4	1.6e5	9.9e4	3.3e5	3.8e4	.	.	.	.	.
1e−8	4	1.6e5	1.0e5	3.4e5	3.9e4	.	.	.	.	.

Δf	f_4 **in 5-D**, N=15, mFE=50430					f_4 **in 20-D**, N=15, mFE=200376				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	8.1e2	5.7e2	1.1e3	8.1e2	0	*14e+0*	*13e+0*	*16e+0*	1.1e5
1	0	*37e−1*	*30e−1*	*40e−1*	1.8e4	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_5 **in 5-D**, N=15, mFE=106					f_5 **in 20-D**, N=15, mFE=410				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.8e1	4.2e1	5.4e1	4.8e1	15	2.5e2	2.3e2	2.7e2	2.5e2
1	15	6.6e1	6.0e1	7.2e1	6.6e1	15	3.0e2	2.8e2	3.2e2	3.0e2
1e−1	15	6.8e1	6.2e1	7.4e1	6.8e1	15	3.1e2	2.9e2	3.3e2	3.1e2
1e−3	15	6.9e1	6.4e1	7.5e1	6.9e1	15	3.1e2	2.9e2	3.3e2	3.1e2
1e−5	15	6.9e1	6.4e1	7.5e1	6.9e1	15	3.1e2	2.9e2	3.3e2	3.1e2
1e−8	15	6.9e1	6.4e1	7.5e1	6.9e1	15	3.1e2	2.9e2	3.3e2	3.1e2

Δf	f_6 **in 5-D**, N=15, mFE=2650					f_6 **in 20-D**, N=15, mFE=24422				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.2e2	2.6e2	3.9e2	3.2e2	15	2.2e3	2.1e3	2.3e3	2.2e3
1	15	5.3e2	4.7e2	6.1e2	5.3e2	15	3.4e3	3.2e3	3.5e3	3.4e3
1e−1	15	7.5e2	6.8e2	8.3e2	7.5e2	15	4.6e3	4.4e3	4.7e3	4.6e3
1e−3	15	1.3e3	1.2e3	1.3e3	1.3e3	15	7.0e3	6.8e3	7.2e3	7.0e3
1e−5	15	1.7e3	1.6e3	1.7e3	1.7e3	15	1.0e4	9.3e3	1.1e4	1.0e4
1e−8	15	2.2e3	2.1e3	2.2e3	2.2e3	15	1.4e4	1.3e4	1.5e4	1.4e4

Δf	f_7 **in 5-D**, N=15, mFE=3288					f_7 **in 20-D**, N=15, mFE=46966				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.4e2	1.2e2	1.7e2	1.4e2	15	3.1e3	2.6e3	3.6e3	3.1e3
1	15	5.8e2	5.0e2	6.6e2	5.8e2	15	1.7e4	1.5e4	2.0e4	1.7e4
1e−1	15	1.4e3	1.2e3	1.7e3	1.4e3	15	2.3e4	2.0e4	2.6e4	2.3e4
1e−3	15	1.9e3	1.6e3	2.1e3	1.9e3	15	2.5e4	2.2e4	2.8e4	2.5e4
1e−5	15	1.9e3	1.6e3	2.1e3	1.9e3	15	2.5e4	2.2e4	2.8e4	2.5e4
1e−8	15	2.0e3	1.8e3	2.3e3	2.0e3	15	2.5e4	2.2e4	2.9e4	2.5e4

Δf	f_8 **in 5-D**, N=15, mFE=5788					f_8 **in 20-D**, N=15, mFE=28922				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.6e2	2.1e2	3.1e2	2.6e2	15	1.1e4	1.1e4	1.1e4	1.1e4
1	15	1.6e3	1.3e3	1.9e3	1.6e3	15	2.1e4	2.1e4	2.1e4	2.1e4
1e−1	15	2.3e3	2.1e3	2.5e3	2.3e3	15	2.3e4	2.2e4	2.3e4	2.3e4
1e−3	15	2.7e3	2.5e3	2.9e3	2.7e3	15	2.4e4	2.4e4	2.5e4	2.4e4
1e−5	15	2.8e3	2.6e3	3.1e3	2.8e3	15	2.5e4	2.5e4	2.6e4	2.5e4
1e−8	15	3.1e3	2.9e3	3.3e3	3.1e3	15	2.6e4	2.6e4	2.7e4	2.6e4

Δf	f_9 **in 5-D**, N=15, mFE=4914					f_9 **in 20-D**, N=15, mFE=55924				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.4e2	1.9e2	4.9e2	3.4e2	15	1.2e4	1.1e4	1.2e4	1.2e4
1	15	1.4e3	1.2e3	1.6e3	1.4e3	15	2.2e4	1.9e4	2.4e4	2.2e4
1e−1	15	2.1e3	2.0e3	2.3e3	2.1e3	15	2.4e4	2.1e4	2.6e4	2.4e4
1e−3	15	2.5e3	2.3e3	2.7e3	2.5e3	15	2.5e4	2.3e4	2.7e4	2.5e4
1e−5	15	2.7e3	2.5e3	2.8e3	2.7e3	15	2.6e4	2.4e4	2.8e4	2.6e4
1e−8	15	2.9e3	2.8e3	3.1e3	2.9e3	15	2.7e4	2.5e4	2.9e4	2.7e4

Δf	f_{10} **in 5-D**, N=15, mFE=4042					f_{10} **in 20-D**, N=15, mFE=30386				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.6e3	2.5e3	2.7e3	2.6e3	15	2.3e4	2.2e4	2.4e4	2.3e4
1	15	2.8e3	2.7e3	2.9e3	2.8e3	15	2.5e4	2.4e4	2.6e4	2.5e4
1e−1	15	2.9e3	2.8e3	3.0e3	2.9e3	15	2.6e4	2.6e4	2.7e4	2.6e4
1e−3	15	3.1e3	3.0e3	3.2e3	3.1e3	15	2.7e4	2.7e4	2.8e4	2.7e4
1e−5	15	3.3e3	3.2e3	3.3e3	3.3e3	15	2.8e4	2.7e4	2.8e4	2.8e4
1e−8	15	3.5e3	3.4e3	3.6e3	3.5e3	15	2.9e4	2.8e4	2.9e4	2.9e4

Δf	f_{11} **in 5-D**, N=15, mFE=4226					f_{11} **in 20-D**, N=15, mFE=31418				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.2e3	1.9e3	2.5e3	2.2e3	15	2.0e4	1.9e4	2.0e4	2.0e4
1	15	2.9e3	2.8e3	3.0e3	2.9e3	15	2.2e4	2.2e4	2.3e4	2.2e4
1e−1	15	3.2e3	3.1e3	3.3e3	3.2e3	15	2.3e4	2.3e4	2.4e4	2.3e4
1e−3	15	3.4e3	3.4e3	3.5e3	3.4e3	15	2.5e4	2.4e4	2.5e4	2.5e4
1e−5	15	3.6e3	3.5e3	3.7e3	3.6e3	15	2.6e4	2.5e4	2.6e4	2.6e4
1e−8	15	3.8e3	3.8e3	3.9e3	3.8e3	15	2.7e4	2.6e4	2.7e4	2.7e4

Δf	f_{12} **in 5-D**, N=15, mFE=9042					f_{12} **in 20-D**, N=15, mFE=58382				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.3e3	9.4e2	1.7e3	1.3e3	15	6.1e3	4.0e3	8.2e3	6.1e3
1	15	2.9e3	2.6e3	3.2e3	2.9e3	15	1.3e4	9.9e3	1.6e4	1.3e4
1e−1	15	3.4e3	2.9e3	3.8e3	3.4e3	15	1.8e4	1.4e4	2.1e4	1.8e4
1e−3	15	3.9e3	3.4e3	4.6e3	3.9e3	15	2.5e4	2.2e4	2.8e4	2.5e4
1e−5	15	4.5e3	3.8e3	5.2e3	4.5e3	15	3.0e4	2.7e4	3.3e4	3.0e4
1e−8	15	5.1e3	4.3e3	5.9e3	5.1e3	15	3.5e4	3.2e4	3.8e4	3.5e4

Δf	f_{13} **in 5-D**, N=15, mFE=6308					f_{13} **in 20-D**, N=15, mFE=105488				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.2e3	8.5e2	1.6e3	1.2e3	15	3.8e3	2.6e3	5.0e3	3.8e3
1	15	2.1e3	1.7e3	2.4e3	2.1e3	15	1.1e4	8.4e3	1.3e4	1.1e4
1e−1	15	3.0e3	2.8e3	3.2e3	3.0e3	15	2.1e4	1.7e4	2.5e4	2.1e4
1e−3	15	3.7e3	3.6e3	3.8e3	3.7e3	15	3.3e4	2.9e4	3.6e4	3.3e4
1e−5	15	4.4e3	4.3e3	4.5e3	4.4e3	15	4.7e4	4.4e4	5.0e4	4.7e4
1e−8	15	5.5e3	5.4e3	5.7e3	5.5e3	15	6.2e4	5.7e4	6.8e4	6.2e4

Δf	f_{14} **in 5-D**, N=15, mFE=3634					f_{14} **in 20-D**, N=15, mFE=31850				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.6e1	1.2e1	2.0e1	1.6e1	15	2.5e2	2.3e2	2.7e2	2.5e2
1	15	1.2e2	1.1e2	1.4e2	1.2e2	15	6.1e2	5.8e2	6.5e2	6.1e2
1e−1	15	2.1e2	1.9e2	2.3e2	2.1e2	15	9.8e2	9.4e2	1.0e3	9.8e2
1e−3	15	7.4e2	6.3e2	8.5e2	7.4e2	15	6.4e3	6.1e3	6.7e3	6.4e3
1e−5	15	2.4e3	2.3e3	2.5e3	2.4e3	15	1.7e4	1.6e4	1.7e4	1.7e4
1e−8	15	3.3e3	3.3e3	3.4e3	3.3e3	15	3.0e4	2.9e4	3.0e4	3.0e4

Δf	f_{15} **in 5-D**, N=15, mFE=50430					f_{15} **in 20-D**, N=15, mFE=200340				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	8.3e2	5.1e2	1.2e3	8.3e2	15	3.3e4	2.8e4	3.8e4	3.3e4
1	15	9.3e3	7.7e3	1.1e4	9.3e3	15	1.5e5	1.3e5	1.6e5	1.5e5
1e−1	14	1.9e4	1.5e4	2.4e4	1.8e4	9	3.1e5	2.4e5	4.5e5	1.8e5
1e−3	14	2.0e4	1.6e4	2.5e4	1.9e4	9	3.2e5	2.5e5	4.3e5	1.9e5
1e−5	14	2.1e4	1.6e4	2.6e4	1.9e4	6	4.9e5	3.5e5	7.5e5	2.0e5
1e−8	14	2.2e4	1.7e4	2.7e4	2.0e4	1	2.9e6	1.4e6	>3e6	2.0e5

Δf	f_{16} **in 5-D**, N=15, mFE=32578					f_{16} **in 20-D**, N=15, mFE=200340				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.6e2	2.0e2	3.1e2	2.6e2	15	4.3e3	2.3e3	6.3e3	4.3e3
1	15	3.1e3	2.5e3	3.7e3	3.1e3	15	2.7e4	2.1e4	3.3e4	2.7e4
1e−1	15	9.3e3	7.2e3	1.2e4	9.3e3	15	7.7e4	6.4e4	9.1e4	7.7e4
1e−3	15	1.0e4	8.4e3	1.3e4	1.0e4	9	2.6e5	2.0e5	3.5e5	1.6e5
1e−5	15	1.2e4	9.7e3	1.4e4	1.2e4	7	3.7e5	2.6e5	6.0e5	1.6e5
1e−8	15	1.2e4	1.0e4	1.4e4	1.2e4	7	3.8e5	2.7e5	6.3e5	1.7e5

Δf	f_{17} **in 5-D**, N=15, mFE=12542					f_{17} **in 20-D**, N=15, mFE=159622				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.0e1	1.4e1	2.5e1	2.0e1	15	1.8e2	1.5e2	2.1e2	1.8e2
1	15	7.2e2	1.8e2	1.3e3	7.2e2	15	4.1e3	1.4e3	6.9e3	4.1e3
1e−1	15	1.2e3	6.8e2	1.7e3	1.2e3	15	1.2e4	9.8e3	1.5e4	1.2e4
1e−3	15	3.8e3	3.2e3	4.5e3	3.8e3	15	3.1e4	2.7e4	3.5e4	3.1e4
1e−5	15	6.9e3	5.8e3	7.9e3	6.9e3	15	5.6e4	5.0e4	6.2e4	5.6e4
1e−8	15	8.3e3	7.1e3	9.3e3	8.3e3	15	9.5e4	8.4e4	1.1e5	9.5e4

Δf	f_{18} **in 5-D**, N=15, mFE=21008					f_{18} **in 20-D**, N=15, mFE=171622				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.4e2	1.1e2	1.6e2	1.4e2	15	6.2e2	5.7e2	6.7e2	6.2e2
1	15	1.9e3	1.0e3	2.9e3	1.9e3	15	4.1e3	2.0e3	6.1e3	4.1e3
1e−1	15	6.3e3	5.1e3	7.6e3	6.3e3	15	2.2e4	1.9e4	2.5e4	2.2e4
1e−3	15	9.6e3	8.2e3	1.1e4	9.6e3	15	6.8e4	5.8e4	7.8e4	6.8e4
1e−5	15	1.1e4	9.6e3	1.2e4	1.1e4	15	1.3e5	1.2e5	1.4e5	1.3e5
1e−8	15	1.3e4	1.2e4	1.4e4	1.3e4	15	1.5e5	1.4e5	1.6e5	1.5e5

Δf	f_{19} **in 5-D**, N=15, mFE=50122					f_{19} **in 20-D**, N=15, mFE=200096				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.2e1	1.8e1	2.6e1	2.2e1	15	1.5e2	1.3e2	1.8e2	1.5e2
1	15	1.9e3	9.5e2	3.0e3	1.9e3	15	2.7e4	1.8e4	3.6e4	2.7e4
1e−1	12	3.9e4	3.1e4	5.0e4	3.0e4	1	3.0e6	1.5e6	>3e6	2.0e5
1e−3	2	3.6e5	1.9e5	>7e5	5.0e4	0	*29e−2*	*18e−2*	*51e−2*	2.0e5
1e−5	2	3.6e5	1.9e5	>7e5	5.0e4	.	.	.	.	.
1e−8	2	3.7e5	1.9e5	>7e5	5.0e4	.	.	.	.	.

Δf	f_{20} **in 5-D**, N=15, mFE=50502					f_{20} **in 20-D**, N=15, mFE=200316				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	5.3e1	4.4e1	6.1e1	5.3e1	15	3.6e2	3.4e2	3.9e2	3.6e2
1	15	5.6e3	3.8e3	7.4e3	5.6e3	5	6.0e5	4.2e5	1.0e6	2.0e5
1e−1	7	8.9e4	6.2e4	1.4e5	3.7e4	0	*11e−1*	*91e−2*	*12e−1*	1.3e5
1e−3	7	9.1e4	6.4e4	1.5e5	3.8e4	.	.	.	.	.
1e−5	7	9.2e4	6.5e4	1.5e5	3.8e4	.	.	.	.	.
1e−8	7	9.4e4	6.6e4	1.4e5	3.9e4	.	.	.	.	.

Δf	f_{21} **in 5-D**, N=15, mFE=50246					f_{21} **in 20-D**, N=15, mFE=200714				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.5e2	4.5e1	2.6e2	1.5e2	15	8.4e3	1.6e3	1.6e4	8.4e3
1	13	1.6e4	9.7e3	2.3e4	1.4e4	6	3.3e5	2.2e5	5.5e5	1.5e5
1e−1	13	1.7e4	1.0e4	2.3e4	1.5e4	3	8.2e5	4.6e5	2.6e6	1.4e5
1e−3	13	1.7e4	1.1e4	2.5e4	1.5e4	3	8.2e5	4.6e5	2.5e6	1.4e5
1e−5	13	1.7e4	1.1e4	2.5e4	1.5e4	3	8.2e5	4.6e5	2.5e6	1.4e5
1e−8	13	1.8e4	1.1e4	2.6e4	1.6e4	3	8.2e5	4.6e5	2.5e6	1.4e5

Δf	f_{22} **in 5-D**, N=15, mFE=50672					f_{22} **in 20-D**, N=15, mFE=200762				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	5.3e2	2.4e2	8.4e2	5.3e2	15	2.9e3	1.5e3	4.4e3	2.9e3
1	14	9.0e3	4.8e3	1.4e4	8.8e3	10	1.3e5	8.4e4	1.9e5	8.7e4
1e−1	8	5.6e4	3.8e4	9.0e4	2.8e4	0	*69e−2*	*69e−2*	*51e−1*	1.3e4
1e−3	8	5.8e4	3.9e4	9.3e4	2.8e4	.	.	.	.	.
1e−5	8	5.8e4	3.9e4	9.2e4	2.8e4	.	.	.	.	.
1e−8	8	5.9e4	3.9e4	9.4e4	2.8e4	.	.	.	.	.

Δf	f_{23} **in 5-D**, N=15, mFE=50064					f_{23} **in 20-D**, N=15, mFE=200096				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	9.2e0	6.9e0	1.2e1	9.2e0	15	1.6e1	1.1e1	2.1e1	1.6e1
1	15	5.0e3	3.3e3	6.9e3	5.0e3	15	2.8e4	2.1e4	3.6e4	2.8e4
1e−1	9	5.3e4	4.0e4	7.5e4	3.3e4	9	2.6e5	1.9e5	3.6e5	1.4e5
1e−3	9	5.5e4	4.2e4	7.7e4	3.4e4	0	*81e−3*	*38e−3*	*20e−2*	1.6e5
1e−5	9	5.5e4	4.2e4	7.7e4	3.4e4	.	.	.	.	.
1e−8	9	5.6e4	4.3e4	8.0e4	3.5e4	.	.	.	.	.

Δf	f_{24} **in 5-D**, N=15, mFE=50228					f_{24} **in 20-D**, N=15, mFE=200158				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.9e3	2.2e3	3.7e3	2.9e3	2	1.4e6	7.3e5	>3e6	2.0e5
1	3	2.2e5	1.3e5	6.8e5	4.1e4	0	*23e+0*	*93e−1*	*24e+0*	1.3e5
1e−1	0	*53e−1*	*32e−2*	*56e−1*	2.5e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathrm{RT}_{\mathrm{succ}}$). If $f_{\mathrm{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

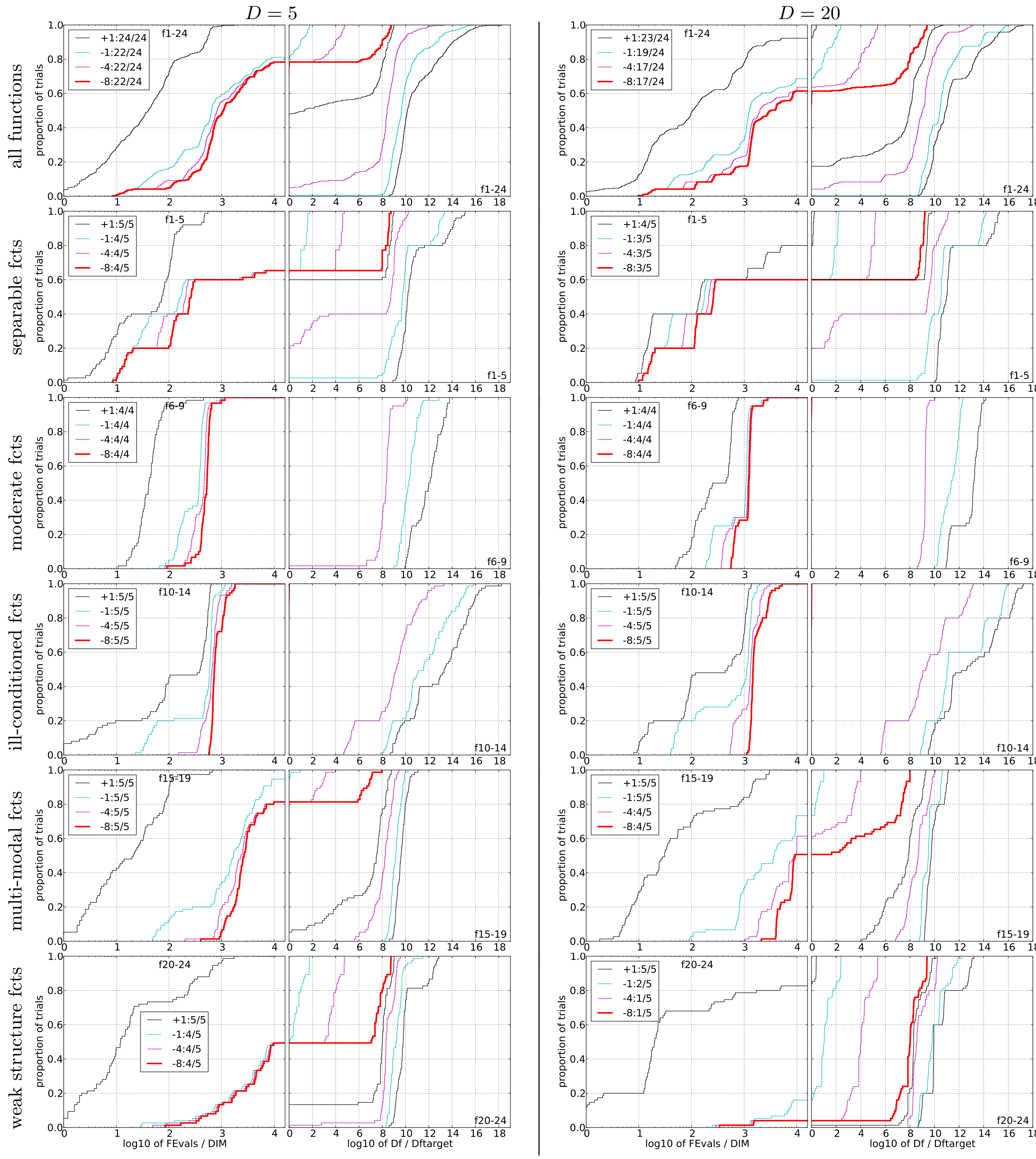

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D\ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

modal separable functions as expected. Its performances on multimodal functions, even separable ones such as f_3 and f_4, are limited though. Whereas for multimodal functions increasing the maximum number of function evaluations is likely to improve the performances of the algorithm, this should not be the case for f_{24}. For the timing experiment, the proposed algorithm was run on f_8 and restarted until at least 30 seconds have passed (according to Figure 2 in [3]). The experiments were conducted with an Intel Core 2 6700 processor (2.66GHz) with Matlab R2008a on Linux 2.6.24.7. The results were 15, 13, 11, 9.7, 9.9, and 13 $\times 10^{-5}$ seconds per function evaluations in dimension 2, 3, 5, 10, 20, and 40 respectively.

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