Benchmarking the NEWUOA on the BBOB-2009 Function Testbed

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ABSTRACT

The NEWUOA which belongs to the class of Derivative-Free optimization algorithms is benchmarked on the BBOB-2009 noisefree testbed. A multistart strategy is applied with a maximum number of function evaluations of up to 10^5 times the search space dimension resulting in the algorithm solving 11 functions in 20-D. The results of the algorithm using the recommended number of interpolation points for the underlying model and the full model are shown and discussed.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Derivative-free optimization

1. ALGORITHM PRESENTATION

The NEWUOA (New Unconstrained Optimization Algorithm) [4] is a Derivative-Free Optimization (DFO) algorithm using the trust region paradigm. NEWUOA computes a quadratic interpolation of the objective function in the current trust region and performs a truncated conjugate gradient minimization of the surrogate model in the trust region. It then updates either the current best point or the radius of the trust region, based on the a posteriori interpolation error. The time complexity of the algorithm is $\mathcal{O}(m^2n)$ in the worst case but in practice closer to $\mathcal{O}(mn)$, where m is the number of interpolation points used for the determination of the quadratic model and n is the dimension of the search space. The number of interpolation points is a parameter of the algorithm and needs to be chosen in the range $[n+2, \frac{(n+1)(n+2)}{2}]$. Other parameters of the algorithm are the initial and final radii of the trust region, respectively governing the initial 'granularity' and the precision of the search. A simple stochastic independent restart procedure (as advised in [2]) was added to improve the probability of the algorithm reaching a target function value.

GECCO'09, July 8–12, 2009, Montréal Québec, Canada.

2. EXPERIMENTAL PROCEDURE

The implementation used for our experiments is the one provided by Matthieu Guibert[1] which delivers Powell's original Fortran source code of the algorithm. This Fortran code has been adapted to the BBOB experimental paradigm. In this paper, we will test two numbers of interpolation points: $2n+1$ which is recommended in [4] and $\frac{(n+1)(n+2)}{2}$ which is the full model. An intermediate model using a number of interpolation points that is the integer closer to $\sqrt{(n+1/2)(n+1)(n+2)}$ was also tested with results that were in-between those of the two models we are considering. The initial radius ρ_{beg} of the search region has been set to 10, the range of the search space. Preliminary experiments shows very few dependencies on this parameter, given it is not too small (ie. by many orders of magnitude) for the problem considered. A final radius $\rho_{\text{end}} = 10^{-16}$ was chosen close to the limit being the machine precision to prevent numerical errors. The starting point x_0 is chosen uniformly in $[-5, 5]^n$. The multistart strategy was used with at most 100 restarts to reduce the duration of an experiment. For the same reason, the maximum number of function evaluations is $10^5 \times n$ for $m = 2n+1$, $10^4 \times n$ otherwise. An example of the algorithm used is presented in Figure 1. No parameter tuning was done, the CrE [2] is computed to zero.

3. RESULTS AND DISCUSSION

Results from experiments according to [2] on the benchmark functions given in [1, 3] are presented in Figures 2 and 3 and in Table 1 for $m = 2n+1$. The algorithm performs well on the convex quadratic functions f_1. It solves f_2 and f_{11}. The algorithm performs well on functions with low or moderate conditioning.

On multimodal functions, the algorithm fails or only solves 2, 3 and/or 5-D, though it does well on the Gallagher functions. As we can see in Figures 4 and 5 and in Table 2 for the full model, these results cannot be improved by using more

[1] `http://www.inrialpes.fr/bipop/people/guilbert/newuoa/newuoa.html`

Δf	**f_1 in 5-D**, N=15, mFE=139					**f_1 in 20-D**, N=15, mFE=471				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.3e1	4.3e1	4.3e1	4.3e1
1	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.3e1	4.3e1	4.4e1	4.3e1
1e−1	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.3e1	4.3e1	4.4e1	4.3e1
1e−3	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.4e1	4.3e1	4.4e1	4.4e1
1e−5	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.4e1	4.3e1	4.4e1	4.4e1
1e−8	15	1.2e1	1.2e1	1.2e1	1.2e1	15	4.4e1	4.3e1	4.4e1	4.4e1

Δf	**f_2 in 5-D**, N=15, mFE=62427					**f_2 in 20-D**, N=15, mFE=315319				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.8e2	3.1e2	6.6e2	4.8e2	15	7.0e3	6.2e3	7.8e3	7.0e3
1	15	1.9e3	1.5e3	2.3e3	1.9e3	15	1.6e4	1.4e4	1.8e4	1.6e4
1e−1	15	4.0e3	3.4e3	4.6e3	4.0e3	15	2.7e4	2.5e4	3.0e4	2.7e4
1e−3	15	7.6e3	6.8e3	8.4e3	7.6e3	15	4.9e4	4.5e4	5.3e4	4.9e4
1e−5	15	1.2e4	1.1e4	1.3e4	1.2e4	15	6.8e4	6.4e4	7.3e4	6.8e4
1e−8	15	2.6e4	2.1e4	3.2e4	2.6e4	15	1.2e5	1.0e5	1.5e5	1.2e5

Δf	**f_3 in 5-D**, N=15, mFE=25753					**f_3 in 20-D**, N=15, mFE=133533				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.3e3	2.9e3	5.9e3	4.3e3	0	*13e+1*	*11e+1*	*16e+1*	6.3e4
1	1	3.7e5	1.8e5	>4e5	2.5e4	.	.	.	.	.
1e−1	0	*40e-1*	*30e-1*	*80e-1*	1.3e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_4 in 5-D**, N=15, mFE=36591					**f_4 in 20-D**, N=15, mFE=255640				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	12	2.1e4	1.6e4	2.8e4	1.7e4	0	*17e+1*	*13e+1*	*22e+1*	8.9e4
1	1	5.0e5	2.4e5	>5e5	3.5e4	.	.	.	.	.
1e−1	0	*60e-1*	*20e-1*	*11e+0*	2.0e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_5 in 5-D**, N=15, mFE=207					**f_5 in 20-D**, N=15, mFE=806				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.3e1	1.2e1	1.3e1	1.3e1	15	5.0e1	4.8e1	5.2e1	5.0e1
1	15	1.5e1	1.4e1	1.5e1	1.5e1	15	5.9e1	5.5e1	6.3e1	5.9e1
1e−1	15	1.5e1	1.4e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1
1e−3	15	1.5e1	1.5e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1
1e−5	15	1.5e1	1.5e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.1e1	6.5e1
1e−8	15	1.5e1	1.5e1	1.6e1	1.5e1	15	6.5e1	6.1e1	7.0e1	6.5e1

Δf	**f_6 in 5-D**, N=15, mFE=10778					**f_6 in 20-D**, N=15, mFE=25866				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.0e2	1.6e2	2.4e2	2.0e2	15	1.3e3	1.1e3	1.5e3	1.3e3
1	15	5.0e2	4.0e2	6.2e2	5.0e2	15	2.3e3	2.1e3	2.6e3	2.3e3
1e−1	15	1.0e3	7.9e2	1.2e3	1.0e3	15	3.4e3	3.0e3	3.9e3	3.4e3
1e−3	15	1.9e3	1.6e3	2.2e3	1.9e3	15	5.8e3	4.9e3	6.6e3	5.8e3
1e−5	15	2.8e3	2.4e3	3.3e3	2.8e3	15	8.4e3	7.1e3	9.7e3	8.4e3
1e−8	15	4.3e3	3.8e3	4.9e3	4.3e3	15	1.1e4	9.9e3	1.3e4	1.1e4

Δf	**f_7 in 5-D**, N=15, mFE=78650					**f_7 in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.3e2	1.3e2	3.4e2	2.3e2	0	*18e+0*	*15e+0*	*22e+0*	1.3e5
1	15	4.1e3	2.6e3	5.6e3	4.1e3	.	.	.	.	.
1e−1	6	7.1e4	4.7e4	1.3e5	2.7e4	.	.	.	.	.
1e−3	0	*32e-2*	*21e-3*	*47e-2*	7.9e3	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_8 in 5-D**, N=15, mFE=1485					**f_8 in 20-D**, N=15, mFE=10852				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	7.3e1	5.9e1	8.9e1	7.3e1	15	2.0e3	1.9e3	2.2e3	2.0e3
1	15	3.0e2	2.2e2	4.0e2	3.0e2	15	3.9e3	3.2e3	4.6e3	3.9e3
1e−1	15	3.9e2	3.2e2	4.8e2	3.9e2	15	4.0e3	3.3e3	4.8e3	4.0e3
1e−3	15	4.5e2	3.8e2	5.4e2	4.5e2	15	4.2e3	3.5e3	4.9e3	4.2e3
1e−5	15	4.9e2	4.1e2	5.7e2	4.9e2	15	4.4e3	3.7e3	5.1e3	4.4e3
1e−8	15	5.2e2	4.4e2	6.0e2	5.2e2	15	4.5e3	3.8e3	5.3e3	4.5e3

Δf	**f_9 in 5-D**, N=15, mFE=1843					**f_9 in 20-D**, N=15, mFE=10808				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	6.3e1	5.7e1	6.9e1	6.3e1	15	1.8e3	1.7e3	1.8e3	1.8e3
1	15	4.6e2	3.3e2	5.9e2	4.6e2	15	3.1e3	2.5e3	3.8e3	3.1e3
1e−1	15	5.3e2	4.1e2	6.5e2	5.3e2	15	3.3e3	2.7e3	4.0e3	3.3e3
1e−3	15	5.8e2	4.7e2	7.1e2	5.8e2	15	3.5e3	2.8e3	4.2e3	3.5e3
1e−5	15	6.3e2	5.0e2	7.4e2	6.3e2	15	3.6e3	2.9e3	4.3e3	3.6e3
1e−8	15	6.5e2	5.3e2	7.8e2	6.5e2	15	3.8e3	3.2e3	4.5e3	3.8e3

Δf	**f_{10} in 5-D**, N=15, mFE=76895					**f_{10} in 20-D**, N=15, mFE=773382				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.1e3	7.3e2	1.5e3	1.1e3	15	1.3e4	1.2e4	1.4e4	1.3e4
1	15	2.7e3	2.0e3	3.5e3	2.7e3	15	2.3e4	2.1e4	2.5e4	2.3e4
1e−1	15	4.6e3	3.7e3	5.6e3	4.6e3	15	3.6e4	3.3e4	3.9e4	3.6e4
1e−3	15	9.0e3	7.7e3	1.0e4	9.0e3	15	6.0e4	5.6e4	6.3e4	6.0e4
1e−5	15	1.3e4	1.2e4	1.5e4	1.3e4	15	8.1e4	7.7e4	8.4e4	8.1e4
1e−8	15	3.0e4	2.4e4	3.7e4	3.0e4	15	2.3e5	1.7e5	3.0e5	2.3e5

Δf	**f_{11} in 5-D**, N=15, mFE=8585					**f_{11} in 20-D**, N=15, mFE=131357				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	5.0e2	4.4e2	5.6e2	5.0e2	15	1.5e4	1.4e4	1.5e4	1.5e4
1	15	9.5e2	8.3e2	1.1e3	9.5e2	15	2.9e4	2.8e4	3.0e4	2.9e4
1e−1	15	1.4e3	1.3e3	1.5e3	1.4e3	15	3.6e4	3.5e4	3.7e4	3.6e4
1e−3	15	2.1e3	2.0e3	2.2e3	2.1e3	15	6.0e4	5.9e4	6.1e4	6.0e4
1e−5	15	2.9e3	2.8e3	3.0e3	2.9e3	15	8.1e4	8.0e4	8.2e4	8.1e4
1e−8	15	4.7e3	4.2e3	5.2e3	4.7e3	15	1.1e5	1.1e5	1.1e5	1.1e5

Δf	**f_{12} in 5-D**, N=15, mFE=4396					**f_{12} in 20-D**, N=15, mFE=28383				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.7e2	2.6e2	5.1e2	3.7e2	15	3.1e3	2.2e3	4.1e3	3.1e3
1	15	6.9e2	5.0e2	8.9e2	6.9e2	15	5.9e3	4.4e3	7.3e3	5.9e3
1e−1	15	9.2e2	6.9e2	1.2e3	9.2e2	15	8.3e3	6.9e3	9.7e3	8.3e3
1e−3	15	1.2e3	9.2e2	1.5e3	1.2e3	15	1.0e4	9.1e3	1.2e4	1.0e4
1e−5	15	1.5e3	1.1e3	1.8e3	1.5e3	15	1.2e4	1.1e4	1.4e4	1.2e4
1e−8	15	1.8e3	1.4e3	2.2e3	1.8e3	15	1.5e4	1.3e4	1.6e4	1.5e4

Δf	**f_{13} in 5-D**, N=15, mFE=42403					**f_{13} in 20-D**, N=15, mFE=186688				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.2e2	2.8e2	5.6e2	4.2e2	15	6.5e2	2.9e2	1.0e3	6.5e2
1	15	1.8e3	1.3e3	2.4e3	1.8e3	15	6.2e3	3.8e3	8.6e3	6.2e3
1e−1	15	8.7e3	6.3e3	1.1e4	8.7e3	15	2.6e4	1.8e4	3.4e4	2.6e4
1e−3	7	7.0e4	5.0e4	1.1e5	2.9e4	6	3.5e5	2.3e5	6.0e5	1.3e5
1e−5	1	5.9e5	2.9e5	>6e5	4.0e4	0	*43e-4*	*23e-5*	*21e-3*	8.9e4
1e−8	0	*17e-4*	*18e-6*	*51e-4*	2.0e4	.	.	.	.	.

Δf	**f_{14} in 5-D**, N=15, mFE=500000					**f_{14} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.6e1	1.5e1	1.8e1	1.6e1	15	1.1e2	9.8e1	1.3e2	1.1e2
1	15	4.1e1	3.7e1	4.4e1	4.1e1	15	2.4e2	2.1e2	2.6e2	2.4e2
1e−1	15	5.8e1	5.4e1	6.2e1	5.8e1	15	3.0e2	2.8e2	3.3e2	3.0e2
1e−3	15	1.7e2	1.6e2	1.8e2	1.7e2	15	9.3e2	8.9e2	9.7e2	9.3e2
1e−5	15	1.4e3	1.3e3	1.5e3	1.4e3	15	1.5e4	1.4e4	1.5e4	1.5e4
1e−8	0	*12e-8*	*74e-9*	*17e-8*	2.0e5	0	*40e-9*	*31e-9*	*98e-9*	1.4e6

Δf	**f_{15} in 5-D**, N=15, mFE=25959					**f_{15} in 20-D**, N=15, mFE=134552				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.9e3	2.2e3	3.7e3	2.9e3	0	*12e+1*	*11e+1*	*15e+1*	5.0e4
1	1	3.8e5	1.9e5	>4e5	2.5e4	.	.	.	.	.
1e−1	0	*30e-1*	*20e-1*	*40e-1*	8.9e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{16} in 5-D**, N=15, mFE=36861					**f_{16} in 20-D**, N=15, mFE=233591				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	2.6e2	1.2e2	4.0e2	2.6e2	15	2.2e4	1.4e4	3.1e4	2.2e4
1	14	1.8e4	1.3e4	2.3e4	1.5e4	0	*53e-1*	*40e-1*	*63e-1*	1.0e5
1e−1	0	*50e-2*	*23e-2*	*81e-2*	2.0e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{17} in 5-D**, N=15, mFE=76530					**f_{17} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.2e1	9.9e0	1.4e1	1.2e1	15	1.0e3	1.3e2	2.0e3	1.0e3
1	14	8.6e3	5.3e3	1.3e4	6.5e3	0	*38e-1*	*30e-1*	*46e-1*	7.9e5
1e−1	1	5.6e5	2.4e5	>6e5	2.8e4	.	.	.	.	.
1e−3	0	*32e-2*	*10e-2*	*82e-2*	2.0e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{18} in 5-D**, N=15, mFE=500000					**f_{18} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	3.3e3	1.2e3	5.4e3	3.3e3	3	7.4e6	4.4e6	2.2e7	1.6e6
1	4	5.1e5	2.7e5	1.2e6	1.2e5	0	*11e+0*	*93e-1*	*16e+0*	7.1e5
1e−1	0	*11e-1*	*63e-2*	*19e-1*	1.8e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{19} in 5-D**, N=15, mFE=500000					**f_{19} in 20-D**, N=15, mFE=2.00e6				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.4e1	1.3e1	1.5e1	1.4e1	15	7.6e1	5.9e1	9.6e1	7.6e1
1	15	2.7e4	8.4e2	5.3e4	2.7e4	5	4.3e6	2.9e6	7.6e6	1.8e6
1e−1	11	3.4e5	2.7e5	4.2e5	3.3e5	0	*12e-1*	*53e-2*	*41e-1*	2.0e6
1e−3	0	*79e-3*	*35e-3*	*42e-2*	1.6e5	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{20} in 5-D**, N=15, mFE=33639					**f_{20} in 20-D**, N=15, mFE=409596				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.6e1	1.5e1	1.7e1	1.6e1	15	8.2e1	7.4e1	9.0e1	8.2e1
1	15	2.8e3	2.1e3	3.5e3	2.8e3	6	7.1e5	4.6e5	1.2e6	2.2e5
1e−1	0	*43e-2*	*24e-2*	*71e-2*	1.4e4	0	*10e-1*	*88e-2*	*11e-1*	1.6e5
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{21} in 5-D**, N=15, mFE=13857					**f_{21} in 20-D**, N=15, mFE=73635				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.6e1	2.7e1	6.7e1	4.6e1	15	9.8e2	5.9e2	1.4e3	9.8e2
1	15	2.5e3	1.4e3	3.7e3	2.5e3	15	1.4e4	8.5e3	2.0e4	1.4e4
1e−1	15	3.0e3	1.9e3	4.2e3	3.0e3	15	1.7e4	1.0e4	2.3e4	1.7e4
1e−3	15	3.1e3	1.9e3	4.2e3	3.1e3	15	1.7e4	1.1e4	2.4e4	1.7e4
1e−5	15	3.2e3	2.0e3	4.4e3	3.2e3	15	1.7e4	1.1e4	2.4e4	1.7e4
1e−8	15	3.3e3	2.2e3	4.5e3	3.3e3	15	1.8e4	1.1e4	2.4e4	1.8e4

Δf	**f_{22} in 5-D**, N=15, mFE=11796					**f_{22} in 20-D**, N=15, mFE=153554				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.5e2	9.3e1	2.0e2	1.5e2	15	4.7e2	1.9e2	7.7e2	4.7e2
1	15	8.3e2	3.8e2	1.3e3	8.3e2	14	2.7e4	1.6e4	4.2e4	2.0e4
1e−1	15	1.9e3	1.0e3	2.9e3	1.9e3	7	1.6e5	1.1e5	2.5e5	7.4e4
1e−3	15	2.1e3	1.3e3	3.1e3	2.1e3	7	1.6e5	1.1e5	2.6e5	7.4e4
1e−5	15	2.4e3	1.5e3	3.3e3	2.4e3	7	1.6e5	1.1e5	2.7e5	7.5e4
1e−8	15	2.8e3	1.9e3	3.7e3	2.8e3	7	1.6e5	1.1e5	2.7e5	7.5e4

Δf	**f_{23} in 5-D**, N=15, mFE=36919					**f_{23} in 20-D**, N=15, mFE=158817				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	1.9e1	1.6e1	2.1e1	1.9e1	15	3.7e1	2.9e1	4.6e1	3.7e1
1	15	1.3e3	8.3e2	1.7e3	1.3e3	15	5.6e3	4.3e3	7.0e3	5.6e3
1e−1	4	1.0e5	6.5e4	2.2e5	2.8e4	1	2.2e6	1.0e6	>2e6	1.6e5
1e−3	0	*20e-2*	*26e-3*	*30e-2*	1.3e4	0	*39e-2*	*21e-2*	*45e-2*	5.6e4
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{24} in 5-D**, N=15, mFE=30729					**f_{24} in 20-D**, N=15, mFE=168674				
	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$	#	ERT	10%	90%	$\mathrm{RT}_{\mathrm{succ}}$
10	15	4.6e3	3.1e3	6.4e3	4.6e3	0	*83e+0*	*74e+0*	*94e+0*	7.1e4
1	1	4.5e5	2.2e5	>4e5	3.1e4	.	.	.	.	.
1e−1	0	*26e-1*	*14e-1*	*51e-1*	1.0e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: NEWUOA, $2n+1$ interpolation points. Shown are, for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}} + \Delta f$ (ERT, see Figure 2); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathrm{RT}_{\mathrm{succ}}$). If $f_{\mathrm{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 2 for the names of functions.

Δf	**f_1 in 5-D**, N=15, mFE=22					**f_1 in 20-D**, N=15, mFE=236				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.1e1	2.0e1	2.2e1	2.1e1	15	2.3e2	2.3e2	2.3e2	2.3e2
1	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2
1e−1	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.4e2	2.3e2
1e−3	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2
1e−5	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2
1e−8	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.3e2	2.3e2	2.3e2	2.3e2

Δf	**f_2 in 5-D**, N=15, mFE=50000					**f_2 in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	5.8e2	4.2e2	6.9e2	5.8e2	12	1.7e5	1.6e5	1.9e5	1.3e5
1	15	1.6e3	1.3e3	2.0e3	1.6e3	2	1.5e6	1.4e6	1.5e6	1.7e5
1e−1	15	3.2e3	2.6e3	3.8e3	3.2e3	0	*47e-1*	*58e-2*	*22e+0*	2.0e5
1e−3	15	6.2e3	5.8e3	6.7e3	6.2e3	.	.	.	.	.
1e−5	15	9.5e3	9.3e3	1.0e4	9.5e3	.	.	.	.	.
1e−8	14	2.3e4	2.1e4	2.8e4	2.2e4	.	.	.	.	.

Δf	**f_3 in 5-D**, N=15, mFE=37504					**f_3 in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.0e3	2.4e3	3.6e3	3.0e3	0	*88e+0*	*71e+0*	*11e+1*	8.9e4
1	2	2.7e5	2.6e5	2.8e5	3.4e4	.	.	.	.	.
1e−1	0	*20e-1*	*99e-2*	*30e-1*	1.8e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_4 in 5-D**, N=15, mFE=50000					**f_4 in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.0e4	7.9e3	1.2e4	1.0e4	0	*13e+1*	*10e+1*	*16e+1*	1.0e5
1	0	*30e-1*	*20e-1*	*60e-1*	2.2e4	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_5 in 5-D**, N=15, mFE=32					**f_5 in 20-D**, N=15, mFE=313				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.2e1	2.1e1	2.3e1	2.2e1	15	2.5e2	2.5e2	2.6e2	2.5e2
1	15	2.4e1	2.4e1	2.5e1	2.4e1	15	2.6e2	2.5e2	2.7e2	2.6e2
1e−1	15	2.4e1	2.3e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.8e2	2.7e2
1e−3	15	2.4e1	2.3e1	2.5e1	2.4e1	15	2.7e2	2.7e2	2.7e2	2.7e2
1e−5	15	2.4e1	2.3e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.8e2	2.7e2
1e−8	15	2.4e1	2.4e1	2.5e1	2.4e1	15	2.7e2	2.6e2	2.7e2	2.7e2

Δf	**f_6 in 5-D**, N=15, mFE=8034					**f_6 in 20-D**, N=15, mFE=11418				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.3e2	1.1e2	1.7e2	1.3e2	15	1.9e3	1.8e3	1.9e3	1.9e3
1	15	2.1e2	2.0e2	3.0e2	2.1e2	15	2.5e3	2.4e3	2.6e3	2.5e3
1e−1	15	2.8e2	2.2e2	3.2e2	2.8e2	15	3.5e3	3.3e3	3.7e3	3.5e3
1e−3	15	5.8e2	4.7e2	6.7e2	5.8e2	15	5.2e3	5.0e3	5.4e3	5.2e3
1e−5	15	1.0e3	7.2e2	1.3e3	1.0e3	15	6.7e3	6.5e3	6.8e3	6.7e3
1e−8	15	2.2e3	1.5e3	3.3e3	2.2e3	15	9.4e3	9.1e3	9.6e3	9.4e3

Δf	**f_7 in 5-D**, N=15, mFE=43917					**f_7 in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.4e1	2.3e1	2.4e1	2.4e1	15	6.3e3	5.1e3	7.9e3	6.3e3
1	15	3.7e2	3.0e2	5.0e2	3.7e2	1	3.0e6	3.0e6	3.0e6	2.0e5
1e−1	15	4.7e3	2.9e3	6.2e3	4.7e3	0	*27e-1*	*15e-1*	*33e-1*	7.1e4
1e−3	11	3.7e4	3.3e4	4.3e4	2.7e4	.	.	.	.	.
1e−5	11	3.7e4	3.4e4	4.2e4	2.7e4	.	.	.	.	.
1e−8	11	3.7e4	3.1e4	4.2e4	2.7e4	.	.	.	.	.

Δf	**f_8 in 5-D**, N=15, mFE=1514					**f_8 in 20-D**, N=15, mFE=15124				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.2e2	8.6e1	1.3e2	1.2e2	15	2.8e3	2.7e3	3.0e3	2.8e3
1	15	2.7e2	1.9e2	4.6e2	2.7e2	15	6.0e3	4.3e3	7.0e3	6.0e3
1e−1	15	3.4e2	2.5e2	5.1e2	3.4e2	15	6.7e3	5.2e3	8.4e3	6.7e3
1e−3	15	3.9e2	2.7e2	4.6e2	3.9e2	15	7.2e3	6.1e3	8.5e3	7.2e3
1e−5	15	4.1e2	3.1e2	4.9e2	4.1e2	15	7.4e3	6.1e3	8.9e3	7.4e3
1e−8	15	4.2e2	3.3e2	5.8e2	4.2e2	15	7.5e3	6.1e3	8.6e3	7.5e3

Δf	**f_9 in 5-D**, N=15, mFE=1278					**f_9 in 20-D**, N=15, mFE=15428				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	9.1e1	7.2e1	1.0e2	9.1e1	15	3.2e3	2.9e3	3.5e3	3.2e3
1	15	3.4e2	2.8e2	4.0e2	3.4e2	15	6.9e3	6.1e3	9.0e3	6.9e3
1e−1	15	4.1e2	3.4e2	5.3e2	4.1e2	15	7.6e3	7.0e3	9.3e3	7.6e3
1e−3	15	4.6e2	3.4e2	4.9e2	4.6e2	15	8.1e3	6.9e3	9.0e3	8.1e3
1e−5	15	4.8e2	3.7e2	5.5e2	4.8e2	15	8.3e3	7.4e3	9.3e3	8.3e3
1e−8	15	4.9e2	4.4e2	5.6e2	4.9e2	15	8.4e3	7.1e3	9.9e3	8.4e3

Δf	**f_{10} in 5-D**, N=15, mFE=50000					**f_{10} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.2e3	1.0e3	1.5e3	1.2e3	10	2.5e5	2.4e5	2.7e5	1.6e5
1	15	3.1e3	2.8e3	3.3e3	3.1e3	0	*67e-1*	*38e-1*	*60e+0*	2.0e5
1e−1	15	5.0e3	4.7e3	5.4e3	5.0e3	.	.	.	.	.
1e−3	15	8.2e3	7.8e3	8.8e3	8.2e3	.	.	.	.	.
1e−5	15	1.1e4	1.1e4	1.2e4	1.1e4	.	.	.	.	.
1e−8	12	3.5e4	2.7e4	3.9e4	2.6e4	.	.	.	.	.

Δf	**f_{11} in 5-D**, N=15, mFE=50000					**f_{11} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.5e3	1.3e3	1.6e3	1.5e3	15	5.8e4	5.5e4	6.6e4	5.8e4
1	15	2.9e3	2.6e3	3.0e3	2.9e3	15	1.0e5	9.3e4	1.1e5	1.0e5
1e−1	15	4.2e3	3.8e3	4.6e3	4.2e3	15	1.3e5	1.3e5	1.4e5	1.3e5
1e−3	15	6.3e3	6.0e3	6.7e3	6.3e3	8	3.5e5	3.5e5	3.6e5	1.8e5
1e−5	15	8.6e3	7.9e3	8.9e3	8.6e3	0	*75e-5*	*32e-6*	*36e-4*	2.0e5
1e−8	13	2.2e4	1.8e4	2.9e4	1.8e4	.	.	.	.	.

Δf	**f_{12} in 5-D**, N=15, mFE=6362					**f_{12} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.0e2	2.0e2	7.9e2	4.0e2	15	1.1e4	8.2e3	1.5e4	1.1e4
1	15	7.0e2	4.6e2	9.4e2	7.0e2	15	2.9e4	2.5e4	3.7e4	2.9e4
1e−1	15	1.0e3	6.2e2	1.3e3	1.0e3	14	7.2e4	5.5e4	8.8e4	6.9e4
1e−3	15	1.5e3	1.3e3	2.1e3	1.5e3	12	1.6e5	1.4e5	1.9e5	1.3e5
1e−5	15	2.1e3	1.7e3	2.7e3	2.1e3	6	4.4e5	4.2e5	4.7e5	1.8e5
1e−8	15	2.8e3	2.0e3	3.4e3	2.8e3	0	*42e-6*	*19e-9*	*19e-4*	1.4e5

Δf	**f_{13} in 5-D**, N=15, mFE=50000					**f_{13} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.4e2	1.3e2	3.3e2	2.4e2	15	1.2e3	1.1e3	1.2e3	1.2e3
1	15	1.3e3	7.7e2	1.8e3	1.3e3	15	1.2e4	9.1e3	1.6e4	1.2e4
1e−1	15	5.6e3	4.0e3	8.7e3	5.6e3	15	5.1e4	4.1e4	7.1e4	5.1e4
1e−3	5	1.3e5	1.1e5	1.4e5	4.7e4	5	4.8e5	4.1e5	5.2e5	2.0e5
1e−5	0	*19e-4*	*16e-5*	*11e-3*	2.5e4	1	2.8e6	2.7e6	3.0e6	2.0e5
1e−8	.	.	.	.	.	0	*30e-4*	*68e-6*	*31e-3*	1.4e5

Δf	**f_{14} in 5-D**, N=15, mFE=50000					**f_{14} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.7e1	2.5e1	2.9e1	2.7e1	15	4.4e2	4.1e2	4.6e2	4.4e2
1	15	4.3e1	4.0e1	4.6e1	4.3e1	15	7.2e2	6.9e2	7.5e2	7.2e2
1e−1	15	6.5e1	6.3e1	6.7e1	6.5e1	15	1.1e3	1.1e3	1.1e3	1.1e3
1e−3	15	1.4e2	1.3e2	1.5e2	1.4e2	15	2.4e3	2.3e3	2.5e3	2.4e3
1e−5	15	8.1e2	7.5e2	8.9e2	8.1e2	15	3.2e4	3.2e4	3.3e4	3.2e4
1e−8	0	*48e-9*	*38e-9*	*59e-9*	3.2e4	0	*12e-7*	*11e-7*	*12e-7*	2.0e5

Δf	**f_{15} in 5-D**, N=15, mFE=35310					**f_{15} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.2e3	2.1e3	4.0e3	3.2e3	0	*77e+0*	*66e+0*	*11e+1*	1.4e5
1	1	5.1e5	5.0e5	5.2e5	3.4e4	.	.	.	.	.
1e−1	0	*50e-1*	*20e-1*	*70e-1*	1.1e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{16} in 5-D**, N=15, mFE=48848					**f_{16} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.2e2	1.8e2	3.8e2	3.2e2	15	6.4e3	2.7e3	8.4e3	6.4e3
1	15	7.1e3	5.6e3	9.1e3	7.1e3	1	2.9e6	2.8e6	3.0e6	2.0e5
1e−1	7	7.7e4	6.4e4	8.9e4	3.5e4	0	*25e-1*	*12e-1*	*44e-1*	3.2e4
1e−3	0	*12e-2*	*35e-4*	*62e-2*	2.5e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{17} in 5-D**, N=15, mFE=50000					**f_{17} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.5e1	2.3e1	2.9e1	2.5e1	15	8.0e2	5.7e2	1.2e3	8.0e2
1	15	5.4e3	3.0e3	7.0e3	5.4e3	0	*37e-1*	*26e-1*	*48e-1*	8.9e4
1e−1	7	6.8e4	5.3e4	8.0e4	3.0e4	.	.	.	.	.
1e−3	0	*13e-2*	*18e-3*	*53e-2*	7.1e3	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{18} in 5-D**, N=15, mFE=50000					**f_{18} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.1e3	4.9e2	2.0e3	1.1e3	4	5.9e5	4.9e5	6.5e5	1.1e5
1	11	3.2e4	1.4e4	4.0e4	2.4e4	0	*12e+0*	*61e-1*	*18e+0*	2.8e4
1e−1	2	3.6e5	3.4e5	3.8e5	3.4e4	.	.	.	.	.
1e−3	0	*78e-2*	*80e-3*	*54e-1*	1.8e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{19} in 5-D**, N=15, mFE=50000					**f_{19} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.1e1	2.8e1	3.8e1	3.1e1	15	4.8e2	4.0e2	5.9e2	4.8e2
1	14	1.1e4	5.8e3	1.7e4	1.0e4	0	*21e-1*	*11e-1*	*33e-1*	1.8e5
1e−1	3	2.1e5	1.6e5	2.5e5	3.5e4	.	.	.	.	.
1e−3	0	*19e-2*	*35e-3*	*84e-2*	1.4e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{20} in 5-D**, N=15, mFE=33205					**f_{20} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	2.2e1	2.2e1	2.2e1	2.2e1	15	2.5e2	2.5e2	2.7e2	2.5e2
1	15	5.4e3	3.7e3	6.4e3	5.4e3	1	2.9e6	2.9e6	3.0e6	2.0e5
1e−1	0	*47e-2*	*24e-2*	*71e-2*	1.4e4	0	*12e-1*	*10e-1*	*13e-1*	1.4e5
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{21} in 5-D**, N=15, mFE=25651					**f_{21} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	9.8e1	3.3e1	1.4e2	9.8e1	15	4.1e3	2.3e3	5.8e3	4.1e3
1	15	2.7e3	1.4e3	4.2e3	2.7e3	15	2.3e4	1.5e4	2.4e4	2.3e4
1e−1	15	4.7e3	2.0e3	8.4e3	4.7e3	14	6.4e4	3.7e4	8.5e4	6.2e4
1e−3	15	4.7e3	2.5e3	6.6e3	4.7e3	14	6.4e4	4.1e4	8.3e4	6.2e4
1e−5	15	4.7e3	2.0e3	7.8e3	4.7e3	14	6.4e4	4.6e4	1.1e5	6.3e4
1e−8	15	4.8e3	2.9e3	5.5e3	4.8e3	14	6.5e4	3.4e4	9.1e4	6.3e4

Δf	**f_{22} in 5-D**, N=15, mFE=12614					**f_{22} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	3.1e2	1.2e2	6.2e2	3.1e2	15	1.0e3	6.4e2	1.3e3	1.0e3
1	15	1.4e3	7.2e2	1.5e3	1.4e3	15	6.8e4	4.9e4	1.0e5	6.8e4
1e−1	15	2.8e3	1.9e3	3.5e3	2.8e3	2	1.4e6	1.2e6	1.5e6	2.0e5
1e−3	15	3.0e3	1.4e3	4.6e3	3.0e3	2	1.4e6	1.3e6	1.5e6	2.0e5
1e−5	15	3.1e3	2.1e3	4.5e3	3.1e3	2	1.4e6	1.3e6	1.5e6	2.0e5
1e−8	15	3.4e3	1.9e3	4.3e3	3.4e3	2	1.4e6	1.3e6	1.5e6	2.0e5

Δf	**f_{23} in 5-D**, N=15, mFE=50000					**f_{23} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	1.6e1	1.3e1	2.4e1	1.6e1	15	4.6e1	1.6e1	7.1e1	4.6e1
1	15	1.0e3	5.9e2	1.3e3	1.0e3	15	1.1e4	6.8e3	1.5e4	1.1e4
1e−1	9	5.4e4	4.9e4	6.1e4	3.8e4	1	3.0e6	3.0e6	3.0e6	2.0e5
1e−3	0	*87e-3*	*13e-3*	*16e-2*	2.5e4	0	*25e-2*	*12e-2*	*39e-2*	7.9e4
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	**f_{24} in 5-D**, N=15, mFE=34643					**f_{24} in 20-D**, N=15, mFE=200000				
	#	ERT	10%	90%	$\mathrm{RT_{succ}}$	#	ERT	10%	90%	$\mathrm{RT_{succ}}$
10	15	4.0e3	3.2e3	5.0e3	4.0e3	0	*71e+0*	*59e+0*	*80e+0*	1.0e5
1	2	2.5e5	2.4e5	2.6e5	3.4e4	.	.	.	.	.
1e−1	0	*31e-1*	*80e-2*	*68e-1*	2.0e4	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 2: NEWUOA, full model. Shown are, for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\mathrm{opt}} + \Delta f$ (ERT, see Figure 2); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value ($\mathbf{RT_{succ}}$). If $f_{\mathrm{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 2 for the names of functions.

Figure 1: Multistart NEWUOA, the number of interpolation points is two times the dimension plus one.

```
#include <stdlib.h>
#include <math.h>
#include <stdio.h>
#include "bbobStructures.h"

/* Call to the Fortran function */
extern void newuoa_(unsigned int* n, int* m, double* x0, double* rhobeg,
                    double* rhoend, int* verbose, int* maxfun,
                    double* W, double* ftarget);

/* The Multistart NEWUOA */
void newuoa(unsigned int dim, unsigned int maxfunevals, double ftarget)
{
    int m, iprint = 0, curmaxfun;
    double * x = malloc(sizeof(double) * dim);
    unsigned int iter = 0, i;
    double rhobeg = 10, rhoend = 1e-16;
    /* internal variable of NEWUOA */
    double * w = malloc(1000000 * sizeof(double));

    m = 2 * dim + 1;

    curmaxfun = maxfunevals - fgeneric_evaluations();
    while (curmaxfun > 0 && fgeneric_best() > ftarget && iter < 100)
    {
        /* Generate a starting point */
        for (i = 0; i < dim; i++)
             x[i] = 10. * ((double)rand() / RAND_MAX) - 5.;
        /* Call NEWUOA */
        newuoa_(&dim, &npt, x, &rhobeg, &rhoend, &iprint, &curmaxfun, w, &ftarget);
        /* Update */
        curmaxfun = maxfunevals - fgeneric_evaluations();
        iter++;
    }
    free(x);
    free(w);
}
```

points on the interpolation of the model. To the contrary, the performances only seem to scale only worse resulting in failures in larger dimensions, for instance on f_2 or f_{12} with the exception of f_7 which the full model NEWUOA solves in 5-D.

4. CPU TIMING EXPERIMENT

The proposed algorithm was run on f_8 and restarted until at least 30 seconds have passed. The experiments were conducted with an Intel Core 2 6700 processor (2.66GHz) on Linux 2.6.24.7. The results were 130, 73, 45, 18, 2.2, 26 for $m = 2n + 1$ and 200, 86, 45, 7.9, 3.7, 36 $\times 10^{-3}$ seconds per function evaluations for the full model in dimension 2, 3, 5, 10, 20 and 40 respectively.

Acknowledgments

The first author would like to acknowledge the support, help, and work of the BBOB team with particular kudos to Anne Auger, Steffen Finck and Nikolaus Hansen.

5. REFERENCES

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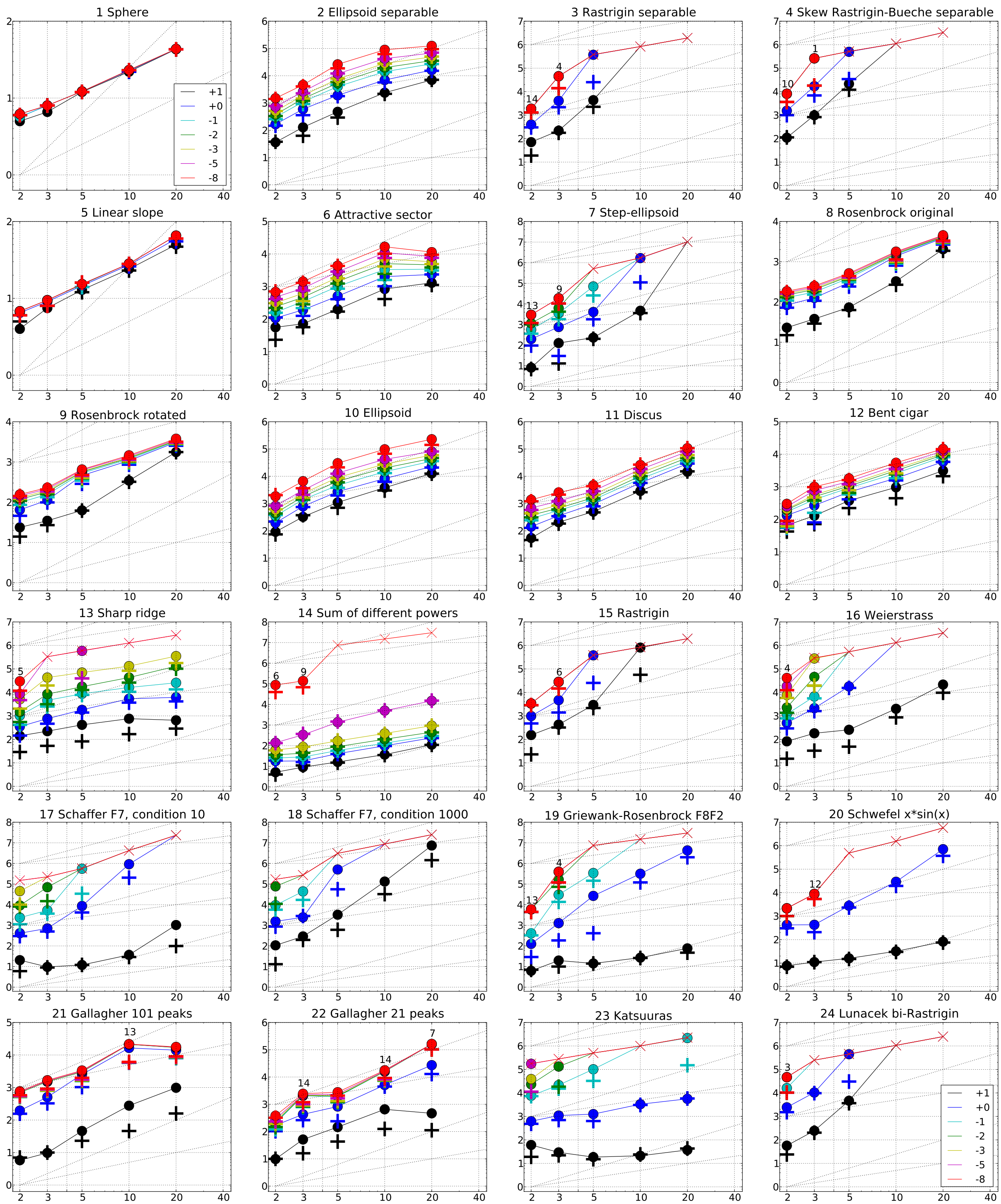

Figure 2: NEWUOA, $2n+1$ interpolation points. Expected Running Time (ERT, ●) to reach $f_{\mathrm{opt}}+\Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. The $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\mathrm{opt}}+\Delta f$ was surpassed during the trial. The $\#\mathrm{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\mathrm{opt}}+\Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses ($\times$) indicate the total number of function evaluations $\#\mathrm{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

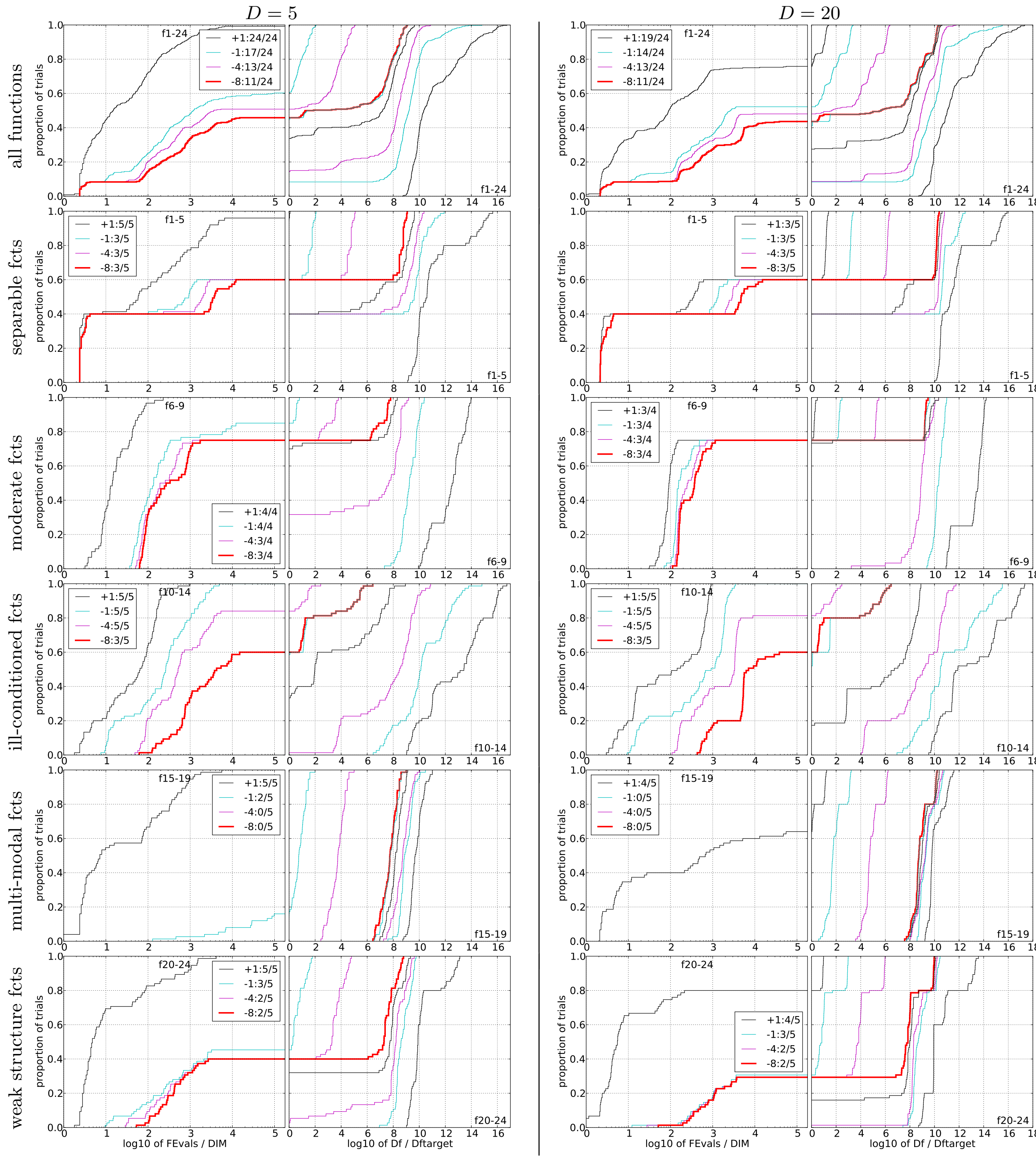

Figure 3: NEWUOA, $2n + 1$ interpolation points. Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\rm opt} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D\ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

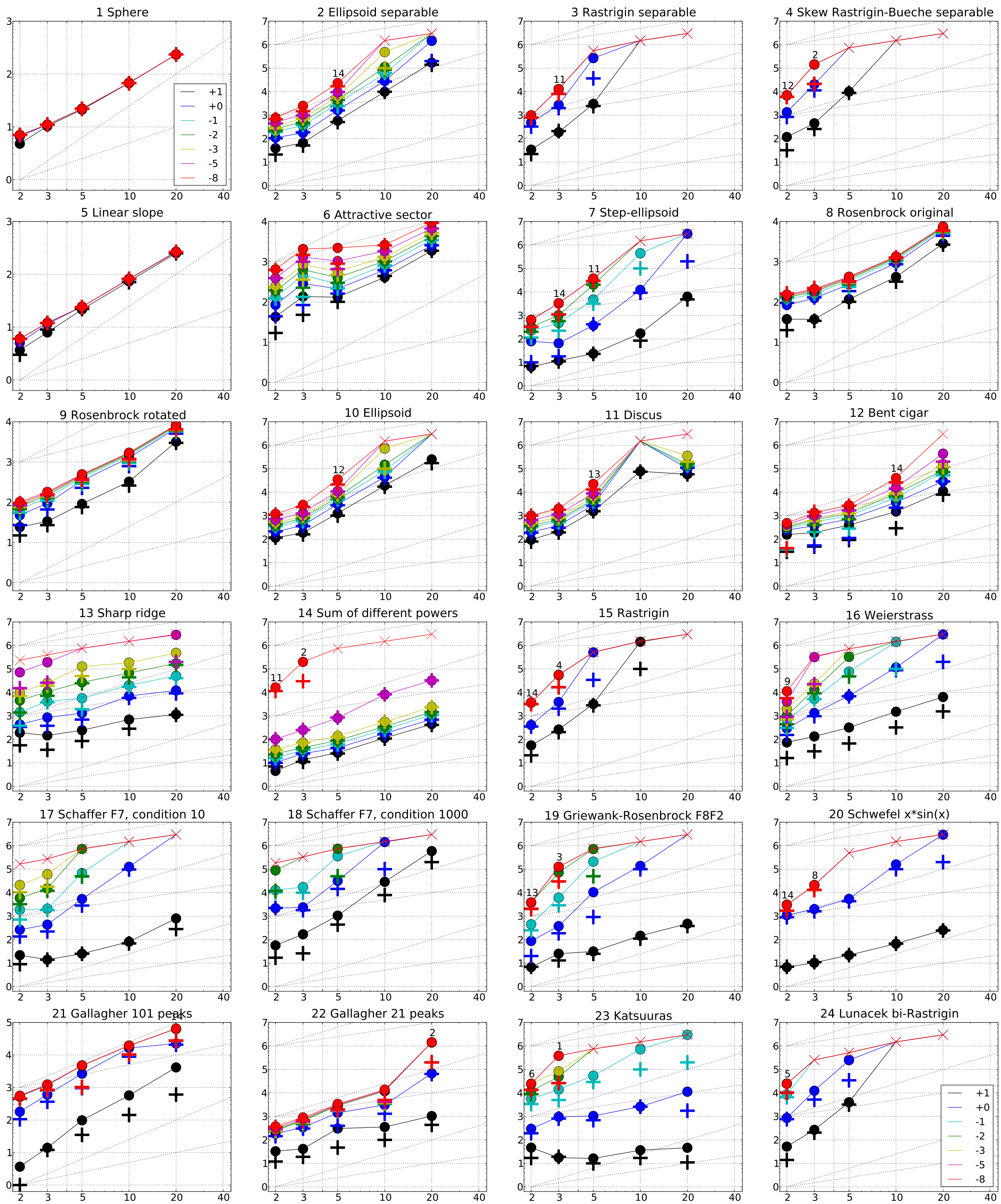

Figure 4: NEWUOA, full model. Expected Running Time (ERT, ●) to reach $f_{\mathrm{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_1 and f_{24}) versus dimension in log-log presentation. The $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\mathrm{opt}} + \Delta f$ was surpassed during the trial. The $\#\mathrm{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\mathrm{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses ($\times$) indicate the total number of function evaluations $\#\mathrm{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

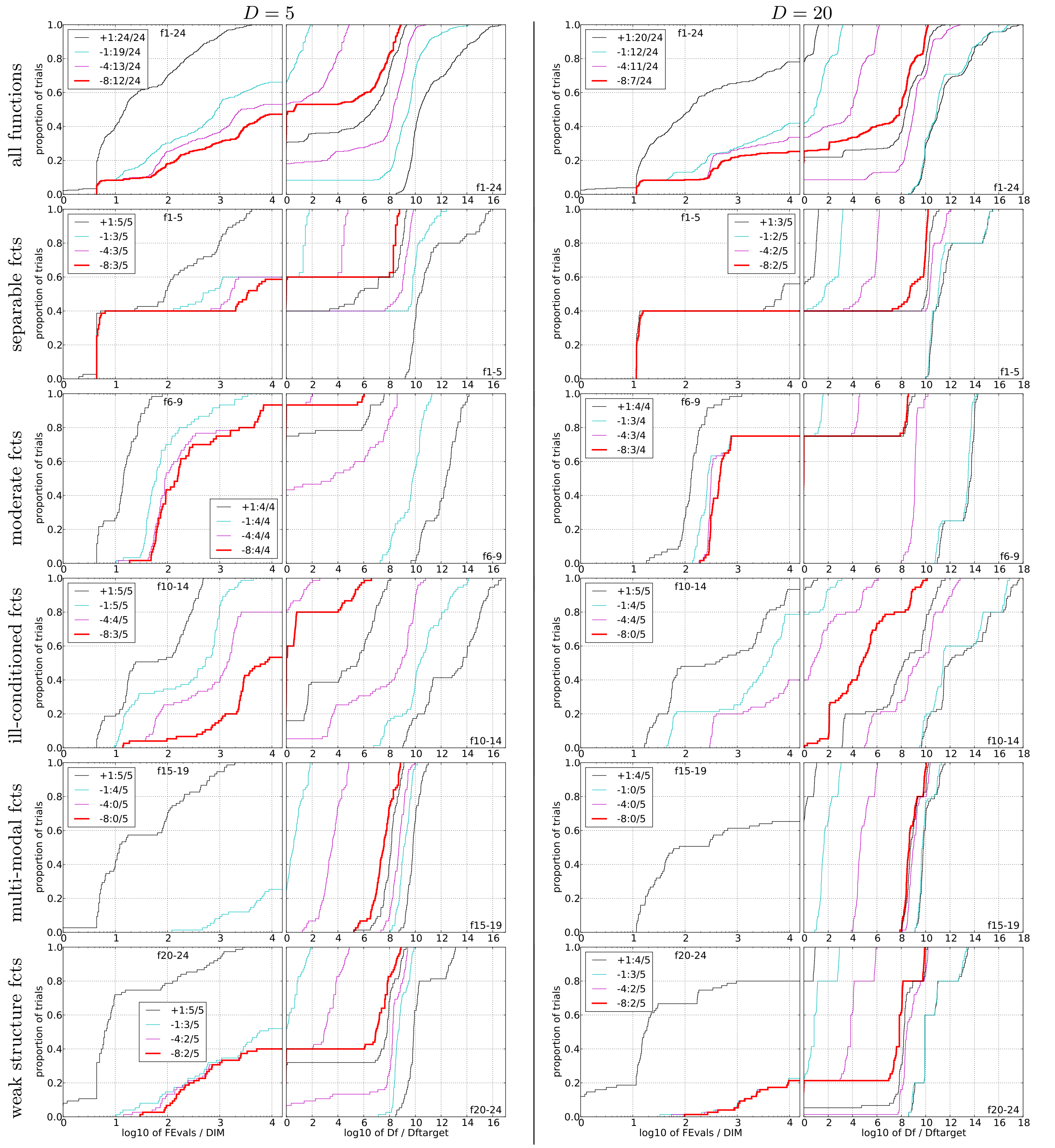

Figure 5: NEWUOA, full model. Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\text{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.