Benchmarking the BFGS Algorithm on the BBOB-2009 Noisy Testbed

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ABSTRACT

The BFGS quasi-Newton method is benchmarked on the noisy BBOB-2009 testbed. A multistart strategy is applied with a maximum number of function evaluations of about 10^4 times the search space dimension.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, BFGS, Quasi-newton

1. INTRODUCTION

The Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [1, 3, 4, 7] is a real-parameter unconstrained non-linear optimization method. The BFGS method belongs to the class of quasi-Newton methods which, by supposing that the objective function can be locally approximated by a quadratic function near its optimum, tries to approximate the Hessian matrix of this quadratic function. The BFGS method is tested here in its default setting on a testbed of noisy functions.

2. ALGORITHM PRESENTATION

Quasi-Newton methods address the problem of unconstrained black-box optimization by the determination of the stationary point of a function using a second-order approximation. The Hessian matrix is iteratively approximated by finding a search direction using the gradient of the current point and operating a line search to find the step size. For each iteration $D + 1$ function evaluations (where D denotes the problem dimension) or double if the step size changes, are done to evaluate the gradient and the function value at the current point. A simple stochastic independent restart procedure (as advised in [5]) was added.

3. EXPERIMENTAL PROCEDURE

The Matlab implementation of the BFGS method was used. It is accessible using the generic function `fminunc` (revision 1.1.6.3) that proposes, among others, the BFGS method for the update of the Hessian Matrix. The starting point is chosen uniformly in $[-5, 5]^D$. The stopping criteria were chosen such that a restart occurs due to numerical errors. The multistart strategy was used with at most 100 restarts to reduce the duration of an experiment. For the same reason, a run is limited to at most $10^4 \times D$ function evaluations. The entire experiment took roughly 7 hours. The algorithm used is presented in Figure 1. No parameter tuning was done, the CrE [5] is computed to zero.

4. RESULTS AND DISCUSSION

Results from experiments according to [5] on the benchmarks functions given in [2, 6] are presented in Figures 2 and 3 and in Tables 1 and 2. The proposed algorithm only solves 5, 3, 2, 2, 2 and 1 out of 30 functions in dimension 2, 3, 5, 10, 20 and 40 respectively. The functions solved use the Cauchy noise model only. The Rosenbrock function and Gallagher functions are solved in dimension 2. The sphere with the Cauchy noise is solved for all dimensions tested, though the algorithm scales comparatively worse than on the noiseless sphere. A more severe Cauchy noise model decreases the probability of success of the algorithm. No success was obtained on the multimodal functions with severe noise.

5. CPU TIMING EXPERIMENT

For the timing experiment, the proposed algorithm was run on f_8 and restarted until at least 30 seconds have passed (according to Figure 2 in [5]). The experiments were conducted with an Intel Core 2 6700 processor (2.66GHz) with Matlab R2008a on Linux 2.6.24.7. The results were 6.0, 4.7, 3.7, 3.0, 2.9, 2.9 and 2.8 $\times 10^{-4}$ seconds per function evaluations in dimension 2, 3, 5, 10, 20, 40 and 80 respectively.

6. CONCLUSION

The results of a quasi-Newton method algorithm with restarts were presented. They show the limits of a default Quasi-Newton method on noisy testbeds.

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Δf	f_{101} in 5-D, N=15, mFE=18156 #	ERT	10%	90%	RT_{succ}	f_{101} in 20-D, N=15, mFE=65163 #	ERT	10%	90%	RT_{succ}
10	12	8.2e3	5.7e3	1.0e4	5.7e3	0	*11e+1*	*85e+0*	*15e+1*	2.0e4
1	0	*70e−1*	*35e−1*	*11e+0*	7.9e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{102} in 5-D, N=15, mFE=18522 #	ERT	10%	90%	RT_{succ}	f_{102} in 20-D, N=15, mFE=58506 #	ERT	10%	90%	RT_{succ}
10	12	9.9e3	8.1e3	1.3e4	8.6e3	0	*12e+1*	*87e+0*	*15e+1*	2.5e4
1	0	*59e−1*	*28e−1*	*11e+0*	8.9e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{103} in 5-D, N=15, mFE=654 #	ERT	10%	90%	RT_{succ}	f_{103} in 20-D, N=15, mFE=11970 #	ERT	10%	90%	RT_{succ}
10	15	8.2e1	5.0e1	1.1e2	8.2e1	15	3.0e3	2.0e3	3.8e3	3.0e3
1	15	1.2e2	5.1e1	1.5e2	1.2e2	15	3.5e3	2.3e3	4.6e3	3.5e3
1e−1	15	1.2e2	4.3e1	1.9e2	1.2e2	15	3.7e3	2.5e3	4.3e3	3.7e3
1e−3	15	1.2e2	5.1e1	2.2e2	1.2e2	15	3.7e3	2.5e3	4.7e3	3.7e3
1e−5	15	1.2e2	4.2e1	1.4e2	1.2e2	15	3.7e3	2.9e3	4.6e3	3.7e3
1e−8	15	1.2e2	7.5e1	1.6e2	1.2e2	15	3.7e3	2.1e3	4.2e3	3.7e3

Δf	f_{104} in 5-D, N=15, mFE=8814 #	ERT	10%	90%	RT_{succ}	f_{104} in 20-D, N=15, mFE=22659 #	ERT	10%	90%	RT_{succ}
10	0	*61e+1*	*24e+1*	*22e+2*	4.0e3	0	*13e+4*	*85e+3*	*21e+4*	1.0e4
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{105} in 5-D, N=15, mFE=7668 #	ERT	10%	90%	RT_{succ}	f_{105} in 20-D, N=15, mFE=21462 #	ERT	10%	90%	RT_{succ}
10	0	*82e+1*	*39e+1*	*21e+2*	3.2e3	0	*15e+4*	*10e+4*	*22e+4*	1.0e4
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{106} in 5-D, N=15, mFE=26574 #	ERT	10%	90%	RT_{succ}	f_{106} in 20-D, N=15, mFE=85008 #	ERT	10%	90%	RT_{succ}
10	15	1.8e3	1.5e3	2.3e3	1.8e3	0	*91e+1*	*34e+1*	*23e+2*	3.2e4
1	9	2.9e4	2.5e4	3.3e4	1.7e4	.	.	.	.	.
1e−1	3	1.1e5	9.6e4	1.2e5	1.8e4	.	.	.	.	.
1e−3	0	*67e−2*	*20e−3*	*28e−1*	1.4e4	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{107} in 5-D, N=15, mFE=11610 #	ERT	10%	90%	RT_{succ}	f_{107} in 20-D, N=15, mFE=26712 #	ERT	10%	90%	RT_{succ}
10	12	5.9e3	5.2e3	7.2e3	3.4e3	0	*11e+1*	*89e+0*	*14e+1*	1.3e4
1	0	*61e−1*	*17e−1*	*10e+0*	5.6e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{108} in 5-D, N=15, mFE=4530 #	ERT	10%	90%	RT_{succ}	f_{108} in 20-D, N=15, mFE=16443 #	ERT	10%	90%	RT_{succ}
10	10	4.9e3	4.4e3	5.4e3	2.7e3	0	*11e+1*	*95e+0*	*14e+1*	5.0e3
1	0	*84e−1*	*44e−1*	*16e+0*	2.8e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{109} in 5-D, N=15, mFE=4422 #	ERT	10%	90%	RT_{succ}	f_{109} in 20-D, N=15, mFE=48111 #	ERT	10%	90%	RT_{succ}
10	15	4.3e2	3.9e2	5.3e2	4.3e2	3	2.0e5	2.0e5	2.2e5	2.9e4
1	15	7.2e2	5.3e2	1.0e3	7.2e2	2	3.1e5	2.7e5	3.4e5	2.7e4
1e−1	15	7.4e2	5.7e2	9.6e2	7.4e2	1	6.4e5	6.0e5	6.8e5	6.5e3
1e−3	15	9.5e2	6.5e2	1.2e3	9.5e2	1	6.4e5	6.1e5	6.8e5	6.5e3
1e−5	15	9.5e2	5.9e2	1.2e3	9.5e2	1	6.4e5	6.0e5	6.9e5	6.5e3
1e−8	15	9.5e2	4.9e2	1.1e3	9.5e2	1	6.4e5	5.6e5	6.9e5	6.5e3

Δf	f_{110} in 5-D, N=15, mFE=5352 #	ERT	10%	90%	RT_{succ}	f_{110} in 20-D, N=15, mFE=12390 #	ERT	10%	90%	RT_{succ}
10	0	*73e+1*	*39e+1*	*18e+2*	1.4e3	0	*14e+4*	*96e+3*	*22e+4*	5.0e3
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{111} in 5-D, N=15, mFE=2988 #	ERT	10%	90%	RT_{succ}	f_{111} in 20-D, N=15, mFE=9366 #	ERT	10%	90%	RT_{succ}
10	0	*10e+2*	*13e+1*	*33e+2*	1.8e3	0	*15e+4*	*88e+3*	*20e+4*	5.0e3
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{112} in 5-D, N=15, mFE=14580 #	ERT	10%	90%	RT_{succ}	f_{112} in 20-D, N=15, mFE=45444 #	ERT	10%	90%	RT_{succ}
10	2	9.8e4	9.3e4	1.0e5	1.3e4	0	*14e+4*	*30e+3*	*22e+4*	2.0e4
1	0	*92e+0*	*68e−1*	*45e+1*	5.6e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{113} in 5-D, N=15, mFE=7968 #	ERT	10%	90%	RT_{succ}	f_{113} in 20-D, N=15, mFE=22911 #	ERT	10%	90%	RT_{succ}
10	4	2.4e4	2.1e4	2.7e4	6.1e3	0	*58e+1*	*46e+1*	*85e+1*	8.9e3
1	0	*24e+0*	*54e−1*	*42e+0*	2.5e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{114} in 5-D, N=15, mFE=4074 #	ERT	10%	90%	RT_{succ}	f_{114} in 20-D, N=15, mFE=14406 #	ERT	10%	90%	RT_{succ}
10	0	*28e+0*	*12e+0*	*42e+0*	2.2e3	0	*67e+1*	*39e+1*	*10e+2*	4.5e3
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{115} in 5-D, N=15, mFE=10314 #	ERT	10%	90%	RT_{succ}	f_{115} in 20-D, N=15, mFE=46410 #	ERT	10%	90%	RT_{succ}
10	1	1.4e5	1.4e5	1.5e5	6.5e3	0	*62e+1*	*42e+1*	*11e+2*	2.5e4
1	0	*24e+0*	*11e+0*	*54e+0*	5.0e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{116} in 5-D, N=15, mFE=5106 #	ERT	10%	90%	RT_{succ}	f_{116} in 20-D, N=15, mFE=13944 #	ERT	10%	90%	RT_{succ}
10	0	*12e+2*	*45e+1*	*31e+2*	2.2e3	0	*55e+3*	*40e+3*	*71e+3*	7.9e3
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{117} in 5-D, N=15, mFE=2970 #	ERT	10%	90%	RT_{succ}	f_{117} in 20-D, N=15, mFE=10038 #	ERT	10%	90%	RT_{succ}
10	0	*91e+1*	*25e+1*	*26e+2*	1.6e3	0	*49e+3*	*26e+3*	*68e+3*	3.2e3
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{118} in 5-D, N=15, mFE=14562 #	ERT	10%	90%	RT_{succ}	f_{118} in 20-D, N=15, mFE=47691 #	ERT	10%	90%	RT_{succ}
10	0	*31e+1*	*69e+0*	*69e+1*	8.9e3	0	*46e+3*	*30e+3*	*78e+3*	1.8e4
1	.	.	.	.	.	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{119} in 5-D, N=15, mFE=13068 #	ERT	10%	90%	RT_{succ}	f_{119} in 20-D, N=15, mFE=28077 #	ERT	10%	90%	RT_{succ}
10	15	2.6e3	1.6e3	3.7e3	2.6e3	0	*43e+0*	*29e+0*	*50e+0*	1.3e4
1	2	8.2e4	7.9e4	8.6e4	1.2e4	.	.	.	.	.
1e−1	0	*50e−1*	*83e−2*	*76e−1*	7.9e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{120} in 5-D, N=15, mFE=4584 #	ERT	10%	90%	RT_{succ}	f_{120} in 20-D, N=15, mFE=16884 #	ERT	10%	90%	RT_{succ}
10	15	6.4e2	5.6e2	1.1e3	6.4e2	0	*34e+0*	*24e+0*	*44e+0*	1.3e4
1	0	*37e−1*	*14e−1*	*76e−1*	8.9e2	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for functions f_{101}-f_{120} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\text{opt}} + \Delta f$ (ERT, see Figure 2); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\text{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 2 for the names of functions.

Figure 1: Matlab code: Multistart BFGS

```
function [x, ilaunch] = MY_OPTIMIZER(FUN, D, ftarget, maxfunevals)
% minimizes FUN in D dimensions by independent restarts of fminunc (BFGS).
% ftarget and maxfunevals are additional external termination conditions.
% Search space is [-5, 5]^D

  % set options, make sure we always terminate
  options = optimset('fminunc');
  options = optimset(options, 'LargeScale', 'off'); % BFGS algorithm
  options = optimset(options, 'MaxIter', inf, 'Tolfun', 1e-11, 'TolX', 0, ...
                     'OutputFcn', @callback, 'Display', 'off');

  maxfunevals = min(1e4*DIM, maxfunevals);

  % multistart such that ftarget is reached with reasonable prob.
  for ilaunch = 1:100;  % relaunch optimizer up to 100 times
    options = optimset(options, 'MaxFunEvals', ...
                       maxfunevals - feval(FUN, 'evaluations'));
    x = fminunc(FUN, 10*rand(DIM,1)-5, options);
    if (feval(FUN, 'fbest') < ftarget || ...
        feval(FUN, 'evaluations') >= maxfunevals)
      break;
    end
  end

  function stop = callback(x, optimValues, state)
    stop = false;
    if optimValues.fval < ftarget
      stop = true;
    end
  end
end % function
```


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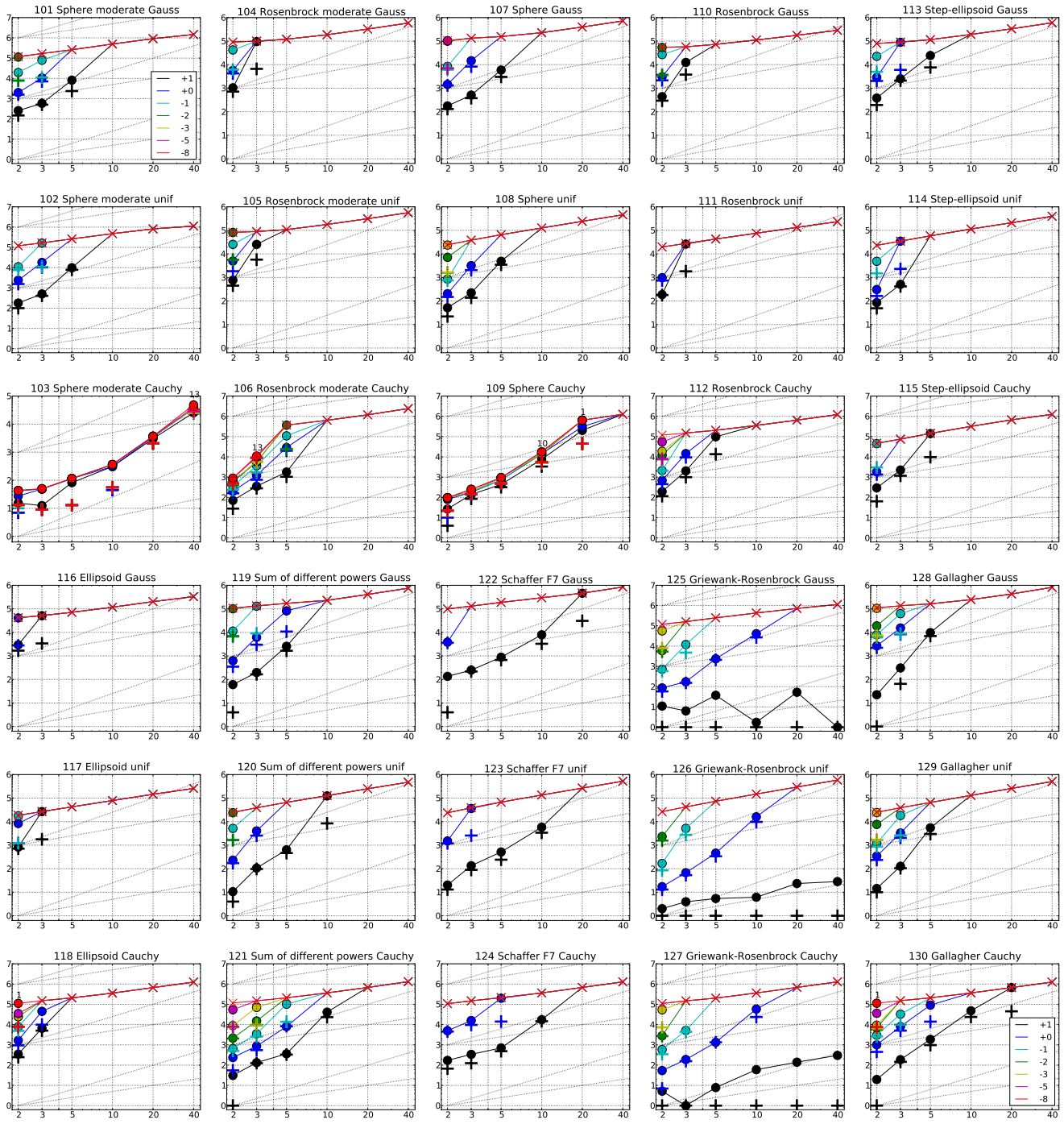

Figure 2: Expected Running Time (ERT, ●) to reach $f_{\mathrm{opt}} + \Delta f$ and median number of function evaluations of successful trials (+), shown for $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ (the exponent is given in the legend of f_{101} and f_{130}) versus dimension in log-log presentation. The $\mathrm{ERT}(\Delta f)$ equals to $\#\mathrm{FEs}(\Delta f)$ divided by the number of successful trials, where a trial is successful if $f_{\mathrm{opt}} + \Delta f$ was surpassed during the trial. The $\#\mathrm{FEs}(\Delta f)$ are the total number of function evaluations while $f_{\mathrm{opt}} + \Delta f$ was not surpassed during the trial from all respective trials (successful and unsuccessful), and f_{opt} denotes the optimal function value. Crosses (×) indicate the total number of function evaluations $\#\mathrm{FEs}(-\infty)$. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.

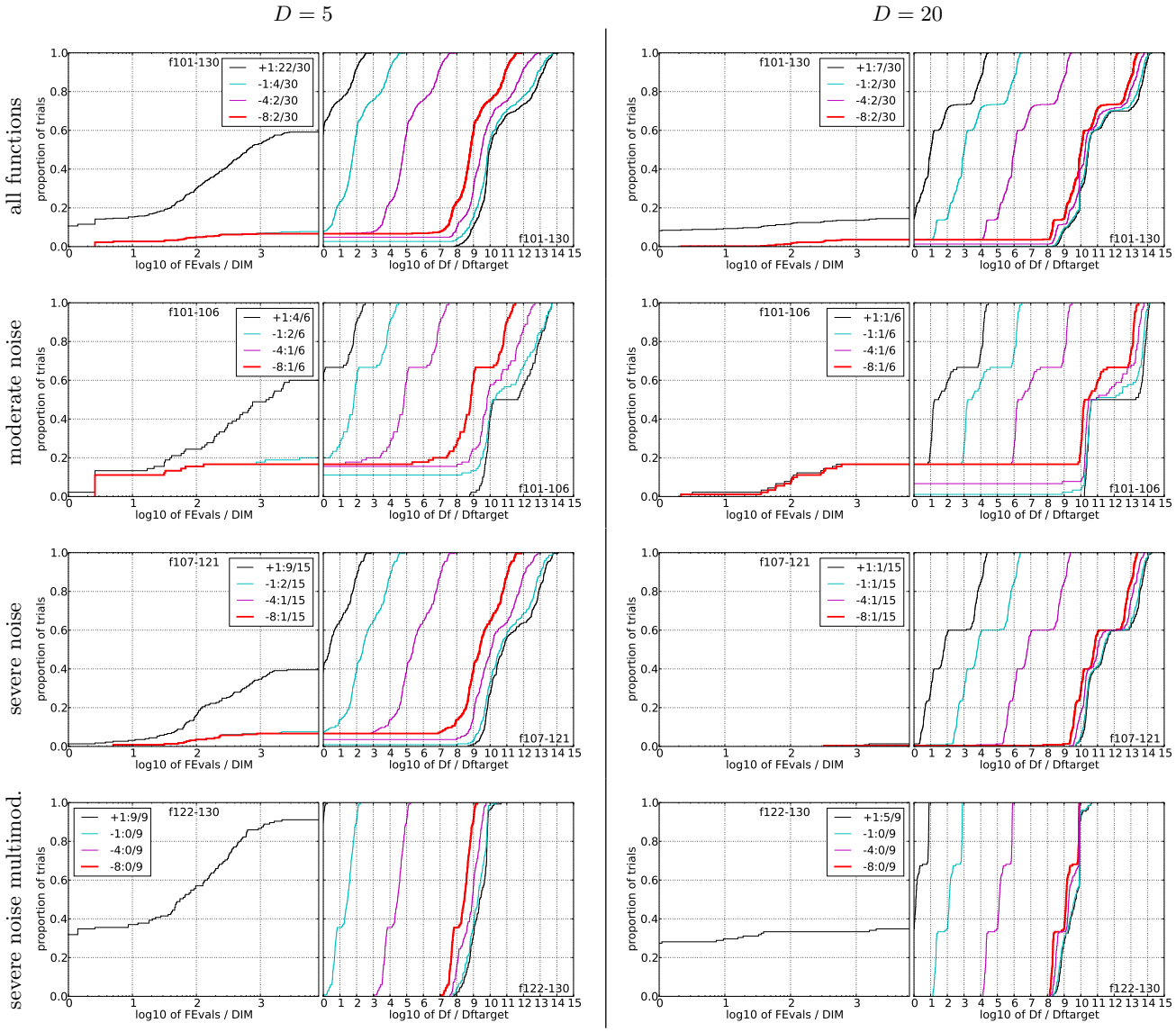

Figure 3: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left subplots) or versus Δf (right subplots). The thick red line represents the best achieved results. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: moderate noise functions; third row: severe noise functions; fourth row: severe noise and highly-multimodal functions. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{121} in 5-D, N=15, mFE=14544					**f_{121} in 20-D**, N=15, mFE=48426				
10	15	3.6e2	3.3e2	4.4e2	3.6e2	0	*33e+0*	*23e+0*	*49e+0*	2.0e4
1	14	7.9e3	6.8e3	9.5e3	7.9e3	.	.	.	.	.
1e−1	2	1.0e5	9.9e4	1.0e5	1.4e4	.	.	.	.	.
1e−3	0	*50e−2*	*99e−3*	*92e−2*	7.9e3	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{123} in 5-D, N=15, mFE=4764					**f_{123} in 20-D**, N=15, mFE=18333				
10	15	5.1e2	3.0e2	7.0e2	5.1e2	0	*14e+0*	*10e+0*	*21e+0*	7.9e3
1	0	*39e−1*	*24e−1*	*49e−1*	2.0e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{125} in 5-D, N=15, mFE=17160					**f_{125} in 20-D**, N=15, mFE=53130				
10	15	3.7e1	1.0e0	5.1e1	3.7e1	15	5.3e1	1.0e0	1.6e2	5.3e1
1	15	2.4e3	1.8e3	3.5e3	2.4e3	0	*21e−1*	*16e−1*	*23e−1*	2.2e4
1e−1	0	*46e−2*	*33e−2*	*54e−2*	5.6e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{127} in 5-D, N=15, mFE=14124					**f_{127} in 20-D**, N=15, mFE=49329				
10	15	7.8e0	1.0e0	7.8e0	7.8e0	15	1.4e2	9.9e1	1.8e2	1.4e2
1	15	1.3e3	9.9e2	1.8e3	1.3e3	0	*20e−1*	*16e−1*	*22e−1*	3.2e4
1e−1	0	*38e−2*	*18e−2*	*63e−2*	6.3e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{129} in 5-D, N=15, mFE=4518					**f_{129} in 20-D**, N=15, mFE=17346				
10	8	5.5e3	5.0e3	6.2e3	3.0e3	0	*76e+0*	*71e+0*	*80e+0*	5.6e3
1	0	*76e−1*	*54e−1*	*13e+0*	2.5e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	#	ERT	10%	90%	RT_{succ}	#	ERT	10%	90%	RT_{succ}
	f_{122} in 5-D, N=15, mFE=14370					**f_{122} in 20-D**, N=15, mFE=35028				
10	15	8.8e2	7.1e2	1.2e3	8.8e2	1	4.6e5	4.6e5	4.8e5	2.9e4
1	0	*36e−1*	*27e−1*	*69e−1*	4.0e3	0	*13e+0*	*11e+0*	*22e+0*	1.6e4
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{124} in 5-D, N=15, mFE=14898					**f_{124} in 20-D**, N=15, mFE=48426				
10	15	7.1e2	5.6e2	9.8e2	7.1e2	0	*13e+0*	*11e+0*	*19e+0*	2.5e4
1	1	2.0e5	2.0e5	2.1e5	1.4e4	.	.	.	.	.
1e−1	0	*39e−1*	*15e−1*	*61e−1*	4.0e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{126} in 5-D, N=15, mFE=4998					**f_{126} in 20-D**, N=15, mFE=20181				
10	15	5.4e0	1.4e0	1.3e1	5.4e0	15	2.3e1	1.0e0	3.6e1	2.3e1
1	15	4.6e2	3.0e2	6.3e2	4.6e2	0	*17e−1*	*14e−1*	*21e−1*	1.0e4
1e−1	0	*49e−2*	*26e−2*	*63e−2*	1.8e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{128} in 5-D, N=15, mFE=11808					**f_{128} in 20-D**, N=15, mFE=28938				
10	10	9.8e3	7.7e3	1.0e4	5.8e3	0	*75e+0*	*70e+0*	*79e+0*	1.3e4
1	0	*94e−1*	*35e−1*	*12e+0*	4.0e3	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.
	f_{130} in 5-D, N=15, mFE=14472					**f_{130} in 20-D**, N=15, mFE=47733				
10	15	1.9e3	1.5e3	2.6e3	1.9e3	1	6.7e5	6.6e5	6.8e5	4.3e4
1	2	9.1e4	7.7e4	9.9e4	7.3e3	0	*75e+0*	*32e+0*	*78e+0*	2.8e4
1e−1	0	*20e−1*	*50e−2*	*57e−1*	3.2e3	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 2: Shown are, for functions f_{121}-f_{130} and for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\text{opt}} + \Delta f$ (ERT, see Figure 2); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\text{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 2 for the names of functions.