A Stigmergy-Based Algorithm for Black-Box Optimization: Noiseless Function Testbed

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ABSTRACT

In this paper, we present a stigmergy-based algorithm for solving optimization problems with continuous variables, labeled Differential Ant-Stigmergy Algorithm (DASA). The performance of the DASA is evaluated on the set of benchmark problems provided for Black-Box Optimization Benchmarking (BBOB) 2009, a GECCO Workshop for Real-Parameter Optimization. Benchmarking for noiseless function testbed is presented.

Categories and Subject Descriptors

G.1.6 [**Numerical Analysis**]: Optimization, Global Optimization, Unconstrained Optimization; F.2.1 [**Analysis of Algorithms and Problem Complexity**]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, Black-box optimization, Stigmergy

1. INTRODUCTION

Numerical optimization problems are an important field of research and have a wide number of applications, from the mathematical optimization of functions to the real-world engineering problems. Many engineering problems involve choosing the best configuration of a set of parameters to achieve a specified objective. Numerical optimization refers to the case when these parameters take continuous values, as opposed to combinatorial optimization, which deals with discrete values.

In this paper we consider the following numerical minimization problem:

$$\min(f(\vec{x})), \vec{B}_l \leq \vec{x} \leq \vec{B}_u$$

where $\vec{x} = (x_1, x_2, \ldots, x_D)$ is the variable vector in $\mathbb{R}^D$, $f(\vec{x})$ denotes the cost function to minimize and $\vec{B}_l = (B_{l1}, B_{l2}, \ldots, B_{lD})$, $\vec{B}_u = (B_{u1}, B_{u2}, \ldots, B_{uD})$ represent, respectively, the lower and the upper bound of the variables, such that $x_i \in [B_{li}, B_{ui}]$.

Usually, numerical optimization problems are black-box optimizations, in which the cost function's form as well as its derivatives are unknown. Normally, this occurs when the cost function is computed using a complex simulation about which the optimization algorithm has no information. Executing a black-box simulation in order to evaluate a candidate solution is usually very expensive and can take up to several minutes or even hours. This is particularly problematic because optimization algorithms for black-box problems are necessarily blind search algorithms that must repeatedly sample points in a solution space, evaluate them by running the simulation, and apply various heuristics in order to choose the next points to sample.

In the past two decades, many bio-inspired optimization algorithms have been proposed to solve this kind of optimization problem, e.g., real-parameter genetic algorithm [10], evolution strategies [3], differential evolution [9], particle swarm optimization [7], immunological algorithm [2], ant-colony optimization [1], etc. These algorithms have been used to solve problems in several research fields due to the fact that do not require previous considerations regarding the problem to be optimized and offers a high degree of parallelism.

Although ant-based optimization has been proven to be one of the best metaheuristics in some combinatorial optimization problems, the application to the numerical optimizations appears more challenging, since the pheromone laying method is not straightforward. There are several possibilities to use ants for numerical optimization. We can use simplified direct simulation of real ants' behavior or we can extend method to explore continuous spaces. This extension can be done by the suitable discretization of a search space or by probabilistic sampling. In this way a fine-grained discrete form of continuous domain is created. With it we are able to represent this problem as a graph, which enables the use of ant-based approach for solving numerical optimization problems.

The remainder of this paper is organized as follows: Section 2 introduces the optimization algorithm. Sections 3 and 4 present the experimental procedure and black-box optimization benchmarking for noiseless function testbed, respectively. CPU timing experiment is presented in Section 5. Finally, Section 6 concludes the paper.

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2. STIGMERGY-BASED ALGORITHM

2.1 Parameter Differences

Let x_i' be the current value of the i-th parameter. During the searching for the optimal parameter value, the new value, x_i, is assigned to the i-th parameter as follows:

$$x_i = x_i' + \delta_i. \tag{1}$$

Here, δ_i is the so-called *parameter difference* and is chosen from the set

$$\Delta_i = \Delta_i^- \cup \{0\} \cup \Delta_i^+,$$

where

$$\Delta_i^- = \{\delta_{i,k}^-|\ \delta_{i,k}^- = -b^{k+B_{li}-1}, k = 1, 2, \ldots, d_i\}$$

and

$$\Delta_i^+ = \{\delta_{i,k}^+|\ \delta_{i,k}^+ = b^{k+B_{li}-1}, k = 1, 2, \ldots, d_i\}.$$

Here, $d_i = B_{ui} - B_{li} + 1$. Therefore, for each parameter x_i, the parameter difference, δ_i, has a range from $b^{B_{li}}$ to $b^{B_{ui}}$, where b is the so-called *discrete base*, $B_{li} = \lfloor \lg_b(\epsilon_i) \rfloor$, and $B_{ui} = \lfloor \lg_b(\max(x_i) - \min(x_i)) \rfloor$. With the parameter ϵ_i, the maximum precision of the parameter x_i is set. The precision is limited by the computer's floating-point arithmetics. To enable a more flexible movement over the search space, the weight ω is added to Eq. 1:

$$x_i = x_i' + \omega\delta_i \tag{2}$$

where $\omega = \text{RandomInteger}(1, b-1)$.

2.2 Graph Representation

From all the sets Δ_i, $1 \le i \le D$, where D represents the number of parameters, the so-called *differential graph* $\mathcal{G} = (V, E)$ with a set of vertices, V, and a set of edges, E, between the vertices is constructed. Each set Δ_i is represented by the set of vertices, $V_i = \{v_{i,1}, v_{i,2}, \ldots, v_{i,2d_i+1}\}$, and $V = \bigcup_{i=1}^{D} V_i$. Then we have that

$$\Delta_i = \{\delta_{i,d_i}^-, \ldots, \delta_{i,d_i-j+1}^-, \ldots, \delta_{i,1}^-, 0, \delta_{i,1}^+, \ldots, \delta_{i,j}^+, \ldots, \delta_{i,d_i}^+\}$$

corresponds to

$$V_i = \{v_{i,1}, \ldots, v_{i,j}, \ldots, v_{i,d_i+1}, \ldots, v_{i,d_i+1+j}, \ldots, v_{i,2d_i+1}\},$$

where

$$\begin{aligned} v_{i,j} &\xrightarrow{\delta} \delta_{i,d_i-j+1}^-, \\ v_{i,d_i+1} &\xrightarrow{\delta} 0, \\ v_{i,d_i+1+j} &\xrightarrow{\delta} \delta_{i,j}^+ \end{aligned}$$

and $j = 1, 2, \ldots, d_i$. Each vertex of the set V_i is connected to all the vertices that belong to the set V_{i+1}. Therefore, this is a directed graph, where each path $\vec{p}$ from the starting vertex, $v_1 \in V_1$, to any of the ending vertices, $v_D \in V_D$, is of equal length and can be defined with v_i as $\nu = (v_1 v_2 \ldots v_i \ldots v_D)$, where $v_i \in V_i$, $1 \le i \le D$.

2.3 Algorithm Implementation

The optimization consists of an iterative improvement of the temporary best solution, $\vec{x}^{\,\text{tb}}$, by constructing an appropriate path $\vec{p}$. New solutions are produced by applying $\vec{p}$ to $\vec{x}^{\,\text{tb}}$ (Eq. 2).

First a solution $\vec{x}^{\,\text{tb}}$ is randomly chosen by uniform sampling and evaluated. Then a search graph is created and an initial amount of pheromone is deposited on search graph according to the Cauchy probability density function

$$C(z) = \frac{1}{s\pi(1 + (\frac{z-l_i}{s})^2)},$$

where l_i is the location offset for the i-th parameter and

$$s = s_{\text{global}} - s_{\text{local}}$$

is the scale factor. For an initial pheromone distribution the Cauchy distribution with $s_{\text{global}} = 10$, $s_{\text{local}} = 0$, and $l_i = 0, i = 1, 2, \ldots, D$ is used and each parameter vertices are equidistantly arranged between $z = [-4, 4]$.

There are m ants in a colony, all of which begin simultaneously from the starting vertex. Ants use a probability rule to determine which vertex will be chosen next. The rule is based on a simple ACO. More specifically, ant a in step i moves from a vertex in set V_{i-1} to vertex $v_{i,j} \in V_i$ with a probability given by:

$$\text{prob}(a, v_{i,j}) = \frac{\tau(v_{i,j})}{\sum_{1 \le k \le 2d_i+1} \tau(v_{i,k})},$$

where $\tau(v_{i,k})$ is the amount of pheromone in vertex $v_{i,k}$.

The ants repeat this action until they reach the ending vertex. For each ant i, path $\vec{p_i}$ is constructed. If for some predetermined number of tries (in our case m^2 for all ants) we get $\vec{p_i} = \mathbf{0}$ the search process is reset by randomly choosing new $\vec{x}^{\,\text{tb}}$ and pheromone re-initialization. Otherwise, a new solution $\vec{x_i}$ is constructed.

After all ants have created solutions, they are being evaluated with a calculation of $y_i = f(\vec{x_i})$. The information about the best among them is stored as currently best information ($\vec{x}^{\,\text{cb}}$, $\vec{p}^{\,\text{cb}}$, and y_i^{cb}).

The current best solution, $\vec{x}^{\,\text{cb}}$ is compared to the temporary best solution $\vec{x}^{\,\text{tb}}$. If $y^{\,\text{cb}}$ is better than $y^{\,\text{tb}}$, then temporally best information is replaced with currently best information. In this case s_{global} is increased according to the global scale increase factor, s_+:

$$s_{\text{global}} \leftarrow (1 + s_+) s_{\text{global}},$$

s_{local} is set to

$$s_{\text{local}} = \frac{1}{2} s_{\text{global}}$$

and pheromone amount is redistributed according to the associated path $\vec{p}^{\,\text{cb}}$, where $l_i = z(p_i^{\text{cb}})$, so that the peak of Cauchy distribution is over with path selected vertex. Furthermore, if new $y^{\,\text{tb}}$ is better then global best $y^{\,\text{b}} = f(x^{\,\text{b}})$, then globally best information is replaced with temporally best information. So, global best solution is stored. If no better solution is found s_{global} is decreased according to the global scale decrease factor, s_-:

$$s_{\text{global}} \leftarrow (1 - s_-) s_{\text{global}}.$$

Pheromone evaporation is defined by some predetermined percentage ρ. The probability density function $C(z)$ is changed in the following way:

$$l_i \leftarrow (1 - \rho) l_i$$

and

$$s_{\text{local}} \leftarrow (1 - \rho) s_{\text{local}}.$$

Here we must emphasize that $\rho > s_-$, because otherwise we might get negative scale factor.

The whole procedure is then repeated until some ending condition is met.

The pseudocode of the Differential Ant-Stigmergy Algorithm (DASA) is presented as follows:

Algorithm 1 The DASA

1: $\vec{x}^{\,\mathrm{tb}} = \texttt{Rnd\_Solution}()$
2: $y^{\,\mathrm{b}} = f(\vec{x}^{\,\mathrm{tb}})$
3: $y^{\,\mathrm{tb}} = \inf$
4: $\mathcal{G} = \texttt{Graph\_Initialization}(\vec{x}^{\,\mathrm{tb}}, \vec{\epsilon})$
5: $\texttt{Pheromone\_Initialization}(\mathcal{G})$
6: **while** not ending condition met **do**
7: $k = 0$
8: **for all** m ants **do**
9: **repeat**
10: $\vec{p_i} = \texttt{Find\_Path}(\mathcal{G})$
11: $k = k + 1$
12: **if** $k > m^2$ **then**
13: $\vec{x}^{\,\mathrm{tb}} = \texttt{Rnd\_Solution}()$
14: $\texttt{Pheromone\_Initialization}(\mathcal{G})$
15: **goto** line 7
16: **end if**
17: **until** $(\vec{p_i} = \mathbf{0})$
18: $\omega = \texttt{Random\_Integer}(1, b-1)$
19: $\vec{x_i} = \vec{x}^{\,\mathrm{tb}} + \omega\delta(\vec{p})$
20: **end for**
21: $y^{\,\mathrm{cb}} = \inf$
22: **for all** m ants **do**
23: $y = f(\vec{x_i})$
24: **if** $y < y^{\,\mathrm{cb}}$ **then**
25: $y^{\,\mathrm{cb}} = y$
26: $\vec{p}^{\,\mathrm{cb}} = \vec{p_i}$
27: $\vec{x}^{\,\mathrm{cb}} = \vec{x_i}$
28: **end if**
29: **end for**
30: **if** $y^{\,\mathrm{cb}} < y^{\,\mathrm{tb}}$ **then**
31: $y^{\,\mathrm{tb}} = y^{\,\mathrm{cb}}$
32: $\vec{x}^{\,\mathrm{tb}} = \vec{x}^{\,\mathrm{cb}}$
33: $s = \texttt{Update\_Scales}(s_{\mathrm{global}}, s_{\mathrm{local}})$
34: $\texttt{Pheromone\_Redistribution}(\vec{p}^{\,\mathrm{cb}}, s)$
35: **if** $y^{\,\mathrm{tb}} < y^{\,\mathrm{b}}$ **then**
36: $y^{\,\mathrm{b}} = y^{\,\mathrm{tb}}$
37: $\vec{x}^{\,\mathrm{b}} = \vec{x}^{\,\mathrm{tb}}$
38: **end if**
39: **else**
40: $\texttt{Update\_Scale}(s_{\mathrm{global}})$
41: **end if**
42: $\texttt{Pheromone\_Evaporation}(\mathcal{G}, \rho)$
43: **end while**

3. EXPERIMENTAL PROCEDURE

The DASA includes six parameters: the number of ants, m, the pheromone dispersion factor, ρ, the global scale-increasing factor, s_+, the global scale-decreasing factor, s_-, the maximum parameter precision, ε, and the discrete base, b. The single setting for parameters' was used for all functions, i.e., the crafting effort was zero. We set $m = 30$ and $\rho = 0.2$, while other parameters' settings are standard [8]: $s_+ = 0.01$, $s_- = 0.02$, $\varepsilon = 1.0 \times 10^{-15}$, and $b = 10$.

Maximal number of restarts was set to 1000 and maximal number of FEs was set to $D \times 10^6$.

4. RESULTS

Results from experiments according to [5] on the benchmark functions given in [4, 6] are presented in Figures 1 and 2 and in Table 1.

5. CPU TIMING EXPERIMENT

For the the timing experiment the DASA was run on f_8. The computer platform used to perform the experiments was based on Intel Core i7 processors at 3.5 GHz, 6 GB of RAM, and the Microsoft Windows Vista x64 operating system. The DASA was implemented in Borland Delphi and for the function testbed the original implementation of functions in C were used in a form of dynamic link library. The results were 1.0; 1.1; 2.0; 3.4; 6.5 and 11.6 $\times$ 10^{-6} seconds per function evaluation in dimension 2; 3; 5; 10; 20 and 40, respectively. We can see a proportional dependency of CPU time on the search space dimensionality.

6. CONCLUSIONS

In this paper a stigmergy-based algorithm called Differential Ant-Stigmergy Algorithm (DASA) for solving optimization problems with continuous variables was presented. The performance of the DASA was evaluated on the noiseless function testbed provided for Black-Box Optimization Benchmarking (BBOB).

The DASA performs very good on separable ($f_1, \ldots, f_5$) and moderate functions ($f_6, \ldots, f_9$) with exception of Step-ellipsoid function (f_7). With weak structured functions (f_{20}, $\ldots, f_{24}$) it has troubles with higher dimensions for functions f_{20}, f_{23}, and f_{24}. It performs poorly on ill-conditioned ($f_{10}, \ldots, f_{14}$) and multi-modal ($f_{15}, \ldots, f_{19}$) functions with exception of Discus function (f_{11}).

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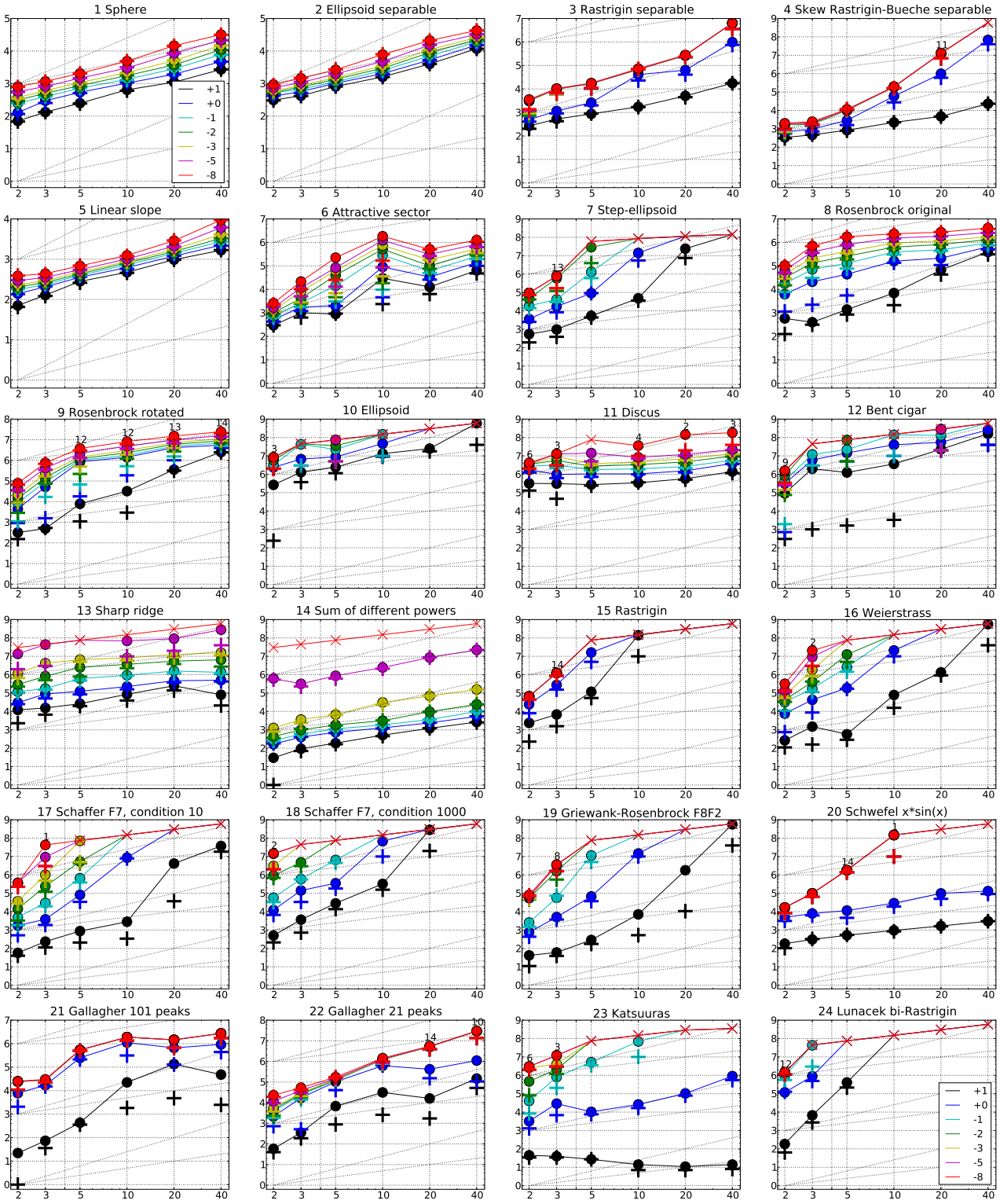

Figure 1: Expected Running Time (ERT, ●) **to reach** $f_{\text{opt}} + \Delta f$ **and median number of function evaluations of successful trials** (+), **shown for** $\Delta f = 10, 1, 10^{-1}, 10^{-2}, 10^{-3}, 10^{-5}, 10^{-8}$ **(the exponent is given in the legend of** f_1 **and** f_{24}**) versus dimension in log-log presentation. The** $\text{ERT}(\Delta f)$ **equals to** $\#\text{FEs}(\Delta f)$ **divided by the number of successful trials, where a trial is successful if** $f_{\text{opt}} + \Delta f$ **was surpassed during the trial. The** $\#\text{FEs}(\Delta f)$ **are the total number of function evaluations while** $f_{\text{opt}} + \Delta f$ **was not surpassed during the trial from all respective trials (successful and unsuccessful), and** f_{opt} **denotes the optimal function value. Crosses (**×**) indicate the total number of function evaluations** $\#\text{FEs}(-\infty)$**. Numbers above ERT-symbols indicate the number of successful trials. Annotated numbers on the ordinate are decimal logarithms. Additional grid lines show linear and quadratic scaling.**

Δf	f_1 in 5-D, N=15, mFE=2473 #	ERT	10%	90%	RT_{succ}	f_1 in 20-D, N=15, mFE=16589 #	ERT	10%	90%	RT_{succ}
10	15	2.5e2	2.0e2	3.0e2	2.5e2	15	1.1e3	1.0e3	1.2e3	1.1e3
1	15	5.4e2	4.8e2	6.0e2	5.4e2	15	1.9e3	1.8e3	2.0e3	1.9e3
1e−1	15	7.2e2	6.7e2	7.8e2	7.2e2	15	2.8e3	2.7e3	2.9e3	2.8e3
1e−3	15	1.1e3	1.0e3	1.1e3	1.1e3	15	5.1e3	5.0e3	5.2e3	5.1e3
1e−5	15	1.5e3	1.4e3	1.5e3	1.5e3	15	8.5e3	8.2e3	8.8e3	8.5e3
1e−8	15	2.0e3	2.0e3	2.1e3	2.0e3	15	1.5e4	1.4e4	1.5e4	1.5e4

Δf	f_2 in 5-D, N=15, mFE=3014 #	ERT	10%	90%	RT_{succ}	f_2 in 20-D, N=15, mFE=22468 #	ERT	10%	90%	RT_{succ}
10	15	8.1e2	7.5e2	8.6e2	8.1e2	15	4.0e3	3.9e3	4.1e3	4.0e3
1	15	1.0e3	9.5e2	1.1e3	1.0e3	15	5.3e3	5.2e3	5.4e3	5.3e3
1e−1	15	1.1e3	1.1e3	1.2e3	1.1e3	15	7.0e3	6.9e3	7.2e3	7.0e3
1e−3	15	1.5e3	1.5e3	1.6e3	1.5e3	15	1.1e4	1.1e4	1.1e4	1.1e4
1e−5	15	2.0e3	1.9e3	2.1e3	2.0e3	15	1.5e4	1.5e4	1.6e4	1.5e4
1e−8	15	2.6e3	2.6e3	2.7e3	2.6e3	15	2.1e4	2.1e4	2.1e4	2.1e4

Δf	f_3 in 5-D, N=15, mFE=54700 #	ERT	10%	90%	RT_{succ}	f_3 in 20-D, N=15, mFE=1.03e6 #	ERT	10%	90%	RT_{succ}
10	15	8.7e2	8.1e2	9.3e2	8.7e2	15	5.1e3	4.7e3	5.5e3	5.1e3
1	15	2.6e3	1.9e3	3.3e3	2.6e3	15	6.3e4	4.0e4	8.6e4	6.3e4
1e−1	15	1.7e4	1.1e4	2.2e4	1.7e4	15	2.7e5	1.8e5	3.6e5	2.7e5
1e−3	15	1.7e4	1.2e4	2.3e4	1.7e4	15	2.7e5	1.8e5	3.6e5	2.7e5
1e−5	15	1.7e4	1.2e4	2.3e4	1.7e4	15	2.7e5	1.9e5	3.6e5	2.7e5
1e−8	15	1.8e4	1.2e4	2.3e4	1.8e4	15	2.8e5	1.9e5	3.7e5	2.8e5

Δf	f_4 in 5-D, N=15, mFE=34629 #	ERT	10%	90%	RT_{succ}	f_4 in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	8.4e2	7.3e2	9.5e2	8.4e2	15	4.7e3	4.4e3	5.1e3	4.7e3
1	15	2.9e3	2.0e3	3.8e3	2.9e3	15	9.9e5	7.1e5	1.3e6	9.9e5
1e−1	15	1.0e4	7.0e3	1.3e4	1.0e4	11	1.3e7	9.2e6	1.6e7	1.0e7
1e−3	15	1.0e4	7.1e3	1.3e4	1.0e4	11	1.3e7	9.2e6	1.7e7	1.0e7
1e−5	15	1.1e4	7.7e3	1.4e4	1.1e4	11	1.3e7	9.2e6	1.7e7	1.0e7
1e−8	15	1.1e4	8.4e3	1.4e4	1.1e4	11	1.3e7	9.3e6	1.6e7	1.0e7

Δf	f_5 in 5-D, N=15, mFE=864 #	ERT	10%	90%	RT_{succ}	f_5 in 20-D, N=15, mFE=4064 #	ERT	10%	90%	RT_{succ}
10	15	2.8e2	2.5e2	3.1e2	2.8e2	15	9.7e2	9.2e2	1.0e3	9.7e2
1	15	3.6e2	3.3e2	3.9e2	3.6e2	15	1.2e3	1.1e3	1.2e3	1.2e3
1e−1	15	4.0e2	3.7e2	4.3e2	4.0e2	15	1.4e3	1.3e3	1.5e3	1.4e3
1e−3	15	4.9e2	4.6e2	5.1e2	4.9e2	15	1.8e3	1.7e3	1.8e3	1.8e3
1e−5	15	5.5e2	5.3e2	5.8e2	5.5e2	15	2.1e3	2.0e3	2.2e3	2.1e3
1e−8	15	6.8e2	6.5e2	7.1e2	6.8e2	15	2.9e3	2.7e3	3.0e3	2.9e3

Δf	f_6 in 5-D, N=15, mFE=2.13e6 #	ERT	10%	90%	RT_{succ}	f_6 in 20-D, N=15, mFE=1.02e6 #	ERT	10%	90%	RT_{succ}
10	15	8.9e2	7.5e2	1.0e3	8.9e2	15	1.3e4	9.4e3	1.6e4	1.3e4
1	15	1.9e3	1.8e3	2.1e3	1.9e3	15	3.4e4	2.7e4	4.1e4	3.4e4
1e−1	15	1.4e4	2.9e3	2.5e4	1.4e4	15	6.1e4	5.0e4	7.4e4	6.1e4
1e−3	15	6.3e4	7.0e3	1.2e5	6.3e4	15	1.8e5	1.5e5	2.2e5	1.8e5
1e−5	15	8.4e4	1.5e4	1.5e5	8.4e4	15	3.1e5	2.5e5	3.8e5	3.1e5
1e−8	15	2.2e5	3.0e4	4.2e5	2.2e5	15	5.1e5	4.2e5	5.9e5	5.1e5

Δf	f_7 in 5-D, N=15, mFE=4.13e6 #	ERT	10%	90%	RT_{succ}	f_7 in 20-D, N=15, mFE=7.81e6 #	ERT	10%	90%	RT_{succ}
10	15	5.3e3	3.3e3	7.4e3	5.3e3	4	2.4e7	2.1e7	2.7e7	6.1e6
1	15	9.1e4	6.7e4	1.2e5	9.1e4	0	*13e+0*	*79e−1*	*17e+0*	2.0e6
1e−1	14	1.3e6	8.5e5	1.8e6	1.3e6	.	.	.	.	.
1e−3	0	*16e−3*	*10e−3*	*79e−3*	2.0e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_8 in 5-D, N=15, mFE=2.15e6 #	ERT	10%	90%	RT_{succ}	f_8 in 20-D, N=15, mFE=3.51e6 #	ERT	10%	90%	RT_{succ}
10	15	1.4e3	9.5e2	1.9e3	1.4e3	15	6.7e4	4.4e4	9.2e4	6.7e4
1	15	4.4e4	2.0e4	7.0e4	4.4e4	15	2.1e5	1.4e5	2.7e5	2.1e5
1e−1	15	1.3e5	8.4e4	1.8e5	1.3e5	15	4.9e5	3.8e5	5.9e5	4.9e5
1e−3	15	4.1e5	3.3e5	4.9e5	4.1e5	15	1.2e6	1.0e6	1.3e6	1.2e6
1e−5	15	8.2e5	7.4e5	9.0e5	8.2e5	15	1.8e6	1.7e6	1.9e6	1.8e6
1e−8	15	1.7e6	1.7e6	1.8e6	1.7e6	15	2.7e6	2.6e6	2.9e6	2.7e6

Δf	f_9 in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_9 in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	7.8e3	1.7e3	1.4e4	7.8e3	15	3.6e5	3.1e5	4.0e5	3.6e5
1	14	8.5e5	3.0e5	1.4e6	8.5e5	15	4.1e6	2.7e6	5.9e6	4.1e6
1e−1	14	1.0e6	4.9e5	1.6e6	1.0e6	15	4.9e6	3.3e6	6.6e6	4.9e6
1e−3	14	1.4e6	8.9e5	2.1e6	1.4e6	15	7.2e6	5.6e6	8.9e6	7.2e6
1e−5	13	2.2e6	1.6e6	2.8e6	2.1e6	15	9.7e6	8.0e6	1.1e7	9.7e6
1e−8	12	3.8e6	3.1e6	4.3e6	3.2e6	13	1.5e7	1.3e7	1.7e7	1.3e7

Δf	f_{10} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{10} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	12	2.4e6	1.8e6	3.1e6	2.2e6	10	2.5e7	2.3e7	2.7e7	1.7e7
1	6	8.8e6	7.2e6	1.0e7	5.0e6	0	*72e−1*	*34e−1*	*59e+0*	2.0e7
1e−1	3	2.2e7	2.0e7	2.4e7	5.0e6	.	.	.	.	.
1e−3	1	7.3e7	7.1e7	7.5e7	5.0e6	.	.	.	.	.
1e−5	1	7.4e7	7.4e7	7.5e7	5.0e6	.	.	.	.	.
1e−8	0	*47e−1*	*75e−4*	*20e+0*	4.5e6	.	.	.	.	.

Δf	f_{11} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{11} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	2.7e5	2.1e5	3.4e5	2.7e5	15	5.7e5	5.0e5	6.4e5	5.7e5
1	15	1.0e6	8.5e5	1.3e6	1.0e6	15	1.5e6	1.3e6	1.8e6	1.5e6
1e−1	15	1.8e6	1.5e6	2.0e6	1.8e6	15	2.6e6	2.2e6	3.0e6	2.6e6
1e−3	13	3.8e6	3.4e6	4.2e6	3.3e6	15	7.1e6	5.9e6	8.3e6	7.1e6
1e−5	5	1.4e7	1.4e7	1.5e7	4.5e6	14	1.2e7	1.1e7	1.4e7	1.1e7
1e−8	0	*17e−6*	*14e−7*	*12e−4*	4.5e6	2	1.4e8	1.4e8	1.5e8	1.7e7

Δf	f_{12} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{12} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	12	1.3e6	4.2e5	2.1e6	1.3e6	7	2.3e7	1.4e7	2.9e7	5.7e6
1	4	1.4e7	1.2e7	1.7e7	3.8e6	4	5.5e7	4.5e7	6.5e7	1.0e7
1e−1	3	2.2e7	1.9e7	2.4e7	3.3e6	2	1.3e8	1.1e8	1.5e8	1.0e7
1e−3	0	*32e−1*	*96e−3*	*13e+0*	4.5e6	1	2.8e8	2.6e8	3.0e8	2.0e7
1e−5	.	.	.	.	.	1	2.8e8	2.6e8	3.0e8	2.0e7
1e−8	.	.	.	.	.	0	*13e+0*	*44e−3*	*31e+0*	2.0e7

Δf	f_{13} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{13} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	2.6e4	1.8e4	3.6e4	2.6e4	15	2.5e5	1.4e5	3.6e5	2.5e5
1	15	1.3e5	8.7e4	1.7e5	1.3e5	15	4.7e5	3.4e5	6.1e5	4.7e5
1e−1	15	6.1e5	3.4e5	9.1e5	6.1e5	15	1.6e6	1.2e6	2.1e6	1.6e6
1e−3	7	6.6e6	5.2e6	8.2e6	3.2e6	12	1.3e7	1.0e7	1.5e7	9.5e6
1e−5	0	*23e−4*	*25e−6*	*14e−3*	1.4e6	3	9.1e7	8.4e7	9.6e7	2.0e7
1e−8	.	.	.	.	.	0	*51e−6*	*43e−7*	*29e−4*	7.9e6

Δf	f_{14} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{14} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	1.9e2	1.5e2	2.3e2	1.9e2	15	1.2e3	1.1e3	1.3e3	1.2e3
1	15	7.4e2	6.3e2	8.5e2	7.4e2	15	2.4e3	2.3e3	2.5e3	2.4e3
1e−1	15	1.1e3	1.0e3	1.3e3	1.1e3	15	3.6e3	3.5e3	3.8e3	3.6e3
1e−3	15	6.7e3	5.6e3	7.8e3	6.7e3	15	7.0e4	6.7e4	7.4e4	7.0e4
1e−5	15	8.7e5	7.0e5	1.1e6	8.7e5	15	8.6e6	8.0e6	9.3e6	8.6e6
1e−8	0	*12e−7*	*18e−8*	*25e−7*	4.5e6	0	*43e−7*	*31e−7*	*55e−7*	2.0e7

Δf	f_{15} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{15} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	1.2e5	7.1e4	1.7e5	1.2e5	0	*11e+1*	*85e+0*	*14e+1*	1.1e7
1	4	1.6e7	1.4e7	1.8e7	4.7e6	.	.	.	.	.
1e−1	0	*20e−1*	*99e−2*	*30e−1*	2.0e6	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{16} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{16} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	5.8e2	2.7e2	8.8e2	5.8e2	15	1.4e6	9.8e5	1.7e6	1.4e6
1	15	1.9e5	1.4e5	2.4e5	1.9e5	0	*44e−1*	*32e−1*	*69e−1*	1.1e7
1e−1	13	2.6e6	2.0e6	3.3e6	2.4e6	.	.	.	.	.
1e−3	0	*24e−3*	*38e−4*	*10e−2*	2.0e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{17} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{17} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	8.8e2	4.1e2	1.3e3	8.8e2	13	4.1e6	1.7e6	7.2e6	3.9e6
1	15	8.2e4	4.3e4	1.3e5	8.2e4	0	*57e−1*	*36e−1*	*11e+0*	2.0e7
1e−1	15	6.6e5	4.2e5	8.9e5	6.6e5	.	.	.	.	.
1e−3	1	7.0e7	6.6e7	7.5e7	5.0e6	.	.	.	.	.
1e−5	0	*82e−4*	*19e−4*	*20e−3*	2.5e6	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{18} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{18} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	2.8e4	1.6e4	4.3e4	2.8e4	1	2.8e8	2.6e8	3.0e8	2.0e7
1	15	3.5e5	2.1e5	5.0e5	3.5e5	0	*25e+0*	*12e+0*	*40e+0*	2.0e7
1e−1	8	6.5e6	5.4e6	7.5e6	3.4e6	.	.	.	.	.
1e−3	0	*64e−3*	*25e−3*	*20e−2*	2.8e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{19} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{19} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	2.9e2	1.8e2	4.1e2	2.9e2	14	1.8e6	2.6e5	3.4e6	1.8e6
1	15	6.7e4	3.8e4	1.0e5	6.7e4	0	*58e−1*	*40e−1*	*96e−1*	2.0e7
1e−1	5	1.1e7	9.6e6	1.3e7	5.0e6	.	.	.	.	.
1e−3	0	*13e−2*	*79e−3*	*28e−2*	2.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{20} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{20} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	5.2e2	4.5e2	5.8e2	5.2e2	15	1.7e3	1.6e3	1.7e3	1.7e3
1	15	1.1e4	6.9e3	1.6e4	1.1e4	15	9.8e4	6.2e4	1.4e5	9.8e4
1e−1	14	1.8e6	1.3e6	2.3e6	1.8e6	0	*40e−2*	*35e−2*	*46e−2*	6.3e6
1e−3	14	1.8e6	1.4e6	2.3e6	1.8e6	.	.	.	.	.
1e−5	14	1.8e6	1.4e6	2.3e6	1.8e6	.	.	.	.	.
1e−8	14	1.8e6	1.3e6	2.3e6	1.8e6	.	.	.	.	.

Δf	f_{21} in 5-D, N=15, mFE=1.51e6 #	ERT	10%	90%	RT_{succ}	f_{21} in 20-D, N=15, mFE=8.83e6 #	ERT	10%	90%	RT_{succ}
10	15	4.3e2	3.3e2	5.4e2	4.3e2	15	1.4e5	6.1e4	2.2e5	1.4e5
1	15	2.5e5	1.9e5	3.2e5	2.5e5	15	6.6e5	3.3e5	1.0e6	6.6e5
1e−1	15	5.4e5	4.2e5	6.6e5	5.4e5	15	1.4e6	7.3e5	2.3e6	1.4e6
1e−3	15	5.4e5	4.3e5	6.7e5	5.4e5	15	1.5e6	7.5e5	2.2e6	1.5e6
1e−5	15	5.4e5	4.3e5	6.6e5	5.4e5	15	1.5e6	7.7e5	2.2e6	1.5e6
1e−8	15	5.4e5	4.3e5	6.6e5	5.4e5	15	1.5e6	7.1e5	2.2e6	1.5e6

Δf	f_{22} in 5-D, N=15, mFE=377653 #	ERT	10%	90%	RT_{succ}	f_{22} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	6.8e3	3.4e3	1.0e4	6.8e3	15	1.6e4	6.0e3	2.7e4	1.6e4
1	15	1.0e5	7.1e4	1.4e5	1.0e5	15	4.2e5	2.7e5	5.7e5	4.2e5
1e−1	15	1.2e5	8.9e4	1.5e5	1.2e5	14	5.1e6	3.5e6	7.0e6	5.0e6
1e−3	15	1.3e5	9.6e4	1.7e5	1.3e5	14	5.2e6	3.4e6	7.0e6	5.1e6
1e−5	15	1.5e5	1.1e5	1.8e5	1.5e5	14	5.3e6	3.5e6	7.0e6	5.2e6
1e−8	15	1.7e5	1.4e5	2.1e5	1.7e5	14	5.5e6	3.8e6	7.3e6	5.4e6

Δf	f_{23} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{23} in 20-D, N=15, mFE=1.99e7 #	ERT	10%	90%	RT_{succ}
10	15	2.7e1	2.1e1	3.3e1	2.7e1	15	1.1e1	7.4e0	1.4e1	1.1e1
1	15	1.0e4	5.9e3	1.5e4	1.0e4	15	1.0e5	7.3e4	1.4e5	1.0e5
1e−1	9	5.2e6	4.0e6	6.2e6	3.7e6	0	*31e−2*	*20e−2*	*37e−2*	7.1e6
1e−3	0	*88e−3*	*17e−3*	*19e−2*	3.2e6	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Δf	f_{24} in 5-D, N=15, mFE=5.00e6 #	ERT	10%	90%	RT_{succ}	f_{24} in 20-D, N=15, mFE=2.00e7 #	ERT	10%	90%	RT_{succ}
10	15	4.1e5	2.4e5	6.1e5	4.1e5	0	*16e+1*	*13e+1*	*17e+1*	1.1e7
1	0	*32e−1*	*14e−1*	*82e−1*	2.2e6	.	.	.	.	.
1e−1	.	.	.	.	.	.	.	.	.	.
1e−3	.	.	.	.	.	.	.	.	.	.
1e−5	.	.	.	.	.	.	.	.	.	.
1e−8	.	.	.	.	.	.	.	.	.	.

Table 1: Shown are, for a given target difference to the optimal function value Δf: the number of successful trials (#); the expected running time to surpass $f_{\text{opt}} + \Delta f$ (ERT, see Figure 1); the 10%-tile and 90%-tile of the bootstrap distribution of ERT; the average number of function evaluations in successful trials or, if none was successful, as last entry the median number of function evaluations to reach the best function value (RT_{succ}). If $f_{\text{opt}} + \Delta f$ was never reached, figures in *italics* denote the best achieved Δf-value of the median trial and the 10% and 90%-tile trial. Furthermore, N denotes the number of trials, and mFE denotes the maximum of number of function evaluations executed in one trial. See Figure 1 for the names of functions.

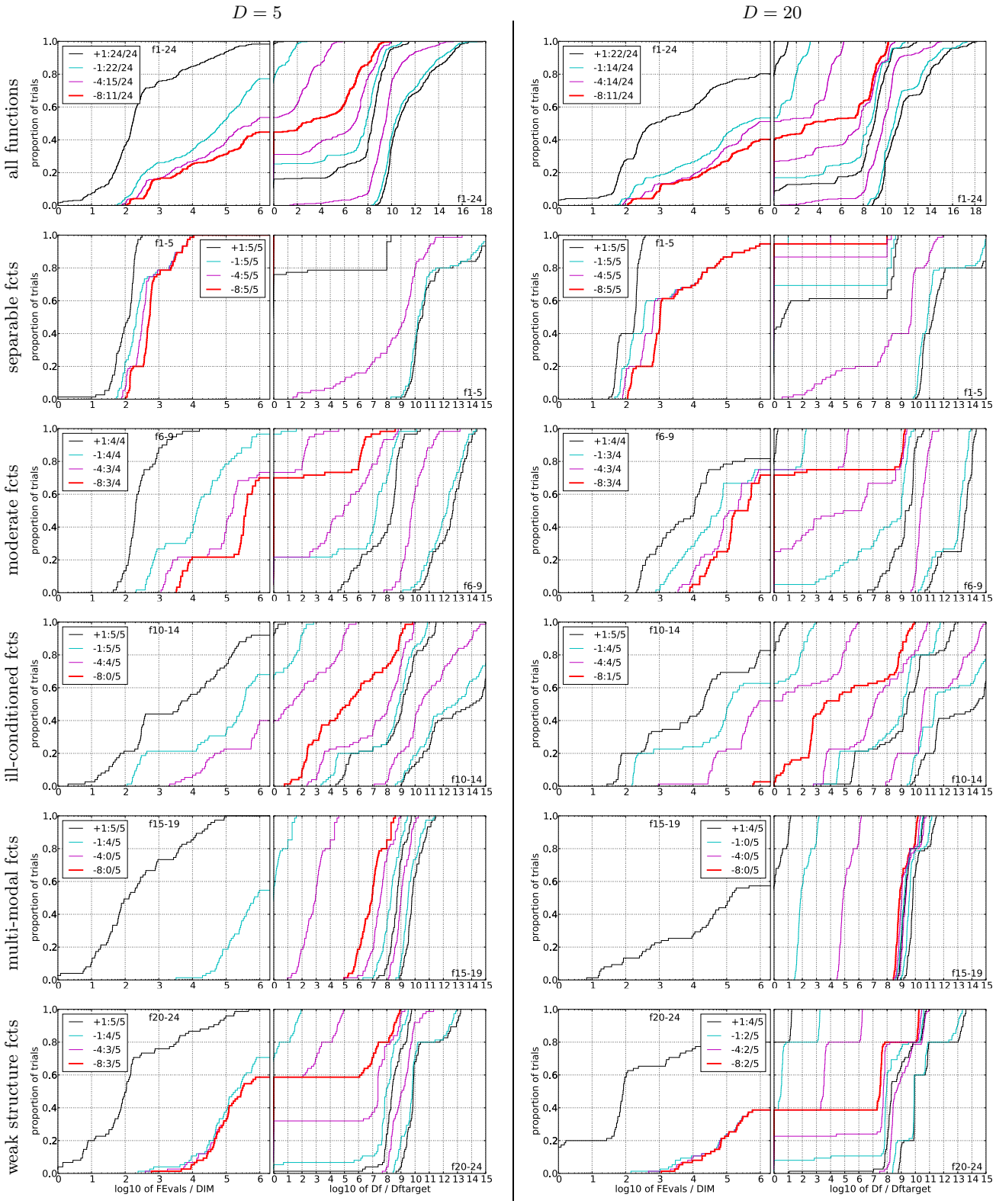

Figure 2: Empirical cumulative distribution functions (ECDFs), plotting the fraction of trials versus running time (left) or Δf. Left subplots: ECDF of the running time (number of function evaluations), divided by search space dimension D, to fall below $f_{\mathrm{opt}} + \Delta f$ with $\Delta f = 10^k$, where k is the first value in the legend. Right subplots: ECDF of the best achieved Δf divided by 10^k (upper left lines in continuation of the left subplot), and best achieved Δf divided by 10^{-8} for running times of $D, 10\,D, 100\,D \ldots$ function evaluations (from right to left cycling black-cyan-magenta). Top row: all results from all functions; second row: separable functions; third row: misc. moderate functions; fourth row: ill-conditioned functions; fifth row: multi-modal functions with adequate structure; last row: multi-modal functions with weak structure. The legends indicate the number of functions that were solved in at least one trial. FEvals denotes number of function evaluations, D and DIM denote search space dimension, and Δf and Df denote the difference to the optimal function value.

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