AN APPROACH TO INTERACTIVE SOLUTION OF MULTICRITERIAL OPTIMIZATION PROBLEMS WITH LINGUISTIC MODELING OF PREFERENCES

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The problem of interactive multicriterial optimization of nonlinear functions and constraints is formulated. The problem of linguistic evaluation and synthesis of fuzzy linguistic regulators for controlling exchange between criteria is solved. A general method of interactive solution of multicriterial optimization problems with linguistic modeling of preferences is proposed. Characteristics of software implementation of the method on a local-area network are given.

INTRODUCTION

A characteristic feature of multicriterial optimization problems is that they utilize two diverse systems: a clear-cut, nonfuzzy system of obtaining the set of noninferior solutions, and a fuzzy "humanistic" system of evaluation and selection, by the decision-maker (DM), of optimally desirable solutions from among the set of noninferior solutions [1]. The large amount of effort involved in obtaining and evaluating the solutions in each of these systems, and, in particular, the organization of their interaction, have substantially retarded the extensive use of multicriterial optimization in practical applications. The main sources of difficulties have been the following aspects of the solution of multicriterial optimization problems.

1. Calculation of large numbers of elements of noninferior Pareto-optimal solutions; when the number of criteria is greater than three, and the function to be optimized and the constraints are nonlinear, this entails large amounts of computation.

2. The DM is required to be highly qualified in the case of a priori specification of information regarding criterion preferences.

3. The DM is required to model his fuzzy "humanistic" system, with a high degree of accuracy, in terms of a nonfuzzy system.

To reduce the overall amount of effort involved in solving multicriterial optimization problems, so as to be able to utilize them more extensively in practical applications, we propose an interactive approach to linguistic modeling of preferences (or priorities). In our approach, the overall amount of computation involved in obtaining the optimal desirable solution is greatly reduced; the qualification requirements on the DM are lowered through training in the process of solution and allowance for evolution of judgment regarding preferences; and the interaction of the diverse systems is simplified by making it possible to describe the preferences of the DM in what are for him the natural fuzzy linguistic terms of the humanistic system.

1. INITIAL INTERACTIVE MULTICRITERIAL OPTIMIZATION ALGORITHM

We represent our multicriterial optimization problem in the form of a scalar optimization problem:

$$\min U[f_1(x),\ldots,f_l(x),\ldots,f_n(x)];\quad h_j(x)=0;\quad j=\overline{1,m};\quad g_j(x)\geqslant 0;$$
$$j=\overline{(m+1),p}, \tag{1}$$

where $f_i(x)$, $h_j(x)$, $g_j(x)$ are nonlinear functions; $U[f_1(x),\ldots,f_i(x),\ldots,f_n(x)]=\sum_{i=1}^{n}\lambda_i f_i(x)$ is the convolution of criteria for the coefficients of relative importance, satisfying the normalization condition $\lambda\in\Lambda=\{\lambda_i|\lambda_i\geqslant 0;\ (\sum_{i=1}^{n}\lambda_i^2)^{0,5}=1\}$. Solution of problem (1) reduces to the solution of the following sequence of problems:

1) determination of noninferior Pareto-optimal solutions;

2) generation of desirable preferences by the DM;

3) search for optimal desirable solution from among noninferior solutions of effective boundary R.

Of the variety of solution methods for the first problem, we will employ the nonlinear programming method with penalty functions [2]; this method is efficient and convenient for computer implementation. Then problem (1) can be represented as follows:

$$\min\left(\sum_{i=1}^{n}\lambda_i f_i(x^k)+\sum_{j=1}^{m}\rho_j^k H(h_j(x^k))+\sum_{j=m+1}^{p}\rho_j^k G(g_j(x^k))\right);$$

$$\left(\sum_{i=1}^{n}\lambda_i^2\right)^{0,5}=1, \tag{2}$$

where $\rho_j^k\geqslant 0$ are the weighting factors of the penalty contribution; $H(h_j(x^k)$, $G(g_j(x^k))$ are respectively functionals of $h_j(x)$, $g_j(x)$; $k = 0, 1, 2, 3, \ldots$ is the number of completed stages of calculations.

To consider ways of solving problem (2), we represent it in the form

$$\min P(U(f(x^k)),\rho_j^k) \tag{3}$$

and we prove the following theorem.

Theorem. Assume that the $\lambda_i\in\Lambda$ are locally known and constant. Then the optimal solution $P^0(U(f^0(x^k)),\rho_j^k)$ of problem (3) on the Pareto-effective boundary R coincides with the solution $U^0(f^0(x^k))$.

Proof. In accordance with the rules for choosing functionals in the penalty-function method [2], the following conditions must be met:

$$\lim_{k\to\infty}\sum_{j=1}^{m}\rho_j^k G(g_j(x^k))=0; \tag{4}$$

$$\lim_{k\to\infty}\sum_{j=m+1}^{p}\rho_j^k H(h_j(x^k))=0; \tag{5}$$

$$\lim_{k\to\infty}|P(U(f(x^k)),\ \rho_j^k)-U(f(x^k))|=0. \tag{6}$$

According to these conditions, in the process of seeking the minimum the adjoint functions are attenuated and at the minimum point, in the limit, they vanish altogether; the minimum of P coincides with that of U when the constraints $h_j(x^k), g_j(x^k)$ are observed. We will prove the fact that the optimal solution $P^0(U(f^0(x^k)),\ \rho_j^k)$ belongs to the effective boundary R in indirect fashion. Assume that there exists a solution $P'(U(f'(x^k)),\rho_j^k)$, that dominates the solution $P^0(U(f^0(x^k)),\rho_j^k)$ then

$$P'(U(f'(x^k)),\rho_j^k)<P^0(U(f^0(x^k)),\ \rho_j^k), \tag{7}$$

and this contradicts the fact that solution $P^0(U(f^0(x^k)),\rho_j^k)$ of problem (3) is a minimum point. Consequently, the solution $P^0(U(f^0(x^k)),\rho_j^k)$ belongs to the effective boundary R. From this we obtain, as $k \to \infty$, that the solution $U^0(f^0(x^k))$ coincides with the solution $P^0(U(f^0(x^k)),\rho_j^k)$ on R, QED.

For minimization of P-functions in the penalty-function method, we employ the reduced-gradient projection method; the rest of what follows will refer to this method. We should note that other methods of finding the minimum involving derivatives can also be employed; the rate of convergence in the search process is somewhat degraded as a result, but the proposed approach as a whole is not altered.

Now, proceding to the second problem in the sequence, let us consider it from the standpoint of reducing the number of comparisons the DM must make in generating the relative importance of the criteria. For this, let us analyze some properties of P-functions on the boundary R. The local behavior of functions (2) in the neighborhood of each point of criterial space can be represented in the form of the gradient of a complex function:

$$\nabla_x P(U(f(x^k)),\ \rho_j^d) = \sum_{i=1}^{n} \left(\frac{\partial P}{\partial f_i}\right)^d \nabla_x f_i(x^d), \tag{8}$$

where $\left(\frac{\partial P}{\partial f_i}\right)^d$ is the i-th partial derivative, calculated at point $f_i(x^d)$: $\nabla_x = f_i(x^d)$ is the gradient at point x^d; and d is the number of the Pareto-optimal solution. Here $\frac{\partial P}{\partial f_i}$ estimates the relative importance λ_i of particular criteria f_i; to generate $\forall\lambda_i \in \Lambda$ the DM must perform $\frac{n(n-1)}{2}$ comparisons of estimates of criteria. To reduce the number of comparisons, we employ the concept of reference criterion [3]. In this case the gradient (8) is transformed as follows:

$$\tilde{\nabla}_x P(U(f(x^d)),\rho_j^d) = \sum_{i=1}^{n} \omega_i^d \nabla_x f_i(x^d), \tag{9}$$

where $\omega_i^d = \left(\frac{\partial P}{\partial f_i}\right)^d \Big/ \left(\frac{\partial P}{\partial f_1}\right)^d$ is the "don't-care" line of the DM, or, as it is also called, the marginal rate of exchange between the first and i-th criteria. Criterion f_1 is chosen as the reference criterion. An increment in this critierion by 1 is completely compensated for by a reduction in the i-th criterion by Δf_i. In other words, we choose a Δf_i such that the solutions $(f_1^0, f_2^0, \dots, f_n^0)$ and $(f_1^0+1, \dots, f_i^0-\Delta f_i, \dots, f_n^0)$ are equivalent for the DM. Here, we convert from mutual comparisons between criteria to comparison of i - 1 criteria with the first one, thus reducing the number of comparisons to (n - 1).

In considering the third problem, we note that the changeover from mutual comparisons to comparison with the first criterion alters the form of the convolution, and problem (2) is converted to a problem of the form

$$\min \left(\sum_{i=1}^{n} \omega_i^d f_i(x^d) + \sum_{j=1}^{m} \rho_j^d H(h_j(x^d)) + \sum_{j=m+1}^{p} \rho_j^d G(g_j(x^d))\right);$$

$$\omega^d \in \Omega = \left\{\omega_i^d \mid \omega_i^d \geqslant 0;\ \ i=\overline{2,n};\ \ \omega_1^d = 1;\ \ \left(\sum_{i=1}^{n} (\omega_i^d)^2\right)^{0,5} = 1\right\}. \tag{10}$$

In problem (10) a new solution $(f_1^0+1, \dots, f_i^0-\Delta f_i, \dots, f_n^0)$ is obtained from the old one $(f_1^0, f_2^0, \dots, f_i^0, \dots, f_n^0)$ as a result of alteration of the marginal rate of exchange ω_i^d by the DM. Let Let us express this change as a percentage s_i^d of the original marginal rate of exchange.

Then the new value can be determined from the expression

$$\omega_i^{d+1} = \omega_i^d \cdot s_i^d. \tag{11}$$

The DM continues to change the marginal rate of exchange until the Pareto-optimal solution coincides with the optimally desirable one, or, equivalently, until the desired gradient $\sum_{i=1}^{n} \omega_i^{d+1} \nabla_x f_i(x^d)$ coincides with the permissible one $\sum_{i=1}^{n} \omega_i^d \nabla_x f_i(x^d)$. On the basis of the above properties, we can set up an interactive solution algorithm for multicriterial optimization of problem (1), which we will call the initial algorithm.

Step 1. Set $\forall \omega_i^d = 1,\ d = 1,\ u^d = 0$.

Step 2. Solve scalar optimization problem (10) and obtain the solution vector $f^d = (f_1^0, f_2^0, \ldots, f_n^0)$ and gradient $\sum_{i=1}^{n} \omega_i^d \nabla_x f_i(x^d)$.

Step 3. Interact with the DM to estimate f^d and to obtain s_i^d.

Step 4. Calculate the desired rate of exchange $\omega_i^{d+1} = \omega_i^d \cdot s_i^d$ and the desired gradient $\sum_{i=1}^{n} \omega_i^{d+1} \nabla_x f(x^d)$.

Step 5. Verify that the permissible and desired gradients are collinear:

$$u^d = \left[\sum_{i=1}^{n} \omega_i^d \nabla_x f_i(x^d) \Big/ \left\| \sum_{i=1}^{n} \omega_i^d \nabla_x f_i(x^d) \right\| \right]^T \times$$
$$\times \left[\sum_{i=1}^{n} \omega_i^{d+1} \nabla_x f_i(x^d) \Big/ \left\| \sum_{i=1}^{n} \omega_i^{d+1} \nabla_x f_i(x^d) \right\| \right]. \tag{12}$$

If the vectors of the desired and permissible gradients are collinear and $1 - u^d \leqslant E$, where E is an arbitrarily specified small quantity, then stop. The resultant solution is the optimal desirable one. Otherwise, go to Step 2. The initial algorithm is the basis for setting up an interactive multicriterial optimization algorithm utilizing the fuzzy information on preferences that is obtained from the humanistic system of the DM.

2. LINGUISTIC ESTIMATION AND CONTROL OF EXCHANGE BETWEEN CRITERIA

In the initial multicriterial optimization algorithm, the DM evaluates the Pareto-optimal solutions f_i^0 and, in accordance with these estimates, he effects control s_i of the rate of exchange between criteria f_1 and f_i. Numerical estimation and control is difficult and unnatural for the humanistic system of the DM [4]. What is most natural and convenient for the DM is linguistic evaluation and control, a distinctive feature of which is its fuzzy content. To describe these, we will employ the concepts of linguistic variable [5] and of fuzzy linguistic algorithm [6]. We evaluate noninferior Pareto-optimal solutions f_i^0 using the structured linguistic variable ESTIMATE, which assumes the following name values: GOOD (G), ACCEPTABLE (A), BAD (B), UNACCEPTABLE (U). The semantics of ESTIMATE are evaluated on a fuzzy universal set $A \in [0, 1]$, while the semantics of the name values (or, as they are also called, primary terms) is evaluated on fuzzy subsets A_t of fuzzy set A. Fuzzy subsets A_t of fuzzy set A are ordered sequentially with respect to t = G, A, B, and U (Fig. 1) and can be specified in the form [7]

$$A_t = \{\alpha_j^d | \mu_{A_t}(\alpha_i^d) \in [0, 1],\ t \neq j \neq d\}, \tag{13}$$

where α_j^d is the subjective degree to which the DM is satisfied with solution f_i^0 at step d; $\mu_{A_t}(\alpha_j^d)$ is a subjectively produced function of α_j^d belonging to A_t. To control the

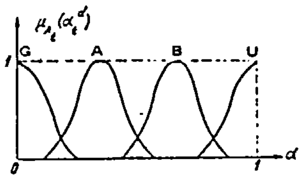

Fig. 1

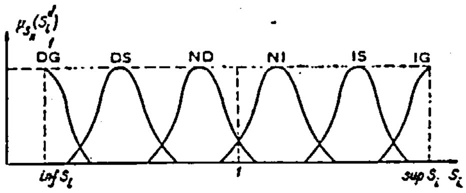

Fig. 2

Fig. 1. Domain of semantic values of the linguistic variable ESTIMATE.

Fig. 2. Domain of semantic values of the linguistic variable EXCHANGE.

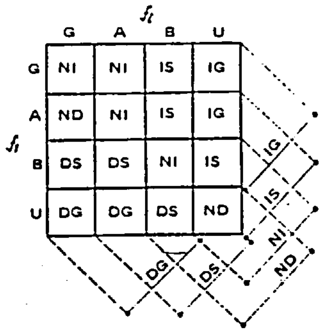

Fig. 3. Decision table of linguistic regulator with projections of fuzzy course of control.

rate of exchange between criteria f_1 and f_i, we employ the linguistic variable EXCHANGE, which assumes the following name values: INCREASES GREATLY (IG), INCREASES SLIGHTLY (IS), DOES NOT INCREASE (NI), DOES NOT DECREASE (ND), DECREASES SLIGHTLY (DS), DECREASES GREATLY (DG). The semantics of EXCHANGE are evaluated on a bounded fuzzy set of percentage change s, whose upper and lower bounds sup s_i and inf s_i depend on the form of the functions f_i and can be obtained through computational experiments. Fuzzy set s is a set of subsets s_k that are successively ordered with respect to k = IG, IS, NI, ND, DS, DG (Fig. 2), each of which represents an ensemble of pairs:

$$s_k = \{s_i^d \mid \mu_{s_k}(s_i^d) \in [0, 1],\ k \neq j \neq d\}, \tag{14}$$

where s_i^d is the percentage of exchange between criteria f_1 and f_i at step d; $\mu_{s_k}(s_i^d)$ is the subjective function that the meaning of the k-th name value belongs to the percentage of exchange.

Analysis of the DM's actions in controlling the exchange process between criteria f_1 and f_i shows that, in developing the "course" of exchange control, the DM adheres to the following heuristic rules: if estimates f_1 and f_i are equally good, then adhere to the current course of control; if estimates f_1 and f_i are equally bad, then "pumping" is called for, i.e., slight increases or decreases in exchange between criteria; if estimates f_1 degrade while estimates f_i improve, then increase exchange; if estimates f_1 improve while estimates f_i degrade, then decrease exchange. The presence of linguistic estimates and of a heuristically generated linguistic course of control makes it possible to synthesize a linguistic regulator [8] for controlling exchange s_i in the form of a decision table (Fig. 3). The projections of the course of control of this regulator onto the axis perpendicular to the principal diagonal of the table form segments of the course of control whose sequence coincides with the order specified by the sequence of k fuzzy sets s_k.

Overlaps of segments characterize the degree of fuzziness of the course of control, and are associated with the level of qualification of the DM. Obviously, increasing the name values of the linguistic variable ESTIMATE, which is equivalent to a less fuzzy course of control of exchange between f_1 and f_i, will enhance the quality of control. However, this leads to an increase in the name values, and renders the DM's work more difficult. Another way of reducing the fuzziness of the course of control involves the use of a training phase for the particular control situation (proposed by MacVicar-Whelan in [8]), which can be implemented by the DM using linguistic indeterminacies that "concentrate" and "deconcentrate" the fuzzy set of estimates [5]. We employ the following linguistic operators as the linguistic indeterminacies in question: EXTREMELY (E), VERY (V), MORE OR LESS (ML), NOT. Following [5, 7], we translate the linguistic operators into operations over fuzzy sets of values: EXTREMELY $(t)\rightarrow(A_t)^4$, VERY $(t)\rightarrow(A_t)^2$, MORE OR LESS $(t)\rightarrow(A_t)^{0.5}$, NOT $(t)\rightarrow \rightarrow(1-A_t)$ the use of linguistic operators makes possible the training by the DM of a fuzzy linguistic regulator, taking into account the evolution of his judgments regarding preferences, and thus reducing the fuzziness of control and enhancing the convergence to the optimally desirable solution from among the noninferior Pareto-optimal solutions. We employ the decomposition rule [9] to convert from fuzzy values of the percentage of exchange s_i to scalar values of exchange. According to this rule, the most representative values are those that have a maximum of the "belonging" functions. If there are several maximum values or a plateau, the mean value s_i is taken. Thus, the use of linguistic estimation and control results in substantial simplification of the work of the DM and the interaction between the clear-cut nonfuzzy and humanistic systems.

3. INTERACTIVE MULTICRITERIAL OPTIMIZATION ALGORITHM WITH LINGUISTIC ESTIMATION AND CONTROL

The above initial algorithm, together with linguistic evaluation and control of exchange, ensure completeness of the solution of the multicriterial optimization problem, and lead to the following solution algorithm.

Step 1. Specify equal exchange between criteria:

$$\forall\omega_i^d=1, \quad d=1.$$

Step 2. Solve problem (10), determining the solution vector $f^0=(f_1^0, f_2^0, \ldots, f_n^0)$ and gradient $\sum_{i=1}^{n}\omega_i^d\nabla_x f_i(x^d)$.

Step 3. Perform interaction with the DM to obtain linguistic estimates of the values of the EXCHANGE variable in the form of linguistic variables and operators.

Step 4. Using heuristic rules, synthesize linguistic regulators for all pairs f_1 and f_i (performed only at step d = 1), and using them, determine the most representative values s_i^d.

Step 5. Check the stopping criterion (12). If $1-u^d\leq E$, then stop. Otherwise $\omega^{d+1}= =(1, \omega_2^d\cdot s_2^d, \ldots, \omega_i^d\cdot s_i^d, \ldots, \omega_n^d\cdot s_n^d)$ and go to Step 2.

CONCLUSIONS

The above approach to the development of an interactive multicriterial optimization algorithm with linguistic estimation and control was implemented in FORTRAN as part of a distributed automated-design system. The "belonging" functions were provided by standard functions from [9]. The semantics of the linguistic variable EXCHANGE varied in the range $0.2 \leq s_i \leq 0.8$. The maximum number of criteria was 20, the dimension of the control variables x running up to 100. A feature of the software is that it is implemented in the form of a package that implements the initial algorithm, and a package of linguistic evaluation and exchange control. Both packages are switched via a local-area network. The first package is implemented by a master computer, which calculates (on a time interval acceptable for interaction) the values $f_i(x)$, $h_j(x)$, $g_j(x)$ of the Pareto-optimal solution. Values of the estimates of criteria $f_i(x)$ are transmitted via the local-area network to an intelligent interactive terminal. This terminal is used for interactive linguistic estimation, and generates the control of the exchange s_i, which is transmitted over the network to the master computer. Network implementation of the interactive multicriterial optimization algorithm also permitted a high rate of interaction at each step of the d-cycle, even in the case of control variable x of large dimension and complex functions $f_i(x)$; in conjunction with linguistic evaluation, network implementation can provide a "friendlier" relationship between computer and designer.

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