A Consensus Model for Multiperson Decision Making With Different Preference Structures

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***Abstract*—In this paper, we present a consensus model for multiperson decision making (MPDM) problems with different preference structures based on two consensus criteria: 1) a consensus measure which indicates the agreement between experts' opinions and 2) a measure of proximity to find out how far the individual opinions are from the group opinion. These measures are calculated by comparing the positions of the alternatives between the individual solutions and collective solution. In such a way, the consensus situation is evaluated in each moment in a more realistic way. With these measures, we design a consensus support system that is able to substitute the actions of the moderator. In this system, the consensus measure is used to guide the consensus process until the final solution is achieved while the proximity measure is used to guide the discussion phases of the consensus process. The consensus support system has a feedback mechanism to guide the discussion phases based on the proximity measure. This feedback mechanism is based on simple and easy rules to help experts change their opinions in order to obtain a degree of consensus as high as possible. The main improvement of this consensus model is that it supports consensus process automatically, without moderator, and, in such a way, the possible subjectivity that the moderator can introduce in the consensus process is avoided.**

***Index Terms*—Consensus, fuzzy preference relations, multiperson decision making (MPDM), multiplicative preference relations, preference orderings, utility functions.**

I. Introduction

In multiperson decision making (MPDM) problems there are two processes to carry out before obtaining a final solution [7], [23]: 1) *the consensus process* and 2) *the selection process*. The first process refers to how to obtain the maximum degree of consensus or agreement between the set of experts on the solution set of alternatives, while the second process consists of how to obtain the solution set of alternatives from the opinions on the alternatives given by the experts. Clearly, it is preferable that the set of experts reach a high degree of consensus on the solution set of alternatives.

We consider an MPDM problem where the information about the alternatives provided by the experts can be represented using preference orderings, utility functions, fuzzy preference relations, and multiplicative preference relations. The selection process to such an MPDM problem is presented in [4] and [5]. In this paper, we present a consensus process for this MPDM problem.

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Consensus has become a major area of research in MPDM [1]–[3], [7]–[9], [11], [13], [14], [17], [26]. Naturally, at the beginning of every MPDM problem, experts' opinions may differ substantially. Therefore, it is necessary to develop a consensus process in an attempt to obtain a solution of consensus. Classically, consensus is defined as the full and unanimous agreement of all the experts regarding all the possible alternatives. This definition is inconvenient for our purposes for two reasons.

1) First, it only allows us to differentiate between two states, namely, the existence and absence of consensus.
2) Second, the chances for reaching such a full agreement are rather low.

Furthermore, complete agreement is not necessary in real life. This has led to the use and definition of a new concept of consensus degree, which is called "soft" consensus degree [7], [9], [11].

On the one hand, using such a soft consensus measure, the consensus process is defined as a dynamic and iterative group discussion process, coordinated by a moderator, who helps the experts to make their opinions closer. In each step of this process, the moderator knows the actual level of consensus between the experts, by means of the consensus measure, which establishes the distance to the ideal state of consensus. If the consensus level is not acceptable, that is, if it is lower than a specified threshold, which means that there exists a great discrepancy between the experts' opinions, then the moderator would urge the experts to discuss their opinions further in an effort to make them closer. On the contrary, when the consensus level is acceptable, the moderator would apply the selection process in order to obtain the final consensus solution to the MPDM problem [2], [26]. In this framework, a question to solve is how to substitute the actions of the moderator in the group discussion process in order to model automatically the whole consensus process.

On the other hand, soft consensus measures are usually calculated by using only the opinions given by the experts [7], [9], [10], [17], [26] or the choice degrees of alternatives obtained from those opinions [2]. In such a case, a soft consensus measure is defined by measuring the coincidence or the distance between them calculated, e.g., by means of the Euclidean distance. The problem of these consensus approaches is that the use of the opinions or choice degrees to calculate the consensus measure can withhold information on the real consensus situation. For example, we can find that different opinions or choice degrees can lead to a low consensus degree although they represent the same solution set of alternatives.

The aim of this paper is to present a consensus model for MPDM problems under different preference structures (preference orderings, utility functions, fuzzy preference relations, and multiplicative preference relations) that overcomes the aforementioned drawbacks. We propose a consensus model based on two consensus criteria.

1) *A consensus measure.* This measure evaluates the agreement of all the experts. It is used to guide the consensus process until the final solution is achieved.

2) *A proximity measure.* This measure evaluates the agreement between the experts' individual opinions and the group opinion. It is used to guide the group discussion in the consensus process.

Both measures are based on the comparison of the individual solutions and the collective solution. This comparison is done by comparing not the opinions or choice degrees but the position of the alternatives in each solution, what allows us to reflect the real consensus situation in each moment of the consensus process. This means that the first thing to do in each step of the consensus process is to apply the selection process to obtain a temporary collective solution, and measure how close the individual solutions are to it. Furthermore, in this consensus model, we define a consensus support system which substitutes the moderator's actions in the group discussion process. This system is based on both the consensus measure and the proximity measure. The system checks in each step of the group discussion process the consensus situation by means of the consensus measure. As part of it, a feedback mechanism is given to help the experts change their opinions on the alternatives and know the direction of that change in the group discussion process. The proximity measure is used as the main feedback information in the control system of the consensus process. The feedback mechanism consists of simple and easy rules that generate the recommendations in the group discussion process. In such a way, we obtain a consensus process that is controlled automatically without using the moderator.

This paper is organized as follows. The MPDM problem with different preference structures is briefly described in Section II. Section III deals with the consensus model. A practical example is given in Section IV. Finally, in Section V, we draw our conclusions.

II. MPDM Problem With Different Preference Structures

This section briefly describes the MPDM problem with multiple preference structures and the resolution process used to obtain the solution set of alternatives.

A. MPDM Problem

Let $X = \{x_1, \ldots, x_n\}$ be a finite set of alternatives. These alternatives have to be classified from best to worst, using the information given by a finite set of experts $E = \{e_1, e_2, \ldots, e_m\}$. As each expert $e_k \in E$ has their own ideas, attitudes, motivations, and personality, it is quite natural to consider that different experts will give their preferences in a different way. This leads us to assume that the experts' preferences over the set of alternatives X may be represented in one of the following four ways.

1) Preference Ordering of the Alternatives: In this case, an expert e_k gives his preferences on X as an individual preference ordering $O^k = \{o^k(1), \ldots, o^k(n)\}$, where $o^k(\cdot)$ is a permutation function over the index set $\{1, \ldots, n\}$ [4], [21]. Therefore, according to this point of view, an ordered vector of alternatives, from best to worst, is given.

2) Fuzzy Preference Relation: In this case, the expert's preferences on X are described by a fuzzy preference relation $P^k \subset X \times X$, with membership function $\mu_{P^k}\colon X \times X \longrightarrow [0, 1]$, where $\mu_{P^k}(x_i, x_j) = p_{ij}^k$ denotes the preference degree or intensity of the alternative x_i over x_j [8], [12], [22]: $p_{ij}^k = 1/2$ indicates indifference between x_i and x_j, $p_{ij}^k = 1$ indicates that x_i is unanimously preferred to x_j, and $p_{ij}^k > 1/2$ indicates that x_i is preferred to x_j. It is usual to assume that $p_{ij}^k + p_{ji}^k = 1$ and $p_{ii}^k = 1/2$ [18], [22].

3) Multiplicative Preference Relation: In this case, the expert's preferences on X are described by a positive preference relation $A^k \subset X \times X$, $A^k = (a_{ij}^k)$, where a_{ij}^k indicates a ratio of the preference intensity of alternative x_i to that of x_j, i.e., it is interpreted as x_i is a_{ij}^k times as good as x_j. According to Miller's study [16], Saaty suggests measuring a_{ij}^k using a ratio scale, and in particular, the 1 to 9 scale [20]: $a_{ij}^k = 1$ indicates indifference between x_i and x_j, $a_{ij}^k = 9$ indicates that x_i is unanimously preferred to x_j, and $a_{ij}^k \in \{2, 3, \ldots, 8\}$ indicates intermediate evaluations. It is usual to assume the multiplicative reciprocity property $a_{ij}^k \cdot a_{ji}^k = 1 \, \forall \, i, j$.

4) Utility Function: In this case, an expert e_k gives his preferences on X as a set of n utility values $U^k = \{u_i^k;\ i = 1, \ldots, n\}$, $u_i^k \in [0, 1]$, where u_i^k represents the utility evaluation given by the expert e_k to the alternative x_i [15], [22].

B. Resolution Process of the MPDM Problem

In this context, the resolution process of the MPDM problem consists of obtaining a set of solution alternatives, $X_{\text{sol}} \subset X$, from the preferences given by the experts. As we assume that the experts give their preferences in different ways, the first step must be to obtain a uniform representation of the preferences. As was pointed out in [4] and [5], we consider fuzzy preference relation as the base to uniform the information. Once this uniform representation has been achieved, we can apply a selection process to obtain the solution set of alternatives. This resolution process is represented in Fig. 1.

This resolution process is developed in the following two steps [4]:

Step 1) *making the information uniform;*
Step 2) *the application of a selection process.*

1) Making the Information Uniform: As was aforementioned, due to their apparent merits, we propose to use fuzzy preference relations as the base element of the uniform representation. The use of fuzzy preference relations in decision making situations to represent an expert's opinion about a set of alternatives appears to be a useful tool in modeling decision processes, especially when we want to aggregate experts' preferences into group preferences. To make the information uniform, it is necessary to obtain transformation functions which relate the different preference structures with fuzzy preference relations. These transformation functions derive

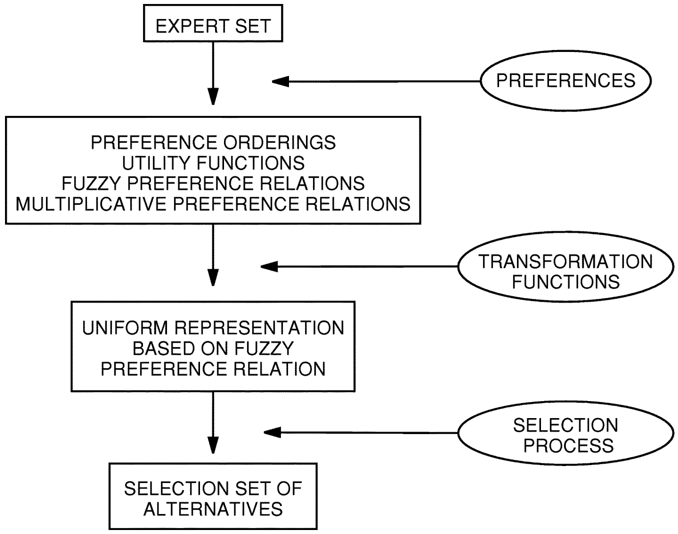

Fig. 1. Diagram of the MPDM resolution process.

an individual fuzzy preference relation from each preference structure. In [4] we studied the transformation function of preference ordering and utility values into fuzzy preference relations. This study can be summarized in the following proposition.

Proposition 1: Suppose that we have a set of alternatives $X = \{x_1, \ldots, x_n\}$, and λ_i^k represents an evaluation associated with alternative x_i, indicating the performance of that alternative according to a point of view (expert or criteria) e_k. Then, the intensity of preference of alternative x_i over alternative x_j, p_{ij}^k, for e_k is given by the following transformation function:

$$p_{ij}^k = \varphi\left(\lambda_i^k, \lambda_j^k\right) = \tfrac{1}{2}\left[1 + \psi\left(\lambda_i^k, \lambda_j^k\right) - \psi\left(\lambda_i^k, \lambda_j^k\right)\right]$$

where ψ is a function verifying

1) $\psi(z, z) = 1/2, \forall\, z \in R$
2) ψ is nondecreasing in the first argument and nonincreasing in the second argument.

For example, if $\lambda_i^k = o_i^k$ represents the preference ordering of the alternative x_i and $\psi(x, y) = ((y-x)/2(n-1))$, then $p_{ij}^k = f^1(o_i^k, o_j^k) = (1/2)(1 + (o_j^k - o_i^k)/(n-1))$, and if $\lambda_i^k = u_i^k$ represents the utility value of the alternative x_i and $\psi(x, y) = (x^2/x^2+y^2)$, then $p_{ij}^k = f^2(u_i^k, u_j^k) = (u_i^k)^2/((u_i^k)^2+(u_j^k)^2)$.

In [5], we obtained the transformation function of multiplicative preference relation into fuzzy preference relations. The result obtained is summarized in the following proposition.

Proposition 2: Suppose that we have a set of alternatives $X = \{x_1, \ldots, x_n\}$, and associated with it a multiplicative preference relation $A^k = (a_{ij}^k)$. Then, the corresponding additive fuzzy preference relation $P^k = (p_{ij}^k)$, associated with A^k is given as follows:

$$p_{ij}^k = f\left(a_{ij}^k\right) = \tfrac{1}{2}\left(1 + \log_9 a_{ij}^k\right).$$

2) Application of a Selection Process: Once the information is uniformed, we have a set of m individual fuzzy preference relations and then we apply a selection process which has two phases [4], [19], [23]:

1) *aggregation;*
2) *exploitation.*

1. Aggregation Phase

This phase defines a collective preference relation $P^c = (p_{ij}^c)$ obtained by means of the aggregation of all individual fuzzy preference relations $\{P^1, P^2, \ldots, P^m\}$, and indicates the global preference between every ordered pair of alternatives according to the majority of experts' opinions. The aggregation operation is carried out by means of an OWA operator ϕ_Q [24]

$$p_{ij}^c = \phi_Q\left(p_{ij}^1, \ldots, p_{ij}^m\right) = \sum_{k=1}^{n} w_k \cdot p_{ij}^k$$

where Q is a fuzzy linguistic quantifier [27] that represents the concept of fuzzy majority and it is used to calculate the weighting vector of ϕ_Q, $W = (w_1, \ldots, w_n)$ such that $w_k \in [0, 1]$ and $\sum_{k=1}^n w_k = 1$, according to the following expression [24]: $w_k = Q(k/n) - Q((k-1)/n),\ k = 1, \ldots, n$. Some examples of linguistic quantifiers are *"most," "at least half," "as many as possible,"* defined by the parameters (a, b), (0.3, 0.8), (0, 0.5), and (0.5, 1), respectively, according to the following expression:

$$Q(r) = \begin{cases} 0, & \text{if } r < a \\ \dfrac{r-a}{b-a}, & \text{if } a \le r \le b \\ 1, & \text{if } r > b \end{cases}$$

with $a, b, r \in [0, 1]$.

2. Exploitation Phase

This phase transforms the global information about the alternatives into a global ranking of them, from which the set of solution alternatives is obtained. The global ranking is obtained applying two choice degrees of alternatives to the collective fuzzy preference relation: the *quantifier guided dominance degree* and the *quantifier guided nondominance degree*.

1) *Quantifier Guided Dominance Degree:* For the alternative x_i we calculate the quantifier guided dominance degree $QGDD_i$, used to quantify the dominance that alternative x_i has over all the others in a fuzzy majority sense as follows:

$$QGDD_i = \phi_Q\left(p_{ij}^c,\ j = 1, \ldots, n\right).$$

2) *Quantifier Guided Nondominance Degree:* We also calculate the quantifier guided nondominance degree $QGNDD_i$, according to the following expression:

$$QGNDD_i = \phi_Q\left(1 - p_{ji}^s,\ j = 1, \ldots, n\right)$$

where $p_{ji}^s = \max\{p_{ji}^c - p_{ij}^c, 0\}$ represents the degree to which x_i is strictly dominated by x_j. In our context, $QGNDD_i$ gives the degree in which each alternative is not dominated by a fuzzy majority of the remaining alternatives.

Finally, the solution X_{sol} is obtained by applying these two choice degrees, and thus, choosing those alternatives with maximum choice degrees.

III. Consensus Model

In this section, we present a consensus model defined for MPDM problems with different preference structures, which is defined assuming the consensus as a measurable parameter whose highest value corresponds to unanimity and lowest one to complete disagreement. This model presents the following main characteristics:

1) It is based on two soft consensus criteria: a *consensus measure* and a *proximity measure*.
2) Both consensus criteria are defined by comparing the individual solutions with the collective solution using as comparison criterion the positions of the alternatives in each solution.
3) A *consensus support system* is defined using the above consensus criteria and a *feedback mechanism* which is able to substitute the moderators actions in the consensus reaching process.

Initially, in this consensus model we consider that in any nontrivial MPDM problem, the experts disagree in their opinions so that consensus has to be viewed as an iterate process, which means that agreement is obtained only after many rounds of consultation. Then, in each round, the consensus support system calculates two consensus parameters: a consensus measure and a proximity measure. To do so, the consensus model takes into account the selection process to obtain the individual solutions and the collective solution from the different experts' preferences. The consensus measure guides the consensus process and the proximity measure supports the group discussion phase of the consensus process. The main problem is finding a way of making individual positions converge and, therefore, how to support the experts in obtaining and agreeing with a particular solution. To do this, a consensus level (CL) required for that solution is fixed in advance. When the consensus measure reaches this level, the decision making session is finished and the solution is obtained. If that is not the case, the experts' opinions must be modified. This is done in a group discussion session in which the consensus support system uses a proximity measure to propose a feedback mechanism based on simple rules of generation of recommendations which supports the experts in changing their opinions. In order to avoid that the collective solution does not converge after several discussion rounds we incorporate in the consensus support system a maximum number of rounds to develop, MAXCYCLE, which was done in the consensus model proposed in [2].

This consensus model for MPDM problems with the four different preference structures is presented in Fig. 2. It will be described in further detail in the following subsections.

A. Consensus and Proximity Measures

Each consensus parameter requires the use of a dissimilarity function $d(V^i, V^c)$ to obtain the level of agreement between the individual solution of expert e_i, $V^i = (V_1^i, \ldots, V_n^i)$, where V_j^i is the position of alternative x_j for the ith expert, and the collective solution $V^c = (V_1^c, \ldots, V_n^c)$, where V_j^c is the position of alternative x_j in that collective solution. Several measures have been proposed, including the Euclidean distance, L-1-norm distance, the cosine and sine of the angle between the vectors, etc. Such measures were applied to the degrees associated with the alternatives [26]. As was mentioned earlier, we are not using these degrees to obtain our consensus indicators, but their actual position in the preference vector, because identical rankings of alternatives can have different choice degree vectors associated

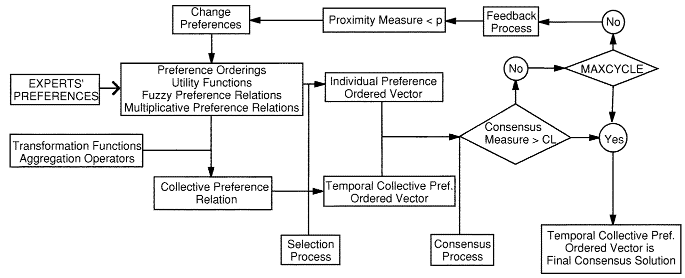

Fig. 2. Consensus model for MPDM with different preference structures.

with them. For example, if we had to compare the following two ordered vectors of alternatives [(3,0.8),(1,1),(2,0.9),(4,0.4)] and [(3,0.4),(1,0.8),(2,0.5),(4,0.1)], where (3,0.4) in the last ordered vector of alternatives means that the first alternative is ranked in position 3 with a choice degree value of 0.4, then both vectors have the alternatives in the same positions, but with different choice degrees. If we use the choice degrees to compare both solutions, then consensus is not obtained in the maximum degree, although we would consider this situation as a full consensus one. That is why in this MPDM, with these four different preference structures, we use the position of alternatives in the solution vectors of alternatives to calculate both the consensus measure and the proximity measure rather than the choice degrees.

Therefore, we define consensus indicators by comparing positions of alternatives in two preferences vectors as follows.

1) Due to the fact that we have different preference structures, we use our selection process described in the previous section, to obtain a collective ordered vector of alternatives ("temporary" collective solution) V^c.

2) We calculate the ordered vector of alternatives (individual solution) for every expert $\{V^i;\ i = 1, \ldots, m\}$. This is obvious when preferences are given as a preference ordering or utility values. When preferences are given as a fuzzy preference relation then we apply the same selection process that was applied to obtain the collective solution. When preferences are given by a multiplicative preference relation, we transform it into a fuzzy preference relation and then we act as explained before.

Remark 1: We point out that when at the end of the selection process we have alternatives with equal choice degrees then we would assign those alternatives the same position in the ordered vector of alternatives. In such a way, we model the indifference situations among alternatives.

3) We calculate the proximity of each expert for each alternative, called $p_i(x_j)$ by comparing the position of that alternative in the experts' individual solution and in the collective solution. This comparison has to be done by using a function $p_i(x_j) = p(V^i, V^c)(x_j) = f(|V_j^c - V_j^i|)$ that reflects the proximity of both positions. This implies that function f must be an increasing function. As a general dissimilarity function, we consider $f(x) = (a \cdot x)^b$, $1 \geq b \geq 0$, and in particular we use that function taking $a = 1/(n-1)$

$$p_i(x_j) = p(V^i, V^c)(x_j) = f(|V_j^c - V_j^i|) = \left(\frac{|V_j^c - V_j^i|}{n-1}\right)^b \in [0, 1].$$

The parameter b controls the rigorousness of the consensus process, in such a way, that values of b close to one decrease the rigorousness and thus the number of rounds to develop in the group discussion process, and values of b close to zero increase the rigorousness and thus, the number of rounds. Appropriate values for b are: 0.5, 0.7, 0.9, 1.

When using this dissimilarity function we observe that the values we obtain are higher when the difference between the position of alternatives in the individual solution and the temporary collective solution increase. We will show this in the next section, where we will calculate consensus using three different values (1, 1/2, 1/3) of constant b.

4) We calculate the consensus degree of all experts on each alternative x_j using the following expression:

$$C(x_j) = 1 - \sum_{i=1}^{m} \frac{p_i(x_j)}{m}.$$

5) The consensus measure over the set of alternatives, called C_X, will be calculated by the aggregation of the above consensus degrees on the alternatives. We consider that it is important to do this aggregation in such a way that the consensus degrees about the solution set of alternatives has to take a more important weight in this aggregation. An aggregation operator that allows this type of aggregation is the S-OWA OR-LIKE operator defined by Yager and Filev [25]

$$\begin{aligned} C_X &= S_{OWA\,OR\text{-}LIKE}(\{C(x_s); x_s \in X_{\text{sol}}\} \\ &\quad \{C(x_t); x_t \in X - X_{\text{sol}}\}), \\ &= (1-\beta) \cdot \sum_{t=1}^{\nu} \frac{C(x_t)}{\nu} + \beta \cdot \sum_{s=1}^{\gamma} \frac{C(x_s)}{\gamma} \end{aligned}$$

where γ is the cardinal of the set X_{sol}; ν is the cardinal of the set $X - X_{\text{sol}}$; and $\beta \in [0, 1]$. β is a parameter to control the OR-LIKE behavior of the aggregation operator. In our case it is used to control the influence of the consensus degrees of the solution alternatives over the consensus measure on the set of alternatives. The higher the value of β, the higher the influence of the consensus degrees of the solution alternatives on the global consensus degree. Some adequate values for β are 0.7, 0.8, and 0.9.

Obviously, the value of C_X depends on the choice of the OWA operator applied in the selection process and of the S-OWA OR-LIKE operator applied to obtain it, especially in the first steps of the consensus process, i.e., when the difference between experts' preferences is high, but we will omit any explicit reference to them in the notation of C_X.

6) The proximity measure of ith expert's individual solution to the collective temporary solution, called P_X^i, is calculated by aggregating the proximity of that expert in the alternatives, doing this aggregation in a similar way as in the calculation of the consensus measure, i.e., using an S-OWA OR-LIKE operator

$$\begin{aligned} P_X^i = S_{OWA\,OR-LIKE}(&\{1 - |p_i(x_s)|; x_s \in X_{\text{sol}}\}, \\ &\{1 - |p_i(x_t)|; x_t \in X - X_{\text{sol}}\}). \end{aligned}$$

When the proximity value associated with ith expert is close to one, this means that his contribution to the consensus is high (positive), while if it is close to zero, then that expert has a negative contribution to consensus.

B. Feedback Mechanism

When the consensus measure C_X has not reached the consensus level required and the number of rounds has not reached MAXCYCLE, then the experts' opinions must be modified. As was stated earlier, we are using the proximity measures $(p_i(x_j), P_X^i)$ to build a feedback mechanism so that experts can change their opinions in order to get closer opinions between them. This feedback mechanism will be applied when the consensus level is not satisfactory and MAXCYCLE is not reached, and will be ceased when a satisfactory consensus level is reached or the number of rounds reaches MAXCYCLE.

The rules of this feedback mechanism will be easy to understand and to apply, and will be expressed in the following form: "If proximity of alternative $p_i(x_j)$ is positive then its evaluation will decrease," and it will be carried out in the following way:

1) Each expert e_i is classified from first to last by associating them to their respective total proximity measure P_X^i. Each expert is given his position and his proximity in each alternative.

2) If the expert's position in the ranking is high (first, second, etc.) then that expert does not change his opinion much, but if it is low (last) then that expert has to change his opinion substantially. In other words, the first experts to change their opinions are those whose individual solutions are furthest from the collective temporary solution. At this point, we have to decide a threshold to calculate how many experts have to change their opinions, i.e., we need a rule like: "If $P_X^i < p$, $p \in [0, 1]$ then change your opinion."

3) The opinions will be changed using the following three rules:

R.1. If $V_j^c - V_j^i < 0$, then increase evaluations associated with alternative x_j.

R.2. If $V_j^c - V_j^i = 0$, do not change evaluations associated with alternative x_j.

R.3. If $V_j^c - V_j^i > 0$, then decrease evaluations associated with alternative x_j.

Obviously, the consensus reaching process will depend on the size of the group of experts as well as on the size of the set of alternatives, so that when these sizes are small and when opinions are homogeneous, the consensus level required is easier to obtain. On the other hand, we note that the change of opinion can produce a change in the temporary collective solution, especially when the experts opinions are quite different, i.e., in the early stages of the consensus process. In fact, when experts opinions are close, i.e., when the consensus measure approaches the consensus level required, changes in experts' opinions will not affect the temporary collective solution; it will only affect the consensus measure. This is a convergent process to the collective solution, once the consensus measure is high "enough." This will be illustrated with a practical example in the next section.

IV. Practical Example

One of the biggest problems present today in the classroom is misbehavior. To find out the causes of this misbehavior and the influence these have on it is of interest to teachers and, in general, to anyone involved in education (the Education Department, parents, etc.). Cohen *et al.* [6] quote a study in which a sample of teachers in different English comprehensive schools were asked to rate a few given causes of disruptive behavior. Among these causes are

- C_1 unsettled home environment;
- C_2 lack of interest in subject or general disinterest in school;
- C_3 pupil psychological or emotional instability;
- C_4 lack of self-esteem;
- C_5 dislike of teacher;
- C_6 use of drugs.

This list was presented to a group of eight Spanish secondary school teachers who were asked to give their opinions about them. Four different questionnaires were prepared, one for each different structure of preference. Teachers e_1 and e_2 gave their opinions by preference orderings, e_3 and e_4 by utility values, e_5 and e_6 by fuzzy preference relations and, finally, e_7 and e_8 by multiplicative preference relations.

Teachers' opinions were as follows:

$$e_1\colon O^1 = \{2, 1, 3, 6, 4, 5\},\ e_2\colon O^2 = \{1, 3, 4, 2, 6, 5\}$$

$$e_3\colon U^3 = \{0.3, 0.2, 0.8, 0.6, 0.4, 0.1\}$$

$$e_4\colon U^4 = \{0.3, 0.9, 0.4, 0.2, 0.7, 0.5\}$$

$$e_5\colon P^5 = \begin{pmatrix} 0.5 & 0.55 & 0.45 & 0.25 & 0.7 & 0.3 \\ 0.45 & 0.5 & 0.7 & 0.85 & 0.4 & 0.8 \\ 0.55 & 0.3 & 0.5 & 0.65 & 0.7 & 0.6 \\ 0.75 & 0.15 & 0.35 & 0.5 & 0.95 & 0.6 \\ 0.3 & 0.6 & 0.3 & 0.05 & 0.5 & 0.85 \\ 0.7 & 0.2 & 0.2 & 0.4 & 0.15 & 0.5 \end{pmatrix}$$

$$e_6\colon P^6 = \begin{pmatrix} 0.5 & 0.7 & 0.75 & 0.95 & 0.6 & 0.85 \\ 0.3 & 0.5 & 0.55 & 0.8 & 0.4 & 0.65 \\ 0.25 & 0.45 & 0.5 & 0.7 & 0.6 & 0.45 \\ 0.05 & 0.2 & 0.3 & 0.5 & 0.85 & 0.4 \\ 0.4 & 0.6 & 0.4 & 0.15 & 0.5 & 0.75 \\ 0.15 & 0.35 & 0.55 & 0.6 & 0.25 & 0.5 \end{pmatrix}$$

$$e_7\colon A^7 = \begin{pmatrix} 1 & 1/2 & 1/3 & 4 & 3 & 5 \\ 2 & 1 & 1/3 & 1/4 & 4 & 6 \\ 3 & 3 & 1 & 7 & 6 & 9 \\ 1/4 & 4 & 1/7 & 1 & 1/2 & 3 \\ 1/3 & 1/4 & 1/6 & 2 & 1 & 4 \\ 1/5 & 1/6 & 1/9 & 1/3 & 1/4 & 1 \end{pmatrix}$$

$$e_8\colon A^8 = \begin{pmatrix} 1 & 1/5 & 1/4 & 1/2 & 3 & 1/6 \\ 5 & 1 & 2 & 4 & 6 & 1/3 \\ 4 & 1/2 & 1 & 3 & 5 & 4 \\ 2 & 1/4 & 1/3 & 1 & 3 & 6 \\ 1/3 & 1/6 & 1/5 & 1/3 & 1 & 8 \\ 6 & 3 & 1/4 & 1/6 & 1/8 & 1 \end{pmatrix}.$$

First Stage in the Consensus Reaching Process

A) Consensus Measure

1) Using transformation functions $f^1(o_i^k, o_j^k) = 1/2(1 + (o_j^k - o_i^k/n-1))$ of preference ordering into fuzzy preference relation $f^2(u_i^k, u_j^k) = ((u_i^k)^2/(u_i^k)^2 + (u_j^k)^2)$ of utility values into fuzzy preference relations and $f^3(a_{ij}^k) = 1/2(1 + \log_9 a_{ij}^k)$ of multiplicative preference relation into fuzzy preference relation, to make the information uniform, we have

$$P^1 = \begin{pmatrix} 0.5 & 0.4 & 0.6 & 0.9 & 0.7 & 0.8 \\ 0.6 & 0.5 & 0.7 & 1 & 0.8 & 0.9 \\ 0.4 & 0.3 & 0.5 & 0.8 & 0.6 & 0.7 \\ 0.1 & 0 & 0.2 & 0.5 & 0.3 & 0.4 \\ 0.3 & 0.2 & 0.4 & 0.7 & 0.5 & 0.6 \\ 0.2 & 0.1 & 0.3 & 0.6 & 0.4 & 0.5 \end{pmatrix}$$

$$P^2 = \begin{pmatrix} 0.5 & 0.7 & 0.8 & 0.6 & 1 & 0.9 \\ 0.3 & 0.5 & 0.6 & 0.4 & 0.8 & 0.7 \\ 0.2 & 0.4 & 0.5 & 0.3 & 0.7 & 0.6 \\ 0.4 & 0.6 & 0.7 & 0.5 & 0.9 & 0.8 \\ 0 & 0.2 & 0.3 & 0.1 & 0.5 & 0.4 \\ 0.1 & 0.3 & 0.4 & 0.2 & 0.6 & 0.5 \end{pmatrix}$$

$$P^3 = \begin{pmatrix} 0.5 & 0.69 & 0.12 & 0.2 & 0.36 & 0.9 \\ 0.31 & 0.5 & 0.06 & 0.1 & 0.2 & 0.8 \\ 0.88 & 0.94 & 0.5 & 0.64 & 0.8 & 0.98 \\ 0.8 & 0.9 & 0.36 & 0.5 & 0.69 & 0.97 \\ 0.64 & 0.8 & 0.2 & 0.31 & 0.5 & 0.94 \\ 0.1 & 0.2 & 0.02 & 0.03 & 0.06 & 0.5 \end{pmatrix}$$

$$P^4 = \begin{pmatrix} 0.5 & 0.1 & 0.36 & 0.69 & 0.16 & 0.26 \\ 0.9 & 0.5 & 0.84 & 0.95 & 0.62 & 0.76 \\ 0.64 & 0.16 & 0.5 & 0.8 & 0.25 & 0.39 \\ 0.31 & 0.05 & 0.2 & 0.5 & 0.08 & 0.14 \\ 0.84 & 0.38 & 0.75 & 0.92 & 0.5 & 0.66 \\ 0.74 & 0.24 & 0.61 & 0.86 & 0.34 & 0.5 \end{pmatrix}$$

$$P^7 = \begin{pmatrix} 0.5 & 0.34 & 0.25 & 0.82 & 0.75 & 0.87 \\ 0.66 & 0.5 & 0.25 & 0.18 & 0.82 & 0.91 \\ 0.75 & 0.75 & 0.5 & 0.94 & 0.91 & 1 \\ 0.18 & 0.82 & 0.065 & 0.5 & 0.34 & 0.75 \\ 0.25 & 0.18 & 0.09 & 0.66 & 0.5 & 0.82 \\ 0.13 & 0.09 & 0 & 0.25 & 0.18 & 0.5 \end{pmatrix}$$

$$P^8 = \begin{pmatrix} 0.5 & 0.13 & 0.18 & 0.34 & 0.75 & 0.09 \\ 0.87 & 0.5 & 0.66 & 0.82 & 0.91 & 0.25 \\ 0.82 & 0.34 & 0.5 & 0.75 & 0.87 & 0.82 \\ 0.66 & 0.18 & 0.25 & 0.5 & 0.75 & 0.91 \\ 0.25 & 0.09 & 0.13 & 0.25 & 0.5 & 0.97 \\ 0.91 & 0.75 & 0.18 & 0.09 & 0.03 & 0.5 \end{pmatrix}.$$

Using the fuzzy majority criterion with the fuzzy quantifier *"most,"* with the pair $(0.3, 0.8)$, and the corresponding OWA operator with the weighting vector $W = [0, 0, 3/20, 1/4, 1/4, 1/4, 1/10, 0]$, the collective fuzzy preference relation is

$$P^c = \begin{pmatrix} 0.5 & 0.439 & 0.373 & 0.556 & 0.649 & 0.644 \\ 0.469 & 0.5 & 0.583 & 0.651 & 0.615 & 0.75 \\ 0.535 & 0.358 & 0.5 & 0.709 & 0.68 & 0.643 \\ 0.332 & 0.228 & 0.26 & 0.5 & 0.603 & 0.598 \\ 0.298 & 0.303 & 0.273 & 0.287 & 0.5 & 0.745 \\ 0.235 & 0.215 & 0.282 & 0.312 & 0.202 & 0.5 \end{pmatrix}.$$

We apply the exploitation process with the fuzzy quantifier *"as many as possible,"* with the pair $(0.5, 1)$, and the corresponding OWA operator with the weighting vector

$W = [0, 0, 0, 1/3, 1/3, 1/3]$. As we have shown in [5], when the information is consistent we obtain the same ordered vector of alternatives using dominance degree and nondominance degree, which are independent of the linguistic quantifier used. However, when the information (fuzzy preference relation or multiplicative preference relation) is not consistent then the application of both choice degrees can give different ordered vectors of alternatives. In a real situation preferences may not be consistent, therefore we apply only one choice degree, the dominance choice degree, to obtain the ordered vector of alternatives. The quantifier guided dominance degree of alternatives acting over the collective fuzzy preference relation supplies the following values:

	C_1	C_2	C_3	C_4	C_5	C_6
$QGDD_i$	0.437	0.517	0.464	0.273	0.286	0.217

These values represent the *dominance that one alternative has over "most" alternatives according to "as many teachers as possible."*

Clearly, the greatest influential cause of student misbehavior, according to this set of teachers, is C_2 and the collective order of causes of misbehavior is $\{C_2, C_3, C_1, C_5, C_4, C_6\}$.

2) On the other hand, the individual orders of causes of misbehavior, calculated using the same quantifier **"as many as possible,"** are the following:

e_1: $\{C_2, C_1, C_3, C_5, C_6, C_4\}$
e_2: $\{C_1, C_4, C_2, C_3, C_6, C_5\}$
e_3: $\{C_3, C_4, C_5, C_1, C_2, C_6\}$
e_4: $\{C_2, C_5, C_6, C_3, C_1, C_4\}$
e_5: $\{C_2, C_3, C_4, C_1, C_6, C_5\}$
e_6: $\{C_1, C_2, C_3, C_5, C_6, C_4\}$
e_7: $\{C_3, C_1, C_2, C_4, C_5, C_6\}$
e_8: $\{C_3, C_2, C_4, C_5, C_1, C_6\}$.

3) The differences between the ranking of causes in the temporary collective solution and the individual solution are as follows:

$V_j^c - V_j^i$	C_1	C_2	C_3	C_4	C_5	C_6
e_1	1	0	−1	−1	0	1
e_2	2	−2	−2	3	−2	1
e_3	−1	−4	1	3	1	0
e_4	−2	0	−2	−1	2	3
e_5	−1	0	0	2	−2	1
e_6	2	−1	−1	−1	0	−1
e_7	1	−2	1	1	−1	0
e_8	−2	−1	1	2	0	0

4) Consensus degrees on alternatives calculated for three different values of b are

	$C(C_1)$	$C(C_2)$	$C(C_3)$	$C(C_4)$	$C(C_5)$	$C(C_6)$
$b = 1$	0.7	0.75	0.775	0.65	0.8	0.825
$b = 1/2$	0.4602	0.6183	0.5624	0.4246	0.6510	0.6796
$b = 1/3$	0.3392	0.5536	0.4503	0.3125	0.5775	0.6022

5) Consensus measure calculated for three different values of b are

	C_X
$b = 1$	0.75
$b = 1/2$	$0.566 + 0.052\beta$
$b = 1/3$	$0.4726 + 0.0811\beta$

There is a great difference in the values obtained when the dissimilarity function is applied using different values of b. If we required a level of consensus of 0.75 then using the easiest dissimilarity function, i.e., $b = 1$, the consensus process would be stopped and this temporary collective solution would be the final consensual solution. In the other two cases, the consensus process should continue. If the individual solutions were observed, it could be deduced that there is a great discrepancy between them and therefore it would not be wise to stop the consensus process at this stage, because the collective collection does not represent the majority of individual solutions. For a β value of 0.8, the total consensus values are 0.75, 0.61, and 0.54, respectively.

B) Proximity Measures

	$b = 1$	$b = 1/2$	$b = 1/3$
	P_X^i	P_X^i	P_X^i
e_1	$0.87 + 0.13\beta$	$0.70 + 0.3\beta$	$0.61 + 0.39\beta$
e_2	0.6	0.37	$0.27 - 0.01\beta$
e_3	$0.67 - 0.47\beta$	$0.50 - 0.39\beta$	$0.41 - 0.34\beta$
e_4	$0.67 + 0.33\beta$	$0.48 + 0.52\beta$	$0.39 + 0.61\beta$
e_5	$0.8 + 0.2\beta$	$0.64 + 0.36\beta$	$0.56 + 0.44\beta$
e_6	0.8	$0.6 - 0.08\beta$	$0.49 - 0.07\beta$
e_7	$0.8 - 0.2\beta$	$0.6 - 0.23\beta$	$0.49 - 0.23\beta$
e_8	0.8	$0.64 - 0.09\beta$	$0.56 - 0.14\beta$

C) Feedback Process

C.1) Classification of Teachers

The ranking of teachers according to the proximity of their individual solutions to the temporary collective solutions is, for a β value of 0.8, the same in any of the three cases: $e_1, e_5, e_4, e_8, e_6, e_7, e_2, e_3$.

C.2) Changing the Opinions

At this point, each teacher is given his proximity value, and the values of the differences of positions between their individual solutions and the collective one. It is clear that teachers changing their preferences have to start in reverse order as the one given above, which means that e_3 was the first one requested to change his/her preferences. Three of the teachers were asked to change their opinions according to the rules proposed in Section III-B. For example:

1) the second teacher must increase his evaluation on C_2 according to rule R.1;
2) the second teacher must decrease his evaluation on C_1 according to rule R.3;
3) the third teacher must not change his evaluation on C_6 according to rule R.2.

Their new preferences are as follows:

$$e_2\colon O^2 = \{2,\, 1,\, 3,\, 4,\, 5,\, 6\}$$
$$e_3\colon U^3 = \{0.45,\, 0.5,\, 0.7,\, 0.4,\, 0.3,\, 0.1\}$$
$$e_7\colon A^7 = \begin{pmatrix} 1 & 2 & 1/3 & 4 & 2 & 4 \\ 1/2 & 1 & 3 & 2 & 5 & 8 \\ 3 & 1/3 & 1 & 7 & 4 & 7 \\ 1/4 & 1/2 & 1/7 & 1 & 2 & 2 \\ 1/2 & 1/5 & 1/4 & 1/2 & 1 & 5 \\ 1/4 & 1/8 & 1/7 & 1/2 & 1/5 & 1 \end{pmatrix}.$$

Second Stage in the Consensus Reaching Process

A) Consensus Measure

1) Using the corresponding transformation functions to make the information uniform and using the same fuzzy quantifier *"most"* as in first step, the collective fuzzy preference relation is

$$P^c = \begin{pmatrix} 0.5 & 0.408 & 0.390 & 0.645 & 0.627 & 0.634 \\ 0.524 & 0.5 & 0.675 & 0.799 & 0.711 & 0.824 \\ 0.544 & 0.301 & 0.5 & 0.733 & 0.683 & 0.693 \\ 0.277 & 0.178 & 0.243 & 0.5 & 0.618 & 0.56 \\ 0.304 & 0.215 & 0.271 & 0.313 & 0.5 & 0.756 \\ 0.248 & 0.124 & 0.236 & 0.359 & 0.194 & 0.5 \end{pmatrix}.$$

Applying the exploitation process with the same fuzzy quantifier *"as many as possible,"* the quantifier guided dominance degree of alternatives acting over the collective fuzzy preference relation supplies the following values:

	C_1	C_2	C_3	C_4	C_5	C_6
$QGDD_i$	0.433	0.566	0.448	0.229	0.263	0.185

Clearly, the greatest influential cause of student misbehavior, according to this set of teachers, is C_2, and the collective order of causes of misbehavior is $\{C_2, C_3, C_1, C_5, C_4, C_6\}$, which is the same temporary collective solution obtained before.

2) The individual solutions for these three new preferences are:

$$e_1\colon \quad \{C_2, C_1, C_3, C_4, C_5, C_6\}$$
$$e_3\colon \quad \{C_3, C_2, C_1, C_4, C_5, C_6\}$$
$$e_7\colon \quad \{C_2, C_3, C_1, C_5, C_4, C_6\}.$$

3) The differences between the ranking of causes in the temporary collective solution and the individual solution are as follows:

$V_j^c - V_j^i$	C_1	C_2	C_3	C_4	C_5	C_6
e_1	1	0	−1	−1	0	1
e_2	1	0	−1	1	−1	0
e_3	0	−1	1	1	−1	0
e_4	−2	0	−2	−1	2	3
e_5	−1	0	0	2	−2	1
e_6	2	−1	−1	−1	0	−1
e_7	0	0	0	0	0	0
e_8	−2	−1	1	2	0	0

4) Consensus degrees on alternatives calculated for three different values of b are

	$C(C_1)$	$C(C_2)$	$C(C_3)$	$C(C_4)$	$C(C_5)$	$C(C_6)$
$b = 1$	0.775	0.925	0.825	0.775	0.85	0.85
$b = 1/2$	0.5951	0.8323	0.6414	0.5624	0.7301	0.7355
$b = 1/3$	0.5044	0.7807	0.5424	0.4503	0.6696	0.6753

5) Consensus measure calculated for three different values of b are

	C_X
$b = 1$	$0.8121 + 0.1129\beta$
$b = 1/2$	$0.6828 + 0.1495\beta$
$b = 1/3$	$0.6038 + 0.1769\beta$

In this case, it is observed that five out of the total eight teachers think that cause C_2 is the most influential in student misbehavior, and this aspect is reflected in the consensus on that alternative, which ranges from a minimum of 0.78 to a maximum of 0.925. However, if we required a level of consensus of 0.75 then using the easiest dissimilarity function, i.e., $b = 1$, the consensus process would be stopped and this temporary collective solution would be the final consensual solution, because for a β value of 0.8, the total consensus values are 0.902 42, 0.8024, and 0.745 32, respectively, this last one being too close to the level of consensus required so that it is not worth having a third step in the consensus process.

As stated earlier, in the early stages of the consensus process, i.e., when the level of consensus is low, the temporary collective solution could change as experts' opinions change, while when the level of consensus is high then this process is a convergent process and the temporary collective solution does not change. In our example, we had the same temporary collective solution, but if teacher e_3 had provided the following utility values $\{0.55, 0.5, 0.6, 0.4, 0.3, 0.1\}$ instead of $\{0.45, 0.5, 0.7, 0.4, 0.3, 0.1\}$, then the temporary collective solution would have been $\{C_2, C_1, C_3, C_5, C_4, C_6\}$, that is, we would have had a different temporary collective solution in the second stage.

V. Conclusions

A consensus model for MPDM with different preference structures, preference orderings, utility values, fuzzy preference relations, and multiplicative preference relations, has been presented. The main improvement of this consensus model is that it presents a consensus support system to model the moderator's actions in the consensus reaching processes which guides the consensus process automatically. To do so, this system is based on two soft consensus criteria: 1) a consensus measure and 2) a proximity measure. Consensus measure evaluates the consensus situation in each moment and it is used to guide the consensus process. The proximity measure evaluates how far the individual experts' opinions are from the collective opinion and it is used to guide the group discussion session. The consensus support system has a feedback mechanism to generate recommendations in the group discussion process. The proximity measure is used to design this feedback mechanism. This mechanism is based on simple rules for changing the individual opinions in order to obtain a higher degree of consensus.

The consensus model has been illustrated using a real and practical example carried out with the collaboration of a group of Spanish secondary school teachers who were given a list of six reasons for disruptive behavior in the classroom.

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