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Building consensus in group decision making with an allocation of information granularity

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Abstract

Consensus is defined as a cooperative process in which a group of decision makers develops and agrees to support a decision in the best interest of the whole. It is a questioning process, more than an affirming process, in which the group members usually modify their choices until a high level of agreement within the group is achieved. Given the importance of forming an accepted decision by the entire group, the consensus problem has attained a great attention as it is a major goal in group decision making. In this study, we propose the concept of the information granularity being regarded as an important and useful asset supporting the goal to reach consensus in group decision making. By using fuzzy preference relations to represent the opinions of the decision makers, we develop a concept of a granular fuzzy preference relation where each pairwise comparison is formed as a certain information granule (say, an interval, fuzzy set, rough set, and alike) instead of a single numeric value. As being more abstract, the granular format of the preference model offers the required flexibility to increase the level of agreement within the group using the fact that we select the most suitable numeric representative of the fuzzy preference relation.
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1. Introduction

Group Decision Making (GDM) is a pervasive and critical activity within companies and organizations both in the public and private sectors [26]. Policies, budget plans, and other organizational tasks frequently involve group

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discussions or meetings due to their effectiveness in making decisions. Research in social psychology on group performance suggests that group tends to be more effective than direct aggregation of individual group members' choices and makes better decisions than the most highly skilled individual in a group [40].

A GDM situation involves multiple decision makers interacting to reach a decision. To do this, decision makers have to convey their preferences or opinions by means of a set of evaluations over a set of possible alternatives. An important issue here is the level of agreement achieved among the group members before making the decision. It is worth noting that when decisions are made by a group of decision makers, it is recommendable that they are engaged in a consensus process [4,37], in which all group members discuss their reasons for making decisions in order to arrive at a sufficient agreement that is acceptable (to the highest possible extent) to all. In essence, consensus aims at attaining the consent, not necessarily the agreement, of the decision makers by accommodating views of all parties involved to accomplish a decision that will yield. This decision will be beneficial to the whole group, not necessarily to the particular decision makers who may give consent to what will not necessarily be their first choice but because, for instance, they wish to cooperate with the group. The full consent, however, does not mean that each decision maker is in full agreement [4]. Therefore, reaching consensus does not assume that everyone must be in complete agreement, a highly unlikely situation in a group of intelligent, creative individuals.

In a GDM situation, a consensus process is usually defined as a negotiation process developed iteratively and composed by several consensus rounds, where the decision makers accept to change their preferences following some advice [4,17,37]. In the first consensus approaches proposed in the literature [9,14,15,22,23], the advice was provided by a moderator, which knows the agreement degree in each round of the consensus process by means of the computation of some consensus measures. However, as the moderator can introduce some subjectivity in the process, new consensus approaches have been proposed in order to make more effective and efficient the decision making process by substituting the moderator figure or providing to the moderator with better analysis tools [7,16,19,24,32]. Either way, several consensus rounds are usually required in order to achieve a sufficient agreement. As a result, the process of building consensus can take a considerable amount of time.

Independently of the source of the advice, it is easy to see that consensus requires that each member of the group has to allow a certain degree of flexibility and be ready to make an adjustment of his/her first choices and, here, information granularity [29–31] may come into play. Information granularity is an important design asset and may offer to each decision maker a real level of flexibility using some initial opinions expressed by each decision maker that can be modified with the intent to reach a higher level of consensus. Assuming that each decision maker expresses his/her preferences using a fuzzy preference relation, this required flexibility is brought into the fuzzy preference relations by allowing them to be granular rather than numeric. That is, we consider the entries of the fuzzy preference relations are not plain numbers but information granules, say intervals, fuzzy sets, rough sets, probability density functions, etc. In summary, information granularity that is present here serves as an important modeling asset, offering an ability of the decision maker to exercise some flexibility to be used in adjusting his/her initial position when becoming aware of the opinions of the other group members. To do so, the fuzzy preference relation is elevated (abstracted) to its granular format.

The aim of this study is to propose an allocation of information granularity as a key component to facilitate the achievement of consensus. In such a way, in the realization of the granular representation of the fuzzy preference relations, we introduce a certain level of granularity supplying the required flexibility to increase the level of consensus among the decision makers. This proposed concept of granular fuzzy preference relation is used to optimize a performance index, which comes as an additive combination of two components: (i) the first one quantifies the level of consensus within the group, and (ii) the second one expresses the level of consistency of the individual decision makers. Given the nature of the required optimization, the ensuing optimization problem is solved by engaging a machinery of population-based optimization, namely Particle Swarm Optimization (PSO) [25].

The study is arranged into five sections. We start with the presentation of the GDM scenario considered in this study. Furthermore, in this section, we describe both the method to obtain the level of consensus reached within the group and the procedure to obtain the consistency level achieved by an individual decision maker when expressing his/her opinions using fuzzy preference relations. Section 3 is concerned with the building of consensus through an allocation of information granularity. In addition, the use of PSO as the underlying optimization tool is described; strong attention is given to the content of the particles utilized in the method and a way in which the information granularity component is used in the adjustment of the single numeric values of the original fuzzy preference relations. To illustrate the method, an experimental study is reported in Section 4. Finally, we offer some conclusions and future works in Section 5.

2. Group decision making

In a classical GDM situation, there is a problem to solve, a solution set of possible alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ ($n \geq 2$), and a group of two or more decision makers, $E = \{e_1, e_2, \ldots, e_m\}$ ($m \geq 2$), characterized by their own motivations, attitudes, ideas and knowledge, who express their opinions about this set of alternatives to achieve a common solution [21]. The objective is to classify the alternatives from best to worst, associating with them some degrees of preference.

Among the different representation formats that decision makers may use to express their opinions, fuzzy preference relations [21,27,33] are one of the most used because of their effectiveness as a tool for modelling decision processes and their utility and easiness of use when we want to aggregate decision makers' preferences into group ones [21,38].

Definition 2.1. A fuzzy preference relation PR on a set of alternatives X is a fuzzy set on the Cartesian product $X \times X$, i.e., it is characterized by a membership function $\mu_{PR} : X \times X \to [0, 1]$.

A fuzzy preference relation PR may be represented by the $n \times n$ matrix $PR = (pr_{ij})$, being $pr_{ij} = \mu_{PR}(x_i, x_j)$ ($\forall i, j \in \{1, \ldots, n\}$) interpreted as the preference degree or intensity of the alternative x_i over x_j: $pr_{ij} = 0.5$ indicates indifference between x_i and x_j ($x_i \sim x_j$), $pr_{ij} = 1$ indicates that x_i is absolutely preferred to x_j, and $pr_{ij} > 0.5$ indicates that x_i is preferred to x_j ($x_i \succ x_j$). Based on this interpretation we have that $pr_{ii} = 0.5$, $\forall i \in \{1, \ldots, n\}$ ($x_i \sim x_i$). Since pr_{ii}'s (as well as the corresponding elements on the main diagonal in some other matrices) do not matter, we will write them as '–' instead of 0.5 [18,21]. When it is assumed that $pr_{ij} + pr_{ji} = 1$ ($\forall i, j \in \{1, \ldots, n\}$) the preference relation is called reciprocal preference relation and it is more easily interpreted as a stochastic relation [8,12,13,34]. However, as it is always not the case [3,18], this assumption is not made in this study.

In what follows, we are going to describe two important aspects which have to be taken into account when dealing with GDM situations: (i) the level of agreement or consensus achieved among the group of decision makers, and (ii) the level of consistency achieved by each decision maker in his/her opinions.

2.1. Level of agreement

Usually, GDM problems are faced by applying two different processes before a final solution can be given [2,23]: (i) the consensus process, which refers to how to obtain the maximum degree of consensus or agreement within the group of decision makers, and (ii) the selection process, which obtains the final solution according to the preferences given by the decision makers. The selection process involves two different steps [5,35]: aggregation of individual preferences and exploitation of the collective preference. Clearly, it is preferable that the decision makers had achieved a high level of consensus concerning their preferences before applying the selection process.

In order to evaluate the agreement achieved among the decision makers, we need to compute coincidence existing among them. Usual consensus approaches determine consensus degrees, which are used to measure the current level of consensus in the decision process, given at three different levels of a preference relation [6,14]: pairs of alternatives, alternatives, and relation. In such a way, once the fuzzy preference relations have been provided by the decision makers, the computation of the consensus degrees is carried out as follows:

1. For each pair of decision makers (e_k, e_l) ($k = 1, \ldots, m-1,\ l = k+1, \ldots, m$) a similarity matrix, $SM^{kl} = (sm^{kl}_{ij})$, is defined as:

$$sm^{kl}_{ij} = 1 - \left|pr^k_{ij} - pr^l_{ij}\right|$$

2. A consensus matrix, $CM = (cm_{ij})$, is calculated by aggregating all the $(m-1) \times (m-2)$ similarity matrices using the arithmetic mean as the aggregation function, ϕ, although different aggregation operators could be used depending on the nature of the GDM problem to solve:

$$cm_{ij} = \phi\left(sm^{kl}_{ij}\right), \quad k = 1, \ldots, m-1,\ l = k+1, \ldots, m$$

3. Once the consensus matrix has been computed, the consensus degrees are obtained at three different levels:

(a) *Consensus degree on pairs of alternatives.* The consensus degree on a pair of alternatives (x_i, x_j), called cp_{ij}, is defined to measure the consensus degree among all the decision makers on that pair of alternatives. In this case, this is expressed by the element of the collective similarity matrix CM:

$$cp_{ij} = cm_{ij}$$

(b) *Consensus degree on alternatives.* The consensus degree on the alternative x_i, called ca_i, is defined to measure the consensus degree among all the decision makers on that alternative:

$$ca_i = \frac{\sum_{j=1; j \neq i}^{n} (cp_{ij} + cp_{ji})}{2(n-1)}$$

(c) *Consensus degree on the relation.* The consensus degree on the relation, called cr, expresses the global consensus degree among all the decision makers' opinions. It is computed as the average of all the consensus degrees for the alternatives:

$$cr = \frac{\sum_{i=1}^{n} ca_i}{n}$$

The consensus degree of the relation, cr, is the value used to control the consensus situation. The closer cr is to 1, the greater the agreement among all the decision makers' opinions.

2.2. Level of consistency

When information is provided by individuals, an important issue to bear in mind is that of consistency [1,10,18]. Due to the complexity of most decision making problems, decision makers' preferences may not satisfy formal properties that fuzzy preference relations are required to verify. Consistency is one of them, and it is associated with the transitivity property.

Definition 2.1 dealing with a preference relation does not imply any kind of consistency property. In fact, preference values of a fuzzy preference relation can be contradictory. However, the study of consistency is crucial for avoiding misleading solutions in GDM [18].

To make a rational choice, properties to be satisfied by such fuzzy preference relations have been suggested [20]. In this paper, we make use of the additive transitivity property which facilitates the verification of consistency in the case of fuzzy preference relations. As it is shown in [20], additive transitivity for fuzzy preference relations can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [36]. The mathematical formulation of the additive transitivity was given by [38]:

$$(pr_{ij} - 0.5) + (pr_{jk} - 0.5) = (pr_{ik} - 0.5), \quad \forall i, j, k \in \{1, \dots, n\} \tag{1}$$

Additive transitivity implies additive reciprocity. Indeed, because $pr_{ii} = 0.5$, $\forall i$, if we make $k = i$ in Eq. (1) then we have: $pr_{ij} + pr_{ji} = 1$, $\forall i, j \in \{1, \dots, n\}$.

Eq. (1) can be rewritten as follows:

$$pr_{ik} = pr_{ij} + pr_{jk} - 0.5, \quad \forall i, j, k \in \{1, \dots, n\} \tag{2}$$

A fuzzy preference relation is considered to be "additively consistent" when for every three options encountered in the problem, say $x_i, x_j, x_k \in X$, their associated preference degrees, $pr_{ij}, pr_{jk}, pr_{ik}$, fulfill Eq. (2).

Given a fuzzy preference relation, Eq. (2) can be used to calculate an estimated value of a preference degree using other preference degrees. Indeed, using an intermediate alternative x_j, the following estimated value of pr_{ik} $(i \neq k)$ can be obtained in three different ways [18]:

- From $pr_{ik} = pr_{ij} + pr_{jk} - 0.5$ we obtain the estimate

$$(ep_{ik})^{j1} = pr_{ij} + pr_{jk} - 0.5 \tag{3}$$

- From $pr_{jk} = pr_{ji} + pr_{ik} - 0.5$ we obtain the estimate

$$(ep_{ik})^{j2} = pr_{jk} - pr_{ji} + 0.5 \tag{4}$$

- From $pr_{ij} = pr_{ik} + pr_{kj} - 0.5$ we obtain the estimate

$$(ep_{ik})^{j3} = pr_{ij} - pr_{kj} + 0.5 \tag{5}$$

Then, we can estimate the value of a preference p_{ik} according to the following expression:

$$ep_{ik} = \frac{\sum^{n}_{\substack{j=1\\ j\neq i,k}} ((ep_{ik})^{j1} + (ep_{ik})^{j2} + (ep_{ik})^{j3})}{3(n-2)} \tag{6}$$

When information provided is completely consistent then $(ep_{ik})^{jl} = pr_{ik}, \forall j, l$. However, because decision makers are not always fully consistent, the assessment made by a decision maker may not verify Eq. (2) and some of the estimated preference degree values $(ep_{ik})^{jl}$ may not belong to the unit interval [0, 1]. We note, from (3)–(5), that the maximum value of any of the preference degrees $(ep_{ik})^{jl}$ $(l \in \{1, 2, 3\})$ is 1.5 while the minimum one is −0.5. Taking this into account, the error between a preference value and its estimated one in [0, 1] is computed as follows [18]:

$$\varepsilon p_{ik} = \frac{2}{3} \cdot |ep_{ik} - pr_{ik}| \tag{7}$$

Thus, it can be used to define the consistency degree cd_{ik} associated to the preference degree pr_{ik} as follows:

$$cd_{ik} = 1 - \varepsilon p_{ik} \tag{8}$$

When $cd_{ik} = 1$, then $\varepsilon p_{ik} = 0$ and there is no inconsistency at all. The lower the value of cd_{ik}, the higher the value of εp_{ik} and the more inconsistent is pr_{ik} with respect to the rest of information.

In the following, we define the consistency degrees associated with individual alternatives and the overall fuzzy preference relation:

- The consistency degree, $cd_i \in [0, 1]$, associated to a particular alternative x_i of a fuzzy preference relation is defined as:

$$cd_i = \frac{\sum^{n}_{k=1; i\neq k} (cd_{ik} + cd_{ki})}{2(n-1)} \tag{9}$$

- The consistency degree, $cd \in [0, 1]$, of a fuzzy preference relation is defined as follows:

$$cd = \frac{\sum^{n}_{i=1} cd_i}{n} \tag{10}$$

When $cd = 1$, the fuzzy preference relation is fully consistent. Otherwise, the lower cd the more inconsistent the fuzzy preference relation is.

3. Building consensus through an allocation of information granularity

Building consensus is about arriving a solution that each decision maker is comfortable with. It is needless to say that this state calls for some flexibility exhibited by all members of the group, who in the name of cooperative pursuits give up their initial opinions and show a certain level of elasticity.

In a GDM problem in which the decision makers communicate their opinions using fuzzy preference relations, these changes of opinions are expressed through alterations of the entries of the fuzzy preference relations. That is, if the pairwise comparisons of the fuzzy preference relations are not treated as single numeric values, which are rigid, but rather as information granules, this will bring the essential factor of flexibility. It means that the fuzzy preference relation is abstracted to its granular format. The notation $\boldsymbol{G}(PR)$ is used to emphasize the fact that we are interested in granular fuzzy preference relations, where $\boldsymbol{G}(.)$ represents a specific granular formalism being used here (for instance, intervals, fuzzy sets, rough sets, probability density functions, and alike). In this manner, we introduce the concept of granular fuzzy preference relation and accentuate a role of information granularity being regarded here as an important conceptual and computational resource which can be exploited as a means to increase the level of consensus achieved among the decision makers. In summary, the level of granularity is treated as synonymous of the level of flexibility injected into the modeling environment, which makes easy the collaboration.

Obviously, the higher level of granularity is offered to the decision maker, the higher the feasibility of arriving at decisions accepted by all members of the group. Here, we appeal to the intuitive concept of granularity by trying to present a qualitative nature of the process in which the asset of granularity is involved. This idea can be formalized depending on the form of information granules being the entries of the fuzzy preference relations. In particular, in this study, the granularity of information is articulated through intervals and, therefore, the length of such intervals (entries of the fuzzy preference relations) can be sought as a level of granularity α. As here we are using interval-valued fuzzy preference relations, $\boldsymbol{G}(PR) = \boldsymbol{P}(PR)$, where $\boldsymbol{P}(.)$ denotes a family of intervals. The flexibility offered by the level of granularity can be effectively used to optimize a certain optimization criterion to capture the essence of the reconciliation of the individual preferences.

The formulation of the optimization problem needs to be now specified so that all technical details are addressed. In what follows, the optimization criterion which has to be optimized is given and its optimization using the PSO framework is described.

3.1. The optimization criterion

In the granular model of fuzzy preference relations, it is supposed that each decision maker feels equally comfortable when selecting any fuzzy preference relation whose values are placed within the bounds established by the fixed level of granularity α, which is used to increase the level of consensus within the group. However, we have to take into account that when the entries of the fuzzy preference relations are adjusting within the bounds offered by the admissible level of granularity in order to increase the level of agreement, it can produce some inconsistencies in the fuzzy preference relations. In particular, the higher the values of α, the higher the potential to reach a significant level of consensus and the higher the potential of producing some quite inconsistent fuzzy preference relations at the level of individual decision maker. Therefore, the level of granularity α is employed in two ways:

- It is used to increase the consensus within the group members by bringing all preferences close to each other. This goal is realized by maximizing the global consensus degree among all the decision makers' opinions, which is quantified in terms of the consensus degree on the relation described in Section 2.1:

$$Q_1 = cr \tag{11}$$

- It is used to increment the consistency of the fuzzy preference relations. This improvement is effectuate at the level of individual decision maker. The following performance index quantifies this effect:

$$Q_2 = \frac{1}{m}\sum_{l=1}^{m} cd_l \tag{12}$$

These are the two objectives to be maximized. If we consider the scalar version of the optimization problem, it arises in the following form:

$$Q = \delta \cdot Q_1 + (1-\delta) \cdot Q_2 \tag{13}$$

being $\delta \in [0, 1]$ a parameter to set up a tradeoff between the consensus obtained within the group and consistency level achieved at the individual decision maker. The higher the value of δ, the more attention is being paid to the consensus at the group level. In the limit, when $\delta = 0$, we are concerned with the consistency achieved at the level of individual decision maker only. Usually, $\delta > 0.5$ will be used to give more importance to the consensus criterion.

The overall optimization problem now reads as follows:

$$\text{Max}_{PR^1, PR^2, \ldots, PR^m \in \boldsymbol{P}(PR)}\, Q \tag{14}$$

The aforementioned maximization problem is carried out for all interval-valued fuzzy preference relations admissible because of the introduced level of information granularity α. This fact is underlined by including a granular form of the fuzzy preference relations allowed in the problem, i.e., $PR^1, PR^2, \ldots, PR^m$, are elements of the family of interval-valued fuzzy preference relations, namely, $\boldsymbol{P}(PR)$.

This optimization task is not an easy one. Because of the nature of the indirect relationship between optimized fuzzy preference relations, which are selected from a quite large search space formed by $\boldsymbol{P}(PR)$, it calls for the use of advanced techniques of global optimization, such as, e.g., genetic algorithms, evolutionary optimization, PSO, simulated annealing, ant colonies, and the like. In particular, here the optimization of the fuzzy preference relations, coming from the space of interval-valued fuzzy preference relations, is realized by means of the PSO, which is a viable optimization alternative for this problem, as it offers a substantial level of optimization flexibility and does not come with a prohibitively high level of computational overhead as this is the case of other techniques of global optimization (say, genetic algorithms). Obviously, one could think of the usage of some other optimization mechanisms as well.

In what follows, we briefly recall the essence of the method and associate the generic representation scheme of the PSO with the format of the problem at hand.

3.2. PSO as a vehicle of optimization of fuzzy preference relations

PSO is a population-based stochastic optimization technique developed by Kennedy and Eberhart [25], which is inspired by social behavior of bird flocking or fish schooling. A particle swarm is a population of particles, which are possible solutions to an optimization problem located in the multidimensional search space [11,25,39]. Each particle explores the search space and during this search adheres to some quite intuitively appealing guidelines navigating the process: (i) it tries to follow its own previous direction, and (ii) it looks back at the best performance reported both at the level of the individual particle as well as the entire population. Based on the history, it changes its velocity and moves to the next position, which looks the most promising. In this search, the algorithm exhibits some societal aspects meaning that there is some collective search of the problem space. The method is equipped with some component of memory (expressed in terms of the previous velocity) incorporated as an integral part of the search mechanism.

The optimization of the fuzzy preference relations coming from the space of interval-valued fuzzy preference relations is realized by means of the PSO. In the following, we elaborate on the fitness function, its realization, and the PSO optimization along with the corresponding formation of the components of the swarm.

3.2.1. Particle

In a PSO algorithm, an important point is finding a suitable mapping between problem solution and the particle's representation. Here, each particle represents a vector whose entries are located in the interval [0, 1]. Basically, if there is a group of m experts and a set of n alternatives, the number of entries of the particle is $m \cdot n(n-1)$.

Starting with the initial fuzzy preference relation provided by the expert and assuming a given level of granularity α (located in the unit interval), let us consider an entry pr_{ij}. The interval of admissible values of this entry of $\boldsymbol{P}(PR)$ implied by the level of granularity is equal to:

$$[a, b] = \left[\max(0, pr_{ij} - \alpha/2), \min(1, pr_{ij} + \alpha/2)\right] \tag{15}$$

Let assume that the entry of interest of the particle is x. It is transformed linearly according to the expression $z = a + (b - a)x$. For example, consider that pr_{ij} is equal to 0.7, the admissible level of granularity $\alpha = 0.1$, and the corresponding entry of the particle is $x = 0.4$. Then, the corresponding interval of the granular fuzzy preference relation computed as given by Eq. (15) becomes equal to $[a, b] = [0.65, 0.75]$. Subsequently, $z = 0.69$, and, therefore, the modified value of pr_{ij} becomes equal to 0.69.

The overall particle is composed of the individual segments, where each of them is concerned with the optimization of the parameters of the fuzzy preference relations.

3.2.2. Fitness function

In the PSO, the performance of each particle during its movement is assessed by means of some performance index (fitness function). Here, the aim of the PSO is the maximization both the consensus achieved among the decision makers and the individual consistency achieved by each decision maker. Therefore, the fitness function, f, associated with the particle is defined as:

$$f = Q \tag{16}$$

being Q the optimization criterion presented in Section 3.1. The higher the value of f, the better the particle is.

3.2.3. Algorithm

In this study, the generic form of the PSO algorithm is used. Here, the updates of the velocity of a particle are realized in the form $\mathbf{v}(t+1) = w \times \mathbf{v}(t) + c_1\mathbf{a} \cdot (\mathbf{z}_p - \mathbf{z}) + c_2\mathbf{b} \cdot (\mathbf{z}_g - \mathbf{z})$ where "t" is an index of the generation and $\cdot$ denotes a vector multiplication realized coordinatewise. $\mathbf{z}_p$ denotes the best position reported so far for the particle under discussion while $\mathbf{z}_g$ is the best position overall and developed so far across the entire population. The current velocity $\mathbf{v}(t)$ is scaled by the inertia weight (w) which emphasizes some effect of resistance to change the current velocity. The value of the inertia weight is kept constant through the entire optimization process and equal to 0.2 (this value is commonly encountered in the existing literature [28]). By using the inertia component, we form the memory effect of the particle. The two other parameters of the PSO, that is **a** and **b**, are vectors of random numbers drawn from the uniform distribution over the [0, 1] interval. These two update components help form a proper mix of the components of the velocity. The second expression governing the change in the velocity of the particle is particularly interesting as it nicely captures the relationships between the particle and its history as well as the history of overall population in terms of their performance reported so far. The next position (in iteration step "t + 1") of the particle is computed in a straightforward manner: $\mathbf{z}(t+1) = \mathbf{z}(t) + \mathbf{v}(t+1)$.

When it comes to the representation of solutions, the particle **z** consists of "$m \cdot n(n-1)$" entries positioned in the [0,1] interval that corresponds to the search space. Finally, one should note that while PSO optimizes the fitness function, there is no guarantee that the result is optimal, rather than that we can refer to the solution as the best one being formed by the PSO.

4. Experimental study

In this section, we report on an experimental study, which helps quantifying the performance of the proposed approach. In particular, we highlight the advantages, which are brought by an effective allocation of information granularity in the building of consensus.

Proceeding with the details of the optimization environment, we set up the values of the parameters, which are typically encountered in the literature. The standard PSO version is being used with the value of the parameters in the update equation for the velocity of the particle set as $c_1 = c_2 = 2$. The population size was set to 100 individuals and the method was run for 300 generations. These values were set up experimentally through a trial-and-error process.

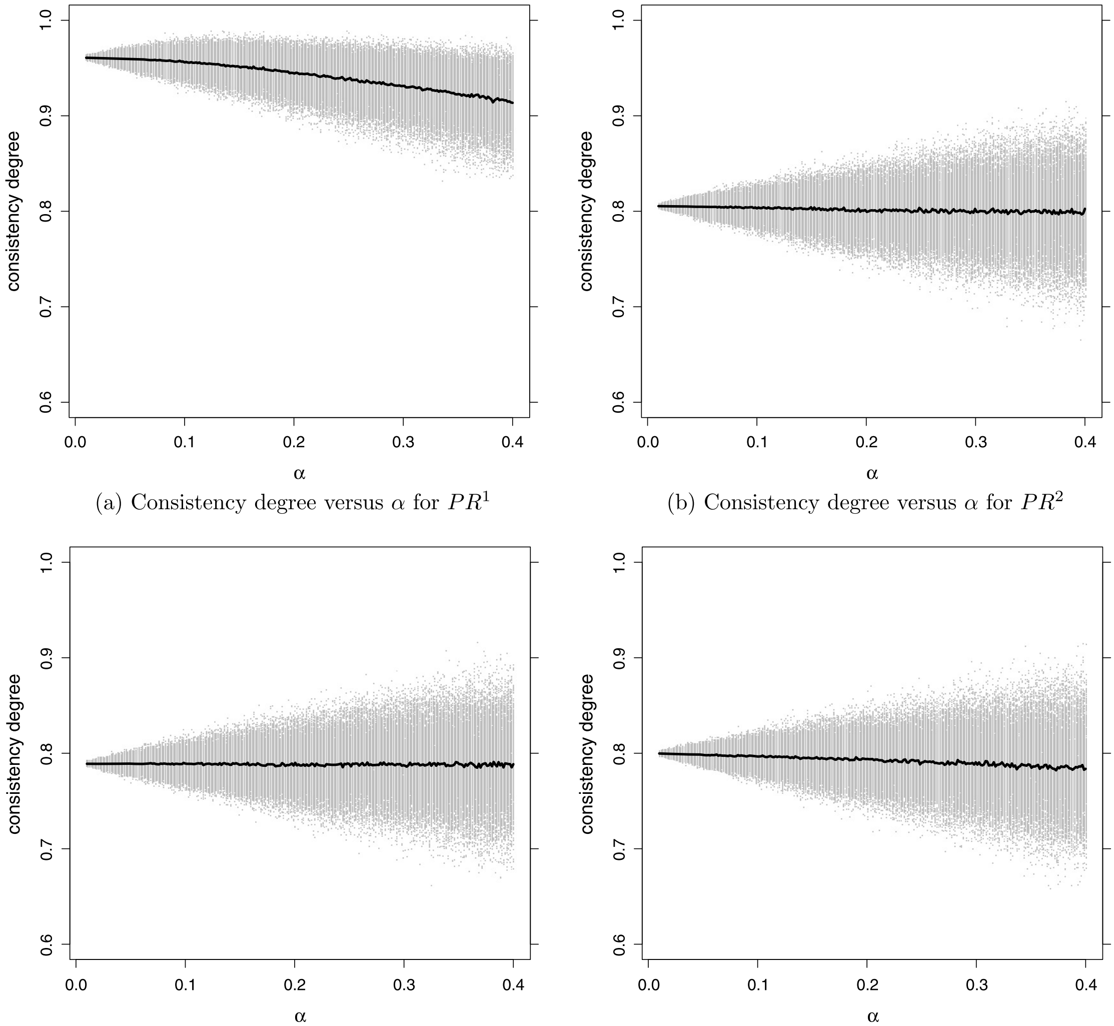
Let us suppose four fuzzy preference relations coming from four decision makers $E = \{e_1, e_2, e_3, e_4\}$. The entries of these fuzzy preference relations are reflective of the pairwise comparisons of four alternatives $X = \{x_1, x_2, x_3, x_4\}$.

$$PR^1 = \begin{pmatrix} - & 0.1 & 0.6 & 0.4 \\ 0.8 & - & 0.8 & 0.7 \\ 0.4 & 0.1 & - & 0.2 \\ 0.6 & 0.3 & 0.7 & - \end{pmatrix} \qquad PR^2 = \begin{pmatrix} - & 0.2 & 0.7 & 0.6 \\ 0.6 & - & 0.9 & 0.3 \\ 0.3 & 0.3 & - & 0.5 \\ 0.1 & 0.7 & 0.5 & - \end{pmatrix}$$

$$PR^3 = \begin{pmatrix} - & 0.7 & 0.5 & 0.3 \\ 0.3 & - & 0.6 & 0.8 \\ 0.5 & 0.4 & - & 0.9 \\ 0.6 & 0.1 & 0.3 & - \end{pmatrix} \qquad PR^4 = \begin{pmatrix} - & 0.8 & 0.2 & 0.6 \\ 0.4 & - & 0.6 & 0.2 \\ 0.8 & 0.4 & - & 0.5 \\ 0.4 & 0.8 & 0.5 & - \end{pmatrix}$$

The corresponding consistency degrees of the four fuzzy preference relations are $cd_1 = 0.96$, $cd_2 = 0.81$, $cd_3 = 0.79$, and $cd_4 = 0.80$. All the fuzzy preference relations exhibit a similar level of consistency degree, with an exception of the fuzzy preference relation PR^1, whose consistency degree is higher than for the rest of the fuzzy preference relations. In case no granularity is admitted, the consensus degree achieved among the group of decision makers is $cr = 0.72$.

Before proceeding with the PSO optimization of the fuzzy preference relations when supplied with the required granularity level, it becomes instructive to analyze an impact of the improvement or deterioration of consistency of the fuzzy preference relations. For a given fuzzy preference relation PR, we allow a certain value of the granularity level α to quantify the effect of the imposed granularity. Then, for this specific value, a fuzzy preference relation is randomly generated coming from a granular representation of PR, $\boldsymbol{P}(PR)$, and its associated consistency degree is computed. The calculations are repeated 500 times for each value of α. The corresponding plots of the consistency degree cd versus the imposed granularity level α are shown in Fig. 1. In addition, in these plots, we visualize average values of the consistency degrees.



(a) Consistency degree versus α for PR^1
(b) Consistency degree versus α for PR^2
(c) Consistency degree versus α for PR^3
(d) Consistency degree versus α for PR^4

Fig. 1. Plots of consistency degrees versus α for the fuzzy preference relations PR^1–PR^4.

On the one hand, the likelihood of arriving at more consistent fuzzy preference relations increases when increasing the values of the granularity level α. It is not surprising as we have inserted some level of flexibility that we intend to take advantage of. On the other hand, the possibility of generating a very inconsistent fuzzy preference relation increases as well. Despite that, the average value of consistency remains pretty steady with respect to increasing values of the granularity level α, as reported for the fuzzy preference relations. However, there is some slight downward trend for higher values of α. In particular, when the consistency degree of the initial fuzzy preference relation provided by the decision maker is very high, it is very common that its average consistency degree decreases for higher values of the granularity level α (see Fig. 1a).

Once we have analyzed the impact of the given granularity level in the improvement or deterioration of the consistency, we run the optimization of the entries of the fuzzy preference relations. Considering a given level of granularity α, Fig. 2 illustrates the performance of the PSO quantified in terms of the fitness function obtained in successive generations. The most notable improvement is noted as the very beginning of the optimization, and afterwards, there is a clearly visible stabilization, where the values of the fitness function remain constant. It is also interesting to analyze the computing time required by the proposed approach in order to measure its efficiency. In this study, the average running time per run of the method is 0.394 seconds and, therefore, the computational cost of our approach is low.

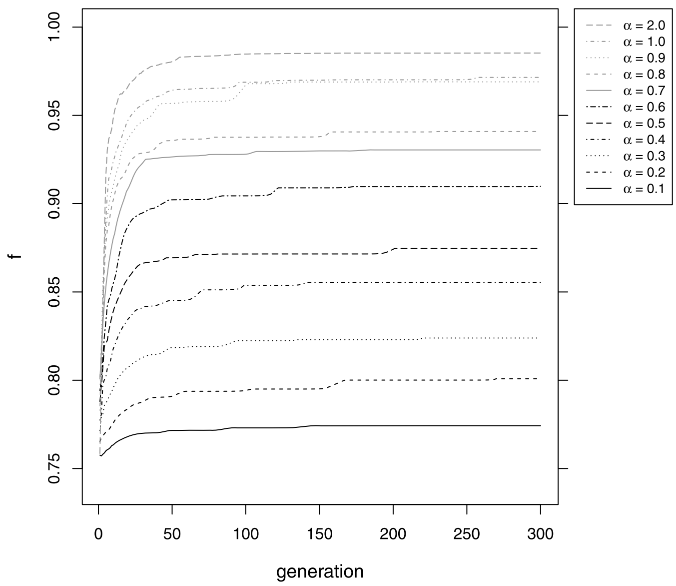
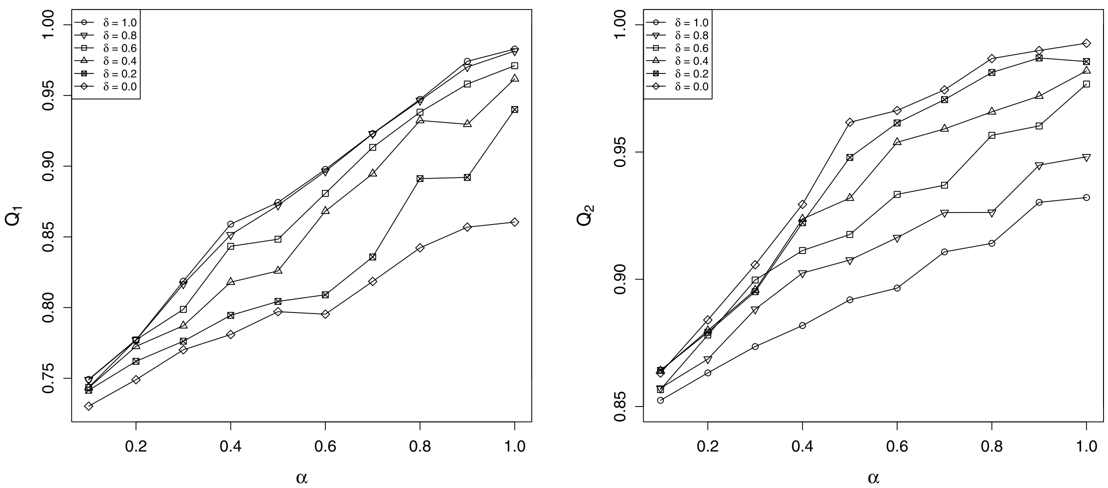


Fig. 2. Fitness function f in successive PSO generations for selected values of α (here $\delta = 0.75$).

To put the obtained optimization results in a certain context, we report the performance obtained when considering the entries of the fuzzy preference relations are single numeric values, that is, when no granularity is allowed ($\alpha = 0$). In such a case, the value of the fitness function f is 0.74 (considering $\delta = 0.75$). Comparing with the values obtained by the PSO, the fitness function f takes on now lower values. As we can see in Fig. 2, the higher the admitted level of granularity α, the higher the values obtained by the fitness function f. It is due to the fact that the higher the level of granularity α, the higher the level of flexibility introduced in the fuzzy preference relations and, therefore, the possibility of realizing decisions with higher level of consensus and consistency increases. In particular, when each entry of the granular preference relation is treated as the whole [0, 1] interval (it occurs when $\alpha = 2.0$), the value of the fitness function is near to the maximum one, which is 1. However, when the level of granularity is very high, the values of the entries of the fuzzy preference relation could be very different in comparison with the original values provided by the decision maker and, therefore, he/she could reject them.

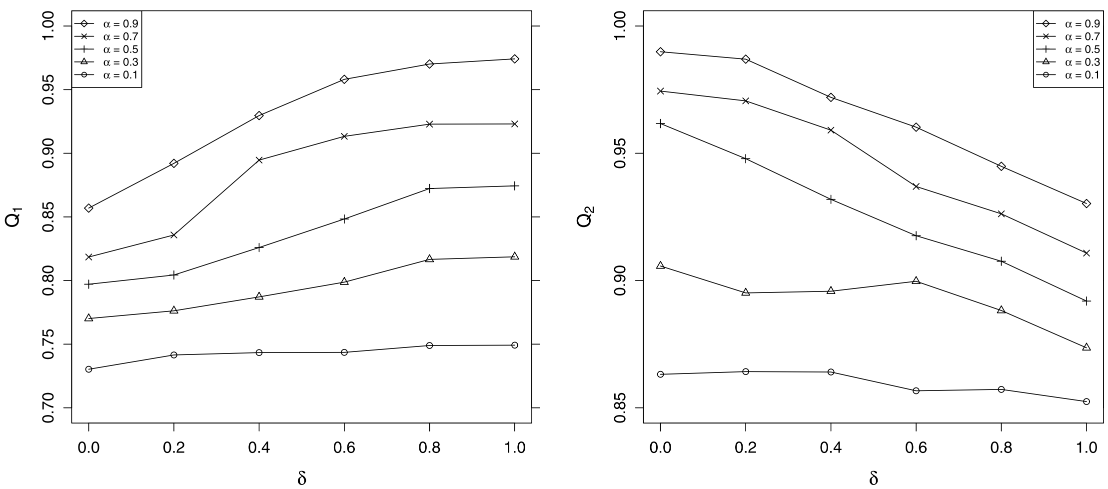
Let us examine an impact of the granularity level α and the parameter δ in the composite fitness function on the performance of the method and the form of the obtained results. For $\delta = 0$, the optimization concerns each of the fuzzy preference relations individually. Here, the increment in the values of α offers more flexibility, which, if wisely used (optimized by the PSO), produces the fuzzy preference relations of higher consistency. This effect is clearly observable in Fig. 3b (the curve for $\delta = 0$). The beneficial effect of granularity is evident: with the increasing values of α, the fuzzy preference relations become more flexible, which results in higher levels of consistency reached by the decision makers. A similar effect is visible when δ takes nonzero values: if there is some interaction, the impact of introduced granularity is positive (the overall level of consistency quantified by Q_2 is an increasing function of α). The strictly monotonic character of this relationship is not maintained for higher values of δ, as it is again shown in Fig. 3b. However, it is not surprising as the performance criterion optimized by PSO is not Q_2 itself but Q, which incorporates also the effect of the level of consensus achieved within the group of decision makers. On the other hand, Fig. 3a includes the progression of the values of Q_1, which shows the consensus within the group. Again, the advantageous effect of granularity is visible, as higher values of α translate into higher values of Q_1. However, now, higher values of δ produce higher values of Q_1 as more important is assigned to Q_1 in the composite criterion Q.

Fig. 4a includes a number of plots of Q_1 regarded as functions of δ for selected levels of granularity α. Once more, the impact of the granularity level is obvious. However, here, for the fixed value of α, there is a visible saturation effect for higher values of δ: when moving beyond a certain point, the values of Q_1 does not increase. On the other hand, the cumulative level of consistency Q_2 drops quickly with the increasing values of δ, as illustrated in Fig. 4b, and this effect is noticeable for different values of α. However, higher values of the granularity level also result in higher consistency levels in this case.



(a) Q_1 versus α for selected values of δ (b) Q_2 versus α for selected values of δ

Fig. 3. Plots of Q_1 and Q_2 versus α for selected values of δ.



(a) Q_1 versus δ for selected values of α (b) Q_2 versus δ for selected values of α

Fig. 4. Plots of Q_1 and Q_2 versus δ for selected values of α.

In summary, as it has been shown in this experimental study, we can conclude that both the level of consensus within the group of decision makers as well as the level of consistency achieved by the individual decision makers have been significantly increased with the use of the method proposed in this study, which speaks to the important role played by information granularity in the building of consensus.

5. Conclusions and future works

In this study, we have developed a method based on an allocation of information granularity as an important asset to increase the consensus achieved within the group of decision makers in group decision making situations. The required flexibility in the opinions provided by the decision makers, which is necessary to increase the level of consensus, was a motivating factor behind the introduction of the concept of granular fuzzy preference relations. Undoubtedly, the granular fuzzy preference relation conveys a far richer representation which can produce numeric realizations so that both the level of consensus and the level of consistency are improved. To do so, the PSO environment has been shown to serve a suitable optimization framework. Using this approach, the consensus is built in a single step rather than running several consensus rounds. On the one hand, it reduces the amount of time required for building consensus. On the other hand, negotiations among the decision makers are not included and, therefore, the decision makers influencing each other are not considered.

In the future, it is worth continuing this research in several directions:

- While the study presented here was focused on interval type of information granulation, different formalisms of information granulation such as fuzzy sets or rough sets can be incorporated into the discussed method.
- In the scenario analyzed in this study, a uniform allocation of granularity has been discussed, where the same level of granularity α has been allocated across all the fuzzy preference relations. However, a nonuniform distribution of granularity could be considered, where these levels are also optimized so that each decision maker might have an individual value of α becoming available to his/her disposal.

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