Social Network Analysis of Co-fired Fuzzy Rules

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Abstract. The popularity of modern online social networks has grown up so quickly in the last few years that, nowadays, social network analysis has become one of the hottest research lines in the world. It is important to highlight that social network analysis is not limited to the analysis of networks connecting people. Indeed, it is strongly connected with the classical methods widely recognized in the context of graph theory. Thus, social network analysis is applied to many different areas like for instance economics, bibliometrics, and so on. This contribution shows how it can also be successfully applied in the context of designing interpretable fuzzy systems. The key point consists of looking at the rule base of a fuzzy system as a fuzzy inference-gram (fingram), i.e., as a social network made of nodes representing fuzzy rules. In addition, nodes are connected through edges that represent the interaction between rules, at inference level, in terms of co-fired rules, i.e., rules fired at the same time by a given input vector. In short, fingram analysis consists of studying the interaction among nodes in the network for the purpose of understanding the structure and behavior of the fuzzy rule base under consideration. It is based on the basic principles of social network analysis which have been properly adapted to the design of fuzzy systems.

1 Introduction

Social networks [40] have existed since humans were aware of the great advantages derived from the fact of collaborating and living together in structured groups. Of course, this happened thousands of years ago. However, in the last few years the popularity of modern social networks has grown up very quickly because of the huge boom of new technologies for telecommunications. Nowadays, some websites like *facebook*[1], *twitter*[2] or *LinkedIn*[3] are widely known all around the world both for fun but also for professional purposes, with millions of users registered. Moreover, users of such social networks consider them as an essential part of their everyday life.

[1] A social utility for connecting with friends online at `http://www.facebook.com`
[2] A social utility for following people online at `http://www.twitter.com`
[3] A professional social network online at `http://www.linkedin.com`

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In consequence, social networks are attracting more and more attention from both industry and academia. Accordingly, lots of researchers have begun to work very actively on issues related to social networks [14] becoming a very flourishing field. There are studies in the context of all kinds of social sciences [25] such as bibliometrics [42], politics [11], medicine [13], economics [19], etc. There are also works dealing with industrial applications, for example supply chain management [27].

This paper introduces a new methodology for visualizing and analyzing fuzzy rule-based systems viewed as social networks. Hence, the main contribution consists in defining the so-called fuzzy inference-grams (*fingrams*).

Since the proposal of Zadeh and Mamdani's seminal ideas [28,43,44], interpretability [1] is widely recognized as one of the strongest points of fuzzy system identification methodologies. It represents the ability of fuzzy systems to model a real system in a human-friendly understandable way. To do so, the knowledge embedded into fuzzy systems is usually expressed in the form of linguistic variables and rules. Thus, the rule base of a fuzzy system becomes the main communication interface to users [31]. Moreover, a fuzzy rule base can be seen as a population made up of a set of individuals (fuzzy rules) which compete and collaborate among them with the aim of yielding both good generality-specificity and interpretability-accuracy trade-offs. In consequence, users can understand the system behavior through checking graphically existent relationships among rules. Fortunately, they can be easily analyzed by looking at the rule base as a fingram, i.e., as a social network made of nodes (representing fuzzy rules) and edges (representing the interaction among rules). Rule interaction is measured at inference level in terms of co-fired rules, i.e., rules fired at the same time by a given input vector.

The main goal of fingram analysis is the understanding of the structure and behavior of a fuzzy rule base under consideration. It is mainly based on the adaptation of given techniques for social network analysis to the design of fuzzy systems. As it will be thoroughly explained along the paper, the analysis of fingrams offers many possibilities: finding out the most significant rules, identifying potential inconsistencies among fuzzy rules, assessing the interpretability of fuzzy systems, etc.

The rest of the contribution is organized as follows. Section 2 starts with a brief overview on visual representation and analysis of fuzzy systems, then it presents some techniques for social network visualization and analysis, and it ends with the introduction of basic aspects related to interpretability assessment. Section 3 goes in detail with the generation and analysis of fingrams. It is important to notice that, as a first step, the general approach is particularized for the analysis of fuzzy rule-based classifiers (FRBCs), i.e., fuzzy rule-based systems for classification purposes. Section 4 summarizes the experiments carried out along with the achieved results. Finally, some conclusions and future work are sketched in Section 5.

2 Preliminaries

2.1 Visual Analysis of Fuzzy Rule-Based Systems

A complete analysis of visualization requirements for fuzzy systems is provided in [35]. It gives an overview on existing methodologies to yield 2D and 3D graphical representations of fuzzy systems. It comprises visualization of fuzzy data, fuzzy partitions, and fuzzy rules. Different alternatives are available depending on the requirements of the end-user. Moreover, requirements may change according to the visualization tasks to perform: interactive exploration; automatic computer-supported exploration; receiving feedback from users; and capturing users' profiles and adaptation.

The most relevant works to obtain visual representations of multi-dimensional fuzzy rules are those developed by Berthold et al. [6,15]. They make a mapping from a high-dimensional feature space onto a two-dimensional space which maintains the pair-wise distances between rules. The established mapping also displays an approximation of the rule spread and overlapping. As a result, it is possible to visualize and explore multi-dimensional fuzzy rule bases in a 2D graphical representation. Authors claim such representation yields a user friendly and interpretable exploratory analysis. However, the complexity of the analysis grows exponentially with the number of features and rules to be displayed. In consequence, in complex problems with many rules the interpretation of the resultant graph is not straightforward.

Unfortunately, there are not many papers tackling with visual analysis of the inference process of fuzzy systems, and most of them are limited to visual descriptions. Probably, this is due to the well-known linguistic expressivity of such systems that gives prominence to linguistic representations. However, when dealing with complex problems, even when the design is made carefully to maximize interpretability, the number of rules can become huge because of the curse of dimensionality characteristic of fuzzy rule-based systems. In those cases, looking for a plausible linguistic explanation of the inferred output, derived from the linguistic description of the fuzzy knowledge base, is not straightforward. Explaining the inferred output as an aggregation of all the involved rules is not easy when many rules are fired at the same time for a given input.

Some authors [22,23] have bet for searching visual explanations of the system output. Ishibuchi et al. established a set of design constraints with the aim of producing groups of rules with only two antecedent conditions that can be plotted in a two-dimensional (2D) space. They look for a visual representation able to explain the output of fuzzy rule-based classifiers to human users. Nevertheless, considering only two antecedents per rule is a strong limitation that may penalize the accuracy of the system.

2.2 Visual Analysis of Social Networks

Although there are several approaches for visualizing different kind of social networks, we will focus on co-citation social networks and the works published by Vargas-Quesada and Moya-Anegón [32,42] which strongly inspired our proposal. Indeed, the term *fingram* was coined by inspiration on the term *scientogram* firstly introduced by Vargas-Quesada and Moya-Anegón [32] as a novel tool for visualizing the structure of science [42].

Scientograms are visual science maps, i.e., visual representations of scientific domains in the form of social networks. They illustrate interactions among authors and papers through the basic notion of paper co-citation, representing the frequency with which documents are jointly cited by pairs. It is possible to group them by author, journal, or categories. Obviously, depending on the kind of regrouping, the information that can be extracted from the generated maps is different.

The standardized co-citation measure, firstly introduced by Salton and Bergmark, is computed by the next equation [38]:

$$MCN(ij) = \frac{Cc(ij)}{\sqrt{c(i) \cdot c(j)}} \quad (1)$$

where Cc means co-citation, c stands for citation, while i and j represent two different entities (authors, documents, journals, categories, institutions, countries, etc.).

In addition, network scaling (NS) is aimed to obtain simplified structures revealing the backbone, i.e., the underlying organization of the original network. NS is based on estimating the proximity between pairs of nodes by means of computing distances, similarities, correlations, and so on. Actually, NS is efficiently carried out by Pathfinder algorithm [8,12] that is essential to make feasible a good visual interpretation. Pathfinder is in charge of pruning the initial network while keeping only the most relevant links into the final Pathfinder networks (PFNETs). It is worthy to remark that the combination of entity co-citation and NS yields high quality, schematic network visualizations in several fields such as psychology (for representing the cognitive structure of a subject [39]), software development (for debugging of multi-agent systems [41]), or scientometrics (for analyzing large scientific domains [9]).

The next step is about the automatic visualization of PFNETs. For this porpuse, the spring embedder family of methods is the most widely used in the area of Information Science. Spring embedders assign coordinates to the nodes with the aim of producing aesthetical pleasant graphs. Vargas-Quesada and Moya-Anegón recommend the use of Kamada-Kawai's algorithm [26] which is one of the most extended methods to perform this task. Starting from a circular position of the nodes, it generates networks following aesthetic criteria: maximizing the use of available space, minimizing the number of crossed links, forcing the separation of nodes, building balanced maps, etc. Notice that, the combination of entities co-citation, PFNETs, and Kamada-Kawai makes the entities that share most sources with the rest, tend to be located toward the center.

Lastly, concerning the analysis of scientograms, according to [42] there are three main measures of centrality that yield useful information with the aim of identifying the most significant nodes of a PFNET: *Degree of Centrality* (regarding the number of direct links gathering in a node), *Centrality of Closeness* (measuring the distance among nodes), and *Centrality of Intermediation or Betweeness* (looking at nodes that act as link between other nodes contained in the shortest path).

2.3 Assessing Interpretability of Fuzzy Rule-Based Systems

Interpretability characterization and evaluation is a very subjective task which strongly depends on the skills and background (experience, preferences, knowledge, etc.) of the specific end-user who interprets the linguistic description of a fuzzy system with the aim of conceiving the significance of the system behavior.

Thus, assessing interpretability remains a trending and hot topic. Gacto et al. [17] have recently published a complete taxonomy about existent interpretability indexes. They identify four groups of indexes coming up from the combination of two different criteria, namely the nature of the index (complexity vs. semantic) and the considered elements (partitions vs. rule base) in the fuzzy system under study:

1. Complexity at partition level.
2. Complexity at rule base level.
3. Semantic-based interpretability at partition level.
4. Semantic-based interpretability at rule base level.

Most previous works [7,20] only deal with the readability of fuzzy systems. Therefore, most indexes correspond to groups (1) and (2). They usually make only basic analysis of complexity, i.e., they only count the number of elements (features, membership functions, rules, premises, etc.) included in the fuzzy system at partition level (group 1) and rule base level (group 2). Hence, they may be deemed as structural-based interpretability indexes.

On the other hand, group (3) contains works regarding structural properties of fuzzy partitions [34] such as distinguishability, coverage, and so on. They generally measure the degree of fulfillment of semantic constraints that should be overimposed during the design process. It is widely admitted that working with the so-called Strong Fuzzy Partitions (SFPs) [37] satisfies all semantic constraints required to have interpretable fuzzy partitions from the structural point of view.

Finally, only a few authors have begun recently to put emphasis on the importance of defining indexes in group (4). They advocate for extending the analysis of readability to evaluate the comprehensibility, i.e., the implicit and explicit semantics embedded in fuzzy systems [16,31]. There are also some papers dealing with the consistency of fuzzy rule bases and with the number of co-fired rules, i.e., rules simultaneously fired from a given input [4,10,30].

3 Proposal

This section thoroughly explains how to visualize and analyze FRBCs by means of fingrams. They represent a novel tool that arises from adopting a social network based approach inspired on the one proposed by Vargas-Quesada and Moya-Anegón for visualizing and analyzing the structure of science [42].

Fingrams are graphs which represent fuzzy rule bases as social networks. They contain nodes representing fuzzy rules and edges showing the interactions among them in terms of co-fired rules.

3.1 Fingram Generation, Scaling and Drawing

Given a fuzzy system containing N rules and an experimental dataset covering most possible situations, the $N \times N$ weight matrix M describes the interactions among the N rules in terms of frequency of co-firing.

$$M = \begin{pmatrix} 0 & m_{12} & \dots & m_{1N} \\ m_{21} & 0 & \dots & m_{2N} \\ \dots & \dots & \dots & \dots \\ m_{N1} & m_{N2} & \dots & 0 \end{pmatrix} \tag{2}$$

The co-firing measure (m_{ij}), inspired on the standardized co-citation measure (Eq. 1) proposed by Salton and Bergmark [38], is defined by the next equation:

$$m_{ij} = \begin{cases} \frac{SFR_{ij}}{\sqrt{FR_i \cdot FR_j}} & , i \neq j \\ 0 & , i = j \end{cases} \tag{3}$$

where SFR_{ij} means the number of data samples for which rules R_i and R_j are simultaneously fired, while FR_i and FR_j count respectively the total number of samples for which the same rules R_i and R_j are fired, without taking care if they are fired together or not. Notice that m_{ij} are normalized and M is symmetrical. Note also that the number of times a rule is fired is computed in an inferential way for all available data samples. Hence, it is extremely dependant on the goodness (quantity and quality) of the available experimental data.

An undirected graph is straightforwardly generated from the weight matrix M. This is made up by connecting N nodes using edges whose weights are directly taken from M. Thus, m_{ij} equals zero means that there is no link between nodes i and j.

Since the initial graph related to the matrix M is likely to be quite dense and difficult to analyze, it is worthy to apply a pruning mechanism before printing and exploring the generated fingram. To do so, a NS method like Pathfinder[4] [8,12] able to discover and keep only the most relevant links in M is very effective. It has already been successfully applied in the context of social networks. As result of running Pathfinder the initial graph representing M is translated into a pruned network called PFNET. This only keeps those links which do not violate the triangle inequality stating that the direct distance between two nodes must be lesser than or equal to the distance between them passing through any group of intermediate and connected nodes. Notice that, thanks to the properties of PFNETs, the pruned fingram preserves the underlying structure with all relevant information at global level in comparison to the original one.

Even though there are many different methods for the automatic visualization of social networks, the spring embedder family has become the most widely used in the area of Information Science. Spring embedders assign coordinates to the nodes in such a way that the final graph will be pleasing to the eye, and that the most important elements are located in the center of the representation. Among them, probably the most famous method is the one proposed by Kamada and Kawai [26]. Starting from a circular position of the nodes, it generates networks with aesthetic criteria such as the maximum use of available space, the minimum number of crossed links, the forced separation of nodes, the generation of balanced maps, etc. Notice that, the combination of rule co-firing, PFNETs, and Kamada-Kawai makes the most relevant rules, those exhibiting the highest interaction with the rest, tend to be located toward the center of the graphical representation.

The visual representation of the resultant graph is what we have called fingram. Furthermore, it can be enhanced with additional relevant information related to the specific problem under consideration. For instance, in the case of classification problems, the nodes represent fuzzy rules of FRBCs. More specifically, each rule is represented by a circular node whose size is proportional to the number of covered instances, and whose color corresponds to the class pointed out by the rule. Each node is labeled with the rule

[4] We have selected a recently published variant of Pathfinder algorithm (MST-PathFinder [36]) able to prune maps in cubic time.

identifier R_i but also with two very informative numbers, the percentage of instances in the dataset that are covered by the rule and the percentage of them matching with the rule output. Moreover, the number of border lines around a node indicates the number of linguistic propositions minus one in the rule description. In addition, each link among two nodes is characterized by an attached label that yields the related co-firing measure. The link thickness is proportional to its value. Furthermore, the link color is informative too. It is green for those rules pointing out the same class, and red in the case of rules pointing out different classes (potential inconsistencies).

Finally, it is important to highlight that our proposal is not affected by the well-known curse of dimensionality problem of fuzzy systems that implies that the number of fuzzy rules grows exponentially with the number of inputs. First, nodes represent directly rules instead of premises. Second, Pathfinder has been successfully applied to the analysis of large scientific domains representing thousands of co-cited entities [42]. In consequence, fingrams are able to display the interactions among thousands of rules in the form of highly interpretable graphs. Hence, even when the number of rules is huge the pruned fingram can be still comfortably viewed by any expert.

3.2 Fingram Exploratory Analysis and Interpretability Assessment

The expert analysis of fingrams can take profit of all tools already available for social network analysis. As a first approach, we advocate for the use of the so-called *Degree of Centrality* [42]. This means that we will point out the most significant rules, those corresponding to the nodes that concentrate the larger number of links in a fingram. Remind that thanks to the specific way scaling and drawing are done, the most salient links and nodes are likely to be placed in the center, and those less relevant in the periphery. Thus, those rules that correspond to nodes located in the periphery of a fingram, especially those connected with a low value (the weight of the associated link is small) to the remaining graph, are good candidates to be deleted. This could have an interesting collateral advantage since removing such rules is likely to increase interpretability while keeping almost the same accuracy. A basic simplification procedure may consist first on ranking rules according to their relevance and then finding out and removing those non-relevant ones, normally located at the periphery of the fingram.

Furthermore, the analysis of fingrams can report very useful information about the analysis and verification, at inference level, of the related fuzzy rule bases. For instance, one can directly analyze its global structure through exploring the number and location of apparent groups of rules, analyze the respective location of the rules coding for different classes, etc. As a result, it is easy to detect potential inconsistencies among fuzzy rules. They turn up when the co-fired rules yield different output classes. In addition, the higher the link weight (co-firing degree computed by Eq. 3) is, the larger the interaction among rules is, and the larger the degree of inconsistency results.

Notice that, even when a rule base is fully consistent at linguistic level, there may arise some possible inconsistencies at inference level because of the rule aggregation procedure made as part of the inference process. Such potential conflicts are difficult to detect mainly because they are partially hidden since they are typically produced by new unknown data samples that were not taken into account during the learning stage. For instance, it may happen that several rules are fired at the same time for a new given input vector and as result several outputs are activated with degrees higher than zero. When two different classes are activated with very similar degrees the situation can be labeled as an ambiguous case. Such situation is not desirable, no matter if the system is (or not) able to yield the right output class, because a slight modification in the input data may yield a wrong output. We can conclude that a FRBC producing many ambiguous cases is a non-reliable system and should be corrected.

With respect to interpretability, we assume that fuzzy partitions are interpretable and the matching among linguistic terms and fuzzy sets is supervised and approved by an expert. Notice that interpretable fuzzy partitions must represent prototypes that are meaningful for the end-user. Then, given a rule format along with an inference mechanism, the system interpretability can be evaluated looking only at rule level. Our assumption is the following: the larger the number of co-fired rules, the smaller the comprehensibility of the FRBC.

Fingrams give us all required information. Eq. 4 formalizes a novel interpretability index:

$$COFCI = \begin{cases} 1 - \sqrt{\frac{\sum_{i=1}^{N}\sum_{j=1}^{N}\left[(P_i+P_j)\cdot m_{ij}\right]}{MaxThr}}, \ if \ \sum_{i=1}^{N}\sum_{j=1}^{N}\left[(P_i+P_j)\cdot m_{ij}\right] \leq MaxThr \\ 0, \ otherwise \end{cases} \quad (4)$$

where $COFCI$ stands for Co-firing Based Comprehensibility Index. N is the total number of rules. P_i and P_j count the number of premises (antecedent conditions) in rules i and j, while m_{ij} is the measure of co-firing for the same rules i and j; it is computed by Eq. 3. In addition, $MaxThr$ is a threshold which represents a maximum value established to get a normalized measure in the interval [0,1]. It should be fixed by the designer of the FRBC, looking at the maximum number of rules that may be acceptable (by an end-user) for each specific problem according to its inherent complexity (number of inputs, output classes, available training data, etc.). According to our experimentations, we suggest setting $MaxThr$ greater or equal than one thousand times the multiplication of the number of classes (C) by the number of inputs (I) by the number of training samples (T):

$$MaxThr \geq 10^3 \cdot C \cdot I \cdot T \quad (5)$$

4 Experimental Analysis

This experimental study deals with an example of medical application where interpretability is of prime importance. Interpretability is a distinguishing capability of fuzzy systems which is really appreciated in most applications. Moreover, it becomes an essential requirement for those applications that involve an extensive interaction with humans. For instance, decision support systems in medicine [33] must be easily understandable, for both physicians and patients, with the aim of being widely accepted and successfully applicable.

We have chosen the well-known Wisconsin Breast Cancer Database (WBCD) [29] for illustrative porpuses. This dataset contains cases from a study that was conducted at the University of Wisconsin Hospitals, Madison, about patients who had undergone surgery for breast cancer. The task is to determine if the detected tumor is benign or malignant. Thus, the dataset contains 683 samples (we have removed the missing values), nine features (*Clump Thickness*, *Cell Size*, *Cell Shape*, *Marginal Adhesion*, *Epithelial Size*, *Bare Nuclei*, *Bland Chromatin*, *Normal Nucleoli*, and *Mitoses*) and one output class (*Benign / Malignant*). The whole dataset is freely available at the KEEL[5] machine-learning repository.

For simplicity, this analysis focuses only on FRBCs that were generated following the HILK (Highly Interpretable Linguistic Knowledge) fuzzy modeling methodology [3,5]. We have chosen HILK because it is especially thought for making easier the design process of interpretable FRBCs. To do so, it imposes several constraints (SFPs, global semantics, Mamdani rules [28], etc.) during the design phase. The rule base is made up of rules of form:

$$\textbf{If } \underbrace{X_a \text{ is } A_a^i}_{Proposition\ P_a} \textbf{ AND} \ldots \textbf{AND } \underbrace{X_z \text{ is } A_z^j}_{Proposition\ P_z} \textbf{ Then } Y \text{ is } C^n$$

where C^n is the selected output class; X_a is the name of the input variable a; and A_a^i represents the label i of such variable. Namely, A_a^i can be one of the elementary terms in the SFP or a composite term defined as a convex hull of adjacent elementary terms corresponding to OR and NOT combinations [21]. These kinds of rules are usually known as DNF rules. Notice that, the absence of an input in a rule means that it is not considered in the evaluation of such rule. This special kind of proposition is usually referred as *Don't care* [24] and it should be interpreted as ANY since it means that it is true no matter the selected linguistic term. Because several output classes can be activated since several fuzzy rules can be fired at the same time by the same input vector, the winner rule fuzzy reasoning mechanism is considered. Furthermore, the well-known minimum and maximum fuzzy operators are taken for conjunction and disjunction.

It is important to notice that HILK methodology is implemented as part of the free software tool GUAJE[6] [2]. Moreover, the new methodology for visualizing and exploring fuzzy rule bases proposed in this paper is also implemented in that tool. The drawing of the graphs themselves is done using another freeware tool named Graphviz[7] [18].

The rest of this section is devoted to show the utility of the new methodology proposed in this paper through some illustrative examples. As a starting point, the entire dataset has been randomly split into two subsets. The 75% of samples are considered as training set while the remaining 25% of samples compose the test set. Please notice that we do not apply cross-validation because, for the sake of clarity, we do not care about finding the best FRBC for the WBCD problem. We are aware that probably there are better rule bases for WBCD in the fuzzy literature, but our goal is to explain the new

[5] KEEL stands for Knowledge Extraction based on Evolutionary Learning. It is a free software tool available online at `http://sci2s.ugr.es/keel/`

[6] A free software tool for generating understandable and accurate fuzzy rule-based systems in a Java environment `http://www.softcomputing.es/guaje`

[7] A free software tool available online at `http://www.graphviz.org/`

methodology with a simple case instead of looking for the best solution for this specific problem.

Thus, we use GUAJE with the aim of building FRBCs automatically extracted from the available training data. Uniform SFPs with three triangular fuzzy sets are initially defined for each input. We are going to consider rules generated with the well-known Wang and Mendel (WM) and Fuzzy Decision Trees (FDT) algorithms both provided by GUAJE[8]. Hence, we generate two first set of rules corresponding to $FRBC_{WM}$ and $FRBC_{FDT}$. Moreover, we have simplified them with the simplification algorithm, also provided by GUAJE, in order to obtain two additional more compact FRBCs. Let's call them $FRBC_{WM-SIMP}$ and $FRBC_{FDT-SIMP}$. Two further simplifications guided by fingram analysis of $FRBC_{FDT-SIMP}$ have been carried out. They are named as $FRBC_{FDT-SIMP-F1}$ and $FRBC_{FDT-SIMP-F2}$.

Table 1 summarizes the main quality indicators characterizing those FRBCs previously generated. On the one hand, each column corresponds to one of the FRBCs under consideration. On the other hand, each row is related to one specific quality indicator.

Table 1. Quality evaluation of the generated FRBCs

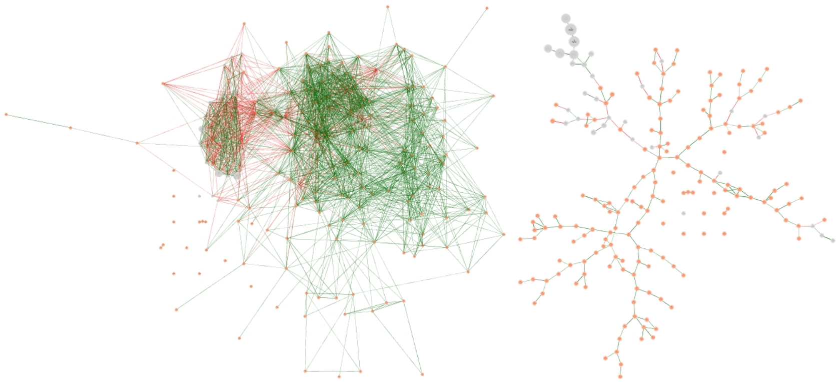
	$FRBC_{WM}$	$FRBC_{WM-SIMP}$	$FRBC_{FDT}$	$FRBC_{FDT-SIMP}$	$FRBC_{FDT-SIMP-F1}$	$FRBC_{FDT-SIMP-F2}$
ACC_{TR}	0.998	0.998	0.975	0.975	0.943	0.939
ACC_{TS}	0.83	0.918	0.947	0.953	0.93	0.918
NR	195	23	35	9	3	2
TRL	1755	155	165	27	6	4
ARL	9	6.739	4.714	3	2	2
AFR_{TR}	6.043	2.977	2.625	1.488	1.133	1.093
AFR_{TS}	6.299	3.047	2.965	1.614	1.155	1.113
AFD_{TR}	0.555	0.797	0.766	0.865	0.859	0.878
AFD_{TS}	0.455	0.776	0.734	0.847	0.823	0.867
$COFCI$	0	0.675	0.510	0.880	0.960	0.969

Firstly, we take care of the achieved accuracy regarding both training (ACC_{TR}) and test (ACC_{TS}). Accuracy is computed as the percentage of samples properly classified. Secondly, we tackle with assessing interpretability. To do so, considering only one index is not enough as it was pointed out in Section 2.3. Therefore, we have considered several structural-based but also semantic-based interpretability indexes at rule base level. NR stands for number of rules. TRL means total rule length, that represents the total number of linguistic propositions into the whole rule base. ARL stands for average rule length, computed as TRL divided by NR. We have also reported the average number of fired rules with respect to both training (AFR_{TR}) and test (AFR_{TS}) sets. One rule is counted as fired by a given data sample only in the case in which it is activated with a confidence firing degree greater or equal than 0.1. In addition, we have computed the average confidence firing degree (AFD) regarding again training (AFD_{TR}) and test (AFD_{TS}) sets. AFD is measured as the firing degree of the winner rule for each

[8] The interested reader is referred to [2,3] for further details about algorithms provided by GUAJE.

data sample and then averaged for the whole dataset. Finally, $COFCI$ is the novel interpretability index proposed in this work. It is computed following Eq. 4 with $MaxThr$ equals 10^4.

Looking carefully to values shown in Table 1, we can draw some interesting conclusions. First, WM generates a lot of complete rules, i.e., each rule takes into account all inputs. In consequence, generated rules are quite specific and they are likely to be simplified. Moreover, rule base is affected by overfitting because $FRBC_{WM}$ exhibits very high ACC_{TR} while ACC_{TS} is not so good. Furthermore, it seems there is a lot of redundancy inside the rule base. Indeed, AFR_{TR} and AFR_{TS} achieve very high values while AFD_{TR} and AFD_{TS} remain quite low that implies a lot of overlapping among rules. Regarding all interpretability indicators (NR, TRL, ARL, AFR, AFD and $COFCI$), $FRBC_{WM}$ can be deemed as not interpretable at all. Such feeling is confirmed when observing the fingrams displayed in Fig. 1. Of course, the rule base is so complex that is not easy to make any useful interpretation neither focusing on the initial network (Fig. 1(a)) nor looking at the scaled one (Fig. 1(b)). Anyway, we can appreciate how the scaling process becomes very effective turning up a quite clear structure that was hidden.

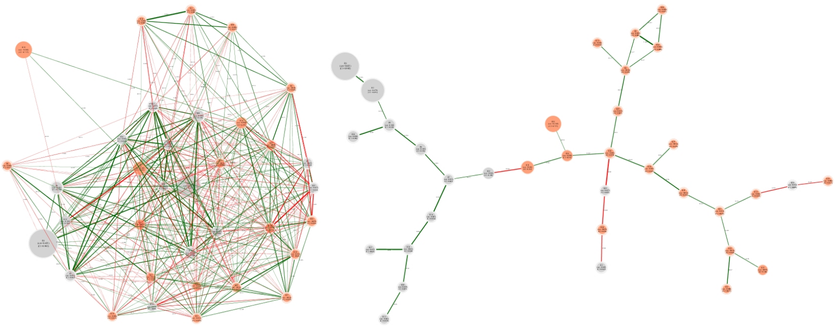


(a) Complete fingram (b) Scaled fingram

Fig. 1. Fingrams related to $FRBC_{WM}$ before and after network scaling with Pathfinder

Second, FDT produces a smaller set of much more general incompletes rules minimizing the overfitting effect. Thus, $FRBC_{FDT}$ yields much closer values for both ACC_{TR} and ACC_{TS}. In comparison with $FRBC_{WM}$, ARL and AFR are significantly decreased while AFD is increased, so $COFCI$ increases accordingly. We can conclude that $FRBC_{FDT}$ yields a better interpretability-accuracy trade-off than $FRBC_{WM}$. Fingrams corresponding to $FRBC_{FDT}$ are depicted in Fig. 2. Obviously, they are much clearer than those ones previously presented in Fig. 1.

With respect to the effect of the initial simplification, not guided by fingrams, we appreciate an improvement in the generalization capabilities of the selected FRBCs. Of course, simplification is made considering only training data. It preserves ACC_{TR} while



(a) Complete fingram (b) Scaled fingram

Fig. 2. Fingrams related to $FRBC_{FDT}$ before and after network scaling with Pathfinder

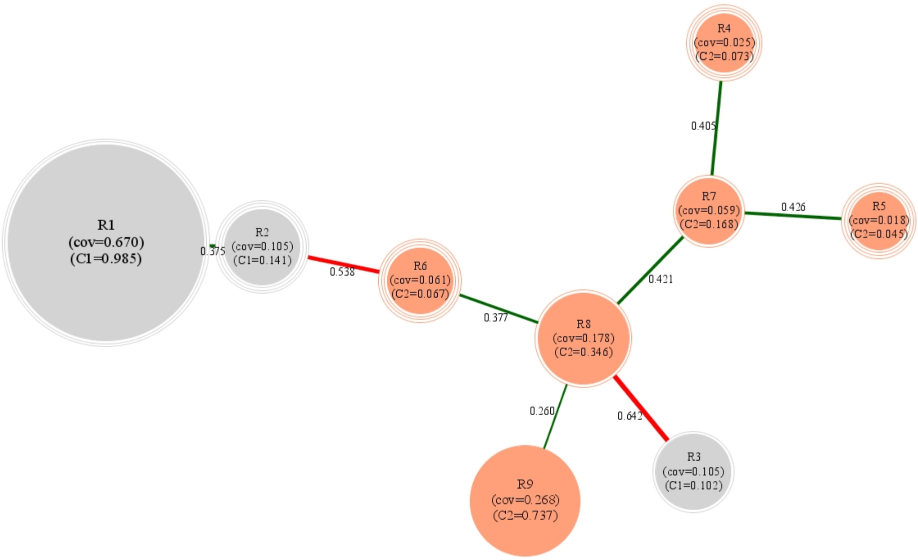


Fig. 3. Scaled fingram related to $FRBC_{FDT-SIMP}$

interpretability is strongly improved. As a side effect, ACC_{TS} is also increased. Furthermore, AFD grows up regarding both training and test. As a result, simplified FRBCs become much more trustworthy. Moreover, making a comparison between the two simplified FRBCs under study ($FRBC_{WM-SIMP}$ and $FRBC_{FDT-SIMP}$), it becomes obvious that $FRBC_{FDT-SIMP}$ yields the best interpretability-accuracy trade-off. $FRBC_{FDT-SIMP}$ is made up of only nine rules so its related fingram, plotted in Fig. 3, becomes very informative.

Each rule is represented by a circular node whose size is proportional to the number of covered instances, and whose color corresponds to the class pointed out by the rule. Each node is labeled with the rule identifier Ri but also with two very informative numbers, the percentage of instances in the dataset that are covered by the rule (cov) and the percentage of them matching with the rule output (Ci). In addition, the number of border lines around a node indicates the number of linguistic propositions minus one in the rule description. Each link between two nodes is characterized by an attached label that yields the related co-firing measure. The link thickness is proportional to its value. Furthermore, the link color is informative too; it is green for those rules pointing out the same class, while it is red in the case of rules pointing out different classes (potential inconsistencies).

From Fig. 3 we appreciate how most samples belonging to class C1 are handled by R1. On the other hand, rules R8 and R9 seem to be the most significant ones for class C2. If we keep only those three rules while removing the remaining, then we generate $FRBC_{FDT-SIMP-F1}$ whose quality indicators are detailed in Table 1. It is a very simple and highly interpretable FRBC, while its accuracy it is not strongly penalized with respect to $FRBC_{FDT-SIMP}$. Finally, looking carefully at rules R8 and R9 they may be merged into only one rule. In that case we obtain $FRBC_{FDT-SIMP-F2}$. Again, interpretability gets better while accuracy is only slightly reduced, as it was shown in Table 1.

5 Conclusions and Future Work

This paper has introduced a new methodology for exploratory analysis of fuzzy rule-based systems. In addition, we have proposed a novel interpretability index that takes into account the comprehensibility of fuzzy systems looking at the correspondence between their linguistic description and their inference process. It deals with semantic-based interpretability at rule base level and it is therefore aimed to cover the lack of such kind of indexes in the fuzzy literature.

We have shown the utility of our proposal in a simple but very illustrative classification problem where interpretability is highly appreciated because it copes with a medical diagnosis application. Achieved results are encouraging. The analysis of fingrams has helped us effectively in the hard task of searching for good interpretability-accuracy trade-offs. Anyway, in the future we will extensively validate the methodology and we will look for other co-firing metrics able to yield additional information about consistency, generality and/or specificity of rules.

Notice that, a software module for fingrams generation and analysis is available with the free software tool GUAJE. It can be freely downloaded at:

http://www.softcomputing.es/guaje

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