# A Decision Aid System to Provide Consistent Linguistic Preference Relations

**S. Alonso, E. Herrera-Viedma, F. Herrera and F.J. Cabrerizo**
Department of Computer Science and Artificial Intelligence
University of Granada, 18071, Granada, Spain
E-mail: {salonso, viedma, herrera}@decsai.ugr.es, cabre@correo.ugr.es
**F. Chiclana**
Centre for Computational Intelligence, School of Computing
De Montfort University, Leicester LE1 9BH, UK.
E-mail: chiclana@dmu.ac.uk

**Abstract:** *In Group Decision Making, the expression of consistent preferences is often a very difficult task for the experts, specially in decision problems with a high number of alternatives. This difficulty increases when experts are asked to provide their preferences in the form of preference relations because the amount of preference values that are required increases exponentially. This usually leads to situations where an expert may not be able to express all his/her preferences properly and consistently (that is, without contradiction). As a result, eventually the information provided can easily be either incomplete or inconsistent. In this paper we develop a transitivity based support system to aid experts to express their linguistic preference relations in a more consistent way. This interactive system provides the expert with recommendations for the preference values that they have not yet expressed. These recommendations are computed with the aim of maintaining the consistency level of the expert as high as possible.*

**Keywords:** Incomplete Information, Consistency, Linguistic Preference Relations

## 1 Introduction

One of the key issues when solving Group Decision Making (GDM) problems is to obtain the preferences of different experts in order to lately combine them and find which of the alternatives of a feasible set $X = \{x_1, ..., x_n\}$ is the best. There exist several different representation formats in which experts can express their preferences; however, Preference Relations (PR) [5, 7, 9] have been widely used because they have proved to be a very expressive representation format and also because they present good properties that allow to operate with them easily [7, 9]. There exist several different types of preference relations such as [1]: fuzzy, multiplicative, interval valued and linguistic preference relations.

Preference relations may also present some disadvantages. As it is required to express a preference degree among all possible pairs of different alternatives, the amount of information that the experts have to provide increases exponentially. Clearly, when the cardinality of the problem is high then we may find situations where the experts do not provide 'good' (consistent and complete) preference relations. In these cases, an expert might choose not to provide all the required preference values, or the expert might provide his/her preferences in an inconsistent way, i.e., his/her preferences might be contradictory. In [1] a procedure to compute the missing values of an incomplete PR (fuzzy, multiplicative, interval valued or linguistic) taking into account the expert's consistency level has been developed. Nevertheless, that procedure could not deal with the initial contradiction that the expert could have introduced in his/her preferences, and what could be worse, the expert might not accept the estimated values (even if they increase the overall consistency level).

Thus, when designing a computer driven model to deal with GDM problems where the informa-

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tion is given in the form of preference relations, software tools to aid the experts to express their preferences avoiding the aforementioned problems should be implemented. As experts might not be familiar with preference relations, the aiding tools should be easy enough to use and they should follow the general principles of interface design [4].

In this paper we present an *interactive support system* to aid experts to express their preferences in the form of *linguistic* preference relations. The system will give recommendations to the expert while he/she is providing the preference values in order to maintain a high level of consistency in the preferences, as well as trying to avoid missing information. Also, the system will provide measures of the current level of consistency and completeness the expert has achieved, which can be used to avoid situations of self contradiction. The system has been programmed using Java technologies, allowing its integration in web-based applications which are increasingly being used in GDM and Decision Support environments [3, 11].

The rest of the paper is set out as follows: In Section 2 we present our preliminaries. In Section 3 we detail the interactive support system. Finally in Section 4 we draw our conclusions.

## 2 Preliminaries

In this section we present the main concepts needed for the rest of the paper: Incomplete Linguistic Preference Relations, the Additive Transitivity Property and how this property is used to estimate missing values in a linguistic preference relation.

### 2.1 Incomplete Linguistic Preference Relations

Preference Relations are one of the most frequently used formats to represent preferences [1, 5, 7, 9]. They present a very high level of expressivity and good properties that allow to operate with them easily [7, 9]. There exist several types of PR, and among them, we can find the Linguistic Preference Relation (LPR). In a previous paper [1], the 2-tuple linguistic model [6] was used to define a LPR.

This linguistic model is based on the symbolic representation model and the concept of symbolic translation to represent the linguistic information by means of a pair of values called linguistic 2-tuple, $(s, \alpha)$, where $s$ is a linguistic term and $\alpha$ is a numeric value representing the symbolic translation.

**Definition 1:** Let $\beta \in [0, g]$ be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set $S = \{s_0, s_1, \ldots, s_{g-1}, s_g\}$, i.e., the result of a symbolic aggregation operation. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-0.5, 0.5)$, then $\alpha$ is called a symbolic translation.

Based on the symbolic translation concept, the linguistic representation model of information by means of 2-tuples $(s_i, \alpha_i)$, $s_i \in S$ and $\alpha_i \in [-0.5, 0.5)$, was developed in [6]. This model defines a set of transformation functions between numerical values and 2-tuples $(\Delta)$ and $(\Delta^{-1})$ (the reader is referred to [1] for more details). A linguistic term can be seen as a linguistic 2-tuple by adding to it the value 0 as symbolic translation, $s_i \in S \equiv (s_i, 0)$, and thus, this linguistic model can be used to provide linguistic preference relations:

**Definition 2 [1]:** A linguistic preference relation $P$ on a set of alternatives X is a set of 2-tuples on the product set $X \times X$, i.e., it is characterized by a membership function

$$\mu_P: X \times X \longrightarrow S \times [-0.5, 0.5)$$

If it is not possible to give the 2-tuples for every pair of alternatives then we have an *incomplete LPR*.

### 2.2 Additive Transitivity Property

For GDM problems with preference relations, some properties about are usually assumed desirable to avoid contradictions within the preferences expressed by the experts, that is, to avoid inconsistent opinions. One of them is the *additive transitivity*, which was defined for fuzzy preference relations [7, 10] as:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \quad \forall i, j, k \in \{1, \dots, n\} \tag{1}$$

This additive transitivity property can be rewritten for linguistic PR as:

$$(\Delta^{-1}(p_{ij}) - g/2) + (\Delta^{-1}(p_{jk}) - g/2) = (\Delta^{-1}(p_{ik}) - g/2) \quad \forall i, j, k \in \{1, \dots, n\} \tag{2}$$

### 2.3 Estimating Missing Values Using Additive Transitivity

Expression (2) can be used to calculate an estimated value of a preference degree using other preference degrees in a linguistic preference relation. Indeed, from Expression (2) the preference value $p_{ik}$ $(i \neq k)$ can be estimated using an intermediate alternative $x_j$ in three different ways:

$$(cp_{ik})^{j1} = \Delta(\Delta^{-1}(p_{ij}) + \Delta^{-1}(p_{jk}) - g/2) \tag{3}$$

$$(cp_{ik})^{j2} = \Delta(\Delta^{-1}(p_{jk}) - \Delta^{-1}(p_{ji}) + g/2) \tag{4}$$

$$(cp_{ik})^{j3} = \Delta(\Delta^{-1}(p_{ij}) - \Delta^{-1}(p_{kj}) + g/2) \tag{5}$$

As we have already said, and expert can choose not to provide complete preference relations, thus, the above equations may not be possible to be applied for every 3-tuple of alternative $(x_i, x_k, x_j)$. If expert $e_h$ provides an incomplete LPR $P^h$, the following sets are defined [8]:

$$\begin{array}{lcl} A = \{(i,j) \mid i,j \in \{1,\dots,n\} \wedge i \neq j\} & ; & H_{ik}^{h1} = \left\{ j \neq i,k \mid (i,j),(j,k) \in EV^h \right\} \\ MV^h = \left\{ (i,j) \in A \mid p_{ij}^h \ is\ unknown \right\} & ; & H_{ik}^{h2} = \left\{ j \neq i,k \mid (j,i),(j,k) \in EV^h \right\} \\ EV^h = A \setminus MV^h & ; & H_{ik}^{h3} = \left\{ j \neq i,k \mid (i,j),(k,j) \in EV^h \right\} \end{array} \tag{6}$$

$MV^h$ is the set of pairs of alternatives whose preference degrees are not given by expert $e_h$, $EV^h$ is the set of pairs of alternatives whose preference degrees are given by the expert $e_h$; and $H_{ik}^{h1}$, $H_{ik}^{h2}$, $H_{ik}^{h3}$ are the sets of intermediate alternative $x_j (j \neq i, k)$ that can be used to estimate the preference value $p_{ik}^h$ $(i \neq k)$ using equations (3), (4), (5) respectively.

The *final* estimated value of a preference degree $p_{ik}^h$ $((i,k) \in EV^h)$ can be calculated as:

$$cp_{ik}^h = \Delta \left( \frac{\sum_{j \in H_{ik}^{h1}} \Delta^{-1}((cp_{ik}^h)^{j1}) + \sum_{j \in H_{ik}^{h2}} \Delta^{-1}((cp_{ik}^h)^{j2}) + \sum_{j \in H_{ik}^{h3}} \Delta^{-1}((cp_{ik}^h)^{j3})}{(\#H_{ik}^{h1} + \#H_{ik}^{h2} + \#H_{ik}^{h3})} \right) \tag{7}$$

In the case of being $(\#H_{ik}^{h1} + \#H_{ik}^{h2} + \#H_{ik}^{h3}) = 0$ then the preference value $p_{ik}^h$ $((i,k) \in EV^h)$ cannot be estimated using the rest of known values.

## 3 Interactive Support System to Aid Experts to Express Consistent Linguistic Preference Relations

In this section we describe in detail an interactive support system to aid experts to express their linguistic preference relations in a consistent way. We will enumerate the *Interface Requirements* and *Logical Goals* desired to be implemented along with the actual implementation solutions that have been developed. To do this, a snapshot of the system will be used (*figure 1*), in which every implementation solution will be pointed out.

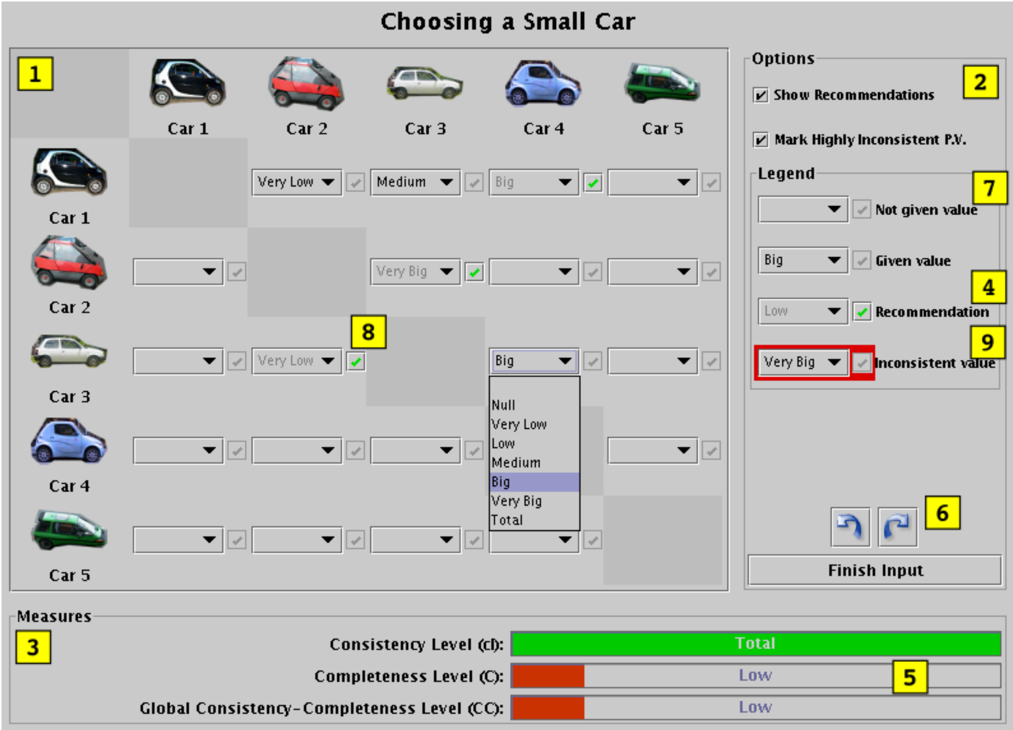


Figure 1: Snapshot of the Support System

### 3.1 Interface Requirements

These requirements deal with the visual representation of the information and the different controls in the system. The system is desired to comply the so called *"Eight Golden Rule"*[4] for interface design:

- **GR 1.** *Strive for consistency:* To comply with this requirement the interface has been homogenised in order to present an easy to understand view of the process which is being carried out. We have introduced 3 main areas: In area number **(1)** we present the *linguistic preference relation* that the expert is providing, as well as a brief description of every alternative. Area number **(2)** contains several global controls to activate/deactivate certain functions, as well as to end the input process. Area number **(3)** contains different measures that show the overall progress (see below).
- **GR 2.** *Enable frequent users to use shortcuts:* Shortcuts have been added to the most frequent options, and the input combo-boxes for the preference values have been ordered to access them easily using the keyboard.
- **GR 3.** *Offer informative feedback:* Our systems provides recommendations **(4)** as well as consistency and completeness measures **(5)** (see below). All controls have tooltips.
- **GR 4.** *Design dialogues to yield closure:* With every change the user makes to his/her preferences the system provides new recommendations and measures.
- **GR 5.** *Offer simple error handling:* As the preferences are given by means of combo-boxes there is no possibility of giving an incorrect value. However, the system provides a way to point out when a preference is highly inconsistent **(9)**.

- **GR 6.** *Permit easy reversal of actions (undo action):* We have introduced *undo* and *redo* buttons **(6)** in the system.
- **GR 7.** *Support internal focus of control (user is in charge):* The user can choose at every moment which preference value wants to give or update, as well as enabling/disabling options.
- **GR 8.** *Reduce short-term memory load of the user:* All information is presented in a single screen, so the user does not have to remember any data.

### 3.2 Logical Goals

- **Goal 1.** *Offer recommendations to the expert to guide him toward a highly consistent and complete linguistic preference relation:* To offer recommendations, the system computes all the missing values that could be estimated by using *equation 7* and it presents them in area **(1)**. As the values are computed taking into account the additive transitivity property, the recommendations tend to increment the overall consistency level. They are presented in a different color (gray) **(4)** to easily differentiate them from the proper expert values **(7)**.
- **Goal 2.** *Recommendations must be given interactively:* When the expert introduces or updates a preference value all possible recommendations are recomputed and presented.
- **Goal 3.** *Recommendations must be simple to understand and to apply:* Recommendations are given in the same manner as the user inputs his/her preferences. There is also a button that enables the user to accept or validate a given recommendation **(8)**.
- **Goal 4.** *The user must be able to refuse recommendations:* A user can choose any value for a particular preference degree ignoring all the recommendations. However, if the introduced preference degree is highly inconsistent this will be pointed out **(9)**.
- **Goal 5.** *The system must provide indicators of the consistency and completeness level achieved in every step:* In a previous paper [2] some measures of the consistency and completeness of fuzzy preference relations **(5)** were provided. These measures have been adapted to the linguistic model in a similar way as it was done with the additive transitivity property. The consistency measure for a particular linguistic PR $P^h$ (called $cl^h$) is based on the error that can be computed between the preference values that the expert provides and the values that can be estimated using *expression 7*. The completeness measure ($C^h$) is obtained as a ratio between the number of values given by the expert and the total number of values os a complete linguistic PR. In our system we also combine these two measures and provide a global consistency/completeness measure that informs the expert of his/her current degree of consistency and completeness.
- **Goal 6.** *The system should be easy to adapt to other types of preference relation:* As the system is programmed following the principles of Object Oriented Programming, its adaptation to new kinds of preference relations is an easy task.
- **Goal 7.** *The system should be easy to incorporate to Web-based GDM models and decision support systems [3, 11]:* As the system is Java based, it is easy to be incorporated into a web-based environment.

## 4 Conclusions and Future Improvements

In this paper we have presented an interactive support system which aids experts to provide consistent preferences and to help them to avoid incomplete information situations in GDM environments where the opinions must be provided in the form of linguistic preference relations. The system provides recommendations simple to understand and to apply at the same time the expert input his/her preference degrees, with the aim of maximizing the consistency of his/her opinions.

In the future we will integrate this support system into a complete consensus reaching process to enrich the preference acquisition step of the process.

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