Additive Consistency as a Tool to Solve Group Decision Making Problems

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Abstract:

This paper shows some of the uses of the the concept of additive consistency in the resolution process of group decision making (GDM) problems where experts express their preferences by means of fuzzy preference relations. A consistency measure for both complete and incomplete fuzzy preference relations is proposed and a particular induced OWA operator based on it, the AC-IOWA operator, is defined. The consistency measure is also used to guide an iterative procedure that estimates the unknown values of an incomplete fuzzy preference relation.

Keywords: group decision making, fuzzy preference relation, additive consistency, induced OWA operator, incomplete information.

1 Introduction

In Group Decision Making (GDM) problems experts have to express their preferences by means of a set of evaluations over the set of alternatives. Many reasons have been provided for *fuzzy preference relations* to be chosen as the preference representation format, among which it is worth noting that of being a useful tool in the aggregation of experts' preferences into group ones [2, 7]. However, there may be cases in which experts do not have an in-depth knowledge of the problem to be solved. In such cases, experts may not have a say on every aspect of the problem, and as a result they may present incomplete preferences, i.e. some values may not be given or may be missing [4, 8].

Due to the complexity of most decision making problems, experts's preferences may not satisfy formal properties that fuzzy preference relations are assumed to verify. One of these properties, consistency, is associated with the *transitivity property*. Many properties have been suggested to model transitivity of fuzzy preference relations and, consequently, consistency may be measured according to which of these different properties is satisfied. One of these properties is the "*additive consistency*", which, as shown in [3], can be seen as the parallel concept of Saaty's consistency property in the case of multiplicative preference relations [6].

In this paper we show some of the uses of the the concept of additive consistency in the resolution process of GDM problems. We propose a new *additive consistency measure* for both complete and incomplete fuzzy preference relations. Based on this measure, a new IOWA operator [10], which we call Additive-Consistency based IOWA (AC-IOWA) operator, and an *iterative procedure* to estimate the *missing values* of an incomplete fuzzy preference relation are presented. Finally, following the choice scheme proposed in [1], i.e., *aggregation* followed by *exploitation*, a resolution process of GDM problems with incomplete fuzzy preference relations, based on fuzzy majority and the IOWA operator presented in this paper, is given.

2 Preliminaries

The problem we deal with is that of choosing the best alternative(s) among a finite set, $X = \{x_1, ..., x_n\}$, $(n \geq 2)$. The alternatives will be classified from best to worst, using the information known according to a set of experts, i.e., $E = \{e_1, ..., e_m\}$, $(m \geq 2)$. Each expert $e_k \in E$, will provide his/her preferences by means of a *fuzzy preference relation*:

Definition 1 *A fuzzy preference relation P on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P\colon X \times X \longrightarrow [0,1]$.*

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ij})$, being $p_{ij} = \mu_P(x_i, x_j)$ $\forall i, j \in \{1, \ldots, n\}$ interpreted as the preference degree or intensity of the alternative x_i over x_j.

Usual decision-making procedures assume that experts are able to provide preference degrees between any pair of possible alternatives. This is not always possible, which makes missing information a problem that has to be dealt with. In order to model these situations, in the following definitions the concept of an incomplete fuzzy preference relation is expressed:

Definition 2 *A function $f\colon X \longrightarrow Y$ is partial when not every element in the set X necessarily maps to an element in the set Y. When every element from the set X maps to one element of the set Y then we have a total function.*

Definition 3 *An incomplete fuzzy preference relation P on a set of alternatives X is a fuzzy set on the product set $X \times X$ that is characterized by a partial membership function.*

As per this definition, a fuzzy preference relation is complete when its membership function is a total one. Clearly, the usual definition of a fuzzy preference relation (definition 1) includes both definitions of complete and incomplete fuzzy preference relations. However, as there is no risk of confusion between a complete and an incomplete fuzzy preference relation, in this paper we refer to the first type as simply fuzzy preference relations.

3 Additive Consistency as a Tool for GDM

As shown in [3], additive transitivity for fuzzy preference relations can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [6]. The mathematical formulation of the *additive transitivity* was given by Tanino [7]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5) \;\; \forall i, j, k \in \{1, \ldots, n\} \tag{1}$$

In this paper, we will consider a fuzzy preference relation to be "*additive consistent*" when for every three options in the problem $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik} fulfil (1). An additive consistent fuzzy preference relation will be referred as consistent throughout the paper.

3.1 Additive Consistency Measure

Expression (1) can be rewritten as:

$$p_{ij} + p_{jk} - 0.5 = p_{ik} \;\; \forall i, j, k \in \{1, \ldots, n\} \tag{2}$$

This expression allows a preference degree p_{ik} to be calculated using other preference degrees. Indeed, let us denote

$$cp_{ik}^{j} = p_{ij} + p_{jk} - 0.5$$

where cp_{ik}^{j} means the calculated value of p_{ik} via j, that is, using p_{ij} and p_{jk}. Obviously, if the information provided in a fuzzy preference relation is completely consistent then cp_{ik}^{j}, $\forall j \in \{1, \ldots, n\}$, and p_{ik} coincide. However, because experts are not always fully consistent, the information given by an expert may not verify (2). In these cases, the value

$$\varepsilon p_{ik} = \frac{\sum_{\substack{j=1 \\ j \neq i,k}}^{n} \left| cp_{ik}^{j} - p_{ik} \right|}{n-2} \tag{3}$$

can be used to measure the error expressed in a preference degree between two options. This error can be interpreted as the consistency level between the preference degree p_{ik} and the rest of preference values of the fuzzy preference relation. When $\varepsilon p_{ik} = 0$ there is no inconsistency at all, and the higher the value of εp_{ik} the more inconsistent is p_{ik} with respect to the rest of information.

The *consistency level* for the whole fuzzy preference relation P is defined as follows:

$$CL_P = \frac{\sum_{\substack{i,k=1 \\ i \neq k}}^{n} \varepsilon p_{ik}}{n^2 - n} \tag{4}$$

When $CL_P = 0$ the preference relation P is fully consistent, otherwise, the higher CL_P the more inconsistent P.

When working with an incomplete fuzzy preference relation, equation (3) cannot be used to estimate preference values. To cover these cases, we define:

$$A = \{(i,j) \mid i,j \in \{1, \ldots, n\} \wedge i \neq j\}$$

$$MV = \{(i,j) \mid p_{ij} \text{ unknown }, (i,j) \in A\}$$

$$EV = A \setminus MV$$

$$H_{ik} = \{j \mid (i,j), (j,k) \in EV\} \ \forall i \neq k$$

$$CE_P = \{(i,k) \in EV \mid \exists j : (i,j), (j,k) \in EV\}$$

$$\varepsilon p_{ik} = \frac{\sum_{j \in H_{ik}} \left| cp_{ik}^{j} - p_{ik} \right|}{\# H_{ik}}$$

$$CL_P = \frac{\sum_{(i,k) \in CE_P} \varepsilon p_{ik}}{\# CE_P}$$

We call CE_P the *computable error* set because it contains all the elements for which we can compute every εp_{ik}. This redefinition of CL_P is an extension of expression (4), because when P is complete both CE_P and A coincide and thus $\# CE_P = n^2 - n$.

3.2 Additive Consistency based IOWA Operator

A rational assumption in the resolution process of a GDM is that of associating more importance to those experts that provide the more *consistent* information. This assumption implies that GDM problems should be viewed as heterogeneous problems. Indeed, in any GDM problem with fuzzy preference relations, each expert e_h can have associated its fuzzy preference relation consistency index value (CL_{P^h}), and therefore Yager's procedure to evaluate the overall satisfaction of Q important criteria (experts) by an alternative can be applied. This procedure associates a zero weight to those expert with zero importance degree (see [9] for more details). In our case, we may as well implement these consistency level values by an alternative approach, which consists of using them to induce the ordering of the IOWA operator to be applied in the aggregation phase of the resolution process [5, 10]. Indeed, the set of consistency levels may be used not just to associate 'importance' values to the experts but also to establish the ordering of the preference values to be aggregated by ordering the experts from most to least consistent one. In this case we obtain an IOWA operator that we call the additive-consistency IOWA (AC-IOWA) operator and denote it as Φ_W^{AC}.

Definition 4 *If a set of experts, $E = \{e_1, \ldots, e_m\}$, provides preferences about a set of alternatives, $X = \{x_1, \ldots, x_n\}$, by means of the fuzzy preference relations, $\{P^1, \ldots, P^m\}$, then the AC-IOWA operator of dimension m, Φ_W^{AC}, is an IOWA operator whose set of order inducing values is $\{1 - CL_{P^1}, ..., 1 - CL_{P^m}\}$.*

4 Estimation of Missing Values in Incomplete Fuzzy Preference Relations Using Additive Consistency

Usual procedures for GDM problems correct the lack of knowledge of a particular expert using the information provided by the rest of the experts in conjunction with aggregation procedures [4,8]. Our proposal estimates missing information in an expert's incomplete fuzzy preference relation using only the rest of preference values provided by that particular expert. By doing this, we assure that the reconstruction of the incomplete fuzzy preference relation is compatible with the rest of the information provided by that expert. In fact, our procedure is guided by the expert's consistency level measured taking into account only the provided preference values, because an important objective is to maintain experts' consistency levels. To develop the iterative procedure to estimate missing values two different tasks have to be carried out:

4.1 Elements to be estimated in step h

The subset of missing values MV that can be estimated in step h is denoted by EMV_h (*estimated missing values*) and defined as follows:

$$EMV_h = \left\{ (i,k) \in MV \setminus \bigcup_{l=0}^{h-1} EMV_l \;\middle|\; \exists j : (i,j),(j,k) \in EV \cup \left(\bigcup_{l=0}^{h-1} EMV_l \right) \right\}$$

with $EMV_0 = \emptyset$.

When $EMV_{maxIter} = \emptyset$ with $maxIter > 0$ the procedure stops because there will not be any more missing values to be estimated. Furthermore, if $\bigcup_{l=0}^{maxIter} EMV_l = MV$ then all missing values are estimated and consequently the procedure is said to be successful in the completion of the fuzzy preference relation.

4.2 Expression to estimate a particular value p_{ik}

In iteration h, to estimate a particular value p_{ik} with $(i,k) \in EMV_h$, the following three steps function is applied:

function estimate_p(i,k)

1. $I_{ik} = \left\{ j \;\middle|\; (i,j),(j,k) \in EV \cup \left(\bigcup_{l=0}^{h-1} EMV_l \right) \right\}$

2. Calculate $cp'_{ik} = \dfrac{\sum_{j \in I_{ik}} cp^j_{ik}}{\# I_{ik}}$

3. Make $p_{ik} = cp'_{ik} + z$ with $z \in [-CL_P, CL_P]$ randomly selected, subject to $0 \le cp'_{ik} + z \le 1$

end function

Therefore, a missing value p_{ik} can be estimated when there is at least one chained pair of known preference values (p_{ij}, p_{jk}) that allow the application of expression (3), in which case the average of the values obtained using it, cp'_{ik}, is calculated. The estimation of p_{ik} is obtained by adding a random value $z \in [-CL_P, CL_P]$ to this average value. This is done in order to maintain the consistency level of the expert, and is subject to the condition of being the final estimated value in the range of fuzzy preference values $[0, 1]$.

5 Resolution Process of a GDM with Incomplete Fuzzy Preference Relations

In this context, to obtain a set of solution alternatives $X_{sol} \subset X$, the first step of a resolution process of GDM problems with incomplete fuzzy preference relations might be the application of the iterative procedure to estimate the missing values. Therefore, the resolution process presents the scheme given in fig. 1.

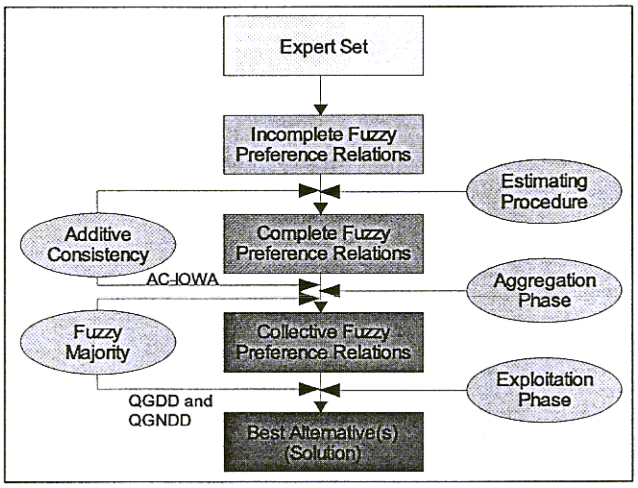

Figure 1: Resolution Process of a GDM with Incomplete FPR

Once the experts provide their (incomplete) preference relations, two main steps are applied: (1) *Estimation of missing information*, and (2) *Application of a selection process*

1. *Estimation of missing information.* In this step, incomplete fuzzy preference relations are completed by using the iterative procedure presented in section 4.

2. *Application of a selection process*, which is carried out in two sequential phases:

(a) *Aggregation phase.* A collective fuzzy preference relation is obtained by aggregating all the individual fuzzy preference relations. This aggregation is carried out by applying the AC-IOWA operator guided by a linguistic quantifier representing the concept of *fuzzy majority* (of experts) desired to implement in the resolution process.

(b) *Exploitation phase.* Using again the concept of fuzzy majority (of alternatives), two choice degrees of alternatives are used: the *quantifier-guided dominance degree (QGDD)* and the *quantifier-guided non-dominance degree (QGNDD)* [1]. These choice degrees will act over the collective preference relation resulting in a global ranking of the alternatives, from which the set of solution alternatives will be obtained.

6 Conclusions

Additive consistency property can be used as a tool to solve GDM problems with complete or incomplete fuzzy preference relations. In the last case, an iterative procedure to estimate missed preference values, using only the preference values provided by that particular expert, has been presented. This is guided by the expert's additive consistency level, for which an additive consistency measure has been defined. Based on this iterative procedure and on the additive consistency property, we have presented a new decision model to solve GDM problems with incomplete fuzzy preference relations. In this decision model, a new IOWA operator is used, the AC-IOWA operator. This operator permits the aggregation of experts' preferences in such a way that more importance is associated to the most consistent ones.

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