Managing Incomplete Information in Consensus Processes

S. Alonso
Dept. of Computer Science
& Artificial Intelligence
University of Granada
salonso@decsai.ugr.es

E. Herrera-Viedma
Dept. of Computer Science
& Artificial Intelligence
University of Granada
viedma@decsai.ugr.es

F. Chiclana
Div. of Inf. Management,
School of Computing
De Montfort University
chiclana@dmu.ac.uk

F. Herrera
Dept. of Computer Science
& Artificial Intelligence
University of Granada
herrera@decsai.ugr.es

Resumen

In the resolution of a Multiperson Decision Making (MPDM) problem is usually important to assure that there is at least a certain level of agreement between the experts on the final solution. In this paper we present a *Consensus Reaching Process* for MPDM where the information given by the experts is based on incomplete fuzzy preference relations (FPRs). In this process we define a method to generate advice by means of easy rules about how experts should change their opinions to reach a solution with a high consensus degree and with a high consistency level on the experts preferences. To do so we use a new Consistency / Consensus IOWA operator that handles FPRs and some consistency and consensus measures about them to aggregate the information provided by the experts.

Keywords: Incomplete Information, Consensus, Additive Transitivity, Fuzzy Preference Relations, IOWA operator.

1. Introduction

Multiperson Decision Making (MPDM) problems involve choosing a solution set of alternatives over a feasible set $X = \{x_1, ..., x_n\}$ according to the preferences provided by different experts $E = \{e_1, ..., e_m\}$. We assume that experts express their preferences about the alternatives using fuzzy preference relations. The resolution of a MPDM problem usually involves two different processes: a *Consensus Reaching Process* (CRP), where the experts should change their preferences in order to achieve a certain degree of consensus and a *Selection Process* where the best options are derived from the previously consensued experts' preferences. It is a desirable property for the CRPs that during the different steps that they involve, not only a high level of consensus is reached, but a certain amount of consistency of the information given by the experts is achieved and maintained.

A problem that has also to be addressed when dealing with real MPDM problems is the lack of information. Sometimes, experts are not able to provide enough information about their preferences, i.e., they may not be able to measure their preference degrees over some alternatives, or they don't have enough knowledge about part of the problem presented, so they prefer not to guess it and not to give part of the information required to solve the problem.

In this paper, we present a CRP where the information given by the experts is based on *incomplete* fuzzy preference relations (IFPR). This process uses a feedback mechanism to give recommendations (expressed as easy rules) that will help the experts to reach a solution that fulfills two different criterions. The first criterion (a *global* one) is to obtain a solution with a high consensus degree between experts. The second one (*individual* criterion) is to obtain a high level of consistency on the experts preferences. Another feature of the process is that it is able to give advice on how to complete the missing information on the IFPRs. To do that, the CRP uses some consistency and consensus measures among with a new IOWA operator (the Consistency / Consensus (CC) IOWA operator) that is able to aggregate the different FPRs into a global one that represents the preferences of all the experts as a whole.

The paper is set as follows. In Section 2 we present our preliminaries, that is, the basic tools used in the design of the CRP, including the CC-IOWA operator. In Section 3 we show the CRP and finally, in Section 4 we point out some conclusions and future works.

2. Preliminaries

In this section, we present the tools to design the CRP, that is, the concept of IFPR, consistency measures, and the new CC-IOWA operator.

2.1. Incomplete Fuzzy Preference Relations

There exist several representation formats in which the experts can express their opinions. One of the most used are *fuzzy preference relations* [2, 7, 9, 10] because of their effectiveness as a tool for modelling decision processes and their utility and easiness of use when we want to aggregate experts' preferences into group preferences [4, 7, 11].

Definition 1. A Fuzzy Preference Relation (FPR) P on a set of alternatives X is a fuzzy set on the product set $X \times X$, i.e., it is characterized by a membership function $\mu_P: X \times X \longrightarrow [0,1]$.

When cardinality of X is small, the preference relation may be conveniently represented by the $n \times n$ matrix $P = (p_{ik})$, being $p_{ik} = \mu_P(x_i, x_k)$ ($\forall i,k \in \{1,\ldots,n\}$) interpreted as the preference degree or intensity of the alternative x_i over x_k: $p_{ik} = 1/2$ indicates indifference between x_i and x_k ($x_i \sim x_k$), $p_{ik} = 1$ indicates that x_i is absolutely preferred to x_k, and $p_{ik} > 1/2$ indicates that x_i is preferred to x_k ($x_i \succ x_k$). Based on this interpretation we have that $p_{ii} = 1/2 \ \forall i \in \{1,\ldots,n\}$ ($x_i \sim x_i$).

Usual models to solve MPDM problems assume that experts are always able to provide all the preferences required, that is, to provide all p_{ik} values. This situation is not always possible to achieve. Experts could have some difficulties in giving their preferences due to lack of knowledge about part of the problem, or due to that expert not being able to quantify his/her degree of preference.

In order to model such situations, we define the concept of an *incomplete fuzzy preference relation*:

Definition 2. A function $f: X \longrightarrow Y$ is *partial* when not every element in the set X necessarily maps onto an element in the set Y. When every element from the set X maps onto one element of the set Y then we have a *total* function.

Definition 3. [6] An *Incomplete Fuzzy Preference Relation* P on a set of alternatives X is a fuzzy set on the product set $X \times X$ that is characterized by a *partial* membership function.

From an IFPR we define the following sets [6]:

- $A = \{(i,j) \mid i,j \in \{1,\ldots,n\} \wedge i \neq j\}$,
- $MV_h = \left\{(i,j) \in A \mid p_{ij}^h \ \textit{is unknown}\right\}$,
- $EV_h = A \setminus MV_h$
- $EV_h^i = \{(a,b) \mid (a,b) \in EV_h \wedge (a = i \vee b = i)\}$

where MV_h is the set of pairs of alternatives for which the preference degree of the first alternative over the second one is not given by expert e_h, EV_h is the set of pairs of alternatives for which the expert e_h provides preference values and EV_h^i is the set of preferences about pairs of alternatives given by an expert e_h involving alternative x_i.

2.2. Consistency Measures

In real MPDM problems with fuzzy preference relations some properties about the preferences expressed by the experts are usually assumed desirable to avoid contradiction between their own opinions (that is, to avoid to have low degree of *consistency*). One of this properties is the *transitivity* property, which represents the idea that the preference value obtained by directly comparing two alternatives should be equal to or greater than the preference value between those two alternatives obtained using an indirect chain of alternatives. There are several possible characterizations for transitivity (see [4]). In this paper we make use of the *Additive Transitivity* property. The mathematical formulation of this property was given by Tanino [11] and can be written as [4, 6]:

$$p_{ik} = p_{ij} + p_{jk} - 0{,}5 \quad \forall i,j,k \in \{1,\dots,n\} \quad (1)$$

A FPR will be considered additive consistent when expression (1) is satisfied $\forall p_{ik}, p_{ij}, p_{jk} \mid i \neq j \neq k$. Moreover, expression (1) can be used to calculate an estimated value cp_{ik} for every p_{ik} as:

$$cp_{ik} = \frac{\sum_{j=1;i\neq k\neq j}^{n} cp_{ik}^{j1} + cp_{ik}^{j2} + cp_{ik}^{j3}}{3(n-2)} \quad (2)$$

where

$$cp_{ik}^{j1} = p_{ij} + p_{jk} - 0{,}5,$$
$$cp_{ik}^{j2} = p_{jk} - p_{ji} + 0{,}5,$$
$$cp_{ik}^{j3} = p_{ij} - p_{kj} + 0{,}5$$

Note that the cp_{ik} and p_{ik} will not usually coincide unless the preference relation P fully satisfies additive consistency, and thus, we can define an error between p_{ik} and cp_{ik} [6]:

$$\varepsilon p_{ik} = (2/3) \cdot |cp_{ik} - p_{ik}|.$$

Definition 4. [6] The *consistency measure* for every pair of alternatives in a FPR P_h is computed as:

$$cl_{ik}^{h} = (1-\alpha_{ik}^{h})\cdot(1-\varepsilon p_{ik}^{h}) + \alpha_{ik}^{h} \cdot \frac{C_h^i + C_h^k}{2} \quad (3)$$

where $C_h^i = \frac{\#EV_h^i}{2(n-1)}$ is the completeness level for the alternative x_i given by the expert e_h and $\alpha_{ik}^{h} \in [0,1]$ is a parameter to control the influence of completeness in the evaluation of the consistency levels. We proposed a simple linear solution to obtain that parameter:

$$\alpha_{ik} = 1 - \frac{\#EV_h^i + \#EV_h^k}{4(n-1)}.$$

Definition 5. The consistency measure for a given alternative x_i is computed as:

$$cl_i^h = \frac{\sum_{k=1;k\neq i}^{n} (cl_{ik}^h + cl_{ki}^h)}{2\cdot(n-1)},$$

Definition 6. The consistency measure for a whole preference relation is computed as:

$$cl^h = \frac{\sum_{i=1}^{n} (cl_i^h)}{n}.$$

2.3. CC-IOWA Operator

In [12] Yager defined the IOWA operator:

Definition 5. [12] An IOWA operator of dimension n is a function $\Phi_W: (\Re \times \Re)^n \rightarrow \Re$, to which a set of weights or weighting vector is associated, $W = (w_1, \dots, w_n)$, with $w_i \in [0,1]$, $\Sigma_i w_i = 1$, and it is defined to aggregate the set of second arguments of a list of n 2-tuples $\{\langle u_1, p_1\rangle, \dots, \langle u_n, p_n\rangle\}$ according to the following expression,

$$\Phi_W(\langle u_1, p_1\rangle, \dots, \langle u_n, p_n\rangle) = \sum_{i=1}^{n} w_i \cdot p_{\sigma(i)}$$

being σ a permutation of $\{1,\dots,n\}$ such that $u_{\sigma(i)} \geq u_{\sigma(i+1)}, \forall i = 1,\dots,n-1$, i.e., $\langle u_{\sigma(i)}, p_{\sigma(i)}\rangle$ is the 2-tuple with $u_{\sigma(i)}$ the highest value in the set $\{u_1, \dots, u_n\}$. The set of values $\{u_1, \cdots, u_n\}$ are called the values of an *order inducing variable*.

IOWA operators can be used to aggregate different FPR $\{P^1, \cdots, P^m\}$ into a global one P^c by defining a proper order inducing variable [6]. We define the Consensus / Consistency IOWA (CC-IOWA) operator to aggregate FPRs as follows:

$$p_{ik}^c = \Phi_W(\langle z_{ik}^1, p_{ik}^1\rangle, \cdots, \langle z_{ik}^m, p_{ik}^m\rangle)$$

where the set of inducing variables $\{z_{ik}^1, \dots, z_{ik}^m\}$ will be computed as

$$z_{ik}^h = (1-\delta)\cdot cl_{ik}^h + \delta \cdot co_{ik}^h,$$

being co_{ik}^h a consensus measure for the preference value p_{ik} expressed by expert e_h and $\delta \in [0,1]$ a parameter to control the weight of both the consistency and consensus in the decision process. Usually $\delta > 0{,}5$ will be used to give more importance to the most consensued values against the most consistent ones.

3. Consensus Reaching Process

In this section we present a CRP for MPDM based on IFPR. Contrary to other previously given CRPs in the literature [1, 3, 5, 8], the CRP that we present is guided by two kinds of measures: consistency and consensus measures. We design it trying to obtain the maximum possible consensus level while trying to achieve a high level of consistency among experts' preferences. We should point out that the consistency search often leads to reduce the consensus level and viceversa. Thus, we try to maintain a balance between both. Moreover, not only the CRP is able to achieve a solution with certain consensus and consistency degrees simultaneously, but it is able to deal with IFPR, giving advice to the experts on how to complete them.

The steps of the CRP are the following:

1. Computing Missing Information
2. Computing Consistency Measures
3. Computing Consensus Measures
4. Controlling the Consensus/Consistency State
5. Feedback Process

3.1. Computing Missing Information

In [6] we presented an iterative procedure capable of completing every IFPR based on the additive consistency of the self preference relation and using equations derived from expression (2). In this step, we compute every $p^h_{ik} \in MV_h$ and thus, we obtain a reconstructed preference relation P'_h for every IFPR P_h.

3.2. Computing Consistency Measures

In this step, for every expert e_h we compute their respective consistent matrices $CP_h = \left(cp^h_{ik}\right)$ according to expression (2). From every P'_h and CP_h we are able to compute the different consistency measures presented in section 2.2, i.e., cl^h_{ik}, cl^h_i, cl^h $\forall i,k \in \{1,...,n\}$. Then, we define a global consistency measure among all experts to control the global consistency status:

$$CL = \frac{\sum_{h=1}^{m} cl^h}{m}.$$

3.3. Computing Consensus Measures

The CRP also needs some consensus and proximity measures about the experts' preferences. In [5] these measures were given on three different levels of a FPR: pairs of alternatives, alternatives and relations. We use this measure structure on the CRP.

Firstly, for each pair of experts e_h, e_l $(h < l)$ we define a similarity matrix $SM^{hl} = \left(sm^{hl}_{ik}\right)$ where

$$sm^{hl}_{ik} = 1 - |p^{h'}_{ik} - p^{l'}_{ik}|.$$

A *consensus matrix*, $CM = (cm_{ik})$ is obtained by aggregating all the similarity matrices using the arithmetic mean as the aggregation function ϕ:

$$cm_{ik} = \phi(sm^{hl}_{ik}) \;;\; \forall h,l = 1,...,m \mid h < l.$$

We can now compute the *consensus degrees* in the different levels:

1. **Level 1.** *Consensus on pairs of alternatives.* The consensus degree on a pair of alternatives (x_i, x_k), called cop_{ik} is defined to measure the consensus degree amongst all the experts on that pair of alternatives:

$$cop_{ik} = cm_{ik}.$$

2. **Level 2.** *Consensus on alternatives.* The consensus degree on an alternative (x_i), called ca_i is defined to measure the consensus degree amongst all the experts on that alternative:

$$ca_i = \frac{\sum_{k=1;k\neq i}^{n}(cop_{ik} + cop_{ki})}{2n-2}.$$

3. **Level 3.** *Consensus on the relation.* The consensus degree on the relation, called CR is defined to measure the global consensus degree amongst all the experts' opinions:

$$CR = \frac{\sum_{i=1}^{n} ca_i}{n}.$$

Similarly, as we did with the consensus degrees, we have to define some *proximity measures* for each expert. To do so, we need a *collective* FPR, P^c that summarizes the preferences given by all the experts. We will use the

CC-IOWA operator (presented in section 2.3), where the co_{ik}^h values used to calculate the set of inducing variables $\{z_{ik}^1, \ldots, z_{ik}^m\}$ are similarity measures between the experts e_h and the rest of experts assessed for the pair of alternatives x_i and x_k, computed as:

$$co_{ik}^h = \frac{\sum_{l=h+1}^{n} sm_{ik}^{hl} + \sum_{l=1}^{h-1} sm_{ik}^{lh}}{n-1}$$

The proximity measures are then computed as follows:

1. **Level 1.** *Proximity on pairs of alternatives.* The proximity of an expert e_h on a pair of alternatives (x_i, x_k) to the group one, called pp_{ik}^h, is calculated as

$$pp_{ik}^h = 1 - |p_{ik}^{h'} - p_{ik}^c|.$$

2. **Level 2.** *Proximity on alternatives.* The proximity of an expert e_h on an alternative x_i to the group one, called pa_i^h, is calculated as:

$$pa_i = \frac{\sum_{k=1;k\neq i}^{n} (pp_{ik} + pp_{ki})}{2n-2}.$$

3. **Level 3.** *Proximity on the relation.* The proximity of an expert e_h on his/her preference relation to the group one, called pr^h, is calculated as:

$$pr^h = \frac{\sum_{i=1}^{n} pa_i^h}{n}.$$

3.4. Controlling Consensus/Consistency State

The Consensus/Consistency State Control process involves to decide when the feedback mechanism should be applied to give advice to the experts or to redirect the process to the selection phase. It takes into account both the consensus and consistency measures. To do that, we define a new measure or level of satisfaction *(Consensus/Consistency Level, CCL)* that we use as a control parameter:

$$CCL = (1-\delta) \cdot CL + \delta \cdot CR$$

When the CCL is above a certain *minimum satisfaction threshold*, $\gamma \in [0,1]$ then the CRP should end towards a selection process to obtain the final solution for the problem.

Additionally, the system should avoid stagnation, that is, the consensus and consistency measures never reaching an appropiate value. To do so, a maximum number of iterations $maxIter$ should be fixed and compared to the actual number of iterations of the process ($numIter$).

The consensus/consistency control routine pseudocode is shown:

1. If $CCL > \gamma$ or $numIter > maxIter$ then
2. *Go to Selection Process*
3. else
5. $numIter++$
6. Advice the experts (*feedback process*)

3.5. Feedback Process

The feedback process consists on two substeps: *Identification of the preference values* that have to be changed and *Generation of advice*

3.5.1. Identification of the Preference Values

We must identify which experts and preference values are contributing less to reach a high consensus/consistency state. We call the *Advice Preferences Set* (APS) to the set of (h, i, k) whose p_{ik}^h values should be changed because they are negativelly contributing to that state. To calculate APS, we apply a three step identification process that uses proximity and consistency measures previously defined.

Step 1. We identify the set of experts $EXPCH$ that should recieve advice on how to change some of their preference values. The experts that should change their opinions are those whose satisfaction degree on the relation is lower than the satisfaction threshold γ, i.e.,

$$EXPCH = \{h \mid (1-\delta) \cdot cl^h + \delta \cdot pr^h < \gamma\}.$$

Step 2. We identify the alternatives that the above experts should consider to change.

This set of alternatives is denoted as ALT. To do this, we select the alternatives whose satisfaction degree is lower than the satisfaction threshold γ, i.e.,

$$ALT = \{(h,i) \mid (1-\delta)\cdot cl_i^h + \delta \cdot ca_i^h < \gamma \text{ and } e_h \in EXPCH\}.$$

Step 3. Finally, we identify which of the preference values for every alternative and expert (x_i ; $e_h \mid (h,i) \in ALT$) should be changed according to their proximity and consistency measures on the pairs of alternatives. Then we have

$$APS = \{(h,i,k) \mid (h,i) \in ALT \text{ and } (1-\delta)\cdot cl_{ik}^h + \delta \cdot pp_{ik}^h < \gamma\}.$$

Additionally the feedback process must provide rules missing preference values. To do so, it has to add to the APS all missing values that were not provided by the experts, i.e.

$$APS' = APS \cup \{(h,i,k) \mid p_{ik}^h \in MV_h\}.$$

3.5.2. Generation of Advice

In this last step, recomendations are generated to the experts based on easy rules that the CRP provides.

The rules not only tell experts which preference values should they change, but they propose particular values for each preference to reach a solution of high consensus/consistency.

To calculate these particular values we use a weighed mean between the cp_{ik}^h value previously computed and the p_{ik}^c value:

$$rp_{ik}^h = (1-\delta)\cdot cp_{ik}^h + \delta \cdot p_{ik}^c,$$

where rp_{ik}^h will be the value that will be used in the rule to the expert e_h to change the preference value about alternatives x_i and x_k. As previously mentioned, with $\delta > 0{,}5$ the CRP directs the experts towards a consensus solution more than to increase their own consistency levels.

Finally, we should only differenciate two cases: if the rule has to be given because a preference value is far from the consensus/consistency state or because the expert did not provide the preference value. Therefore there are two kinds of recommendation rules:

1. If $p_{ik}^h \in EV_h$ the rule generated for the expert e_h is: *"You should change your preference value (i,k) to a value close to rp_{ik}^h."*
2. If $p_{ik}^h \in MV_h$ the rule generated for the expert e_h is: *"You should provide a value for (i,k) close to rp_{ik}^h."*

Once experts receive the recommendations the CRP should begin again, with the experts giving their new IFPRs which should be closer to a consensus solution with higher level of consistency on the individual experts' preferences (if they have followed the provided rules).

4. Conclusions and Future Works

In this paper we have presented a new Consensus Reaching Process capable of handling Incomplete Fuzzy Preference Relations. It uses some consistency, consensus and proximity measures among with a new IOWA operator (CC-IOWA operator) to give advice to the experts by means of easy rules that would direct them towards a more consensued solution, and also achieving a high consistency degree on the experts' preferences. It is even able to give recommendations to reconstruct the missing information on the IFPRs in a consistent way.

In the future, we will refine and extend this CRP to accept different kinds of preference relations.

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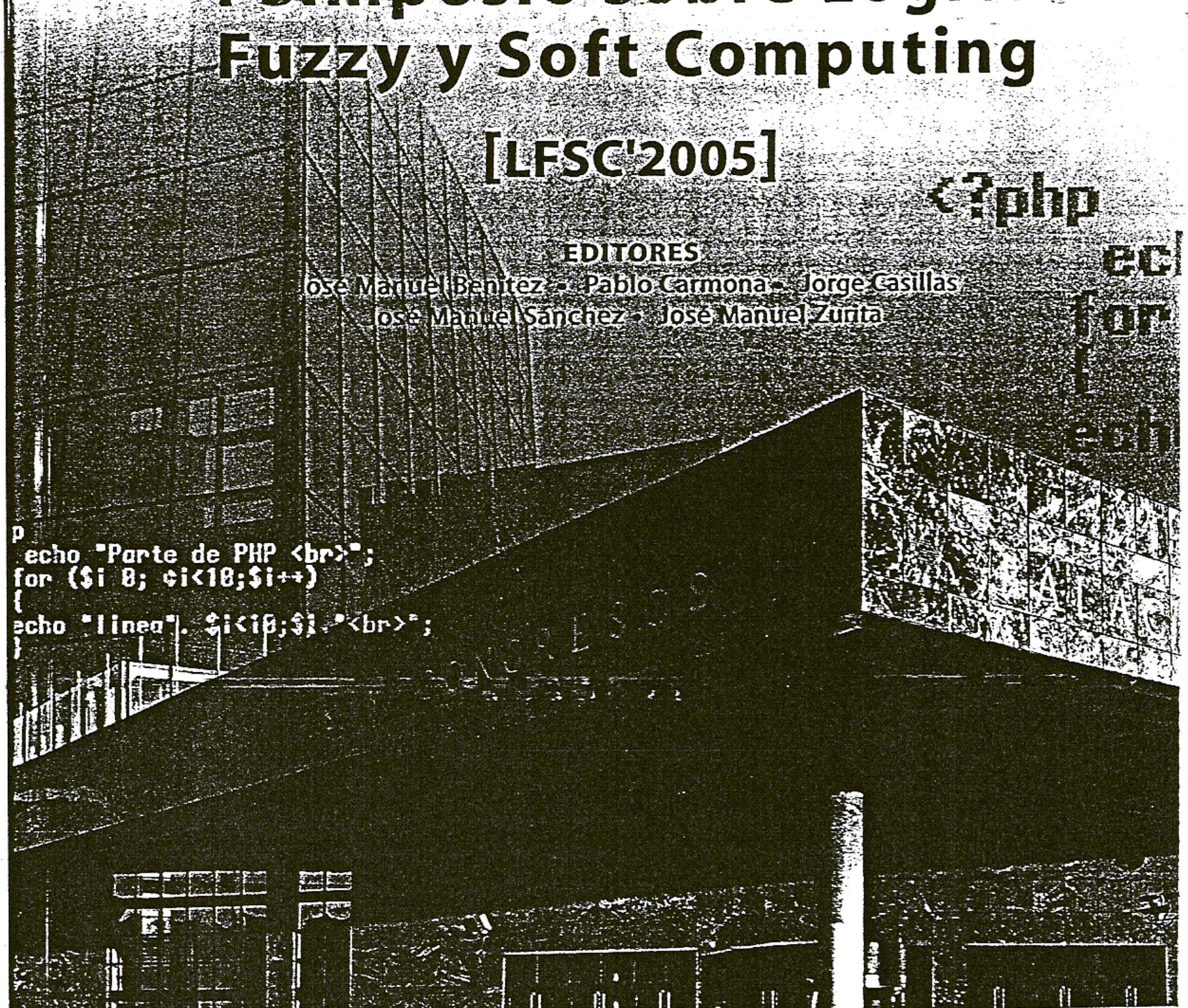

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