AN APPROACH FOR COMBINING LINGUISTIC AND NUMERICAL INFORMATION BASED ON THE 2-TUPLE FUZZY LINGUISTIC REPRESENTATION MODEL IN DECISION-MAKING

F. HERRERA

Department of Computer Sciences and A.I.
University of Granada, 18071 - Granada, Spain
E-mail: herrera@decsai.ugr.es

L. MARTINEZ

Department of Computer Science
University of Jaén, 23071 - Jaén, Spain
E-mail: martin@ujaen.es

In this paper we shall develop a procedure for combining numerical and linguistic information without loss of information in the transformation processes between numerical and linguistic information, taking as base for representing the information the 2-tuple fuzzy linguistic representation model. We shall analyze the conditions to impose the linguistic term set in order to ensure that the combination procedure does not produce any loss of information. Afterwards the aggregation process will be applied to a decision procedure over a multi-attribute decision-making problem dealing with numerical and linguistic information, that is, with qualitative and quantitative attributes.

Keywords: Linguistic modeling, aggregation processes, combining linguistic and numerical information, multi-attribute decision-making.

1. Introduction

Decision-Making is a very important activity widely studied in the literature [1,2]. The most of the proposals for decision-making that can be found are focused in problems that present either quantitative aspects or qualitative aspects. However due to the nature of the decision-making problems, in practice it is common to find problems with both quantitative and qualitative aspects. Usually quantitative aspects are assessed by means of precise numerical values, but qualitative aspects are complex to assess with precise and exact values. In the latter case, the use of the Fuzzy Linguistic Approach has provided very good results, it deals with qualitative

aspects that are presented in qualitative terms by means of linguistic variables, i.e., variables whose values are not numbers but words or sentences in a natural or artificial language [3]. The decision-making problems can deal with information assessed in the numerical and in the linguistic domains due to the fact that different experts of the decision-making problem come from different knowledge areas and each one prefers to express his/her knowledge in a predefined domain, or because in the problem there exist qualitative and quantitative aspects. In this contribution, we focus in Multi-attribute decision-making (MADM) problems with quantitative and qualitative attributes, therefore dealing with numerical and linguistic information.

The decision process is a two-step process [4] where the first step consists of combining the information. Therefore, those decision problems that present linguistic and numerical information need aggregation operators for combining this type of information. In the specialized literature we can find different decision processes for decision-making problems dealing with numerical and linguistic information [5,6], but these approaches present different difficulties in the aggregation process such as the loss of information, results expressed far from the initial domains, complexity etc.

The aim of this paper is the development of an aggregation procedure for easily combining linguistic and numerical information without loss of information. To do so, in first place we shall take as its representation base the 2-tuple fuzzy linguistic representation model [7]. Afterwards we shall define different transformation functions between the linguistic domain, the numerical domain and linguistic 2-tuples, and we shall impose under what conditions these transformations are carried out without any loss of information. Finally, a fusion method based on the 2-tuple linguistic representation model for combining linguistic and numerical information without loss of information is developed.

In order to do so, this paper is structured as follows: in Section 2, we shall provide a brief review of the fuzzy linguistic approach, the 2-tuple fuzzy linguistic representation model and present the scheme of an MADM problem; in Section 3, we present the transformation processes between numerical information assessed in $[0,1]$ and linguistic 2-tuples, afterwards we study under what conditions these processes are carried out without any loss of information; in Section 4, we develop an aggregation procedure for numerical and linguistic information based on the 2-tuple representation model; in Section 5, is presented an example of an MADM problem with both linguistic and numerical information. Some concluding remarks are pointed out in Section 6.

2. Preliminaries

In this section, we briefly review the fuzzy linguistic approach and the 2-tuple fuzzy linguistic representation model. Afterwards, we shall present a general scheme for an MADM problem.

2.1. *Fuzzy Linguistic Approach*

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case, a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [3]. The fuzzy linguistic approach has been applied successfully in different areas, such as, "decision-making" [8,9,10,11,12,13,14,15], "information retrieval" [16], "clinical diagnosis" [17], "marketing" [18], "risk in software development" [19], "technology transfer strategy selection" [20], "educational grading systems" [21], "scheduling" [22,23], "consensus" [24,25], "materials selection" [26], "personnel management" [27], etc.

We have to choose the appropriate linguistic descriptors for the linguistic term set and their semantics. In order to accomplish this objective, an important aspect to analyze is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, where the mid term represents an assessment of "approximately 0.5", and with the rest of the terms being placed symmetrically around it [28]. These classical cardinality values seem to fall into line with Miller's observation regarding the fact that human beings can reasonably manage to bear in mind seven or so items [29].

Once it has been established the cardinality of the linguistic term set, it must be provided a way to generate the linguistic terms and its semantics, there exist different possibilities to accomplish this task [30,16,25,31,32]. One possibility of generating the linguistic term set consists of directly supplying the term set by considering all the terms distributed on a scale on which a total order is defined [31,32]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: $\text{Neg}(s_i) = s_j$ such that $j = g$-i ($g+1$ is the cardinality).

2. $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a minimization and a maximization operator.

The semantics of the terms is given by fuzzy numbers defined in the [0,1] interval, which are usually described by membership functions. A computationally efficient way to characterize a fuzzy number is to use a representation based on parameters of its membership function [28]. Because the linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments, since it may be impossible and unnecessary to obtain more accurate values [8]. This parametric representation is achieved by the 4-tuple (a, b, d, c), where

b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [28]. A specific case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, so we represent this type of membership function by a 3-tuple (a, b, c). For example, we may assign the following semantics to the set of seven terms:

$$P = Perfect = (.83, 1, 1) \quad VH = Very\_High = (.67, .83, 1)$$
$$H = High = (.5, .67, .83) \quad M = Medium = (.33, .5, .67)$$
$$L = Low = (.17, .33, .5) \quad VL = Very\_Low = (0, .17, .33)$$
$$N = None = (0, 0, .17).$$

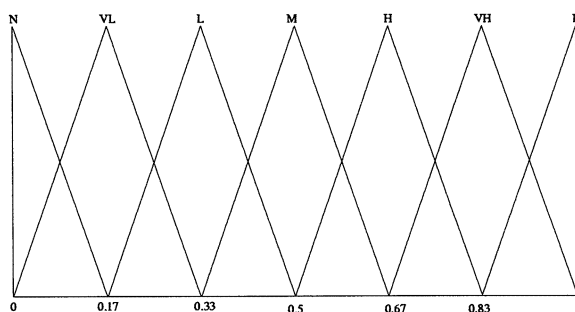which is graphically shown in Figure 1.

Fig. 1. A Set of Seven Terms with its Semantic.

Other authors use a non-trapezoidal representation, e.g., Gaussian functions [16].

2.2. *The 2-tuple Fuzzy Linguistic Representation Model*

This model has been presented in [7,33], in those papers it is shown different advantages of this formalism for representing the linguistic information over classical models, such as:

1. The linguistic domain can be treated as continuous, while in the classical models it is treated as discrete.

2. The linguistic computational model based on linguistic 2-tuples carries out processes of computing with words easily and without loss of information.

3. The results of the processes of computing with words are always expressed in the initial linguistic domain.

Due to these advantages, we shall use this linguistic representation model to accomplish our objective, the development of a fusion procedure for linguistic and numerical information.

The 2-tuple linguistic representation model represents the linguistic information by means of a 2-tuple, (s, α), where s is a linguistic term and α is a numerical value that represents the value of the symbolic translation.

Definition 1. *The Symbolic Translation of a linguistic term $s_i \in S = \{s_0, ..., s_g\}$ consists of a numerical value $\alpha_i \in [-.5, .5)$ that supports the "difference of information" between a counting of information β assessed in $[0, g]$ obtained after a symbolic aggregation operation (acting on the order index of the labels) and the closest value in $\{0, ..., g\}$ that indicates the index of the closest linguistic term in S (s_i).*

From this concept we develop a linguistic representation model which represents the linguistic information by means of 2-tuples (r_i, α_i), $r_i \in S$ and $\alpha_i \in [-.5, .5)$. r_i represents the linguistic label center of the information and α_i is a numerical value that represents the translation from the original result β to the closest index label in the linguistic term set (r_i), i.e., the Symbolic Translation.

This linguistic representation model defines a set of functions to make transformations between linguistic terms, 2-tuples and numerical values:

Definition 2. *Let $s_i \in S$ be a linguistic term, then its equivalent 2-tuple representation is obtained by means of the function θ as:*

$$\theta : S \longrightarrow (S \times [-0.5, 0.5))$$

$$\theta(s_i) = (s_i, 0)/s_i \in S \tag{1}$$

Definition 3. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$

$$\Delta(\beta) = \left\{ \begin{array}{ll} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{array} \right. \tag{2}$$

where round is the usual rounding operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.

Proposition 1. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a function Δ^{-1}, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof.
It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta. \tag{3}$$

In addition, together with this representation model a linguistic computational approach is also defined, in which there exist:

1. *2-tuples comparison operators.*

   The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

   Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, then

   - if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2)
   - if $k = l$ then

     (a) if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information

     (b) if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2)

     (c) if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2)

2. *A 2-tuple negation operator.*

$$Neg((s_i, \alpha)) = \Delta(g - (\Delta^{-1}(s_i, \alpha)))$$

   where $g + 1$ is the cardinality of S, $S = \{s_0, ..., s_g\}$.

3. *A wide range of 2-tuple aggregation operators* [7] has been developed extending classical aggregation operators, such as the LOWA operator, the weighted average operator, the OWA operator, ...

The use of these operators will be useful for the development of any aggregation method to combine the information and its application to a decision process.

2.2.1. *Example*

Here we present a simple use of the linguistic 2-tuple representation model in a process of computation with words to show its operability.

Let $S = \{N, VL, L, M, H, VH, P\}$ be a linguistic term set whose semantics can be seen in Figure 1, and $\{P, L, H, VL\}$ linguistic preferences to be aggregated. In this example we shall use the 2-tuple arithmetic mean for aggregating the linguistic 2-tuples, this operator is defined as:

Definition 4. [7] *Let* $x = \{(r_1, \alpha_1), \ldots, (r_m, \alpha_m)\}$ *be a set of 2-tuples, their arithmetic mean* $\bar{x}$ *is computed as,*

$$\bar{x} = \Delta\left(\sum_{i=1}^{n} \frac{1}{n}\Delta^{-1}(r_i, \alpha_i)\right) = \Delta\left(\frac{1}{n}\sum_{i=1}^{n}\beta_i\right) \tag{4}$$

Therefore, in this case the result will be:

$$\{P, L, H, VL\} \overset{\theta}{\Longrightarrow} \{(P, 0), (L, 0), (H, 0), (VL, 0)\}$$

$$\bar{x} = \Delta\left(\frac{13}{4}\right) = (M, -.25)$$

2.3. *A Multi-Attribute Decision-Making Problem*

Let $A = \{a_1, ..., a_n\}$ be a set of alternatives. Each alternative a_i has a set of attributes $\{y_1, ..., y_k\}$, that must be qualified by means of performance values. This scheme is shown in Table 1:

Table 1. A general MADM problem

Alternatives	*Attributes*			
(a_i)	y_1	y_2	$\cdots$	y_k
a_1	y_{11}	y_{12}	$\cdots$	y_{1k}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_n	y_{n1}	y_{n2}	$\cdots$	y_{nk}

There exists a wide literature on fuzzy MADM problems [35]. In the following we focus in MADM problems with quantitative and qualitative attributes, i.e., dealig with linguistic and numerical information. Let us suppose each alternative has k attributes, with k_1 being the quantitative attributes and k_2 being the qualitative ones, such that, $k_1 + k_2 = k$, see Table 2:

Table 2. MADM problem with linguistic and numerical information

Alternatives	*Attributes*							
	Quantitative Attributes				*Qualitative Attributes*			
(a_i)	y_1	y_2	$\cdots$	y_{k_1}	y_{k_1+1}	y_{k_1+1}	$\cdots$	$y_{k_1+k_2}$
a_1	y_{11}	y_{12}	$\cdots$	y_{1k_1}	l_{11}	l_{12}	$\cdots$	l_{1k_2}
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
a_n	y_{n1}	y_{n2}	$\cdots$	y_{nk_1}	l_{n1}	l_{n2}	$\cdots$	l_{nk_2}

where y_{ij} are the assessments for the quantitative attributes assessed in $[0, 1]$, whilst l_{ij} are the linguistic assessments for the qualitative attributes assessed in a linguistic term set S.

3. Transformation functions between $[0, 1]$ values and linguistic 2-tuples

Several transformation functions are presented here to perform the task of transforming a number $\vartheta \in [0, 1]$ into a linguistic 2-tuple and vice versa.

3.1. *Transforming $[0, 1]$ values into linguistic 2-tuples*

Let $\vartheta \in [0, 1]$ be a numerical value and $S = \{s_0, ..., s_g\}$ a linguistic term set. We want to obtain a linguistic 2-tuple that represents the information from ϑ and that is assessed in S. To do this, we develop a transformation procedure according to the following steps (Graphically, Figure 2):

- Converting ϑ into a fuzzy set in S.

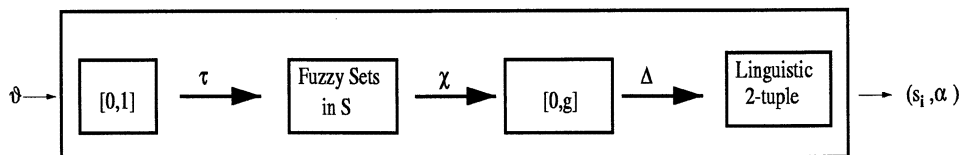- Transforming the above fuzzy set into a linguistic 2-tuple assessed in S.

Fig. 2. Transforming $[0,1]$ value into a linguistic 2-tuple

3.1.1.   *Converting ϑ into a fuzzy set in S*

Let F(S) be the set of fuzzy sets in S, we shall transform a numerical value $\vartheta \in [0,1]$ into a fuzzy set in F(S) computing the membership values of ϑ in the functions associated with the linguistic terms of S.

Definition 5. Let $\vartheta \in [0,1]$ and $S = \{s_0, ..., s_g\}$ be a numerical value and a linguistic term set, respectively. We transform ϑ into a fuzzy set in S by means of τ function defined subsequently:

$$\tau : [0,1] \longrightarrow F(S)$$

$$\tau(\vartheta) = \{(s_0, \omega_0), ..., (s_g, \omega_g)\}, s_i \in S \text{ and } \omega_i \in [0,1], \text{ such that,}$$

$$\alpha_i = \mu_{s_i}(\vartheta) = \begin{cases} 0 & if \ \vartheta \notin Support(\mu_{s_i}(x)) \\ \frac{\vartheta - a_i}{b_i - a_i} & if \ a_i \leq \vartheta \leq b_i \\ 1 & if \ b_i \leq \vartheta \leq d_i \\ \frac{c_i - \vartheta}{c_i - d_i} & if \ d_i \leq \vartheta \leq c_i \end{cases} \tag{5}$$

Let us remember that the semantics of the membership function μ_{s_i} is given by trapezoidal parametric functions whose parameters are (a_i, b_i, d_i, c_i).

Example. Let $\vartheta = .78$ be a numerical value to be transformed into a fuzzy set in S. Here we shall compute four fuzzy sets of ϑ over four different linguistic term sets:

	S_1	S_2	S_3	S_4
L	$(0,0,.3)$	$(0,0,.25)$	$(0,0,0,.1)$	$(0,0,.2)$
VL	$(0,.3,.5)$	$(0,.25,.5)$	$(0,.1,.3,.4)$	$(.1,.2,.4)$
M	$(.2,.5,.8)$	$(.25,.5,.75)$	$(.3,.4,.6,.7)$	$(.3,.5,.7)$
H	$(.5,.7,1)$	$(.5,.75,1)$	$(.6,.7,.9,1)$	$(.6,.8,1)$
VH	$(.7,1,1)$	$(.75,1,1)$	$(.9,1,1,1)$	$(.8,1,1)$

$$\tau_{S_1}(.78) = \{(VL, 0), (L, .0), (M, .06), (H, .73), (VH, .26)\}$$
$$\tau_{S_2}(.78) = \{(VL, 0), (L, .0), (M, .0), (H, .88), (VH, .12)\}$$
$$\tau_{S_3}(.78) = \{(VL, 0), (L, .0), (M, 0), (H, 1), (VH, 0)\}$$
$$\tau_{S_4}(.78) = \{(VL, 0), (L, .0), (M, .0), (H, .9), (VH, .0)\}$$

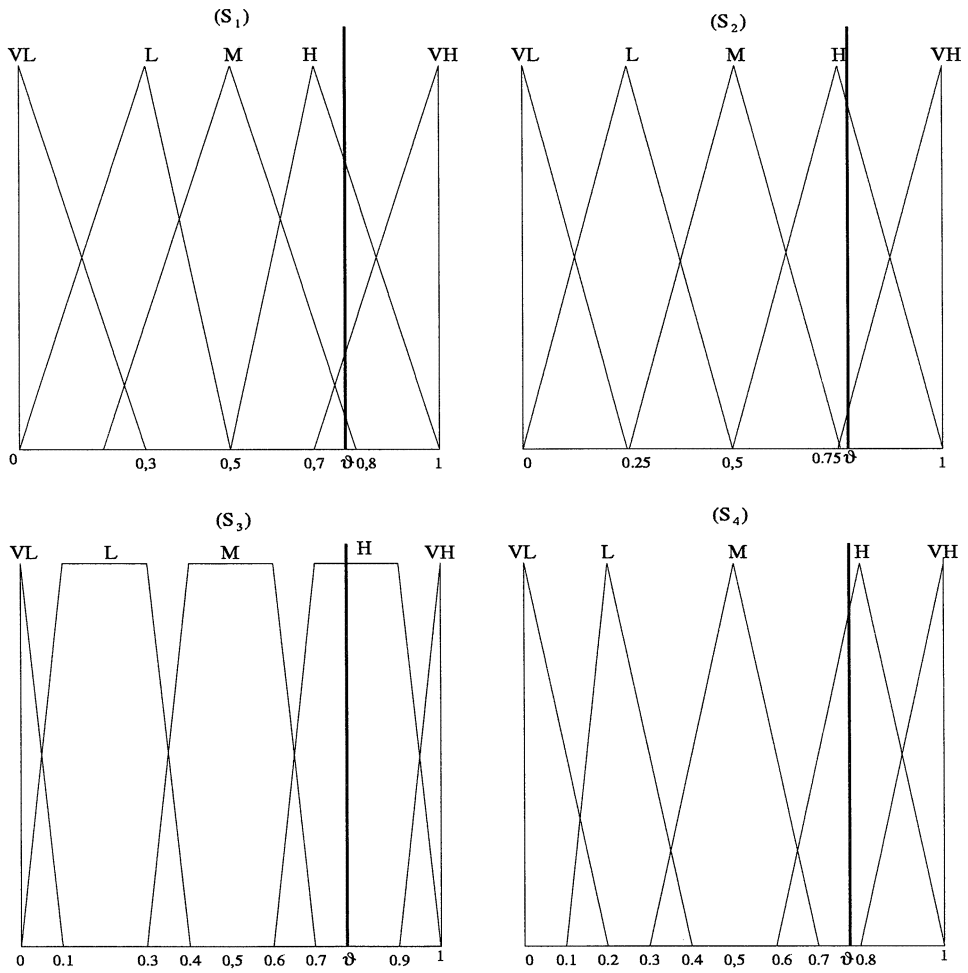The graphical representation is shown in Figure 3.

Fig. 3. Matching between numerical and linguistic values

3.1.2. *Transforming a Fuzzy Set in S into a linguistic 2-tuple assessed in S*

So far, we have been able to convert numerical values into fuzzy sets on S, but our objective is to obtain a linguistic 2-tuple assessed in S. To do so, we use a function χ that computes a central numerical value from the membership degrees of the fuzzy set, $\tau(\vartheta)$, using a weighted average function. This central value will be assessed in the interval of granularity,$[0, g]$, of the linguistic term set S. This function χ is defined as:

Definition 6 [34]. *Let* $\tau(\vartheta) = \{(s_0, \omega_0), \ldots, (s_g, \omega_g)\}$ *be a fuzzy set that represents a numerical value* $\vartheta \in [0, 1]$ *over the linguistic term set* $S = \{s_0, \ldots, s_g\}$. *We obtain a numerical value, that represents the information from the fuzzy set, assessed in the interval* $[0, g]$ *by means of the function* χ:

$$\chi : F(S_T) \longrightarrow [0, g]$$

$$\chi(\tau(\vartheta)) = \chi(\{(s_j, \omega_j), \ j = 0, \ldots, g\}) = \frac{\sum_{j=0}^{g} j\omega_j}{\sum_{j=0}^{g} \omega_j} = \beta \qquad (6)$$

The value β obtained by χ is easily transformed into a linguistic 2-tuple using the function Δ (Eq. 2).

Example. From the fuzzy sets obtained in the above example, we shall obtain the linguistic 2-tuples that represent their information.

$$\chi(\tau_{S_1}(.78)) = \chi((VL, 0), (L, 0), (M, .06), (H, .73), (VH, .26)) = 3.19$$
$$\chi(\tau_{S_2}(.78)) = \chi((VL, 0), (L, 0), (M, 0), (H, .88), (VH, .12)) = 3.12$$
$$\chi(\tau_{S_3}(.78)) = \chi((VL, 0), (L, 0), (M, 0), (H, 1), (VH, 0)) = 3$$
$$\chi(\tau_{S_4}(.78)) = \chi((VL, 0), (L, 0), (M, 0), (H, .9), (VH, 0)) = 2.7$$

The 2-tuples that represent the information of 0.78 in each linguistic term set S_i, are the following

$$S_1 \Longrightarrow \Delta(3.19) = (H, .19) \quad S_2 \Longrightarrow \Delta(3.12) = (H, .12)$$
$$S_3 \Longrightarrow \Delta(3) = (H, 0) \qquad S_4 \Longrightarrow \Delta(2.7) = (H, -.3)$$

3.2. *Transforming linguistic 2-tuples into* $[0, 1]$ *values*

Let (s_i, α) be a linguistic 2-tuple, we want to obtain a value $\vartheta \in [0, 1]$ that supports the information represented by (s_i, α). We present a transformation process according to the scheme shown in Figure 4.
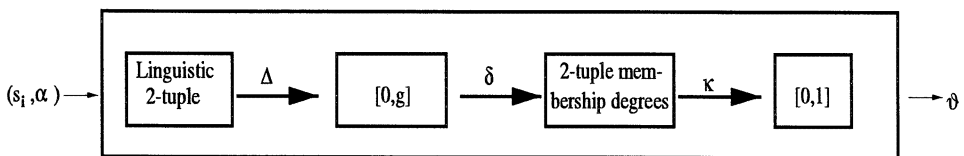

Fig. 4. Transforming a linguistic 2-tuple into a $[0, 1]$ value

We shall use the function δ presented in [34] that transforms a linguistic 2-tuple, (s_i, α), into two 2-tuples based on the membership degree, $\{(s_k, \omega_k), (s_{k+1}, \omega_{k+1})\}$, representing the same counting of information. These two 2-tuples based on the membership degree will be used to obtain the equivalent numerical value assessed in $[0, 1]$.

Definition 7 [34]. *Let (s_k, α) be a linguistic 2-tuple based on the symbolic translation with $s_k \in S = \{s_0, ..., s_g\}$ and $\alpha \in [-.5, .5)$ whose equivalent numerical value is $\Delta^{-1}((s_k, \alpha)) = \beta$ with $\beta \in [0, g]$. The function δ computes two 2-tuples based on the membership degree, from the initial linguistic 2-tuple, that support the same counting of information:*

$$\delta : [0, g] \longrightarrow \{S_T \times [0, 1]\} \times \{S_T \times [0, 1]\}$$

$$\delta(\beta) = \{(s_h, 1 - \gamma), (s_{h+1}, \gamma)\} \tag{7}$$

where

$$h = trunc(\beta)$$
$$\gamma = \beta - h$$

with trunc being the usual trunc operation.

Now, we use the concept of the *characteristic values* associated with a fuzzy number [36,5], that is, we use are crisp values that summarize the information given by a fuzzy set v_i.

Let $F(\mathcal{R})$ be the set of fuzzy numbers defined on $\mathcal{R}$. Each fuzzy number, $v_i \in F(\mathcal{R})$, has an associated membership function, $\mu_{v_i} : F(\mathcal{R}) \longrightarrow [0, 1]$. For each fuzzy number, v_i, we know different characteristic values, $CV(v_i) = \{C_i^1, C_i^2, ..., C_i^z\}$. We shall assume that, $C_i^j \in Supp(v_i) = \{r \epsilon \mathcal{R} \mid \mu_{v_i}(r) > 0\}$. Different *characteristic values* can be found in [36,5], such as the center of gravity, the maximum value, etc.

Hence, from the two 2-tuples based on the membership degree we shall obtain the correspondent numerical value assessed in $[0,1]$ by means of the function κ.

Definition 8. *Let $(s_h, 1 - \gamma)$ and (s_{h+1}, γ) be two 2-tuples based on the degree of membership their equivalent numerical value assessed in [0,1] is obtained using the κ function:*

$$\kappa : \{S_T \times [0, 1]\} \times \{S_T \times [0, 1]\} \longrightarrow [0, 1]$$

$$\kappa((s_h, 1 - \gamma), (s_{h+1}, \gamma)) = CV(s_h)(1 - \gamma) + CV(s_{h+1})(\gamma) \tag{8}$$

where $CV(\cdot)$ is a function providing a characteristic value.

Example. From the 2-tuples based on the symbolic translation obtained from the Example in Section 3.1.2, we shall transform each one into two 2-tuples based on

the membership degree:

$$S_1 \Longrightarrow \delta((H, .19)) = \{(H, .81), (VH, .19)\}$$
$$S_2 \Longrightarrow \delta((H, .12)) = \{(H, .88), (VH, .12)\}$$
$$S_3 \Longrightarrow \delta((H, 0)) = \{(H, 1), (VH, 0)\}$$
$$S_4 \Longrightarrow \delta((H, -.3)) = \{(M, .7), (H, .3)\}$$

and from them, their equivalent numerical values using the *Maximum Value* function[1] as a characteristic value,

$$S_1 \Longrightarrow \kappa((H, .81), (VH, .19)) = .75$$
$$S_2 \Longrightarrow \kappa((H, .88), (VH, .12)) = .78$$
$$S_3 \Longrightarrow \kappa((H, 1), (VH, 0)) = .8$$
$$S_4 \Longrightarrow \kappa((M, .7), (H, .3)) = .6$$

We can see that from the same initial value, $\vartheta = 0.78$, depending on the linguistic term set used, the results can be exact or may have a loss of information. In the following subsection, we study the conditions to be imposed on S, in order to achieve transformation functions that do not produce any loss of information.

3.3. *Conditions for transforming the values in* $[0,1]$ *into linguistic 2-tuples without any loss of information*

Now, we study the conditions to be imposed on the linguistic term set S, for avoiding any loss of information during the transformation processes, that is in order to guarantee the fulfilment of the expression:

$$\kappa(\delta(\chi(\tau(\vartheta)))) = \vartheta. \tag{9}$$

Proposition 2. Let $S = \{s_0, ..., s_g\}$ be a linguistic term set verifying that,

1. *S is a fuzzy partition.* According to Ruspini [37], a finite family $\{s_0, ..., s_g\}$ of fuzzy subsets in the universe X (in our case $X = [0,1]$) is called a fuzzy partition if $\sum_{i=0}^{g} \mu_{s_i}(x) = 1$, $\forall x \in X$.

2. *The membership functions of its terms are triangular*, i.e., $s_i = (a_i, b_i, c_i)$.

3. *The* $CV(s_i) = x \ / \ \mu_{s_i}(x) = 1$, i.e., the characteristic value function used returns the value with the maximum membership degree, b_i.

These conditions are necessary and sufficient for the transformations to be precise between linguistic 2-tuples and $[0,1]$ values, and between $[0,1]$ values and linguistic 2-tuples, i.e., without any loss of information, thus verifying the Eq 9:

$$\kappa(\delta(\chi(\tau(\vartheta)))) = \vartheta$$

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[1]Given a label, s_i, with a membership function, $\mu_{y_{s_i}}(v), v \in V = [0,1]$, its height is defined as $height(s_i) = Sup\{\mu_{y_{s_i}}(v), \forall v\}$.Therefore, the $CV(\cdot)$ Maximum Value is defined as $MV(s_i) = \{v \ / \ \mu_{(s_i)}(v) = height(s_i)\}$

Proof.

1. *Sufficient conditions.*

Let $\vartheta \in [0,1]$ be a value to be transformed into a linguistic 2-tuple in $S = \{s_0, ..., s_g\}$, then we try to prove that the three conditions are sufficient for the expression

$$\kappa(\delta(\chi(\tau(\vartheta)))) = \vartheta$$

The conditions 1 and 2 imply that, $\forall i \in \{0, g-1\}$:

$$\left. \begin{array}{l} s_i = (a_i, b_i, c_i) \\ s_{i+1} = (a_{i+1}, b_{i+1}, c_{i+1}) \end{array} \right\} \Longrightarrow b_i = a_{i+1} \text{ and } c_i = b_{i+1}$$

On the other hand,

$$\tau(\vartheta) = \{(s_i, \omega_i), i = 0, ..., g\}$$
$$\omega_i > 0 \Longleftrightarrow \vartheta \in Support(s_i).$$

Let us suppose, without loss of generality, that $\vartheta \in [b_i, c_i]$ then

$$\vartheta \in [b_i, c_i] \Longleftrightarrow \vartheta \in [a_{i+1}, b_{i+1}], \ \omega_i > 0, \omega_{i+1} > 0 \text{ and } \omega_j = 0 \ \forall j \notin \{i, i+1\}.$$

So far, we have

$$\kappa(\delta(\chi((s_i, \omega_i), (s_{i+1}, \omega_{i+1}))))$$

and we know that

$$\chi((s_i, \omega_i), (s_{i+1}, \omega_{i+1})) = i\omega_i + (i+1)\omega_{i+1}$$

hence, we have

$$\kappa(\delta(i\omega_i + (i+1)\omega_{i+1})).$$

Since S is a fuzzy partition (condition 1),

$$\mu_{s_i}(\vartheta) + \mu_{s_{i+1}}(\vartheta) = 1 \Longrightarrow \omega_i + \omega_{i+1} = 1$$

then,

$$\delta(i\omega_i + (i+1)\omega_{i+1}) = \delta(i(1 - \omega_{i+1}) + (i+1)\omega_{i+1}) = \delta(i + \omega_{i+1})$$

$$\delta(\beta) = \delta(h + \gamma) = \delta(i + \omega_{i+1}) \text{ with } \left\{ \begin{array}{l} h = i \\ \gamma = \omega_{i+1} \end{array} \right.$$

therefore,

$$\delta(i + \gamma) = \{(s_i, (1 - \gamma)), (s_{i+1}, \gamma)\} = \{(s_i, \omega_i), (s_{i+1}, \omega_{i+1})\}.$$

The last step is to prove, that

$$\kappa((s_i, \omega_i), (s_{i+1}, \omega_{i+1})) = \vartheta$$

where
$$\kappa((s_i, \omega_i), (s_{i+1}, \omega_{i+1})) = CV(s_i)\omega_i + CV(s_{i+1})\omega_{i+1}.$$

According to the condition 3, $CV(s_j) = b_j$ / $\mu_{s_j}(b_j) = 1$, and b_j is unique according to condition 2, then

$$\kappa((s_i, \omega_i), (s_{i+1}, \omega_{i+1})) = \omega_i b_i + (1-\omega_i)b_{i+1} = \alpha_i b_i + (1-\alpha_i)c_i = \alpha_i(b_i - c_i) + c_i,$$

because,

$$\omega_i = \frac{c_i - \vartheta}{c_i - b_i}$$

then,

$$\kappa((s_i, \omega_i), (s_{i+1}, \omega_{i+1})) = \omega_i(b_i - c_i) + c_i = \frac{c_i - \vartheta}{c_i - b_i}(b_i - c_i) + c_i = \vartheta.$$

2. *Neccesary conditions.*

(a) Let us suppose that S is not a fuzzy partition, i.e.,

$$\mu_{s_i}(\vartheta) + \mu_{s_{i+1}}(\vartheta) \neq 1 \implies \delta(i\omega_i + (i+1)\omega_{i+1}) \neq \delta(i + \omega_{i+1})$$

then, the expression $\kappa(\cdot) = \vartheta$ is false, as we can see with the transformation process on the linguistic term sets S_1 and S_4 in Section 3.2 (see Figure 3). Contradiction.

(b) If the terms of S are not triangular then $CV(s_i)$ return a value, but there is more than one value, x, such that $\mu_{s_i}(x) = 1$, then the value obtained by κ can be different from the initial one, as occurs in the transformation processes on the linguistic term set S_3 (see Figure 3). Contradiction.

(c) If $CV(s_i)$ does not return the maximum value such that $\mu_{s_i}(x) = 1$, as happened in paragraph (b) and the value x obtained by κ can be different from ϑ. As can be seen in the following example:

Let us suppose the center of gravity as $CV(\cdot)$, where

$$CG(s_i) = \frac{\int_V v\mu_{y_{s_i}}(v)dv}{\int_V \mu_{y_{s_i}}(v)dv},$$

for trapezoidal fuzzy numbers, we obtain:

$$CG(s_i) = \begin{cases} a^i & \text{if } a_i = b_i = d_i = c_i \\ \frac{(c_i)^2 + (d_i)^2 - (b_i)^2 - (a_i)^2 + c_i d_i - a_i b_i}{3(c_i + d_i - b_i - a_i)} & \text{otherwise} \end{cases}$$

Given $\vartheta = .78$ and the following linguistic term set S_5:

$$S_5$$

L	$(0, 0, .4)$
VL	$(0, .4, .5)$
M	$(.4, .5, .9)$
H	$(.5, .9, 1)$
VH	$(.9, 1, 1)$

Then,

$$\tau_{S_5}(.78) = \{(VL, 0), (L, .0), (M, .7), (H, .3), (VH, 0)\}$$
$$\chi(\{(VL, 0), (L, 0), (M, .7), (H, .3), (VH, 0)\}) = 2.3 \Rightarrow \Delta(2.3) = (M, .3)$$
$$\delta(\Delta^{-1}((M, .3))) = \{(M, .7)(H, .3)\} \Rightarrow \kappa((M, .7)(H, .3)) = .66$$

Contradiction, because $\kappa(\cdot) \neq \vartheta$.

We have proven the need for the three conditions, because if we eliminate any of them we find a contradiction.

4. Combining Linguistic and Numerical Information. A fusion method based on the 2-tuple fuzzy linguistic representation model

We assume that the input information is provided using absolute and compatible scales, i.e., all the preferences are assessed either in the numerical domain $[0, 1]$ or in a linguistic term set $S = \{s_0, ..., s_g\}$ with fuzzy membership functions covering the range $[0, 1]$.

The fusion method acts according to the following steps (Graphically, Figure 5):

- First, it unifies the linguistic information assessed in S and the numerical information in $[0, 1]$ into linguistic 2-tuples based on the symbolic translation assessed in a basic linguistic term set (BLTS) denoted as S_T. There are two possibilities when it comes to selecting S_T:
    1. If S verifies Proposition 2, then S_T will be S itself.
    2. If S does not verify it, we shall transform the linguistic terms in S into linguistic 2-tuples assessed in a BLTS, S_T, verifying Proposition 2.

- Subsequently, it combines the linguistic 2-tuples to obtain collective values.

- Finally, it returns the collective values to the original expression domains or expression domains near the original ones.
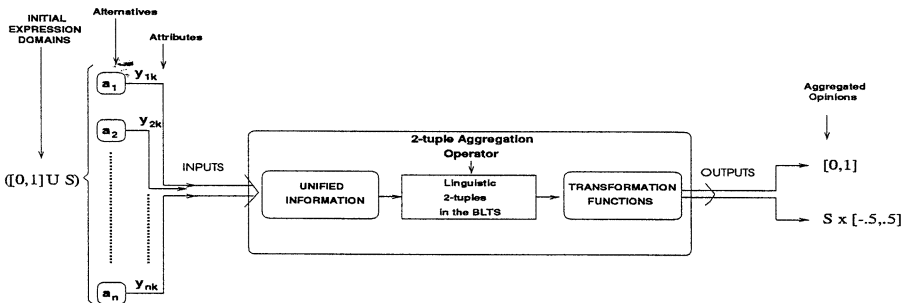

Fig. 5. Aggregation Procedure

4.1. *Unifying the numerical and linguistic information as linguistic 2-tuples*

Without loss of generality, we have supposed that the numerical values used in the decision problem are assessed in $[0, 1]$, to convert these values into linguistic 2-tuples we shall use the process presented in Section 2.1 (see Figure 2).

In the following we shallstudy the process for transforming linguistic terms into linguistic 2-tuples. We can find two possibities:

a) S verifies Proposition 2

Let $S = S_T = \{s_0, ..., s_g\}$ be a linguistic term set verifying Proposition 2. To transform the linguistic terms into linguistic 2-tuples it is easy to carry out using function θ (Eq. 1):

$$s_i \in S \Longrightarrow \theta(s_i) = (s_i, 0)$$

b) S does not verify Proposition 2

In this case, we must transform the linguistic terms in S into linguistic 2-tuples in a BLTS verifying Proposition 2. Before defining a transformation function into this BLTS, S_T, we have to decide how to choose S_T.

S_T must be able to maintain the uncertainty degree associated with the terms in S, i.e., must have at least the same granularity of uncertainty. Besides S_T must maintain the ability of discrimation to express the performance values and finally it must verify Proposition 2. With this goal in mind, we look for a BLTS with a granularity larger than a person is able to discriminate, different psychological studies about this can be found in the literature Miller in [29] says that a expert can manage sets with 7 ± 2 terms while Bonissone in [28] says that in any occasions the experts can manage until 11 or 13 terms. Therefore, we shall choose as BLTS a term set that accomplish the proposition 2 and its granularity is bigger than an expert can reasonably manage. In our case we shall choose a term set with 15 terms and the following semantics (see Figure 6):

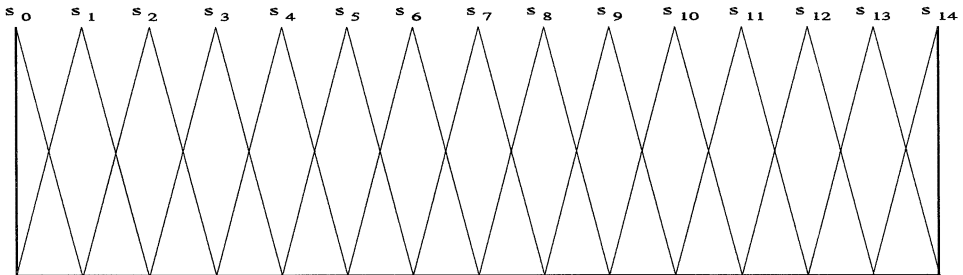|							
s_0	$(0, 0, .07)$	s_1	$(0, .07, .14)$	s_2	$(.07, .14, .21)$	s_3	$(.14, .21, .28)$
s_4	$(.21, .28, .35)$	s_5	$(.28, .35, .42)$	s_6	$(.35, .42, .5)$	s_7	$(.42, .5, .58)$
s_8	$(.5, .58, .65)$	s_9	$(.58, .65, .72)$	s_{10}	$(.65, .72, .79)$	s_{11}	$(.72, .79, .86)$
s_{12}	$(.79, .86, .93)$	s_{13}	$(.86, .93, 1)$	s_{14}	$(.93, 1, 1)$		

Fig. 6. Term set with 15 terms

Remark. We should point out that the justification for this choice is based on the use of a symmetrical term set with a granularity greater than the number of terms that an expert is able to discriminate (11 or 13, see [28,29]).

Now we define a transformation process from linguistic terms in S into linguistic 2-tuples in S_T. Graphically, it is described in Figure 7.
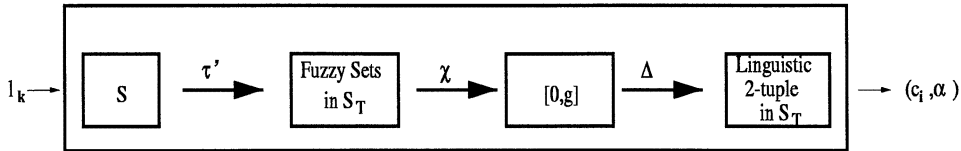

Fig. 7. Obtaining a 2-tuple into S_T from a term in S

We use a transformation function which represents each linguistic performance value as a fuzzy set defined in the BLTS, S_T.

Definition 9 [38]. *Let* $S = \{l_0, \ldots, l_p\}$ *and* $S_T = \{c_0, \ldots, c_g\}$ *be two linguistic term sets, such that,* $g \geq p$. *Then, a multi-granularity transformation function,* τ'_{SS_T} *is defined as*

$$\tau'_{SS_T} : S \longrightarrow F(S_T)$$

$$\tau'_{SS_T}(l_i) = \{(c_k, \alpha_k^i) \ / k \in \{0, \ldots, g\}\}, \ \forall l_i \in S$$

$$\alpha_k^i = \max_y min\{\mu_{l_i}(y), \mu_{c_k}(y)\} \tag{10}$$

where $F(S_T)$ *is the set of fuzzy sets defined in* S_T, *and* $\mu_{l_i}(y)$ *and* $\mu_{c_k}(y)$ *are the membership functions with the fuzzy sets associated to the terms* l_i *and* c_k, *respectively.*

Therefore, the result of $\tau'_{SS_T}(l_i)$ for any linguistic value of S is a fuzzy set defined in the BLTS, S_T. From this fuzzy set in $F(S_T)$, we shall obtain a linguistic 2-tuple in S_T, as in subsection 3.1, using the functions χ and Δ.

4.2. *Combining linguistic 2-tuples*

Decision procedures attempt to rank the alternatives. To do so, the decision process firstly aggregates the input information to obtain collective values for each alternative. In MADM problems the input information is made up by the performance values (linguistic and/or numerical ones) provided for all the attributes and for each alternative. Taking into account the functions presented just above, all the input information is transformed into an unified expression domain by means of linguistic 2-tuples assessed in S_T. Therefore, for combining the input information we shall use a linguistic 2-tuple aggregation operator.

Formally,

$$FO((l_1, \alpha_1), ..., (l_{k_1}, \alpha_{k_1}), ..., (l_{k_1+k_2}, \alpha_{k_1+k_2})) = (l, \alpha)$$

where FO is a 2-tuple aggregation operator, and (l, α) is the aggregated value. In [7] a wide range of these operators were presented, such as the 2-tuple convex combination, the 2-tuple OWA, the 2-tuple OR-LIKE S-OWA, etc.

4.3. *Expression domains for the results*

The input information is assessed in $[0,1]$ and in S, however the results are expressed by means of linguistic 2-tuples in S_T, therefore in some cases it may be convenient to return the results of the aggregation in the initial expression domains to improve the comprehensiveness.

There are several possibilities:

1. If $S_T = S$, then the linguistic 2-tuples assessed in S are near enough in the representation.

2. If the initial expression domain was $[0,1]$, it seems convenient to express the results (linguistic 2-tuples) by means of an equivalent numerical value assessed in $[0,1]$. To do so, we shall use the transformation process presented in Section 3.2 (see Figure 4).

3. If $S_T \neq S$, hence S_T is far away from S, then it can be appropriate to express the linguistic 2-tuples in S_T by means of equivalent 2-tuples assessed in S.

Let $S = \{l_0, ..., l_p\}$ and $S_T = \{c_0, ..., c_g\}$ be two linguistic term sets with $g \geq p$. From (c_i, α) we want to obtain an equivalent 2-tuple (l_i, α). To do so, we apply the following process. Graphycally it is described in Figure 8:
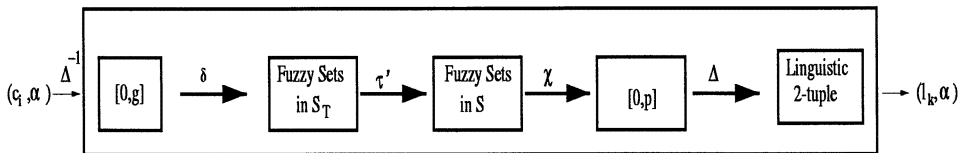

Fig. 8. Obtaining a 2-tuple in S from a 2-tuple in S

(a) We use the functions Δ^{-1} and δ, then we shall obtain

$$\delta(\Delta^{-1}(c_i, \alpha)) = \{(c_h, 1 - \gamma), (c_{h+1}, \gamma)\}$$

(b) A matching procedure using the function $\tau'_{S_T S}$ is applied to c_h and c_{h+1} thereby obtaining two fuzzy sets on S_i.

$$\tau'_{S_T S}(c_h) = \{(l_0, \omega_0), ..., (l_p, \omega_p)\}$$
$$\tau'_{S_T S}(c_{h+1}) = \{(l_0, \omega_0), ..., (l_g, \omega_p)\}$$

(c) The fuzzy sets are converted into numerical values assessed in $[0, p]$ by means of the χ function, obtaining β_h and $\beta_{h+1} \in [0, p]$, such that,

$$\chi(\tau'_{S_T S}(c_h)) = \beta_h$$
$$\chi(\tau'_{S_T S}(c_{h+1})) = \beta_{h+1}$$

(d) To achieve our objective we need to obtain a value, β, assessed in $[0, p]$ that represents the same information that $\{(c_h, 1 - \gamma), (c_{h+1}, \gamma)\}$. We have β_h and $\beta_{h+1} \in [0, p]$, that represent the information supported by c_h and c_{h+1}, now we perform a linear combination using the membership degrees of these labels in their respective 2-tuples based on the membership degree to obtain the value that we are looking for:

$$(\beta_h * (1 - \gamma)) + (\beta_{h+1} * \gamma) = \beta$$

where $\beta \in [0, p]$ and it represents the same information that (c_i, α). Then, by applying Δ to β we shall obtain the linguistic 2-tuple assessed in S that we were looking for:

$$\Delta(\beta) = (l_k, \alpha), \ l_k \in S$$

Remark: The reason for using the 2-tuple linguistic representation as base of the computation processes, is due to proposition 2 guarantees the absence of lack of precision.

5. MADM Problem

Let us consider, a customer who intends to buy a car. Four types of cars are available, "car 1", "car 2", "car 3" and "car 4". The customer takes into account six attributes including both quantitative and qualitative ones to decide which car to buy. Quantitative ones are assessed in $[0, 1]$ and qualitative ones are assessed in S whose semantics can be seen in Figure 1, where S verifies Proposition 2 and therefore S_T will be S itself.

The table of values is described subsequently:

Table 3. A MADM problem

Type of Attributes		Quantitative Attributes			Qualitative Attributes		
Alternatives	Attributes	Fuel economy	Aerod. degree	Price	Comfort	Design	Safety
car 1		.8	.9	.7	VH	H	VH
car 2		.8	.7	.8	H	M	M
car 3		.8	.6	.9	M	H	M
car 4		.85	.8	.8	H	VH	H

5.1. *Decision Process*

We shall use a simple decision procedure [4] to solve the above MADM problem consisting of the following steps:

1. *Aggregation process.* In this step all the preference values of the attributes for each alternative are aggregated to obtain a global degree of preference, GD_i.

   In this example we shall use the 2-tuple arithmetic mean [7] denoted as $\bar{x}$ for aggregating the linguistic 2-tuples and for expressing the results in the numerical domain we select the maximum value (MV) as characteristic value.

2. *Exploitation process.* The objective of this process is to rank the collective values obtaining in the aggregation process, and to select the best alternative/s.

5.2. *Applying the Decision Process with 2-tuples*

The values provided on each alternative for each attribute must be aggregated. To do so, we shall use the fusion method presented in Section 3.

1. Transforming input information into linguistic 2-tuples based on the Symbolic Translation.

   Since S verifies Proposition 2, we need the function θ for linguistic terms and the functions τ, χ and Δ for numerical values:

Table 4.   Attributes preferences in 2-tuples

	Fuel economy	*Aerod. degree*	*Price*	*Comfort*	*Design*	*Security*
car 1	$(VH, -.19)$	$(VH, .41)$	$(H, .19)$	$(VH, 0)$	$(H, 0)$	$(VH, 0)$
car 2	$(VH, -.19)$	$(H, .19)$	$(VH, -.19)$	$(H, 0)$	$(M, 0)$	$(M, 0)$
car 3	$(VH, -.19)$	$(H, -.41)$	$(VH, .41)$	$(M, 0)$	$(H, 0)$	$(M, 0)$
car 4	$(VH, .13)$	$(VH, -.19)$	$(VH, -.19)$	$(H, 0)$	$(VH, 0)$	$(H, 0)$

An example can be:

$$\tau(y_{11}) = \tau(.8) = \{(N, 0), (VL, 0), (L, 0), (M, 0), (H, .19), (VH, .81), (P, 0)\}$$

$$\chi(\{(N, 0), (VL, 0), (L, 0), (M, .0), (H, .19), (VH, .81), (P, 0)\}) = 4.81$$

$$\Delta(4.81) = (VH, -.19)$$

2. Aggregating 2-tuples.

For obtaining the global preference, we shall aggregate the preferences of all the attributes for each alternative. We shall use the 2-tuple arithmetic mean (Eq. 4) as the 2-tuple fusion operator. We shall obtain the following global preferences that are shown in Table 5:

Table 5. Collective preferences obtained with the 2-tuple model

	car 1	car 2	car 3	car 4
Linguistic 2 − tuples	$(VH, -.44)$	$(H, -.03)$	$(H, -.03)$	**$(VH, -.38)$**
Numerical values	.72	.66	.66	**.76**

The numerical values from Table 5 are obtained using functions δ and κ. The κ function uses the Maximum Value as the characteristic value . Example:

$$\delta(VH, -.44) = \{(H, .44), (VH, .56)\}$$

$$\kappa((H, .44), (VH, .56)) = CV(H) * .44 + CV(VH) * .56 = .725$$

Finally, the decision process must select in the exploitation process the best alternative/s. In this example we select as best alternative the one with the maximum global preference. Therefore, from the above results we obtain a_4 (car 4) as the best alternative.

5.3. *Applying the Decision Process with classical linguistic labels*

We have pointed out along the paper that there exists other models to deal with linguistic and numerical environments [5,6]. In Table 6, we present the results obtained in the above MADM problem using the process presented in [5] with classical linguistic labels. In the aggregation process is used the LOWA operator [9] with the same weight for each alternative:

Table 6. Collective preferences obtained with classical linguistic labels

	car 1	car 2	car 3	car 4
Linguistic 2 − tuples	**VH**	*H*	*H*	**VH**
Numerical values	**.83**	.67	.67	**.83**

The solution set of alternatives in this case is composed by $\{x_1, x_4\}$ because both alternatives have the same value and with the classical linguistic representation model we cannot choose between them the best one.

6. Concluding Remarks

In this paper, we have presented a fusion method based on linguistic 2-tuples for combining linguistic and numerical information without any loss of information under certain conditions. We have used an MADM problem to show the use of the fusion method, but there are many type of problems that may present information assessed in different domains, therefore we can use the fusion method in those problems.

Afterwards we have presented a solution over the same MADM problem based on the classical linguistic representation model. We can observe comparing the results, that the solution based on the linguistic 2-tuple model is more precise than the solution based on the classical one. This is due to the 2-tuple representation model operates without loss of information in processes of Computing with Words.

Finally, we should point out that according to the multigranularity model presented in [34], it is easy to design a mix of the fusion method presented in this paper and the fusion method for multi-granularity linguistic information, thereby producing a fusion method for combining information expressed on any scale and in any domain.

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