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THE 2-TUPLE LINGUISTIC COMPUTATIONAL MODEL. ADVANTAGES OF ITS LINGUISTIC DESCRIPTION, ACCURACY AND CONSISTENCY.

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The Fuzzy Linguistic Approach has been applied successfully to different areas. The use of linguistic information for modelling expert preferences implies the use of processes of Computing with Words. To accomplish these processes different approaches has been proposed in the literature: (i) Computational model based on the Extension Principle, (ii) the symbolic one(also called ordinal approach), and (iii) the 2-tuple linguistic computational model. The main problem of the classical approaches, (i) and (ii), is the loss of information and lack of precision during the computational processes. In this paper, we want to compare the linguistic description, accuracy and consistency of the results obtained using each model over the rest ones. To do so, we shall solve a Multiexpert Multicriteria Decision-Making problem defined in a multigranularity linguistic context using the different computational approaches. This comparison helps us to decide what model is more adequated for computing with words.

Keywords: Computing with Words, linguistic preference variables, aggregation.

1. Introduction

The problems present aspects that can belong to different nature, depending on the nature of these aspects the problem will deal with different types of information. Usually, the problems present quantitative aspects that can be easily assessed by means of numerical values, however in other cases they present qualitative aspects that are complex to asess by means of precise values. In the latter case, the use of the fuzzy linguistic approach [1] has provided good results.

Focusing our study in the modelling of expert preferences evaluations, the linguistic preference modelling has been applied successfully to different areas as information retrieval [2,3], economics [4,5], planning [6], consensus reaching processes [7,8], decision-making [9,10,11]. When the problems of these areas deal with multiple criteria or experts, they are solved following a common resolution scheme with two processes [12]:

- *Aggregation process.* Obtains collective preference values.
- *Exploitation process.* Makes a ranking of the collective values.

This resolution scheme implies processes of Computing with Words (CW) and therefore, the need to use linguistic computational techniques to accomplish these processes. In the specialized literature we can find different linguistic computational models for aggregating linguistic preference variables:

- *The approximative computational model based on the Extension Principle* [13,14]. This model uses fuzzy arithmetic based on the Extension Principle to make computations over the linguistic variables.
- *The ordinal linguistic computational model* [15,9]. This symbolic model makes direct computations on labels, using the ordinal structure of the linguistic term sets.
- *The 2-tuple linguistic computational model* [16]. It uses the 2-tuple linguistic representation model and its characteristics to make linguistic computations.

The 2-tuple linguistic computational model is an extension of the ordinal one[16]. Hence, in this paper we shall not study the ordinal linguistic approach because of the 2-tuple one always obtains the same or better results than the ordinal one.

The aim of this paper is to analyse the advantages and disadvantages from the points of view, of linguistic description, of consistency and accuracy of the results, obtained by the approximative linguistic computational model and the 2-tuple one in processes of CW. We shall study both linguistic computational models analysing the results obtained over a Multiexpert Multicriteria Decision-Making (MEMC-DM) problem. To do so, we shall present the *"Technology transfer strategy selection"* problem defined in a multigranular linguistic context. Afterwards, we shall solve it with the above linguistic computational models. Subsequently, we shall study the solutions obtained by both ones.

In order to do that, this contribution is structured as follows: in Section 2 we shall present a brief review of the Fuzzy Linguistic Approach and the computational model based on the Extension Principle, also review the linguistic 2-tuple representation model together with its computational model. In Section 3 we shall present the technology transfer strategy selection problem and solve it using the methods reviewed in Section 2. In Section 4 a study of linguistic computational models in CW processes is presented. Finally, some concluding remarks are included.

2. Fuzzy Linguistic Approach

Usually, we work in a quantitative setting, where the information is expressed by means of numerical values. However, many aspects of different activities in the real world cannot be assessed in a quantitative form, but rather in a qualitative one, i.e., with vague or imprecise knowledge. In that case a better approach may be to use linguistic assessments instead of numerical values. The fuzzy linguistic approach represents qualitative aspects as linguistic values by means of linguistic variables [1]. This approach is adequate in some situations where the information may be unquantifiable due to its nature, and thus, it may be stated only in linguistic terms.

We have to choose the appropriate linguistic descriptors for the term set and their semantics. In order to accomplish this objective, an important aspect to analyse is the *"granularity of uncertainty"*, i.e., the level of discrimination among different counts of uncertainty. Typical values of cardinality used in the linguistic models are odd ones, such as 7 or 9, where the mid term represents an assessment of "approximately 0.5", and with the rest of the terms being placed symmetrically around it [17].

One possibility of generating the linguistic term set consists of directly supplying the term set by considering all terms distributed on a scale on which a total order is defined [9]. For example, a set of seven terms S, could be given as follows:

$$S = \{s_0 = N, s_1 = VL, s_2 = L, s_3 = M, s_4 = H, s_5 = VH, s_6 = P\}.$$

Usually, in these cases, it is required that in the linguistic term set there exist:

1. A negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = g$-i (g+1 is the cardinality).

2. $s_i \leq s_j \Longleftrightarrow i \leq j$. Therefore, there exists a *min* and a *max* operator.

The semantics of the linguistic terms is given by fuzzy numbers defined in the [0,1] interval. A way to characterize a fuzzy number is to use a representation based on parameters of its membership function [17]. The linguistic assessments given by the users are just approximate ones, some authors consider that linear trapezoidal membership functions are good enough to capture the vagueness of those linguistic assessments. The parametric representation is achieved by the 4-tuple (a, b, d, c), where b and d indicate the interval in which the membership value is 1, with a and c indicating the left and right limits of the definition domain of the trapezoidal membership function [17]. A particular case of this type of representation are the linguistic assessments whose membership functions are triangular, i.e., $b = d$, then we represent this type of membership functions by a 3-tuple (a, b, c). An example may be the following (Figure 1) :

$$\begin{array}{llll} N = (0, 0, .17) & VL = (0, .17, .33) & L = (.17, .33, .5) & \\ M = (.33, .5, .67) & H = (.5, .67, .83) & VH = (.67, .83, 1) & P = (.83, 1, 1). \end{array}$$

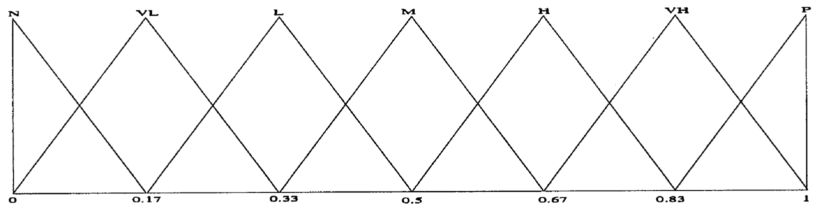

Figure 1: A set of seven linguistic terms with its semantics

Other authors use a non-trapezoidal representation, e.g., Gaussian functions [2].

2.1. *Linguistic Computational Model based on the Extension Principle*

The Extension Principle is a basic concept in the fuzzy sets theory [18] which is used to generalize crisp mathematical concepts to fuzzy sets, i.e., it allows a non-fuzzy function to be fuzzified in the sense that if the function arguments are made into fuzzy sets, then the function value is also a fuzzy set.

The use of the fuzzy arithmetic based on the Extension Principle increases the vagueness of the results. In processes of CW this increasing of vagueness means that fuzzy numbers obtained do not match with any linguistic term in the initial term set, S.

The resolution scheme for linguistic problems, presented in the introduction, is divided in two processes: *aggregation* and *exploitation*. Depending on the process, this computational model can deal with information from different nature.

- *Aggregation process.* The approximative model based on the Extension Principle uses the membership functions of the linguistic terms, μ_{s_i} where $s_i \in S$, for computing collective (aggregated) values that are expressed by means of fuzzy numbers. A formal scheme can be:

$$S^n \xrightarrow{\tilde{F}} F(\mathcal{R})$$

  where S^n symbolizes the n cartesian product of S, $\tilde{F}$ is an aggregation operator based on the Extension Principle, $F(\mathcal{R})$ the set of fuzzy sets over the Real line $\mathcal{R}$.

- *Exploitation process.* The objective of this process is to order the collective values obtained in the aggregation process. To do so, this computational model can act on two different ways:

  1. *Ranking fuzzy numbers obtained on the aggregation process.* Therefore a fuzzy ranking function must be applied to them, obtaining a ranking of alternatives according to the real numbers computed by the fuzzy ranking:

$$F(\mathcal{R}) \xrightarrow{\tilde{O}} \mathcal{R}$$

     In the specialized literature we can find different fuzzy ranking functions [19], in this paper we shall use the following two different ones.

(a) The Kim & Park fuzzy ranking [20]. Let F_j (j=1,...,m) be m fuzzy numbers whose membership functions are $F_j = (a_j, b_j, c_j)$. Define the ranking value $U_T(F_j)$ of fuzzy numbers as:

$$U_T(F_j) = \alpha U_M(F_j) + (1-\alpha)U_G(F_j), 0 \leq \alpha \leq 1$$

The value α is an index of rating attitude.

(b) Another fuzzy ranking was presented by Chang in [21], where:

$$C(\tilde{F_j}) = \frac{1}{6}(c_i - a_i)(a_i + b_i + c_i)$$

Both fuzzy orders compute a real number for each fuzzy number, that will be used to order them.

2. *Applying a linguistic approximation process to the fuzzy numbers obtained in the aggregation process.* This approximation process obtains the linguistic term closest to the unlabelled fuzzy number in the initial term set. The aggregated values are expressed by means of linguistic terms that have an inner order defined by the structure of the linguistic term set:

$$F(\mathcal{R}) \xrightarrow{app_1(\cdot)} S$$

Different linguistic approximation functions, app_1, can be used. In this paper we shall use the following one. Let $F_j = (a_j, b_j, c_j)$, $(j = 1, ..., m)$ be the collective fuzzy numbers obtained in the aggregation process the linguistic approximation process, $app_1(\cdot)$, comes back the closest linguistic term, $s_l \in S$, to the unlabelled fuzzy number, F_j.

$$d(s_l, F_j) = \sqrt{P_1(a_l - a_j)^2 + P_2(b_l - b_j)^2 + P_3(c_l - c_j)^2}$$

being (a_l, b_l, c_l) and (a_j, b_j, c_j) the membership functions of "s_l" and "F_j" respectively, with P_1, P_2, P_3 being the weights that represent the importance of a, b *and* c. Therefore, $app_1(\cdot)$ chooses s_l^* $(app_1(F_j) = s_l^*)$, such that, $d(s_l^*, F_j) \leq d(s_l, F_j)$ $\forall s_l \in S$, where S is the linguistic term set used as expression domain.

2.2. *The 2-tuple Linguistic Representation Model*

This model has been presented in [16], where different advantages of this formalism to represent the linguistic information over classical models are shown, such as:

1. The linguistic domain can be treated as continuous, whilst in the classical models it is treated as discrete.
2. The linguistic computational model based on linguistic 2-tuples carries out processes of "computing with words" easily and without loss of information.
3. The results of the processes of "computing with words" are always expressed in the initial linguistic domain.

This linguistic model takes as a basis the symbolic model and in addition defines the concept of Symbolic Translation and uses it to represent the linguistic information by means of a pair of values called linguistic 2-tuple, (s, α), where s is a linguistic term and α is a numeric value representing the symbolic translation.

Definition 1. *Let β be the result of an aggregation of the indexes of a set of labels assessed in a linguistic term set S, i.e., the result of a symbolic aggregation operation. $\beta \in [0, g]$, being $g + 1$ the cardinality of S. Let $i = round(\beta)$ and $\alpha = \beta - i$ be two values, such that, $i \in [0, g]$ and $\alpha \in [-.5, .5)$ then α is called a Symbolic Translation.*

This model defines a set of transformation functions between numeric values and linguistic 2-tuples.

Definition 2. *Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value representing the result of a symbolic aggregation operation, then the 2-tuple that expresses the equivalent information to β is obtained with the following function:*

$$\Delta : [0, g] \longrightarrow S \times [-0.5, 0.5)$$
$$\Delta(\beta) = (s_i, \alpha), \ \ with \ \begin{cases} s_i & i = round(\beta) \\ \alpha = \beta - i & \alpha \in [-.5, .5) \end{cases}$$

where $round(\cdot)$ is the usual round *operation, s_i has the closest index label to "β" and "α" is the value of the symbolic translation.*

Proposition 1.*Let $S = \{s_0, ..., s_g\}$ be a linguistic term set and (s_i, α) be a 2-tuple. There is always a Δ^{-1} function, such that, from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g] \subset \mathcal{R}$.*

Proof. It is trivial, we consider the following function:

$$\Delta^{-1} : S \times [-.5, .5) \longrightarrow [0, g]$$
$$\Delta^{-1}(s_i, \alpha) = i + \alpha = \beta$$

Remark: From definitions 2 and 3 and from proposition 1, it is obvious that the conversion of a linguistic term into a linguistic 2-tuple consist of adding a value 0 as symbolic translation: $s_i \in S \Longrightarrow (s_i, 0)$

1. Aggregation of 2-tuples

The aggregation of information consists of obtaining a value that summarizes a set of values, therefore, the result of the aggregation of a set of 2-tuples must be a 2-tuple. In [16] we can find some 2-tuple aggregation operators.

2. Comparison of 2-tuples

The comparison of linguistic information represented by 2-tuples is carried out according to an ordinary lexicographic order.

Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, with each one representing a counting of information:

- if $k < l$ then (s_k, α_1) is smaller than (s_l, α_2)
- if $k = l$ then

1. if $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represents the same information
2. if $\alpha_1 < \alpha_2$ then (s_k, α_1) is smaller than (s_l, α_2)
3. if $\alpha_1 > \alpha_2$ then (s_k, α_1) is bigger than (s_l, α_2)

3. Application Problem: Technology transfer strategy selection in Biotechnology

Here we make a brief review of this Multi-Expert Multi-Criteria Decision-Making problem (for a depth description see [13,22]) defined over a multigranular linguistic context and afterwards we shall solve it, using the above computational models.

3.1. *Description. Definition of criteria and strategies*

Brown et al. [22] provided a technique that managers of goverment-sponsored R&D could use to evaluate innovations during their precommercialization stage in order to identify an appropiate technology transfer strategy.

The transfer of technology from its source to commercial application is a very complex process. It is an MEMC-DM problem in ill-structured situations. It is must be made a careful analysis among criteria, alternatives, weights, and decision makers before making a decision. Using crisp decision methods, we always have to find precise data, but under several conditions, we cannot get precise data because the data are from the experience and the judgment of decision makers. In these cases, it is more adequated to use linguistic variables instead of precise data to assess the alternatives versus various criteria and the importance weight of criteria.

A general representation of MEMC-DM problem is introduced. Let us suppose there exists a committee of n experts $(p_1, ..., p_n)$ who assess the appropriateness of m alternatives $(x_1, ..., x_m)$ according to each of k criteria $(C_1, ..., C_k)$ as well as the importance weight of the criteria. Let S_{itj} $(i = 1, ..., m; t = 1, ..., k; j = 1, ..., n)$ be the rating assigned to alternative x_i by the expert p_j according to the criterion C_t. Let W_{tj} be the importance given to C_t by the expert p_j.

In our problem a committee of four experts, $P = \{p_1, p_2, p_3, p_4\}$, has been formed to determined the most appropiate technology transfer strategy. After screening, four selection criteria are considered:

- *(C_1) Technological availability.* This criterion describes the technology R&D by reflecting its likely rate of difusion.
- *(C_2) Market potencial.* It includes considerations as the breadth of possible applications, competitive or concentrated market, size of market, and product life cycle.
- *(C_3) Policy support.* This refers to the level of goverment support, including tax incentives or subsidies, infraestructure projects required for years to implement the R&D.
- *(C_4) Management ability.* This reflects whether the managerial and business functions of the recipient (firm) is effective or not, including manufacturing capability, financial and human resources, and marketing skills.

For selecting one of four commonly used strategies that were successful in transferring technologies in the case studied are described as follows:

- *(x_1) Purchasing.* (firms just buy R&D from research bodies).
- *(x_2) Working with an industrial partner.* (like a joint venture strategy, but do not set up an individual firm, both shared 50-50 the required sources).
- *(x_3) Licensing.* (the lincense has rights to produce the market the product).
- *(x_4) Cooperative R&D.* (the company's cost sharing are 100%. The company is committed to the commercialization process and as a way of enhancing the R&D effort).

3.2. *Decision process*

The process we shall use for solving this problem is structured as follows:

- **Aggregation Process**. The committee has to aggregate the rating S_{itj} of n experts for each alternative x_i versus each criterion C_t to obtain the rating S_{it}. Each pooled S_{it} can further be weighted by weight W_t according to the relative importance of the k criteria. Then, the final score F_i, fuzzy appropriateness index, of alternative x_i can be obtained by aggregating S_{it} and W_t.

- **Exploitation Process**. Finally, rank the final scores F_i, to obtain the most appropiate alternative.

3.3. *Linguistic treatments of the problem*

For describing the attitudes of the experts, they will use linguistic variables. Each one can give his preferences in his own linguistic term set. Therefore, the definition context of the problem is a multigranular linguistic context, due to the fact there exist linguistic terms assessed in different term sets with different granularity and/or semantics. In our particular case, p_1 uses the term set A (7 labels), p_2 *and* p_3 use the term set B (5 labels) and p_4 uses the term set C (7 labels):

	A	B	C
a_0	$(0,0,0)$	$(0,0,.25)$	$(0,0,.16)$
a_1	$(0,0,.25)$	$(0,.25,.5)$	$(0,.16,.33)$
a_2	$(0,.25,.5)$	$(.25,.5,.75)$	$(.16,.33,.5)$
a_3	$(.25,.5,.75)$	$(.5,.75,1)$	$(.33,.5,.67)$
a_4	$(.5,.75,1)$	$(.75,1,1)$	$(.5,.67,.84)$
a_5	$(.75,1,1)$		$(.67,.84,1)$
a_6	$(1,1,1)$		$(.84,1,1)$

The preferences provided by the experts are the following:

	p_1	p_2	p_3	p_4
C_1	a_3	b_4	b_3	c_3
C_2	a_4	b_3	b_2	c_4
C_3	a_5	b_3	b_4	c_4
C_4	a_4	b_2	b_3	c_3

Table 1. The importance of the criteria C_i according to the experts p_i

	p_1				p_2				p_3				p_4			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
x_1	a_4	a_3	a_2	a_4	b_4	b_0	b_0	b_4	b_3	b_1	b_1	b_3	c_4	c_1	c_1	c_4
x_2	a_2	a_3	a_1	a_3	b_2	b_1	b_1	b_2	b_1	b_1	b_0	b_3	c_2	c_4	c_3	c_5
x_3	a_2	a_4	a_3	a_3	b_1	b_3	b_2	b_2	b_1	b_3	b_3	b_2	c_3	c_4	c_4	c_3
x_4	a_3	a_4	a_2	a_3	b_2	b_4	b_2	b_3	b_2	b_3	b_3	b_1	c_1	c_5	c_2	c_3

Table 2. Evaluation of the experts p_i under C_t

3.3.1. *Solution based on the Extension Principle*

To apply this approach to our decision process, we shall use the aggregation process presented in section 2. In the exploitation process we shall use the two sides that presents this model: (i)Dealing with fuzzy numbers, (ii) and dealing with linguistic labels using a linguistic approximation process.

Aggregation Process

There exist a lot of aggregation operators for combining the experts' preferences. We use the mean operator to aggregate the experts' assessments as in [13].

$$\mu_{S_{it}} = (\frac{1}{n}) \otimes (\mu_{S_{it1}} \oplus \mu_{S_{it2}} \oplus ... \oplus \mu_{S_{itn}}), \qquad \mu_{W_t} = (\frac{1}{n}) \otimes (\mu_{W_{t1}} \oplus \mu_{W_{t2}} \oplus ... \oplus \mu_{W_{tn}}).$$

Where S_{it} is the average fuzzy appropriateness rating of alternative x_i under criterion C_t and W_t is the average importance weight of criterion C_t. The results are presented in tables 3 and 4 respectively.

	C_1	C_2	C_3	C_4
x_1	(.5625,.7925,.9600)	(.0625,.2275,.4575)	(0,.1650,.3950)	(.5625,.7925,.9600)
x_2	(.1025,.3325,.5625)	(.1875,.4175,.6475)	(.0825,.1875,.4175)	(.4175,.6475,.8750)
x_3	(.0825,.3125,.5425)	(.5000,.7300,.9600)	(.3750,.6050,.8350)	(.2700,.5000,.7300)
x_4	(.1875,.4150,.6450)	(.6050,.8350,1)	(.2275,.4575,.6875)	(.2700.5000,.7300)

Table 3. S_{it}, Fuzzy appropiateness rating of x_i under C_t

C_1	(.4575,.6875,.8550)
C_2	(.4375,.6675,.8975)
C_3	(.6250,.8550,.9600)
C_4	(.3950,.6250,.8550)

Table 4. W_t, Average importance weight of C_t

Thus, the fuzzy appropiateness index F_i of the ith alternative, x_i, is obtained using:

$$F_i = (\frac{1}{k}) \otimes [(\mu_{S_{i1}} \otimes \mu_{W_1}) \oplus ... \oplus (\mu_{S_{ik}} \otimes \mu_{W_k})].$$

F_1	F_2	F_3	F_4
(.1267, .3332, .6078)	(.0863, .2680, .5527)	(.1493, .3829, .6877)	(.1498, .3865, .6832)

Table 5. Appropiateness indices

Exploitation Process

Here we shall order the collective fuzzy numbers. This ordering will be computed according to the two possiblities that allows this computational model.

1. **Ranking Fuzzy Numbers**

   As it was mentioned, there exist many methods for ranking fuzzy numbers [19], in this case we shall use the Kim and Park [20] and Chang [21] methods, presented in 2.1, to compute the ranking values of fuzzy appropriateness indices under a group of experts. Obtaining the following rankings:

   (a) *Kim & Park.*

Alternatives	x_1	x_2	x_3	x_4
$U_T(F_i)$	.4540	.3828	.5132	**.5151**

Table 6. Ranking indices with Kim & Park.

   The best alternative is x_4, i.e., **"Cooperative R&D"**.

   (b) *Chang.*

Alternatives	x_1	x_2	x_3	x_4
$U_T(F_i)$	.0856	.0705	**.1094**	.1084

Table 7. Ranking indices with Chang.

   The best selection of technology transfer strategy is x_3, i.e., **"Licensing"**.

2. **Linguistic Approximation**

   To express the collective fuzzy numbers by means of linguistic labels we shall use approximation function ($app_1(\cdot)$):

$$d(s_l, F_j) = \sqrt{P_1(a_l - a_j)^2 + P_2(b_l - b_j)^2 + P_3(c_l - c_j)^2}$$

   $app_1(\cdot)$ chooses s_l^* ($app_1(F_j) = s_l^*$), such that, $d(s_l^*, F_j) \leq d(s_l, F_j) \quad \forall s_l \in S$.

   In first place, we must choose what will the expression domain be?. Our problem is defined in a multigranularity linguistic context, therefore, we could choose any of the linguistic term sets used. In this case, we shall approximate the fuzzy numbers to the three term sets used by the experts A, B, C, to make clear the comparison between this method and the 2-tuple one.

   Once known the linguistic term sets used as expression domains, the linguistic approximation process, $app_1(\cdot)$, is applied to the fuzzy numbers of the *Table 5*, with $P_1 = 0.3$, $P_2 = 0.4$, $P_3 = 0.3$. We select these values because of the parameter "b_i" is the most representative of the membership function and "a_i", "c_i" are equally representative. Obtaining the following results:

$\mathbf{F_i \in A}$	a_2	a_2	a_3	a_3
$\mathbf{F_i \in B}$	b_1	b_1	b_2	b_2
$\mathbf{F_i \in C}$	c_2	c_2	c_2	c_2

Table 8. Appropiateness indices

   The linguistic terms have an order defined in the structure of the linguistic term set. Table 9 shows the solution set alternatives selected according to the linguistic term set:

A	B	C
x_3, x_4	x_3, x_4	x_1, x_2, x_3, x_4

Table 9. Solution set of alternatives

From Table 9, we cannot choose an unique alternative as solution because there exist more than one alternative having the maximum collective value.

3.3.2. *Solution based on 2-tuples*

Aggregation Process

To carry out this process using linguistic 2-tuples, we use the process presented in [23] (Fig.2):

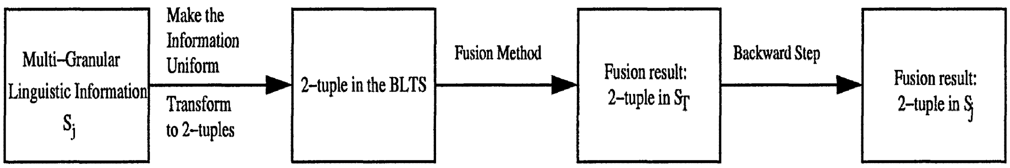

Figure 2: Fusion of multi-granularity linguistic information

The multigranular linguistic information is unified over a Basic Linguistic Term Set (BLTS) denoted as, S_T, in this case we shall use the term set of the Figure 3 whose semantics are the following ones:

s_0	$(0,0,.07)$	s_1	$(0,.07,.14)$	s_2	$(.07,.14,.21)$	s_3	$(.14,.21,.28)$
s_4	$(.21,.28,.35)$	s_5	$(.28,.35,.42)$	s_6	$(.35,.42,.5)$	s_7	$(.42,.5,.58)$
s_8	$(.5,.58,.65)$	s_9	$(.58,.65,.72)$	s_{10}	$(.65,.72,.79)$	s_{11}	$(.72,.79,.86)$
s_{12}	$(.79,.86,.93)$	s_{13}	$(.86,.93,1)$	s_{14}	$(.93,1,1)$		

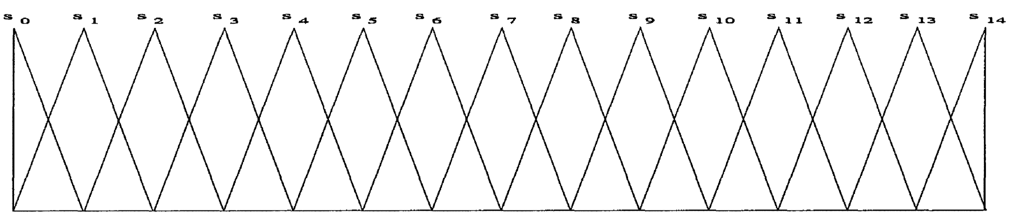

Figure 3: Term set of fifteen terms simmetrically distributed

According to [23] the information is unified into fuzzy sets using the following function:

Definition 3. *Let $A = \{l_0, \ldots, l_p\}$ and $S_T = \{c_0, \ldots, c_g\}$ be two linguistic term sets, such that, $g \geq p$. Then, a multigranular transformation function, τ_{AS_T} is defined as:*

$$\tau_{AS_T} : A \longrightarrow F(S_T)$$

$$\tau_{AS_T}(l_o) = \{(c_k, \alpha_k^o) \ /k \in \{0, \ldots, g\}\}, \ \forall l_j \in A$$

$$\alpha_k^o = \max_x min\{\mu_{l_o}(x), \mu_{c_k}(x)\}$$

where $F(S_T)$ is the set of fuzzy sets defined in S_T, and $\mu_{l_o}(x)$ and $\mu_{c_k}(x)$ are the membership functions of the fuzzy sets associated to the terms l_o and c_k, respectively.

These fuzzy sets are transformed into linguistic 2-tuples by means of:

Definition 4. *Let $\tau_{S_jS_T}(l_o) = \{(c_0, \alpha_0^o), \ldots, (c_g, \alpha_g^o)\}$ be a fuzzy set that represents a linguistic term $l_o \in S_j$ over the basic linguistic term set S_T. We shall obtain a numerical value, that supports the information of the fuzzy set, assessed in the interval $[0, g]$ by means of the following function:*

$$\chi : F(S_T) \longrightarrow [0, g]$$

$$\chi(\tau_{S_jS_T}(l_o)) = \frac{\sum_{k=0}^{g} k\alpha_k^o}{\sum_{k=0}^{g} \alpha_k^o} = \beta$$

This value β is easy to transform into a linguistic 2-tuple using the function Δ.

Therefore applying the functions τ, χ, *and* Δ to the values provided by the experts in Tables 1 and 2. The preferences will be expressed by means of linguistic 2-tuples assessed in the BLTS, S_T, such as it is shown in Tables 10 and 11:

	p_1	p_2	p_3	p_4
C_1	$(s_7, 0)$	$(s_{13}, -.22)$	$(s_{11}, -.5)$	$(s_7, 0)$
C_2	$(s_{11}, -.5)$	$(s_{10}, .5)$	$(s_7, 0)$	$(s_9, .36)$
C_3	$(s_{13}, -.22)$	$(s_{10}, .5)$	$(s_{13}, -.22)$	$(s_9, .36)$
C_4	$(s_{11}, -.5)$	$(s_7, 0)$	$(s_{11}, -.5)$	$(s_7, 0)$

Table 10. Importance of the criteria, W_{tj}

	p_1				p_2			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
x_1	$(s_{11}, -.5)$	$(s_7, 0)$	$(s_3, .23)$	$(s_{11}, -.5)$	$(s_{13}, -.22)$	$(s_1, .25)$	$(s_1, .25)$	$(s_{13}, -.22)$
x_2	$(s_3, .23)$	$(s_7, 0)$	$(s_1, .25)$	$(s_7, 0)$	$(s_7, 0)$	$(s_3, .23)$	$(s_3, .23)$	$(s_7, 0)$
x_3	$(s_3, .23)$	$(s_{11}, -.5)$	$(s_7, 0)$	$(s_7, 0)$	$(s_3, .23)$	$(s_{11}, -.5)$	$(s_7, 0)$	$(s_7, 0)$
x_4	$(s_7, 0)$	$(s_{11}, -.5)$	$(s_3, .23)$	$(s_7, 0)$	$(s_7, 0)$	$(s_{13}, -.22)$	$(s_7, 0)$	$(s_{11}, -.5)$
	p_3				p_4			
	C_1	C_2	C_3	C_4	C_1	C_2	C_3	C_4
x_1	$(s_{11}, -.5)$	$(s_3, .23)$	$(s_3, .23)$	$(s_{11}, -.5)$	$(s_9, .36)$	$(s_2, .36)$	$(s_2, .36)$	$(s_9, .36)$
x_2	$(s_3, .23)$	$(s_3, .23)$	$(s_1, .21)$	$(s_{11}, -.5)$	$(s_5, -.37)$	$(s_9, .36)$	$(s_7, 0)$	$(s_{12}, -.37)$
x_3	$(s_3, .23)$	$(s_{11}, -.5)$	$(s_{11}, -.5)$	$(s_7, 0)$	$(s_7, 0)$	$(s_9, .36)$	$(s_9, .36)$	$(s_7, 0)$
x_4	$(s_7, 0)$	$(s_{11}, -.5)$	$(s_{11}, -.5)$	$(s_3, .23)$	$(s_2, .36)$	$(s_{12}, -.37)$	$(s_5, -.37)$	$(s_7, 0)$

Table 11. Evaluation of the experts p_i under C_t, S_{itj}

Now we are combining the 2-tuples using the arithmetic mean for 2-tuples [16]:

$$S_{it} = \Delta(\frac{1}{n}\sum_{j=1}^{n} \beta_{itj}) = (s_{it}, \alpha_{it}),\ s_{it} \in S_T, \quad W_t = \Delta(\frac{1}{n}\sum_{j=1}^{n} \beta_{tj}) = (w_t, \alpha_t),\ w_t \in S_T.$$

With $\beta_{itj} = \Delta^{-1}(S_{itj})$ and $\beta_{tj} = \Delta^{-1}(W_{tj})$.

Obtaining the following collective values:

S_{it}	C_1	C_2	C_3	C_4
x_1	$(s_{11}, -.21)$	$(s_3, .46)$	$(s_3, -.49)$	$(s_{11}, -.21)$
x_2	$(s_5, -.5)$	$(s_6, -.3)$	$(s_3, .17)$	$(s_9, .03)$
x_3	$(s_4, .17)$	$(s_{10}, .21)$	$(s_8, .46)$	$(s_7, 0)$
x_4	$(s_6, -.16)$	$(s_{11}, .35)$	$(s_6, .34)$	$(s_7, -.07)$

Table 12. Fuzzy appropiatness rating of x_i under C_t

C_1	C_2	C_3	C_4
$(s_9, .32)$	$(s_9, .34)$	$(s_{11}, .35)$	$(s_9, -.25)$

Table 13. W_t, Average importance weight of C_t

The appropiateness index F_i for each alternative x_i is obtained as:

$$F_i = \Delta(\frac{\sum_{t=1}^{k} \beta_{it} \cdot \beta_{W_t}}{\sum_{t=1}^{k} \beta_{W_t}}), \; with \; \beta_{it} = \Delta^{-1}(S_{it}) \; and \; \beta_{w_t} = \Delta^{-1}(W_t).$$

The indices obtained are:

	F_i *en* S_T	F_i *en* A	F_i *en* B	F_i *en* C
x_1	$(s_7, -.41)$	$(a_3, -.15)$	$(b_2, -.15)$	$(c_3, -.2)$
x_2	$(s_5, .42)$	$(a_3, -.43)$	$(b_2, -.43)$	$(c_2, .31)$
x_3	$(s_8, -.48)$	$(a_3, .17)$	$(b_2, .17)$	$(c_3, .24)$
x_4	$(s_8, -.45)$	$(a_3, .19)$	$(b_2, .19)$	$(c_3, .27)$

Table 14. F_i in the BLTS and initial domains.

Exploitation Process

The representation by means of 2-tuples has defined a total order over itself. Therefore, the ranking of the alternatives is independent of the linguistic domain used to express the results [23]. According to the results of the Table 14, the best selection of technology transfer strategy is x_4, i.e., **"Cooperative R&D"**.

4. Analysis of the linguistic computational models in CW processes

Our objective is to analyse the approximative and 2-tuple linguistic computational models from the following points of view: (i) Linguistic description, (ii) consistency and (iii) accuracy of the results.

From the results obtained just above, we shall carry out this analysis comparing the different models.

4.1. *Linguistic description*

A review of the values obtained for the alternatives by the different models can be seen in the following table:

	Ranking Fuzzy Numbers		Ling. Approx.			2-tuples		
	Kim&Park	Chang	A	B	C	A	B	C
x_1	.4540	.0856	a_2	b_1	c_2	$(a_3, -.15)$	$(b_2, -.15)$	$(c_3, -.2)$
x_2	.3828	.0705	a_2	b_1	c_2	$(a_3, -.43)$	$(b_2, -.43)$	$(c_2, .31)$
x_3	.5132	.1094	a_3	b_2	c_2	$(a_3, .17)$	$(b_2, 17)$	$(c_3, .24)$
x_4	.5151	.1084	a_3	b_2	c_2	$(a_3, .19)$	$(b_2, .19)$	$(c_3, .27)$

Table 15. Results.

The approximative model using ranking fuzzy methods obtains real numbers for expressing the appropriatness for each alternative, therefore it uses an expression domain far from the linguistic one lossing linguistic description. However, the linguistic approximative model with linguistic terms and the 2-tuple model express their results in the initial linguistic domain used by the experts being easier to understand the results.

4.2. *Consistency Analysis*

When we talk about consistency, it means to obtain the same solution from the same inputs. Let us review the following table with the solution set of alternatives:

	Ranking Fuzzy Numbers		Ling. Approx.			2-tuples		
	Kim&Park	Chang	A	B	C	A	B	C
x_1	.4540	.0856	a_2	b_1	$\mathbf{c_2}$	$(a_3, -.15)$	$(b_2, -.15)$	$(c_3, -.2)$
x_2	.3828	.0705	a_2	b_1	$\mathbf{c_2}$	$(a_3, -.43)$	$(b_2, -.43)$	$(c_2, .31)$
x_3	.5132	**.1094**	$\mathbf{a_3}$	$\mathbf{b_2}$	$\mathbf{c_2}$	$(a_3, .17)$	$(b_2, 17)$	$(c_3, .24)$
x_4	**.5151**	.1084	$\mathbf{a_3}$	$\mathbf{b_2}$	$\mathbf{c_2}$	$(\mathbf{a_3}, \mathbf{.19})$	$(\mathbf{b_2}, \mathbf{.19})$	$(\mathbf{c_3}, \mathbf{.27})$

Table 16. Solution set of alternatives

The bold typed values in each column represent the solution set of alternatives correspondent to each model. Following, we review the consistency of the different linguistic computational models:

1. *Approximative model with fuzzy numbers.* We can see this model from the same inputs obtains different solution sets depending on the fuzzy ranking method selected to order the collective preference values.

2. *Approximative model with linguistic terms.* The solution set of alternatives depends on the linguistic term set used to express the solution.

3. *2-tuple model.* This model always obtains the same solution set of alternatives indepently of the expression domain. It was proved in [23] the ranking function is inherent to the structure of the 2-tuple and therefore, independent of the linguistic term set used to express the results.

Therefore, we can say that the linguistic computational model based on 2-tuples is more consistent than the approximative one in its two sides, due to the fact, its results does not depend on the linguistic domain neither on the ranking function, i.e., this model always obtains the same solution set from the same inputs.

4.3. *Accuracy analysis*

This analysis studies the capacity of discrimination and the accuracy of the different linguistic computational models.

From Table 16 we observe that the linguistic approximative model with linguistic terms is less precise than the other ones because of among the alternatives with the same linguistic term cannot discern which is better. While the approximative model ranking fuzzy numbers obtains real numbers that are values with a higher granularity than the before one, and it is difficult two alternatives having the same assessment. Finally, the 2-tuple model is able to choose the best alternative among different alternatives with the same linguistic term using the symbolic translation.

Therefore the approximative model with fuzzy numbers and the 2-tuple model obtain results more precise than the other one, since they have a highest capacity of discrimination and hence, a highest accuracy.

4.4. *Conclusions*

This study shows that the 2-tuple computational model is always at least so good as the approximative one in all the characteristics studied, depending on the characteristic and the side of the approximative model. Therefore, we can say that the 2-tuple linguistic computational model is almost always the most adequated model for dealing with linguistic information in processes of CW.

5. Concluding Remarks

In this paper we have presented a study about the improvements of the linguistic 2-tuple computational model over the classical linguistic computational models. We have solved an application for showing that the 2-tuple model is more adequated for dealing with linguistic information, in processes of CW, than classical ones.

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